Garbling an evaluation to retain an advantage*

Ascensión Andina-Díaz† and José A. García-Martínez‡

March 29, 2021

Abstract

We study the effect of introducing interpersonal comparisons on the decisions made by career concerned experts. We consider competition between two experts who may differ in their initial reputation. We obtain that whereas full transmission of the experts' private information is an equilibrium when experts are homogenous, this is not necessarily the case when they are heterogenous. In this case, we identify an incentive for the stronger expert to discard her private information, aiming at garbling the evaluation of the principal to retain her advantage. In equilibrium, this expert may even completely contradict her signal and the other expert's decision.

Keywords: Interpersonal comparisons; heterogenous expertise; career concerns; probability of feedback

JEL: C72; D82; D83

1 Introduction

It's human nature to compare ourselves to others. Sometimes unconsciously, individuals tend to evaluate our own social and personal achievements based on how we stack up against others. We do it on a daily basis and across multiple dimensions, from success and intelligence, to wealth and attractiveness.

Though a regular and well established phenomenon, we do not know much about how interpersonal comparisons affect individual behavior and decision making.[1] This paper aims to contribute to this research question. More precisely, we are interested in understanding the effects that interpersonal

*We thank Gilat Levy, Miguel A. Meléndez-Jiménez, Nicola Pavoni, Raghul Venkatesh and seminar participants at Universidad de Málaga for useful comments. We gratefully acknowledge the financial support from the Ministerio de Ciencia, Innovación y Universidades (MCIU/AEI/FEDER, UE) through projects RTI2018-097620-B-I00 and PGC2018-097965-B-I00, the Junta de Andalucía through project P18-FR-3840, and the Universidad de Málaga through project UMA18-FEDERJA-243. The usual disclaimer applies.

†Dpto. Teoría e Historia Económica, Universidad de Málaga, Spain. E-mail: aandina@uma.es

‡Dpto. Estudios Económicos y Financieros, Universidad Miguel Hernández, Spain. E-mail: jose.garciam@umh.es

[1]This phenomenon, known as the "Social Comparison Theory", was first proposed in 1954 by psychologist Leon Festinger. See Meyer and Vickers (1997), Fershtman et al. (2003, 2006), Luttmer (2005), Clark et al. (2008), Roels and Su (2014), and López-Pintado and Meléndez-Jiménez (2019) for the analysis of decision making with interpersonal comparisons under different contexts.

comparisons have on agents with a career concern for *expertise*.[2] This is a relevant question, as success and reputation is one of the more prominent dimensions on which individuals tend to compare ourselves to others.

To illustrate our research question, consider the following example. Suppose two policy advisors who provide simultaneous advice to a policy maker. Policy advisors have a concern for expertise and care about how the policy maker perceives them in comparison to the other advisor. A reason for this may be that only the policy advisor with the higher relative reputation will be listened in the future, promoted to a fixed position in the politician's cabinet, etc. Alternatively, it may simply illustrate the internal pressure of a policy maker that aims to succeed and prove better than others. Suppose further that policy advisors may have different initial expertise. In this situation, should we expect the two policy advisors to provide the politician with the same advice? Will they differ? Will the answer depend on whether policy advisors are homogenous or heterogeneous? Which advice will be more informative?

The model has the following structure. There are two experts of heterogeneous expertise who face common uncertainty about the state of the world and are imperfectly and asymmetrically informed about it. Each expert can be either of two types: wise or normal, the difference being the quality of the information they receive about the state of the world. Experts differ in their initial expertise, i.e., the probability an expert is believed to be wise. According to this, we talk about the stronger expert and the weaker expert. Upon receiving information on the state, experts take simultaneous actions. We consider experts who have reputational concerns and care about interpersonal comparisons, and refer to this system of evaluation as the Relative Performance Evaluation (RPE) system.[3] To isolate effects, we compare the results under this system with those under the standard system, referred to as the Absolute Performance Evaluation (APE) system, in which experts seek to maximize their individual/absolute reputation. We identify two key variables that determine whether both evaluation systems can yield the same outcome (in terms of the experts' behavior and the information transmitted) or not: the heterogeneity in experts' expertise and the probability of feedback, i.e., the probability that the principal learns the state. Our results show that when experts are homogenous, the behavior and the information revealed by the two experts is the same under the two systems (Corollary 1). However, when experts are rather different in expertise and the probability of feedback is below a certain threshold, both systems no not longer yield the same result. In this case, we obtain that full revelation of the experts' private information is always an equilibrium under the APE system (Proposition 1); however, it is never an equilibrium under the RPE system (Proposition 3). In the latter case, we identify an incentive for the stronger expert to discard her informative private signal and differentiate her advice from that of the weaker expert, aiming at garbling the evaluation of the

[2] With expertise we refer to the ability of an agent to know about an uncertain common variable, such as the state of the world. Other models of career concerns consider agents who can differ in either their ability to exert effort (Harris and Holmström (1982) and Holmström (1999)) or their preference for the implemented policy (Morris (2001)). See Lin (2015) for a recent literature review.

[3] This term was first used by Holmström (1982), who defined it for a model of effort provision in teams with career concerned agents.

principal to retain her advantage. For a sufficiently stronger expert, this incentive may be strong enough to induce her to always contradict her signal and the other expert's advice.

To have an intuition for the result, note that by deviating and contradicting the opponent's action, the stronger expert harms more the posterior of the opponent than her posterior. This is so because whenever actions contradict, a bayesian principal will put higher weight in the informative content of the stronger expert's action, for her being the expert with the ex-ante better signal.[4] The weaker expert, however, cannot do better than following her signal, for fear of the stronger expert being a wise type. To the best of our knowledge, this insight is new in the literature.

Another noteworthy result we derive is that the incentive of the stronger expert to contradict her signal may even increase in the quality of the signal. This is so because the ability of this expert to anticipate the weaker expert's action increases in the signal's quality.[5] Thus, a stronger enough expert (who gains a lot from retaining her initial advantage) may find it optimal to go more often against her signal the more informative it is. Naturally, it requires the probability of feedback not to be very high. This result has an interesting implication. It suggests that when interpersonal comparisons matter, an increase in "familiarity", i.e., the knowledge about how an opponent will behave, may lead to higher experts' differentiation.

The results in this paper have straightforward implications on the effects that interpersonal comparisons have on experts' consensus and dissent. They suggest that whereas the APE system fosters consensus, the RPE system is likely to drive dissent. From our personal experience, we may probably recognize situations in which informed agents (think of tv commentators on economic, environmental or health issues, workplace colleges, friends, relatives, etc), having no different biases or preferences, choose to support different opinions and contradict each other, making consensus difficult to achieve and putting burdens on the principal's capacity to make the correct decision. This work develops a new rationale for this behavior that roots on experts comparing each other and the stronger expert seeking to retain her advantage.

We also consider that our results generate new insights into the human being aspiration of being perceived as good and reliable experts. The point is that an expert that tends to compare herself with others may be willing to give up a higher posterior and accept instead a lower posterior if by accepting a lower posterior she harms more the posterior of the other expert. This behavior may be optimal if the market evaluates experts in relative terms, but it is definitively not optimal otherwise. Our results suggest that paying attention to others may not be a good strategy if one aims at building a reputation for expertise, as rather that helping attain the goal, it may just erode the expert's reputation.

The rest of the paper is organized as follows. After reviewing the literature in Section 2, we formulate the model in Section 3. Section 4 contains all the analysis and the results both under APE

[4] This result requires signals (of the normal type experts) to be either of the same quality, independently of the expert's ex ante reputation, or to be of higher quality for the stronger expert and of lower quality for the weaker expert. For simplicity, in the paper we consider that the quality of a signal is the same for any normal type expert. However, we conjecture that the results in this paper would maintain if stronger experts received better signals.

[5] This result is obtained considering signals that are i.i.d. conditional on the state. However, we predict this result to be stronger if conditional on the state, signals were correlated.

and RPE, and Section 5 discusses some implications and the robustness of the main result. Finally, Section 6 concludes. All the proofs are in the Appendix.

2 Related literature

This paper contributes to the literature about career concerns for expertise and speaks directly to the analysis of reputational concerns in the presence of competing experts (see Ottaviani and Sørensen (2001, 2006*a*,*b*), Gentzkow and Shapiro (2006), Bourjade and Jullien (2011), and Andina-Díaz and García-Martínez (2020)).[6] We contribute to this literature by considering experts with heterogeneous expertise and relative reputational concerns.

A useful comparison is with the work by Ottaviani and Sørensen (2006*b*), who study a career concern model in which experts care about relative reputation (see their Sec 7). They consider experts who are identical and a state of the world that is always revealed, and obtain that introducing relative reputational concerns have no effects on experts' behavior (as compared to the absolute reputation model), nor it induces experts to differentiate their advices. In contrast to this work, we consider experts with heterogenous expertise and a probability of feedback that may be less than one, and show that under this set-up, relative reputational concerns do induce experts to differentiate their advices.

The literature on forecasting contests has shown that concerns for ranking may create an incentive for experts to differentiate their forecasts. Ottaviani and Sørensen (2006*c*) show that in equilibrium forecasters differentiate from their competitors by putting too much weight on their signals. The reason is that by so doing, they increase the probability of not sharing the prize with a large number of forecasters. Lichtendahl et al. (2013) show that the amount of differentiation increases in the number of forecasters and Banerjee (2020) extends the model to allow for conditional correlation in the experts' signals. All these papers assume an exogenous payoff function describing a winner-takes-all context in which the prize is proportional to the number of succeeding forecasters. This assumption introduces an incentive for *all* experts to differentiate their advices, in an attempt to reduce the number of experts with whom to share the prize. Hence, even if experts are identical (as considered in these works), there is an incentive to differentiate. As already discuss, the mechanism in our paper is rather different, as proved by our result that when experts are identical, in equilibrium they do not differentiate.

Our result on experts' dissent rely on two important elements: allowing experts to differ in their initial expertise, and considering that the state of the world may not be revealed. To the best of our knowledge, there is no work where experts have heterogenous expertise and career concerns for expertise. Regarding the second element, a quite standard assumption in the literature is to consider that the state of the world is always revealed (Ottaviani and Sørensen (2001, 2006*a*,*b*)). On the contrary, Prendergast and Stole (1996) propose a dynamic reputational signalling model in which the state is never observed. Our work excludes the analysis of the latter (limit) case as, given the cheap-talk

[6] In a different class of literature, Crawford and Sobel (1982), Krishna and Morgan (2001), Battaglini (2002), Andina-Díaz (2015), and Kartik and van Weelden (2019*a*,*b*) consider competition among partisan (rather than professional) experts. Bourjade and Jullien (2011) introduce reputational concerns about the quality of expertise in a model of partisan experts.

nature of our game, there is multiplicity of equilibria when there is no feedback. Closer to our work on this respect, Canes-Wrone et al. (2001), Prat (2005), Levy (2007), Li and Madarász (2008), Fox and Van Weelden (2012), and Andina-Díaz and García-Martínez (2020, 2021) allow the probability of feedback to vary and identify conditions under which transparency may have a perverse effect. In our case, however, transparency always disciplines, as in Gentzkow and Shapiro (2006).

Last, the insight of our paper has a flavour of the reputational herding and anti-herding literature, initiated by Scharfstein and Stein (1990). This paper shows that when experts make decisions in a sequential order and they do not know their type, there is an incentive for the second expert to herd on the decision of the first expert. Effinger and Polborn (2001) consider a model as in Scharfstein and Stein (1990), but assume an exogenous payoff function such that an expert is most valuable if he is the only smart expert. They show that if the value of being the only smart expert is sufficiently large, anti-herding occurs. Herding and anti-herding results have also been shown to arise when not all the states of the world are equally likely. See Heidhues and Lagerlöf (2003), Cummins and Nyman (2005), Ottaviani and Sørensen (2001, 2006*a*,*b*), and Gentzkow and Shapiro (2006) for models of herding, and Levy (2004) and Panova (2010) for a model of anti-herding, either with one or two experts. In contrast to this literature, the results in this paper are derived under the assumption that all the states of the world are equally likely and competition is simultaneous. This guarantees that herding/anti-herding effects are not behind our results, as there is neither a popular belief nor a first decision to follow.

3 The model

We consider a model between two experts $i \in \{1, 2\}$ (she) with career concerns and one principal (he). There is a binary state of the world $\omega \in \{L, R\}$ and a binary set of actions $a_i \in \{\hat{l}, \hat{r}\}$. We assume that the two states are equally likely.

Experts make simultaneous decisions on the actions to take. Prior to taking an action, expert i receives a private signal $s_i \in \{l, r\}$ on the state of the world. We denote by γ the quality of a signal, with $\gamma = P(l \mid L) = P(r \mid R)$, and assume that the distribution of the quality of a signal depends on the type of the expert, which can be either wise type (W) or normal type (N). Let t_i be the type of an expert, with $t_i \in \{W, N\}$ and $i \in \{1, 2\}$. We assume that a wise type expert always receives a signal that perfectly reveals the state of the world (it has quality 1); whereas a normal type expert receives an imperfect but informative signal of quality $\gamma \in (\frac{1}{2}, 1)$. Types of experts are i.i.d. and signals are i.i.d. conditional on the state. Note that γ can be arbitrarily close to 1, i.e., normal type experts can receive signals of arbitrarily excellent quality. The type of an expert is the expert's private information. The other players (expert j and the principal) have a common prior about the probability that expert i is a wise type. Let $\alpha_i \in (0, 1)$ be this common prior probability; then $1 - \alpha_i$ is the prior probability that expert i is normal type. We assume $\alpha_1 \geq \alpha_2$, i.e., it is common knowledge that, ex ante, expert 1 has a higher (or equal) probability of being wise type than expert 2. Hereafter, we refer to expert 1 as the *stronger* expert and to expert 2 as the *weaker* expert.

We define the strategy of an expert as a mapping that associates with every possible type and signal of the expert a probability distribution over the space of actions. For the sake of simplicity, we denote by $\sigma_t^i(s) \in [0,1]$ the probability that expert i of type t takes the action a that corresponds to her signal s. Thus, $\sigma_t^i(l) = P_t^i(\hat{l} \mid l)$ and $\sigma_t^i(r) = P_t^i(\hat{r} \mid r)$, for $i \in \{1,2\}$ and $t \in \{W,N\}$. Then, $1-\sigma_t^i(l) = P_t^i(\hat{r} \mid l)$ and $1-\sigma_t^i(r) = P_t^i(\hat{l} \mid r)$ is the probability that expert i of type t takes the action a that does not correspond to expert i's signal s.

Let $\mu > 0$ denote the probability that before forming a belief about the type of the experts, the principal receives ex-post verification of the state of the world. We refer to μ as the *probability of feedback*. We denote by $X \in \{L, R, \varnothing\}$ the feedback received by the principal, with $X = \varnothing$ indicating that there is no feedback and $X = L$ indicating that the principal learns that the state is L (analogously for $X = R$). The principal observes the vector of actions (a_1, a_2) and feedback X and, based on this information, updates his beliefs about each of the experts' type. Let $\hat{\alpha}_i(a_1, a_2, X)$ denote the principal's posterior probability that expert i is type W, given (a_1, a_2) and X.

Experts have concerns for expertise and each chooses the action to take seeking to maximize her reputation for looking wise, i.e., for being perceived as type W. We consider experts that care about interpersonal comparisons and aim at maximizing their (relative) reputation for being a wise type. We refer to this system of evaluation as the *Relative Performance Evaluation* (RPE) system and consider that the payoff function of expert i is:

$$\Pi_i^R(t; a_i, a_j, X) = \frac{\hat{\alpha}_i(a_1, a_2, X)}{\hat{\alpha}_i(a_1, a_2, X) + \hat{\alpha}_j(a_1, a_2, X)}. \tag{1}$$

This evaluation system illustrates situations in which either the principal or the experts themselves evaluate an expert's performance based on the comparison with the other expert. When it is the expert who evaluates herself this way, she does it through the eyes of the principal, using the principal's posterior that he assigns to each expert being a wise type. Note that under this evaluation system, an expert may increase her payoff by either increasing her posterior $\hat{\alpha}$ or by decreasing the posterior of the opponent, as it is the distance away from the opponent's posterior that matters. Thus, under this evaluation system an expert might prefer to be perceived as an expert with a small posterior rather than a high posterior, if by accepting a lower posterior she harms more the posterior of the other expert (see Section 5.3 for a discussion of the robustness of our results to the consideration of alternative specifications of the payoff function). This kind of considerations introduce an asymmetry into the game, as the two experts are asymmetrically able to harm the opponent. This asymmetry will turn crucial to the results.

To better isolate the interesting aspects of the results under RPE, we also study a benchmark case in which experts care about their (absolute) relative reputation. We refer to this system of evaluation as the *Absolute Performance Evaluation* (APE) system, which corresponds to the standard system in the literature.

Our equilibrium concept is Perfect Bayesian Equilibrium. We say that $(\sigma_W^i(l)^*, \sigma_W^i(r)^*; \sigma_N^i(l)^*, \sigma_N^i(r)^*)$ is an *equilibrium strategy* of expert i if given the equilibrium strategy of expert j and the players' consistent beliefs, $\sigma_t^i(l)^*$ maximizes the expected payoff of expert i of type t after observing signal l, and $\sigma_t^i(r)^*$ does it after signal r. We denote an equilibrium strategy by $\{(\sigma_W^i(l)^*, \sigma_W^i(r)^*; \sigma_N^i(l)^*, \sigma_N^i(r)^*)\}_{i\in\{1,2\}}$.

4 Analysis

In this section we analyze the equilibrium behavior of the experts. Prior to presenting the results, we introduce some concepts. For a given $i \in \{1, 2\}$ and $t \in \{W, N\}$, we say that the strategy of expert i of type t is *honest* when $(\sigma_t^i(l)^*, \sigma_t^i(r)^*) = (1, 1)$, i.e., the expert takes the action that corresponds to her signal with probability one. When the two types of the two experts use an honest strategy, we say that the equilibrium is honest. Additionally, for a given $i \in \{1, 2\}$ and $t \in \{W, N\}$, we say that the strategy of expert i of type t is *symmetric* when $\sigma_t^i(l) = \sigma_t^i(r) = \sigma_t^i$, i.e., the expert takes the action that corresponds to her signal with the same probability across the two information sets, $s = l$ and $s = r$. When the two types of the two experts use symmetric strategies, we say that the equilibrium is symmetric.[7]

For expositional purposes, the analysis that follows considers that the wise type experts always take the action that corresponds to their signal, i.e., they play an honest strategy, and focuses the attention on the behavior of the normal type experts; hereafter simply referred to as the *experts*. This assumption is relaxed in Part II of the Appendix, where we analyze the model considering that wise type experts are also strategic, and show that most of the results of the paper hold under the more general case (see Propositions 4 and 5).

4.1 The APE system

As a benchmark case, let us start considering that experts aim at maximizing their (absolute) reputation for being a wise type (APE). This case corresponds to the standard approach in the literature of competing experts. The payoff function of player i is then:

$$\Pi_i^A(t; a_i, a_j, X) = \hat{\alpha}_i(a_1, a_2, X).$$

Note that, in this case, the evaluation of an expert is exclusively based on the expert's performance. Nevertheless, to evaluate expert i's performance we may take into account the action taken by expert i's opponent, i.e., a_j. This is so when $X = \emptyset$, in which case action a_j may contain information on the state; hence, it may reveal information on expert i's expertise. We obtain the following result.

Proposition 1. *Consider $\alpha_1 \geq \alpha_2$. Under APE there is always an equilibrium in which both the stronger and the weaker expert follow their signals, i.e., $(\sigma_N^i(l)^*, \sigma_N^i(r)^*) = (1, 1)$, for all $i \in \{1, 2\}$ and $\mu > 0$. Furthermore, if we restrict attention to symmetric strategies, this equilibrium is unique.*

This proposition shows that independently of whether experts' are initially heterogenous or homogenous in expertise, under APE there is always an honest equilibrium in which the two normal type experts follow their signals, for any probability of feedback $\mu > 0$. It further states that if we restrict attention to symmetric strategies, this equilibrium is unique. This result extends the standard positive effect of reputation (e.g. Kreps et al. (1982)) to the consideration of heterogenous experts, suggesting

[7] Note that in our set-up, considering symmetric strategies is quite natural, as the two information sets that correspond to the two possible states of the world are symmetric: both states are equally likely, signals are equally informative across states, and the probability of feedback is fixed and invariant across actions and/or states.

that even if experts are heterogenous in their initial expertise, in the equilibrium of the APE system we should never observe experts discarding their signals.

4.2 The RPE system

In this section we consider that experts care about interpersonal comparisons, as described by equation (1). Our first result shows that also under RPE, when experts are homogenous, in equilibrium they honestly reveal their information.

Proposition 2. *Consider $\alpha_1 = \alpha_2$. Under RPE there is always an equilibrium in which $(\sigma_N^i(l)^*, \sigma_N^i(r)^*) = (1,1)$, for all $i \in \{1,2\}$ and $\mu > 0$. Furthermore, if we restrict attention to symmetric strategies, this equilibrium is unique.*

Two notes on this result. Firstly, it shows that when experts have the same initial reputation, the two evaluation systems, APE and RPE, are equivalent in terms of outcomes. The next corollary formalizes this idea.

Corollary 1. *Suppose $\alpha_1 = \alpha_2$. The honest equilibrium is the unique symmetric equilibrium under both APE and RPE.*

Secondly, it shows that the mechanism driving experts' differentiation in our model is necessarily different from the one posited by existing literature (see Effinger and Polborn (2001), Ottaviani and Sørensen (2006*c*), Lichtendahl et al. (2013), and Banerjee (2020)). The reason is that these papers obtain experts' dissent with identical experts.

The rest of the paper focuses on the more interesting case $\alpha_1 > \alpha_2$, for which the two systems, APE and RPE, do not longer coincide. Proposition 3 below constitutes the main result of the paper. The expressions of the thresholds and the equilibrium probability x are defined in the proof.[8]

Proposition 3. *Consider $\alpha_1 > \alpha_2$. Under RPE there exists μ_1 and μ_2, with $\mu_1 < \mu_2$ and $\mu_2 \in (0,1)$, such that there is an equilibrium in which $(\sigma_N^2(l)^*, \sigma_N^2(r)^*) = (1,1)$ and:*

- *If $\mu > \mu_2$, then $(\sigma_N^1(l)^*, \sigma_N^1(r)^*) = (1,1)$.*
- *If $\mu_1 < \mu < \mu_2$, then $(\sigma_N^1(l)^*, \sigma_N^1(r)^*) = (x,x)$, with $x \in (0,1)$.*
- *If $\mu < \mu_1$, then $(\sigma_N^1(l)^*, \sigma_N^1(r)^*) = (0,0)$, where $\mu_1 > 0$ if and only if $\alpha_1 > \bar{\alpha}$, with $\bar{\alpha} \in (\alpha_2, 1)$.*

Additionally, if we restrict attention to symmetric strategies, the equilibria above are unique.

Proposition 3 identifies different scenarios according to the probability of feedback μ. For each of these scenarios, we describe the equilibrium of the game and show that it is unique if we restrict attention to the use of symmetric strategies.

[8] The probability x is a function of the parameters in the model α_1, α_2, γ, and μ, and it satisfies $\Delta_r^{1,R} = 0$, with $\Delta_r^{1,R} = -\Delta_l^{1,R}$ being defined by expressions (13) and (14). In the proof of this result we also derive the explicit expressions of thresholds μ_1, μ_2 and $\bar{\alpha}$, with μ_1 and μ_2 being a function of parameters α_1, α_2, and γ; and $\bar{\alpha}$ being a function of α_2 and γ.

A common feature to all the scenarios is that the weaker expert always follows her signal. More interesting is the behavior of the stronger expert. We observe that except for the case in which the probability of feedback is sufficiently high (i.e., $\mu > \mu_2$), the RPE introduces an incentive for the stronger expert to differentiate her action from that of the weaker expert, with this incentive decreasing in the probability of feedback. In particular, we obtain that in equilibrium, the stronger expert always sticks to her signal when $\mu > \mu_2$, she does it with positive probability when $\mu_1 < \mu < \mu_2$, and she never does it when the probability of feedback is sufficiently small, i.e., $\mu < \mu_1$. Noteworthy, the latter case only occurs for strong enough experts, i.e., $\alpha_1 > \bar{\alpha}$, which implies that the incentive of the stronger expert to completely discard and contradict her signal only occurs for highly reputed experts.

Note that when $(\sigma_N^1(l)^*, \sigma_N^1(r)^*) = (x, x)$ the stronger expert uses a mixed strategy, whereas when $(\sigma_N^1(l)^*, \sigma_N^1(r)^*) = (0, 0)$ she uses a mirror strategy, i.e., she always chooses the action opposite to her signal. Given that in equilibrium $(\sigma_N^2(l)^*, \sigma_N^2(r)^*) = (1, 1)$ always, we say that there is an *honest equilibrium* when $(\sigma_N^1(l)^*, \sigma_N^1(r)^*) = (1, 1)$, there is a *mixed equilibrium* when $(\sigma_N^1(l)^*, \sigma_N^1(r)^*) = (x, x)$, with $x \in (0, 1)$, and there is a *mirror equilibrium* when $(\sigma_N^1(l)^*, \sigma_N^1(r)^*) = (0, 0)$.[9] See Figure 1 for a graphical representation of these regions. As observed from the figure, ceteris paribus the rest of parameters, an increase in μ makes it more likely than the stronger expert follows her signal, whereas an increase in α_1 never increases this probability.

To see the intuition for the result under the RPE system, and in particular the existence of the mixed equilibrium and the mirror equilibrium, note that when the probability of feedback is sufficiently small, to evaluate an expert it is useful to consider the action taken by the other expert. In this case, taking the same action than the opponent does not substantially affect the principal's updating process. However, taking different actions induces a bayesian principal to put a higher weight on the action taken by the stronger expert. The reason being that the stronger expert is more likely to be wise type, hence more likely to have received a perfect signal. This induces the stronger expert to discard her signal and contradict the weaker expert's action, in an attempt to garble the evaluation of the principal to retain her advantage. For μ sufficiently low, this incentive is so powerful that the stronger expert always discards her signal. The weaker expert, however, cannot do better than sticking to her signal, for her having fear of the stronger expert being a wise type who perfectly informs about the state of the world.

The following exercise may help clarify these ideas. It considers the limit case in which the probability of feedback tends to zero ($\mu \to 0$, hence $X = \varnothing$) and the weaker expert uses the honest strategy, and describes the stronger expert's payoff $\Pi_i^R(N; a_i, a_j, X)$ when she also uses the honest strategy. In this case, the payoff to the stronger expert i if she takes the same action a than the opponent is:

$$\Pi_i^R(N; a, a, \varnothing) = \frac{\alpha_i(\alpha_j(1-\gamma)+\gamma)}{\alpha_j\gamma + \alpha_i(\gamma - 2\alpha_j(\gamma-1))},$$

whereas if she takes action $a' \neq a$, she gets:

$$\Pi_i^R(N; a', a, \varnothing) = \frac{\alpha_i(1-\alpha_j)}{\alpha_j + \alpha_i(1-2\alpha_j)}.$$

[9] The uniqueness result implies that the RPE system cannot produce "reversed" equilibria such as one in which the stronger expert plays the honest strategy and the weaker expert discards her signal with positive probability.

It can be shown that $\Pi_i^R(N; a', a, \varnothing) > \Pi_i^R(N; a, a, \varnothing) \Leftrightarrow \alpha_i > \alpha_j$.[10] Additionally, we also obtain that $\Pi_i^R(N; a', a, \varnothing) > \Pi_i^R(N; a, a, \varnothing) \Leftrightarrow \alpha_i > \alpha_j$ when the stronger expert uses the mirror strategy and the weaker expert uses the honest strategy.[11] These results suggest a preference (and an incentive) for the stronger expert to contradict the weaker expert's action, and a preference (and an incentive) for the weaker expert to make her action coincide with that of the stronger expert. They further suggest that when the probability of feedback is very small, the RPE system shapes the nature of the game for the two players, producing very different incentives to the experts: an incentive to the stronger expert to differentiate her action from that of the weaker expert (as in strategic substitutes games), and an incentive to the weaker expert to take the same action than the stronger expert (as in strategic complements games).

The next corollary presents a comparative static exercise of the equilibrium regions.

Corollary 2. *Thresholds μ_2 and μ_1, which delimit the regions of existence of the different type of equilibria in the RPE system, satisfy:*

1. *With respect to α_1, we have $\frac{\partial \mu_2}{\partial \alpha_1} > 0$ and $\frac{\partial \mu_1}{\partial \alpha_1} > 0$.*
2. *With respect to γ, we have $\frac{\partial \mu_2}{\partial \gamma} > 0$ and the existence of $\hat{\alpha} \in (\bar{\alpha}, 1)$ and $\hat{\gamma} \in (\frac{1}{2}, 1)$ such that if $\alpha_1 > \hat{\alpha}$ and $\gamma < \hat{\gamma}$, then $\frac{\partial \mu_1}{\partial \gamma} > 0$. Otherwise, $\frac{\partial \mu_1}{\partial \gamma} < 0$.*

Figure 1 below presents a graphical description of the results of Corollary 2, where top panels present a comparative static exercise with respect to parameter α_1, and bottom panels present a comparative static exercise with respect to parameter γ.

Figure 1 about here

As stated in Corollary 2, a look at top panels shows that ceteris paribus the rest of parameters, an increase in α_1 increases both μ_2 and μ_1. This implies that the higher the initial reputation of the stronger expert, the smaller the range of values of parameter μ for which the honest equilibrium holds, and the higher the range of values for which the mirror equilibrium holds. We also observe that $\mu_1 \to \mu_2$ when $\alpha_1 \to 1$ (see the last part of the proof of Proposition 3 for a proof of this result), which implies that the higher the initial reputation of the stronger expert, the smaller the region where she uses a mixed strategy. In the limit, she either sticks to her signal or contradicts it. Additionally, we observe that $\mu_2 \to 0$ when $\alpha_1 \to \alpha_2$ (see also the proof of Proposition 3 for a proof of this result), which implies that when experts are very similar in terms of their initial reputation, the slightly stronger expert reveals her signal for most values of parameter μ. In the limit, the two experts are always honest (see Proposition 2).

Regarding bottom panels, on the one hand we observe that μ_2 increases in γ, which means that an increase in the quality of the signal always increases the region where the stronger expert deviates from her signal. On the other hand, we observe that the effect of γ on μ_1 is not always monotonic:

[10] $\Pi_i^R(N; a, a, \varnothing)$ is increasing in γ, and it takes value $\Pi_i^R(N; a, a, \varnothing) = \frac{\alpha_i}{\alpha_i + \alpha_j}$ when $\gamma = 1$. This says that by taking the same action than her opponent, expert i's payoff is simply her initial relative reputation.

[11] The payoffs in this case are $\Pi_i^R(N; a, a, \varnothing) = \frac{\alpha_i(\alpha_j(1-\gamma)+\gamma)}{\alpha_i\gamma + \alpha_j(\alpha_i+1-\gamma)}$ and $\Pi_i^R(N; a', a, \varnothing) = \frac{\alpha_i(\alpha_j-1)(1-\gamma)}{\alpha_i(\alpha_j+\gamma-1)-\alpha_j\gamma}$.

whereas the left-hand side panel represents a situation in which $\frac{\partial \mu_1}{\partial \gamma} < 0$ always, the right-hand side panel represents a situation in which first $\frac{\partial \mu_1}{\partial \gamma} > 0$ and then $\frac{\partial \mu_1}{\partial \gamma} < 0$. According to Corollary 2, in the left-hand side panel we have $\alpha_1 < \hat{\alpha}$ and in the right-hand side panel we have $\alpha_1 > \hat{\alpha}$. To understand the non-monotonicity result and more generally the result that honesty decreases in γ, it is important to distinguish the two effects that γ has on the behavior of the stronger expert. Firstly, when γ increases the stronger expert receives a more precise signal of the state of the world. Thus, ceteris paribus the rest of parameters, an increase in γ increases the cost of deviating from the expert's signal. Secondly, when γ increases the stronger expert makes a more accurate prediction of the action that the weaker expert will take. Thus, ceteris paribus the rest of parameters, an increase in γ increases the gain for deviating from her signal and contradicting the opponent's action. Which effect dominates explains how the equilibrium behavior changes with γ. In particular, we observe that an increase in γ always decreases the region where the honest equilibrium exists, i.e., it decreases honesty. This is due to the second effect. Additionally, we observe that when the stronger expert has a sufficiently high initial reputation ($\alpha_1 > \hat{\alpha}$), an increase in γ can even increase the region where the mirror equilibrium exist. This occurs for any $\gamma < \hat{\gamma}$, in which case $\frac{\partial \mu_1}{\partial \gamma} > 0$ (upward sloping part of μ_1 in the right-bottom panel). The idea is that when α_1 is sufficiently high and γ sufficiently low, the gain from following the signal (seeking to match the state) is not enough to compensate the gain from contradicting it (seeking to contradict the opponent's action). A final comment regarding the effect of parameter γ on the equilibrium regions is that $\mu_1 \to \mu_2$ when $\gamma \to 1/2$ (see also the proof of Proposition 3 for a proof of this result), which implies that the smaller γ, the smaller the region where the mixed equilibrium exists. In the limit, the stronger expert either sticks to her signal or contradicts it.

Last, we would like to draw the attention of the reader to the fact that irrespective of the performance evaluation system, there is no pooling equilibrium in which the stronger expert and the weaker expert take the same action, i.e., there is not an equilibrium in which either $(\sigma_N^i(l)^*, \sigma_N^i(r)^*) = (1, 0)$ or $(\sigma_N^i(l)^*, \sigma_N^i(r)^*) = (0, 1)$, for all $i \in \{1, 2\}$. The reason is that in a pooling equilibrium in which all the experts pool at a, deviating to a' will be interpreted by the principal as the expert being a wise type with probability 1. This result holds for any $\alpha_1 \geq \alpha_2$.

5 Discussion

This section provides a comparison of the two systems in terms of: i) experts' consensus and dissent and ii) building a reputation for expertise. In the last part of the section, we discuss the robustness of our main result to the consideration of i) other formulations of the payoff function and ii) correlated signals.

5.1 Experts' consensus and dissent

Casual evidence suggests that in many occasions experts disagree on the action to take. Existing arguments explain experts disagreement using differences in preferences, access to different information, or a desire to reduce the number of experts with whom to share a prize. The results in this paper propose a new rationalize to explain this behavior that roots on experts comparing each other and the stronger expert seeking to retain her advantage.

To formalize ideas, let us say there is experts' consensus when both experts take the same action with a probability higher than one half; there is dissent otherwise. Because the signal about the state is informative, there is expert's consensus in an honest equilibrium, and there is dissent in a mirror equilibrium. To see this, note that in an honest equilibrium the probability that experts' actions coincide is $\gamma^2 + (1-\gamma)^2 > 1/2$, whereas it is $2\gamma(1-\gamma) < 1/2$ in the mirror equilibrium. Having this idea in mind, a comparison of the results under APE and RPE suggests that a committee composed of homogenous experts will reach consensus, irrespective of the evaluation system. In contrast to this, if experts are heterogenous in expertise and they care about interpersonal comparisons, they outcome is likely to be experts' dissent.

5.2 Building a reputation for expertise

The reader may probably recognize situations in which agents who are not being evaluated by the market in relative terms, feel an internal pressure to prove better than others. The results in our paper have clear implications for this kind of situations.

An important idea here is that an expert who cares about interpersonal comparisons may be willing to give up a higher posterior and accept instead a lower posterior if by accepting a lower posterior she harms more the posterior of the other expert. This effect is what drives the stronger expert to use the mirror strategy and contradict the weaker expert's action. Of course, by accepting a lower posterior the stronger expert does not maximize the probability that the principal perceives her as a wise type, which has important effects if the market evaluates her in absolute terms.

This effect, which is proper to the stronger expert,[12] suggests that a stronger expert who aims to build a reputation for expertise should be careful about paying attention to others as, if misperceiving the market evaluation system, it may just erode her reputation.

5.3 Robustness: other payoff functions

Our results with interpersonal comparisons consider a particular payoff function where each expert's payoff is the expectation of the ratio of the posterior probability that the expert is wise over the sum of the posterior probability that each of the two experts is wise. Other alternative specifications may include, e.g., the difference between the expert's posterior and the opponent's posterior, or the rate between the two posteriors.

Although our results are derived for a particular set-up, we consider that the forces behind them will remain as long as by contradicting the opponent, the stronger expert retains her advantage. Key to this insight is the fact that, with interpersonal comparisons, a stronger expert that contradicts the opponent can harm more the posterior of the opponent than her posterior. It requires two ingredients: a payoff function of an expert that depends negatively on the opponent's posterior, and experts to

[12] Remember that the weaker experts always sticks to her signal, both under APE and RPE.

be heterogenous. The first ingredient introduces an incentive for the expert to increase her posterior over the opponent, which may induce her to hurt the opponent. The second ingredient makes that, in case of contradicting actions, the principal puts higher weight on the informative content of the stronger expert, which results in the stronger expert benefiting while the weaker expert bearing the brunt of contradicting actions. These features are not particular to our payoff function but hold for other specifications, such as when the payoff function is the difference between the expert's posterior and the opponent's posterior, or the rate between the two posteriors. Common to these specifications, they all satisfy that the payoff to an expert i, Π_i, decreases in the opponent's posterior, $\hat{\alpha}_j$. For theses cases, we consider that the insights we derive in this paper would also apply.[13]

5.4 Robustness: correlated signals

In the model we consider that signals are i.i.d. conditional on the state. This is a standard assumption in the literature (see Ottaviani and Sørensen (2001, 2006*a*,*b*,*c*)), which greatly simplifies the analysis. Despite this limitation, we consider that the main result in the paper would maintain (even reinforce) if, conditional on the state, signals were correlated. The reason is that with correlated signals the stronger expert would have a better predictor of the weaker expert's action, which would probably increase her incentive to contradict her signal, aiming at contradicting the weaker expert's action. In this sense, our intuition is that with correlated signals, experts' dissent would increase.

6 Conclusion

We consider a model of career concerns with heterogenous experts who care about interpersonal comparisons. We compare our results under this scenario with those under the standard system, where experts are evaluated in absolute terms. We draw results on when introducing interpersonal comparisons make a difference and what is the source of the difference. Our main result shows that with interpersonal comparisons there is an incentive for the stronger expert to contradict her signal, aiming at garbling the evaluation of the principal to retain her advantage.

[13] Although apparently similar, other systems of evaluation entailing comparisons among agents, such as a *contest* game in which the expert with the higher posterior wins a fixed prize and the expert with the lower posterior wins nothing, may produce different incentives. In our model, for example, if instead of the APE or RPE evaluation system we were to consider a contest as described, it can be shown that there is always an equilibrium in which both the stronger and the weaker expert follow their signals, as in the APE system. To see the difference between this contest and the RPE system, note that whereas in RPE the prize is higher the higher the distance away from the opponent's posterior, in this contest the prize is fixed, which means that the only thing that matters for an expert is to be above the opponent and not the distance. In short, for the incentive we identity in our model to appear a ranking of experts' posteriors where the winner takes all is not enough; distance in posteriors should also matter.

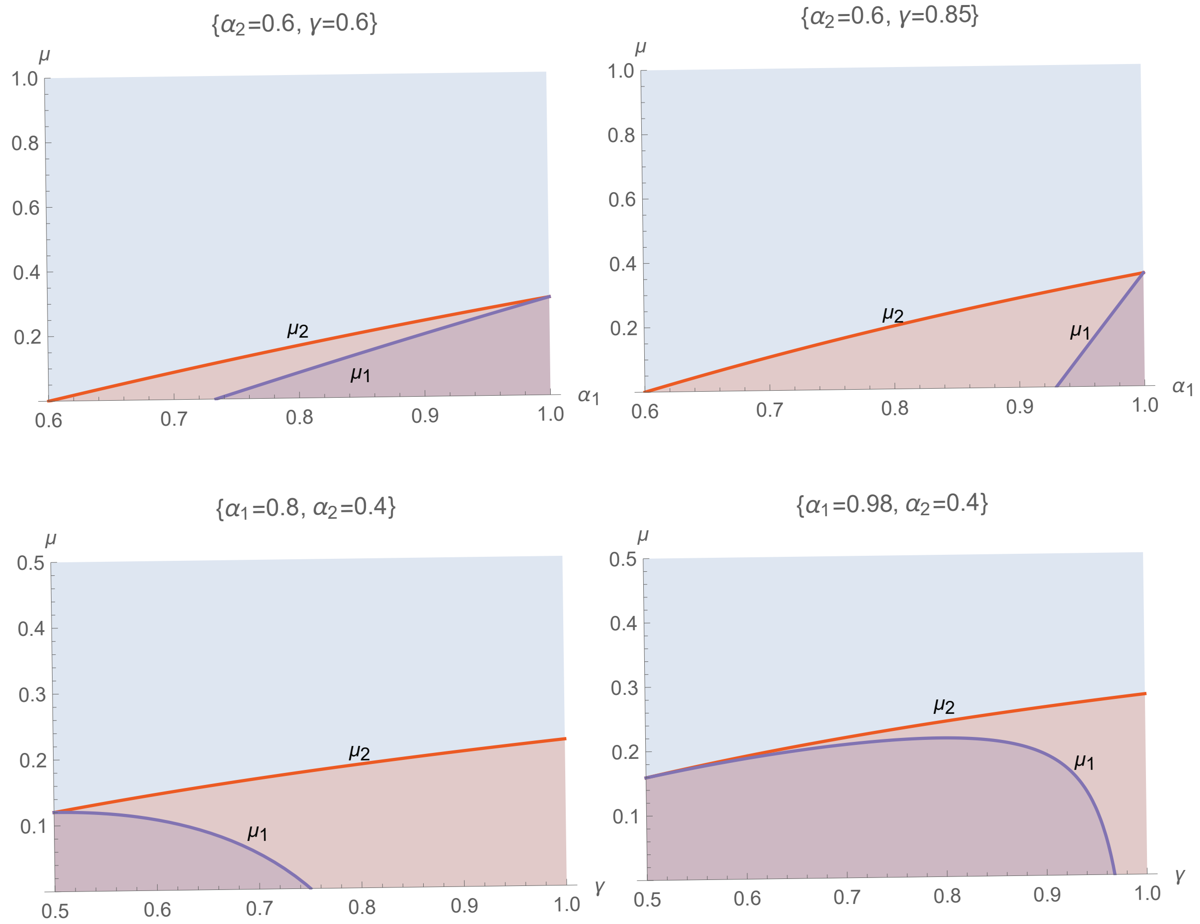

Figure 1: Top panels represent the effect of a change in α_1 on the regions where the honest (blue), the mixed (brown) and the mirror equilibrium (purple) exist. Threshold $\bar{\alpha}$ corresponds to $\mu_1(\alpha_1) = 0$. The top-left panel considers $\alpha_2 = 0.6$ and $\gamma = 0.6$ and the top-right panel considers $\alpha_2 = 0.6$ and $\gamma = 0.85$. Bottom panels represent the effect of a change in γ on the regions where the honest (blue), the mixed (brown) and the mirror equilibrium (purple) exist. The bottom-left panel considers $\alpha_1 = 0.8$ and $\alpha_2 = 0.4$, and the bottom-right panel considers $\alpha_1 = 0.98$ and $\alpha_2 = 0.4$.

A Appendix

We start obtaining the consistent beliefs that the players (the principal and the other expert) place on expert i being a wise type $\hat{\alpha}_i(a_1, a_2, L)$. Note that when $F \neq \varnothing$, $\hat{\alpha}_i(a_1, a_2, F)$ does not depend on a_j, with $i, j \in \{1, 2\}$, $i \neq j$. The beliefs are:

$$\hat{\alpha}_i(l_i, a_j, R) = \hat{\alpha}_i(r_i, a_j, L) = 0, \tag{2}$$

$$\hat{\alpha}_i(l_i, a_j, L) = \frac{\alpha_i}{\alpha_i + (1-\alpha_i)(\gamma\sigma_N^i(l) + (1-\gamma)(1-\sigma_N^i(r)))}, \tag{3}$$

$$\hat{\alpha}_i(r_i, a_j, R) = \frac{\alpha_i}{\alpha_i + (1-\alpha_i)(\gamma\sigma_N^i(r) + (1-\gamma)(1-\sigma_N^i(l)))}, \tag{4}$$

$$\hat{\alpha}_i(l_i, l_j, \varnothing) = \frac{\alpha_i}{\alpha_i + (1-\alpha_i)\left(\gamma\sigma_N^i(l) + (1-\gamma)(1-\sigma_N^i(r)) + (\gamma(1-\sigma_N^i(r)) + (1-\gamma)\sigma_N^i(l))\frac{(1-\alpha_j)(\gamma(1-\sigma_N^j(r)) + (1-\gamma)\sigma_N^j(l))}{\alpha_j + (1-\alpha_j)(\gamma\sigma_N^j(l) + (1-\gamma)(1-\sigma_N^j(r)))}\right)}, \tag{5}$$

$$\hat{\alpha}_i(r_i, r_j, \varnothing) = \frac{\alpha_i}{\alpha_i + (1-\alpha_i)\left(\gamma\sigma_N^i(r) + (1-\gamma)(1-\sigma_N^i(l)) + (\gamma(1-\sigma_N^i(l)) + (1-\gamma)\sigma_N^i(r))\frac{(1-\alpha_j)(\gamma(1-\sigma_N^j(l)) + (1-\gamma)\sigma_N^j(r))}{\alpha_j + (1-\alpha_j)(\gamma\sigma_N^j(r) + (1-\gamma)(1-\sigma_N^j(l)))}\right)}, \tag{6}$$

$$\hat{\alpha}_i(l_i, r_j, \varnothing) = \frac{\alpha_i}{\alpha_i + (1-\alpha_i)\left(\gamma\sigma_N^i(l) + (1-\gamma)(1-\sigma_N^i(r)) + (\gamma(1-\sigma_N^i(r)) + (1-\gamma)\sigma_N^i(l))\frac{\alpha_j + (1-\alpha_j)(\gamma\sigma_N^j(r) + (1-\gamma)(1-\sigma_N^j(l)))}{(1-\alpha_j)(\gamma(1-\sigma_N^j(l)) + (1-\gamma)\sigma_N^j(r))}\right)}, \tag{7}$$

$$\hat{\alpha}_i(r_i, l_j, \varnothing) = \frac{\alpha_i}{\alpha_i + (1-\alpha_i)\left(\gamma\sigma_N^i(r) + (1-\gamma)(1-\sigma_N^i(l)) + (\gamma(1-\sigma_N^i(l)) + (1-\gamma)\sigma_N^i(r))\frac{\alpha_j + (1-\alpha_j)(\gamma\sigma_N^j(l) + (1-\gamma)(1-\sigma_N^j(r)))}{(1-\alpha_j)(\gamma(1-\sigma_N^j(r)) + (1-\gamma)\sigma_N^j(l))}\right)}, \tag{8}$$

for all $i, j \in \{1, 2\}$, $a_j \in \{\hat{l}_j, \hat{r}_j\}$. Note that $\hat{\alpha}_i(l_i, a_j, L) > \hat{\alpha}_i(r_i, a_j, R)$ if and only if $\sigma_N^i(l) < \sigma_N^i(r)$.[14]

Let $EU_i(a_i \mid s_i)$ be the expected payoff (or expected reputation) to the normal type expert i for taking action $a_i \in \{\hat{l}_i, \hat{r}_i\}$ after signal $s_i \in \{l_i, r_i\}$:

$$EU_i(\hat{l}_i \mid s_i) = P(\hat{r}_j \mid \hat{l}_i, s_i)EU_i(\hat{l}_i, \hat{r}_j, F \mid s_i) + P(\hat{l}_j \mid \hat{l}_i, s_i)EU_i(\hat{l}_i, \hat{l}_j, F \mid s_i),$$
$$EU_i(\hat{r}_i \mid s_i) = P(\hat{r}_j \mid \hat{r}_i, s_i)EU_i(\hat{r}_i, \hat{r}_j, F \mid s_i) + P(\hat{l}_j \mid \hat{r}_i, s_i)EU_i(\hat{r}_i, \hat{l}_j, F \mid s_i).$$

with $P(a_j \mid a_i, s_i) = P(a_j \mid s_i)$ for all $a_i \in \{\hat{l}_i, \hat{r}_i\}$ being

$$P(a_j \mid s_i) = P(a_j \mid s_i, L)P(L \mid s_i) + P(a_j \mid s_i, R)P(R \mid s_i),$$

hence:

$$P(\hat{l}_j \mid l_i) = (1-\alpha_j)(\gamma(1-\sigma_N^j(r)) + (1-\gamma)\sigma_N^j(l))(1-\gamma) + (\alpha_j + (1-\alpha_j)(\gamma\sigma_N^j(l) + (1-\gamma)(1-\sigma_N^j(r))))\gamma,$$
$$P(\hat{l}_j \mid r_i) = (1-\alpha_j)(\gamma(1-\sigma_N^j(r)) + (1-\gamma)\sigma_N^j(l))\gamma + (\alpha_j + (1-\alpha_j)(\gamma\sigma_N^j(l) + (1-\gamma)(1-\sigma_N^j(r))))(1-\gamma),$$
$$P(\hat{r}_j \mid l_i) = (1-\alpha_j)(\gamma(1-\sigma_N^j(l)) + (1-\gamma)\sigma_N^j(r))\gamma + (\alpha_j + (1-\alpha_j)(\gamma\sigma_N^j(r) + (1-\gamma)(1-\sigma_N^j(l))))(1-\gamma),$$
$$P(\hat{r}_j \mid r_i) = (1-\alpha_j)(\gamma(1-\sigma_N^j(l)) + (1-\gamma)\sigma_N^j(r))(1-\gamma) + (\alpha_j + (1-\alpha_j)(\gamma\sigma_N^j(r) + (1-\gamma)(1-\sigma_N^j(l))))\gamma.$$

Additionally, let $EU_i(a_i, a_j, F \mid s_i)$ denote the expected payoff to the normal type expert $i \in \{1, 2\}$ for taking action $a_i \in \{\hat{l}_i, \hat{r}_i\}$ after signal $s_i \in \{l_i, r_i\}$ when the other expert takes action $a_j \in \{\hat{l}_j, \hat{r}_j\}$.

[14] For a derivation of beliefs $\hat{\alpha}_i(a_i, a_j, \varnothing)$ see Andina-Díaz and García-Martínez (2020).

When payoffs are absolute (it is absolute performance that matters) we have:

$$
\begin{aligned}
EU_i(\hat{l}_i, \hat{l}_j, F \mid s_i) &= (1-\mu)\hat{\alpha}_i(\hat{l}_i, \hat{l}_j, \varnothing) + \mu P(L \mid s_i, \hat{l}_j)\hat{\alpha}_i(\hat{l}_i, \hat{l}_j, L), \\
EU_i(\hat{l}_i, \hat{r}_j, F \mid s_i) &= (1-\mu)\hat{\alpha}_i(\hat{l}_i, \hat{r}_j, \varnothing) + \mu P(L \mid s_i, \hat{r}_j)\hat{\alpha}_i(\hat{l}_i, \hat{r}_j, L), \\
EU_i(\hat{r}_i, \hat{r}_j, F \mid s_i) &= (1-\mu)\hat{\alpha}_i(\hat{r}_i, \hat{r}_j, \varnothing) + \mu P(R \mid s_i, \hat{r}_j)\hat{\alpha}_i(\hat{r}_i, \hat{r}_j, R), \\
EU_i(\hat{r}_i, \hat{l}_j, F \mid s_i) &= (1-\mu)\hat{\alpha}_i(\hat{r}_i, \hat{l}_j, \varnothing) + \mu P(R \mid s_i, \hat{l}_j)\hat{\alpha}_i(\hat{r}_i, \hat{l}_j, R),
\end{aligned}
$$

as $\hat{\alpha}_i(\hat{l}_i, a_j, R) = \hat{\alpha}_i(\hat{r}_i, a_j, L) = 0$, with $a_j \in \{\hat{l}_j, \hat{r}_j\}$, and $s_i \in \{l_i, r_i\}$.

When payoffs are defined in relative terms (it is relative performance that matters), we have:

$$
\begin{aligned}
EU_i(\hat{l}_i, \hat{l}_j, F \mid s_i) &= (1-\mu)\tfrac{\hat{\alpha}_i(\hat{l}_i,\hat{l}_j,\varnothing)}{\hat{\alpha}_i(\hat{l}_i,\hat{l}_j,\varnothing)+\hat{\alpha}_j(\hat{l}_i,\hat{l}_j,\varnothing)} + \mu P(L \mid s_i, \hat{l}_j)\tfrac{\hat{\alpha}_i(\hat{l}_i,\hat{l}_j,L)}{\hat{\alpha}_i(\hat{l}_i,\hat{l}_j,L)+\hat{\alpha}_j(\hat{l}_i,\hat{l}_j,L)}, \\
EU_i(\hat{l}_i, \hat{r}_j, F \mid s_i) &= (1-\mu)\tfrac{\hat{\alpha}_i(\hat{l}_i,\hat{r}_j,\varnothing)}{\hat{\alpha}_i(\hat{l}_i,\hat{r}_j,\varnothing)+\hat{\alpha}_j(\hat{l}_i,\hat{r}_j,\varnothing)} + \mu P(L \mid s_i, \hat{r}_j)\tfrac{\hat{\alpha}_i(\hat{l}_i,\hat{r}_j,L)}{\hat{\alpha}_i(\hat{l}_i,\hat{r}_j,L)+\hat{\alpha}_j(\hat{l}_i,\hat{r}_j,L)}, \\
EU_i(\hat{r}_i, \hat{r}_j, F \mid s_i) &= (1-\mu)\tfrac{\hat{\alpha}_i(\hat{r}_i,\hat{r}_j,\varnothing)}{\hat{\alpha}_i(\hat{r}_i,\hat{r}_j,\varnothing)+\hat{\alpha}_j(\hat{r}_i,\hat{r}_j,\varnothing)} + \mu P(R \mid s_i, \hat{r}_j)\tfrac{\hat{\alpha}_i(\hat{r}_i,\hat{r}_j,R)}{\hat{\alpha}_i(\hat{r}_i,\hat{r}_j,R)+\hat{\alpha}_j(\hat{r}_i,\hat{r}_j,R)}, \\
EU_i(\hat{r}_i, \hat{l}_j, F \mid s_i) &= (1-\mu)\tfrac{\hat{\alpha}_i(\hat{r}_i,\hat{l}_j,\varnothing)}{\hat{\alpha}_i(\hat{r}_i,\hat{l}_j,\varnothing)+\hat{\alpha}_j(\hat{r}_i,\hat{l}_j,\varnothing)} + \mu P(R \mid s_i, \hat{l}_j)\tfrac{\hat{\alpha}_i(\hat{r}_i,\hat{l}_j,R)}{\hat{\alpha}_i(\hat{r}_i,\hat{l}_j,R)+\hat{\alpha}_j(\hat{r}_i,\hat{l}_j,R)}.
\end{aligned}
$$

with $s_i \in \{l_i, r_i\}$ and $P(\omega \mid s_i, a_j)$ being

$$
P(\omega \mid s_i, a_j) = \tfrac{P(s_i|a_j,\omega)P(a_j|\omega)P(\omega)}{P(s_i|a_j,L)P(a_j|L)P(L)+P(s_i|a_j,R)P(a_j|R)P(R)},
$$

hence:

$$
\begin{aligned}
P(L \mid s_i, \hat{l}_j) &= \frac{P(s_i/L)}{P(s_i/L)+P(s_i/R)\frac{(1-\alpha_j)(\gamma(1-\sigma_N^j(r))+(1-\gamma)\sigma_N^j(l))}{\alpha_j+(1-\alpha_j)(\gamma\sigma_N^j(l)+(1-\gamma)(1-\sigma_N^j(r)))}}, \\
P(L \mid s_i, \hat{r}_j) &= \frac{P(s_i/L)}{P(s_i/L)+P(s_i/R)\frac{\alpha_j+(1-\alpha_j)(\gamma\sigma_N^j(r)+(1-\gamma)(1-\sigma_N^j(l)))}{(1-\alpha_j)(\gamma(1-\sigma_N^j(l))+(1-\gamma)\sigma_N^j(r))}}, \\
P(R \mid s_i, \hat{r}_j) &= \frac{P(s_i/R)}{P(s_i/R)+P(s_i/L)\frac{(1-\alpha_j)(\gamma(1-\sigma_N^j(l))+(1-\gamma)\sigma_N^j(r))}{\alpha_j+(1-\alpha_j)(\gamma\sigma_N^j(r)+(1-\gamma)(1-\sigma_N^j(l)))}}, \\
P(R \mid s_i, \hat{l}_j) &= \frac{P(s_i/R)}{P(s_i/R)+P(s_i/L)\frac{\alpha_j+(1-\alpha_j)(\gamma\sigma_N^j(l)+(1-\gamma)(1-\sigma_N^j(r)))}{(1-\alpha_j)(\gamma(1-\sigma_N^j(r))+(1-\gamma)\sigma_N^j(l))}}.
\end{aligned}
$$

Note that when $a_i = a_j \neq \omega$, there is an indeterminacy in $EU_i(a_i, a_j, F \mid s_i)$, as $\frac{\hat{\alpha}_i(\hat{l}_i,\hat{l}_j,R)}{\hat{\alpha}_i(\hat{l}_i,\hat{l}_j,R)+\hat{\alpha}_j(\hat{l}_i,\hat{l}_j,R)} = \frac{\hat{\alpha}_i(\hat{r}_i,\hat{r}_j,L)}{\hat{\alpha}_i(\hat{r}_i,\hat{r}_j,L)+\hat{\alpha}_j(\hat{r}_i,\hat{r}_j,L)} = \frac{0}{0}$. In this case, as implicit in the formulation, we assume that the relative payoffs to the experts are zero. In the case $a_i \neq a_j'$, with $a_j = \omega$, $\frac{\hat{\alpha}_i(\hat{l}_i,\hat{r}_j,R)}{\hat{\alpha}_i(\hat{l}_i,\hat{r}_j,R)+\hat{\alpha}_j(\hat{l}_i,\hat{r}_j,R)} = \frac{\hat{\alpha}_i(\hat{r}_i,\hat{l}_j,L)}{\hat{\alpha}_i(\hat{r}_i,\hat{l}_j,L)+\hat{\alpha}_j(\hat{r}_i,\hat{l}_j,L)} = 0$ is a result and not an assumption.

Finally, let $\Delta_s^i(\sigma_l^1, \sigma_r^1; \sigma_l^2, \sigma_r^2)$ be the expected gain to the normal type expert i from taking action $\hat{r}_i$ rather than $\hat{l}_i$, after observing signal $s_i \in \{l_i, r_i\}$. Then:

$$
\Delta_s^i(\sigma_l^1, \sigma_r^1; \sigma_l^2, \sigma_r^2) = EU_i(\hat{r}_i \mid s_i) - EU_i(\hat{l}_i \mid s_i). \tag{9}
$$

After some calculations we obtain:

$$
\begin{aligned}
\Delta_s^i(\sigma_l^1, \sigma_r^1; \sigma_l^2, \sigma_r^2) &= P(\hat{r}_j \mid \hat{r}_i, s_i)EU_i(\hat{r}_i, \hat{r}_j, F \mid s_i) + P(\hat{l}_j \mid \hat{r}_i, s_i)EU_i(\hat{r}_i, \hat{l}_j, F \mid s_i) \\
&- \Big(P(\hat{r}_j \mid \hat{l}_i, s_i)EU_i(\hat{l}_i, \hat{r}_j, F \mid s_i) + P(\hat{l}_j \mid \hat{l}_i, s_i)EU_i(\hat{l}_i, \hat{l}_j, F \mid s_i)\Big) \\
&= P(\hat{r}_j \mid s_i)\Big(EU_i(\hat{r}_i, \hat{r}_j, F \mid s_i) - EU_i(\hat{l}_i, \hat{r}_j, F \mid s_i)\Big) \\
&+ P(\hat{l}_j \mid s_i)\Big(EU_i(\hat{r}_i, \hat{l}_j, F \mid s_i) - EU_i(\hat{l}_i, \hat{l}_j, F \mid s_i)\Big).
\end{aligned} \tag{10}
$$

A.1 Part I: Results

In this section we prove the results of the text, which assume that the wise type experts always follow the honest strategy. Part II of the Appendix relaxes this assumption and considers strategic wise type experts.

First, we fully describe the expected gain described by (10) for any $s_i \in \{l_i, r_i\}$, both under the APE and the RPE system. We denote the expected gain Δ_s^i by $\Delta_r^{i,A}$ in the APE system and by $\Delta_r^{i,R}$ in the RPE system. For $i, j \in \{1,2\}$ and $i \neq j$, in the APE system we have:

$$\begin{aligned}
&\Delta_r^{i,A} = \\
&P(\hat{r}_j \mid r_i)\Big((1-\mu)\hat{\alpha}_i(\hat{r}_i,\hat{r}_j,\varnothing) + \mu P(R \mid r_i,\hat{r}_j)\hat{\alpha}_i(\hat{r}_i,\hat{r}_j,R) - (1-\mu)\hat{\alpha}_i(\hat{l}_i,\hat{r}_j,\varnothing) - \mu P(L \mid r_i,\hat{r}_j)\hat{\alpha}_i(\hat{l}_i,\hat{r}_j,L)\Big) + \\
&P(\hat{l}_j \mid r_i)\Big((1-\mu)\hat{\alpha}_i(\hat{r}_i,\hat{l}_j,\varnothing) + \mu P(R \mid r_i,\hat{l}_j)\hat{\alpha}_i(\hat{r}_i,\hat{l}_j,R) - (1-\mu)\hat{\alpha}_i(\hat{l}_i,\hat{l}_j,\varnothing) - \mu P(L \mid r_i,\hat{l}_j)\hat{\alpha}_i(\hat{l}_i,\hat{l}_j,L)\Big),
\end{aligned} \tag{11}$$

$$\begin{aligned}
&\Delta_l^{i,A} = \\
&P(\hat{r}_j \mid l_i)\Big((1-\mu)\hat{\alpha}_i(\hat{r}_i,\hat{r}_j,\varnothing) + \mu P(R \mid l_i,\hat{r}_j)\hat{\alpha}_i(\hat{r}_i,\hat{r}_j,R) - (1-\mu)\hat{\alpha}_i(\hat{l}_i,\hat{r}_j,\varnothing) - \mu P(L \mid l_i,\hat{r}_j)\hat{\alpha}_i(\hat{l}_i,\hat{r}_j,L)\Big) + \\
&P(\hat{l}_j \mid l_i)\Big((1-\mu)\hat{\alpha}_i(\hat{r}_i,\hat{l}_j,\varnothing) + \mu P(R \mid l_i,\hat{l}_j)\hat{\alpha}_i(\hat{r}_i,\hat{l}_j,R) - (1-\mu)\hat{\alpha}_i(\hat{l}_i,\hat{l}_j,\varnothing) - \mu P(L \mid l_i,\hat{l}_j)\hat{\alpha}_i(\hat{l}_i,\hat{l}_j,L)\Big).
\end{aligned} \tag{12}$$

In the REP system the expected gain $\Delta_s^{i,R}$ is given by expressions (11) and (12) with the exception that we substitute $\hat{\alpha}_i(a_i, a_j, X)$ by $\frac{\hat{\alpha}_i(a_i,a_j,X)}{\hat{\alpha}_i(a_i,a_j,X)+\hat{\alpha}_j(a_i,a_j,X)}$. Let $\hat{\alpha}_i^R(a_i, a_j, X) = \frac{\hat{\alpha}_i(a_i,a_j,X)}{\hat{\alpha}_i(a_i,a_j,X)+\hat{\alpha}_j(a_i,a_j,X)}$. Noteworthy, the expressions for $\hat{\alpha}_i^R(a_i, a_j, X)$ greatly simplify when $a_1 \neq a_2$ and $X \neq \varnothing$, in which case $\hat{\alpha}_i^R(a_i, a_j, X) = 1$ for the expert that matches the state and it is 0 for the other expert. For $i, j \in \{1,2\}$ and $i \neq j$, in the RPE system we have:

$$\begin{aligned}
&\Delta_r^{i,R} = \\
&P(\hat{r}_j \mid r_i)\Big((1-\mu)\hat{\alpha}_i^R(\hat{r}_i,\hat{r}_j,\varnothing) + \mu P(R \mid r_i,\hat{r}_j)\hat{\alpha}_i^R(\hat{r}_i,\hat{r}_j,R) - (1-\mu)\hat{\alpha}_i^R(\hat{l}_i,\hat{r}_j,\varnothing) - \mu P(L \mid r_i,\hat{r}_j)\Big) + \\
&P(\hat{l}_j \mid r_i)\Big((1-\mu)\hat{\alpha}_i^R(\hat{r}_i,\hat{l}_j,\varnothing) + \mu P(R \mid r_i,\hat{l}_j) - (1-\mu)\hat{\alpha}_i^R(\hat{l}_i,\hat{l}_j,\varnothing) - \mu P(L \mid r_i,\hat{l}_j)\hat{\alpha}_i^R(\hat{l}_i,\hat{l}_j,L)\Big),
\end{aligned} \tag{13}$$

$$\begin{aligned}
&\Delta_l^{i,R} = \\
&P(\hat{r}_j \mid l_i)\Big((1-\mu)\hat{\alpha}_i^R(\hat{r}_i,\hat{r}_j,\varnothing) + \mu P(R \mid l_i,\hat{r}_j)\hat{\alpha}_i^R(\hat{r}_i,\hat{r}_j,R) - (1-\mu)\hat{\alpha}_i^R(\hat{l}_i,\hat{r}_j,\varnothing) - \mu P(L \mid l_i,\hat{r}_j)\Big) + \\
&P(\hat{l}_j \mid l_i)\Big((1-\mu)\hat{\alpha}_i^R(\hat{r}_i,\hat{l}_j,\varnothing) + \mu P(R \mid l_i,\hat{l}_j) - (1-\mu)\hat{\alpha}_i^R(\hat{l}_i,\hat{l}_j,\varnothing) - \mu P(L \mid l_i,\hat{l}_j)\hat{\alpha}_i^R(\hat{l}_i,\hat{l}_j,L)\Big).
\end{aligned} \tag{14}$$

Proof of Proposition 1

Note that the strategy profile $(\sigma_N^i(l), \sigma_N^i(r)) = (1,1)$ satisfies $\sigma_N^i(l) = \sigma_N^i(r)$, with $\sigma_N^i(l) = \sigma_N^i(r) = \sigma_N^i$ denoting the symmetric strategy of the normal type expert i. Furthermore, note that under symmetric strategies (with $(\sigma_N^i(l), \sigma_N^i(r)) = (1,1)$ being a case of symmetric strategies) an expert is honest with the same probability after either signal $s_i \in \{l_i, r_i\}$. This implies $EU_i(\hat{l}_i \mid l_i) = EU_i(\hat{r}_i \mid r_i)$ and $EU_i(\hat{r}_i \mid l_i) = EU_i(\hat{l}_i \mid r_i)$. Additionally, since $\Delta_r^{i,A} = EU_i(\hat{r}_i \mid r_i) - EU_i(\hat{l}_i \mid r_i)$ and $\Delta_l^{i,A} = EU_i(\hat{r}_i \mid l_i) - EU_i(\hat{l}_i \mid l_i)$, then $\Delta_r^{i,A} = -\Delta_l^{i,A}$.

Next we use σ_N^i, with $i \in \{1,2\}$, and show that $\Delta_r^{i,A} > 0$ for any σ_N^i and σ_N^j, with $i,j \in \{1,2\}$, which implies that $-\Delta_l^{i,A} < 0$. This proves both i) that the strategy profile $(\sigma_N^i(l)^*, \sigma_N^i(r)^*) = (1,1)$ is an equilibrium strategy profile and ii) that this is the unique equilibrium under symmetric strategies.

First, given $i,j \in \{1,2\}$ and $i \neq j$, after some algebra we obtain:

$\Delta_r^{i,A} = \alpha_i \left(\frac{(1-2\gamma)\mu}{(\alpha_i-1)\gamma(2\sigma_N^i-1)-\alpha_i\sigma_N^i+\sigma_N^i-1} + \frac{(\mu-1)(2\gamma-1)(2(\alpha_j-1)(\gamma(2\sigma_N^j-1)-\sigma_N^j)-1)}{f_1} \right)$,

with

$f_1 = 2(\alpha_i-1)(\alpha_j-1)\gamma^2(2\sigma_N^i-1)(2\sigma_N^j-1) + \gamma(-2(\alpha_i-1)\sigma_N^i(4(\alpha_j-1)\sigma_N^j - \alpha_j + 2) + 2\alpha_i\alpha_j\sigma_N^j - 2\alpha_i\sigma_N^j + \alpha_i - 4\alpha_j\sigma_N^j + \alpha_j + 4\sigma_N^j - 2) + (\alpha_j-1)\sigma_N^j(2(\alpha_i-1)\sigma_N^i+1) + (\alpha_i-1)\sigma_N^i + 1$

Second, note that $\Delta_r^{i,A}$ is linear in μ. Then, if $\Delta_r^{i,A}$ is greater than zero both at $\mu = 0$ and $\mu = 1$, then $\Delta_r^{i,A} > 0$. Substituting, it can be shown that:

$\Delta_r^{i,A}\big|_{\mu=1} = \alpha_i \frac{1-2\gamma}{(\alpha_i-1)\gamma(2\sigma_N^i-1)-\alpha_i\sigma_N^i+\sigma_N^i-1} > 0$,

$\Delta_r^{i,A}\big|_{\mu=0} = -\alpha_i \frac{(2\gamma-1)(2(\alpha_j-1)(\gamma(2\sigma_N^j-1)-\sigma_N^j)-1)}{f_1} > 0$,

which concludes the proof. ■

Proof of Proposition 2

The proof follows directly from the fact that when $\alpha_1 = \alpha_2$, threshold μ_2, defined in the proof of Proposition 3, is equal to 0. ■

Proof of Proposition 3.

We focus on the use of symmetric strategies and show that, in this case, the equilibrium is unique. Note that this proves both uniqueness (under symmetric strategies) and existence of the equilibrium (for any strategy of the experts).

The proof requires Lemmas 1-5, which we prove below, and consists of the following steps.

First, by the rationale used in the proof of Proposition 1 to show $\Delta_r^{i,A} = -\Delta_l^{i,A}$, we have $\Delta_r^{i,R} = -\Delta_l^{i,R}$.

Second, by Lemma 1, we learn that the equilibria configurations 4 and 6, which we describe below, are not possible. Third, by Lemma 2, showing $\Delta_r^{1,R} > \Delta_l^{2,R}$, we learn that configurations 1, 3, 7, and 9 below can neither occur. Then, in equilibrium, only configurations 2, 5, and 8 can hold, which imply $\sigma_N^{2*} = 1$. The configurations are the following:

$$\begin{array}{lllll}
1. & \sigma_N^{1*} = 0 & \sigma_N^{2*} = 0 & \Longrightarrow & \Delta_r^{1,R} \leq 0 \quad \Delta_l^{2,R} \geq 0, \\
2. & \sigma_N^{1*} = 0 & \sigma_N^{2*} = 1 & \Longrightarrow & \Delta_r^{1,R} \leq 0 \quad \Delta_l^{2,R} \leq 0, \\
3. & \sigma_N^{1*} = 0 & 0 < \sigma_N^{2*} < 1 & \Longrightarrow & \Delta_r^{1,R} \leq 0 \quad \Delta_l^{2,R} = 0, \\
4. & \sigma_N^{1*} = 1 & \sigma_N^{2*} = 0 & \Longrightarrow & \Delta_r^{1,R} \geq 0 \quad \Delta_l^{2,R} \geq 0, \\
5. & \sigma_N^{1*} = 1 & \sigma_N^{2*} = 1 & \Longrightarrow & \Delta_r^{1,R} \geq 0 \quad \Delta_l^{2,R} \leq 0, \\
6. & \sigma_N^{1*} = 1 & 0 < \sigma_N^{2*} < 1 & \Longrightarrow & \Delta_r^{1,R} \geq 0 \quad \Delta_l^{2,R} = 0, \\
7. & 0 < \sigma_N^{1*} < 1 & \sigma_N^{2*} = 0 & \Longrightarrow & \Delta_r^{1,R} = 0 \quad \Delta_l^{2,R} \geq 0, \\
8. & 0 < \sigma_N^{1*} < 1 & \sigma_N^{2*} = 1 & \Longrightarrow & \Delta_r^{1,R} = 0 \quad \Delta_l^{2,R} \leq 0, \\
9. & 0 < \sigma_N^{1*} < 1 & 0 < \sigma_N^{2*} < 1 & \Longrightarrow & \Delta_r^{1,R} = 0 \quad \Delta_l^{2,R} = 0.
\end{array}$$

Fourth, by Lemma 3, showing $\frac{\partial \Delta_r^{1,R}}{\partial \sigma_N^1} < 0$, we have $\frac{\partial \Delta_l^{1,R}}{\partial \sigma_N^1} > 0$, as $\Delta_r^{1,R} = -\Delta_l^{1,R}$.

Fifth, by Lemma 4, showing $\Delta_r^{1,R}\big|_{\sigma_N^1=1} < 0 \iff \mu > \mu_2$, we have that $\sigma_N^{1*} = 1$ is the unique equilibrium strategy if and only if $\mu > \mu_2$, as $\frac{\partial \Delta_r^{1,R}}{\partial \sigma_N^1} < 0$ and $\frac{\partial \Delta_l^{1,R}}{\partial \sigma_N^1} > 0$.

Last, Lemma 5 below consists of two points. Point 1. shows $\Delta_r^{1,R}\big|_{\sigma_N^1=0} < 0 \iff \alpha_1 > \bar{\alpha}$ and $\mu < \mu_1$. Since $\Delta_r^{1,R} = -\Delta_l^{1,R}$, it implies $\Delta_r^{1,R}\big|_{\sigma_N^1=0} < 0 \iff \Delta_l^{1,R}\Big|_{\sigma_N^1=1} > 0$. Consequently, in the unique equilibrium strategy $\sigma_N^{1*} = 0 \iff \alpha_1 > \bar{\alpha}$ and $\mu < \mu_1$ (as $\frac{\partial \Delta_r^{1,R}}{\partial \sigma_N^1} < 0$ and $\frac{\partial \Delta_l^{1,R}}{\partial \sigma_N^1} > 0$). Finally, point 2. shows $\Delta_r^{1,R}\big|_{\sigma_N^1=0} > 0$ if and only if either $\alpha_1 < \bar{\alpha}$ or both $\alpha_1 > \bar{\alpha}$ and $\mu > \mu_1$ hold. Again, $\Delta_r^{1,R}\big|_{\sigma_N^1=0} > 0 \iff \Delta_l^{1,R}\Big|_{\sigma_N^1=0} < 0$, which further implies that in the unique equilibrium strategy $0 < \sigma_N^{1*} = x < 1$ if and only if either $\alpha_1 < \bar{\alpha}$ or both $\alpha_1 > \bar{\alpha}$ and $\mu > \mu_1$ hold (as $\frac{\partial \Delta_r^{1,R}}{\partial \sigma_N^1} < 0$ and $\frac{\partial \Delta_l^{1,R}}{\partial \sigma_N^1} > 0$), where x is the unique solution of the equation $\Delta_r^{1,R} = 0$, which it is also solution to $\Delta_l^{1,R} = 0$, as $\Delta_r^{1,R} = -\Delta_l^{1,R}$.

Note that, in all the previous cases the equilibrium is unique as $\frac{\partial \Delta_r^{1,R}}{\partial \sigma_N^1} < 0$, $\frac{\partial \Delta_l^{1,R}}{\partial \sigma_N^1} > 0$ and, in all the possible equilibria configurations, $\sigma_N^{2*} = 1$.

Next we show Lemmas 1-5.

Lemma 1. *Under RPE and symmetric strategies, there is not an equilibrium in which* $\sigma_N^{1*} = 1$ *and* $\sigma_N^{2*} < 1$.

Note that function $\Delta_r^{2,R}$ is lineal in μ. Thus, if $\Delta_r^{2,R}\big|_{\mu=0} > 0$ and $\Delta_r^{2,R}\big|_{\mu=1} > 0$, the function will be always positive.

With $\sigma_N^1 = 1$ and $\sigma_N^2 < 1$, using (13), after some calculations we obtain:

$$\Delta_r^{2,R}\big|_{\mu=0} = \frac{\alpha_1\alpha_2(1-2\gamma)(2\alpha_1(\gamma-1)-2\gamma+1)(\alpha_1(\gamma-1)+\gamma(-2\alpha_2\sigma_2+\alpha_2+2\sigma_2-2)+(\alpha_2-1)\sigma_2+1)}{(\alpha_1(\alpha_2-1)(2\gamma-1)\sigma_2-\alpha_1\alpha_2+\alpha_1\gamma-\alpha_2\gamma+\alpha_2)(\alpha_1((\alpha_2-1)(2\gamma-1)\sigma_2-\alpha_2+\gamma-1)-\alpha_2\gamma)} > 0,$$
$$\Delta_r^{2,R}\big|_{\mu=1} = \frac{(2\gamma-1)\big(\alpha_1^2(\gamma-1)((\alpha_2-1)\gamma(2\sigma_2-1)-\alpha_2\sigma_2+\sigma_2-1)-\alpha_1(\gamma-1)((\alpha_2-1)(2\gamma-1)\sigma_2+\alpha_2(-\gamma)-\alpha_2+\gamma-1)-\alpha_2\gamma\big)}{\alpha_1((\alpha_2-1)(2\gamma-1)\sigma_2-\alpha_2+\gamma-1)-\alpha_2\gamma} > 0.$$

Since $\Delta_r^{2,R} > 0$ if $\sigma_N^1 = 1$ and $\sigma_N^2 < 1$, it cannot exist an equilibrium in which $\sigma_N^{1*} = 1$ and $\sigma_N^{2*} < 1$.

Lemma 2. *Under RPE and symmetric strategies,* $\Delta_r^{1,R} > \Delta_l^{2,R}$.

From (13) and (14), $\Delta_r^{1,R}$ and $\Delta_l^{2,R}$ are linear in μ. Given $\sigma_N^2 = 1$ and $\alpha_1 > \alpha_2$, after some algebra it can be shown that:

$$\Delta_r^{1,R} - \Delta_l^{2,R}\Big|_{\mu=0} = \frac{2\alpha_1\alpha_2(1-2\gamma)(\alpha_1(\gamma(2\sigma_1-1)-\sigma_1)-(\alpha_2-1)(2\gamma-1)\sigma_2+\alpha_2\gamma-2\gamma\sigma_1+\sigma_1)^2}{f_2} > 0,$$
$$\Delta_r^{1,R} - \Delta_l^{2,R}\Big|_{\mu=1} = \frac{f_3}{\alpha_1(\gamma(2\alpha_2(\sigma_1+\sigma_2-1)-2\sigma_2+1)-\alpha_2(\sigma_1+\sigma_2)+\sigma_2-1)+\alpha_2(-2\gamma\sigma_1+\gamma+\sigma_1-1)} > 0,$$

with

$$\begin{aligned} f_2 \;=\;& \alpha_1(\gamma(2\alpha_2(\sigma_1+\sigma_2-1)-2\sigma_2+1)-\alpha_2(\sigma_1+\sigma_2)+\sigma_2-1) \\ &+\alpha_2(-2\gamma\sigma_1+\gamma+\sigma_1-1) \\ &(\alpha_1(\gamma(2\alpha_2(\sigma_1+\sigma_2-1)-2\sigma_2+1)-\alpha_2(\sigma_1+\sigma_2)+\sigma_2)+\alpha_2(-2\gamma\sigma_1+\gamma+\sigma_1)), \end{aligned}$$

and

$$
\begin{aligned}
f_3 \;=\;& (2\gamma-1)(\alpha_1^2(\gamma(2\sigma_1-1)-\sigma_1)((\alpha_2-1)\gamma(2\sigma_2-1)-\alpha_2\sigma_2+\sigma_2-1)\\
&+\alpha_1(\alpha_2^2(\gamma(2\sigma_1-1)-\sigma_1)(\gamma(2\sigma_2-1)-\sigma_2)-\alpha_2(2(\gamma^2(2\sigma_1-1)(2\sigma_2-1)\\
&-4\gamma\sigma_1\sigma_2+\gamma+\sigma_1\sigma_2)+\sigma_1+\sigma_2)+(\gamma(2\sigma_1-1)-\sigma_1-1)(\gamma(2\sigma_2-1)-\sigma_2+1))\\
&-\alpha_2(\gamma(2\sigma_1-1)-\sigma_1+1)((\alpha_2-1)\gamma(2\sigma_2-1)-\alpha_2\sigma_2+\sigma_2+1)),
\end{aligned}
$$

which implies $\Delta_r^{1,R} > \Delta_l^{2,R}$. ♦

Lemma 3. *Let $\sigma_N^{2*}=1$. Under RPE and symmetric strategies, $\frac{\partial\Delta_r^{1,R}}{\partial\sigma_N^1}<0$.*

Let $\Delta_r^{1,R}\big|_{\sigma_N^{2*}=1}=\frac{A}{B}$. Then $\frac{\partial\Delta_r^{1,R}}{\partial\sigma_N^1}=\frac{A'B-AB'}{B^2}$, with the sing of the derivative depending on the sign of the numerator $A'B-AB'$.[15] Note that since $\Delta_r^{1,R}$ is linear in μ, the derivative $\frac{\partial\Delta_r^{1,R}}{\partial\sigma_N^1}$ will also be linear in μ. Thus, if $A'B-AB'\big|_{\mu=0}<0$ and $A'B-AB'\big|_{\mu=1}<0$, then $A'B-AB'<0$. It can be shown that:

$$
\begin{aligned}
&A'B-AB'\big|_{\mu=0}=\\
&-(\alpha_1-1)\alpha_1\alpha_2(1-2\gamma)^2(2\alpha_2(\gamma-1)-2\gamma+1)\\
&\big(\alpha_1^2\left(\alpha_2^2(2\gamma-1)(\sigma_N^1-1)((2\gamma-1)\sigma_N^1-1)+\alpha_2(\gamma-1)(2\gamma-1)-\gamma^2+\gamma\right)\\
&-2\alpha_1\alpha_2\left(\alpha_2^2(\gamma-1)((2\gamma-1)\sigma_N^1-1)+\alpha_2(2\gamma-1)\left((2\gamma-1)\left(\sigma_N^1\right)^2-3\gamma\sigma_N^1+\sigma_N^1+1\right)+(\gamma-1)\gamma\right)\\
&+\alpha_2^2\left(\alpha_2(\gamma-1)(2\gamma-1)(2\sigma_N^1-1)+\gamma^2(4(\sigma_N^1-2)\sigma_N^1+3)+\gamma(-4(\sigma_N^1-2)\sigma_N^1-3)+(\sigma_N^1-1)^2)\right)<0,
\end{aligned}
$$

and

$$
\begin{aligned}
&A'B-AB'\big|_{\mu=1}=\\
&(\alpha_1-1)\alpha_1\alpha_2(1-2\gamma)^2(+\alpha_2+\gamma-\alpha_2\gamma)^2((\alpha_1-1)\alpha_2(2\gamma-1)\sigma_N^1+\alpha_1(-\alpha_2)-\alpha_1\gamma+\alpha_1+\alpha_2\gamma)^2<0. \blacklozenge
\end{aligned}
$$

Lemma 4. *Let $\sigma_N^{2*}=1$. Under RPE and symmetric strategies, $\Delta_r^{1,R}\big|_{\sigma_N^1=1}<0 \iff \mu>\mu_2$.*

It can be shown that:

$$
\Delta_r^{1,R}\big|_{\sigma_N^1=1}=\frac{(2\gamma-1)\big((\alpha_2-1)\mu\big(\alpha_1^2\big(\alpha_2(2\alpha_2-1)(\gamma-1)^2-\gamma\big)+\alpha_1\alpha_2\big((\gamma-2)\gamma-3\alpha_2(\gamma-1)^2\big)+\alpha_2^2(\gamma-1)\gamma\big)+\alpha_1\alpha_2(\alpha_1-\alpha_2)(2\alpha_2(\gamma-1)-2\gamma+1)\big)}{(\alpha_1(2\alpha_2-1)-\alpha_2)(\alpha_1(2\alpha_2(\gamma-1)-\gamma)-\alpha_2\gamma)},
$$

which is linear in μ and $\Delta_r^{1,R}\big|_{\sigma_N^1=1}\le 0$ for all $\mu\ge\mu_2$, with

$$
\mu_2=\frac{\alpha_1\alpha_2(\alpha_1-\alpha_2)(2\alpha_2(\gamma-1)-2\gamma+1)}{(1-\alpha_2)\big(\alpha_1^2(\alpha_2(2\alpha_2-1)(\gamma-1)^2-\gamma)+\alpha_1\alpha_2((\gamma-2)\gamma-3\alpha_2(\gamma-1)^2)+\alpha_2^2(\gamma-1)\gamma\big)}. \blacklozenge
$$

Lemma 5. *Let $\sigma_N^{2*}=1$. Under RPE and symmetric strategies:*

1. $\Delta_r^{1,R}\big|_{\sigma_N^1=0}<0 \iff \alpha_1>\bar{\alpha}$ *and* $\mu<\mu_1$,
2. $\Delta_r^{1,R}\big|_{\sigma_N^1=0}>0 \iff$ *either* $\alpha_1<\bar{\alpha}$ *or simultaneously* $\alpha_1>\bar{\alpha}$ *and* $\mu>\mu_1$.

[15] The expressions are too large to be displayed but are available from the authors upon request.

It can be shown that:

$$\Delta_r^{1,R}\Big|_{\sigma_N^1=0} = \frac{(2\gamma-1)\big(-(\alpha_2-1)(\gamma-1)\mu\big(\alpha_1^2(\gamma-\alpha_2(\gamma(\alpha_2+\gamma+1)-1))+\alpha_1\alpha_2\big(\alpha_2((\gamma-1)\gamma+1)+\gamma^2+\gamma\big)-\alpha_2^2(\gamma-1)\gamma\big)-\alpha_1\alpha_2(2\alpha_2(\gamma-1)-2\gamma+1)(\gamma(\alpha_1+\alpha_2-2)-\alpha_2+1)\big)}{(\alpha_2(\alpha_1-\gamma+1)+\alpha_1\gamma)(\alpha_1(\alpha_2+\gamma-1)-\alpha_2\gamma)},$$

which is linear in μ, and $\Delta_r^{1,R}\big|_{\sigma_N^1=0} \leq 0$ for all $\mu \leq \mu_1$ with

$$\mu_1 = \frac{\alpha_1\alpha_2(2\alpha_2(\gamma-1)-2\gamma+1)(\gamma(\alpha_1+\alpha_2-2)-\alpha_2+1)}{(1-\alpha_2)(\gamma-1)\big(\alpha_1^2(\gamma-\alpha_2(\gamma(\alpha_2+\gamma+1)-1))+\alpha_1\alpha_2(\alpha_2((\gamma-1)\gamma+1)+\gamma^2+\gamma)-\alpha_2^2(\gamma-1)\gamma\big)}.$$

Note that $\mu_1 > 0$ if and only if $\alpha_1 > \bar{\alpha}$, with

$$\bar{\alpha} = \frac{\alpha_2\,(1-\gamma)+2\gamma-1}{\gamma}.$$

Then, $\Delta_r^{1,R}\big|_{\sigma_N^1=0} > 0$ if and only if either $\alpha_1 < \bar{\alpha}$ or both $\alpha_1 > \bar{\alpha}$ and $\mu > \mu_1$ hold. In addition, it is straightforward to prove that $\bar{\alpha} = \frac{\alpha_2(1-\gamma)+2\gamma-1}{\gamma}$ is always greater than α_2 and smaller than 1; thus, $\bar{\alpha} \in (\alpha_2, 1)$. It is also directly derived that $\frac{\partial\bar{\alpha}}{\partial\alpha_2} > 0$ and $\frac{\partial\bar{\alpha}}{\partial\gamma} > 0$. ♦

To finish the proof, we show that $\mu_2 > \mu_1$. To this aim, it suffices to obtain
$\frac{\mu_2}{\mu_1} = \frac{(\gamma-1)(\alpha_1-\alpha_2)\big(\alpha_1^2(\gamma-\alpha_2(\gamma(\alpha_2+\gamma+1)-1))+\alpha_1\alpha_2\big(\alpha_2((\gamma-1)\gamma+1)+\gamma^2+\gamma\big)-\alpha_2^2(\gamma-1)\gamma\big)}{(\gamma(\alpha_1+\alpha_2-2)-\alpha_2+1)\big(\alpha_1^2(\alpha_2(2\alpha_2-1)(\gamma-1)^2-\gamma)+\alpha_1\alpha_2((\gamma-2)\gamma-3\alpha_2(\gamma-1)^2)+\alpha_2^2(\gamma-1)\gamma\big)}$,
and to show that this expression is decreasing in α_1, with $\frac{\mu_2}{\mu_1}\Big|_{\alpha_1=1} = 1$ (as for $\alpha_1 = 1$, $\mu_2 = \frac{\alpha_2(2\gamma+2\alpha_2-2\gamma\alpha_2-1)}{\gamma+\alpha_2+\alpha_2^2-\gamma\alpha_2^2} = \mu_1$). This proves $\frac{\mu_2}{\mu_1} > 1$.

Some additional results are the following. They all refer to limit cases.

If $\alpha_1 = 0$, then $\mu_2 = \mu_1 = 0$. If $\gamma = \frac{1}{2}$, then $\mu_2 = \mu_1 = \frac{4\alpha_1\alpha_2^2(\alpha_1-\alpha_2)}{\alpha_1^2\big(2\alpha_2^3-3\alpha_2^2-\alpha_2+2\big)-3\alpha_1\big(\alpha_2^3-\alpha_2\big)-\alpha_2^3+\alpha_2^2}$. If either $\alpha_1 = \alpha_2$ or $\alpha_2 = 0$, then $\mu_2 = \mu_1 = 0$. Hence:

if $\gamma \to \frac{1}{2}$, then $\mu_2 \to \mu_1$

if $\alpha_1 \to 1$, then $\mu_2 \to \mu_1$

if $\alpha_1 \to 0$, then $\mu_2, \mu_1 \to 0$

if $\alpha_2 \to \alpha_1$, then $\mu_2, \mu_1 \to 0$

if $\alpha_2 \to 0$, then $\mu_2, \mu_1 \to 0$

This concludes the proof of Proposition **3.** ■

Proof of Corollary 2

From the definition of thresholds μ_2 and μ_1 (see Lemmas 4 and 5, respectively), we obtain:

$$\frac{\partial\mu_2}{\partial\alpha_1} = \frac{\alpha_2^2(2\alpha_2(\gamma-1)-2\gamma+1)\big(\alpha_1^2\big(2\alpha_2^2(\gamma-1)^2-4\alpha_2(\gamma-1)^2+(\gamma-3)\gamma\big)+2\alpha_1\alpha_2(\gamma-1)\gamma-\alpha_2^2(\gamma-1)\gamma\big)}{(1-\alpha_2)\big(\alpha_1^2\big(2\alpha_2^2(\gamma-1)^2-\alpha_2(\gamma-1)^2-\gamma\big)+\alpha_1\alpha_2((\gamma-2)\gamma-3\alpha_2(\gamma-1)^2)+\alpha_2^2(\gamma-1)\gamma\big)^2} > 0$$

$$\frac{\partial\mu_1}{\partial\alpha_1} = \frac{f_4}{(\alpha_2-1)(\gamma-1)\big(\alpha_1^2(\gamma-\alpha_2(\gamma(\alpha_2+\gamma+1)-1))+\alpha_1\alpha_2(\alpha_2((\gamma-1)\gamma+1)+\gamma^2+\gamma)-\alpha_2^2(\gamma-1)\gamma\big)^2} > 0$$

$$\frac{\partial\mu_2}{\partial\gamma} = \frac{\alpha_1\alpha_2(\alpha_1-\alpha_2)(\alpha_1(2\alpha_2-1)-\alpha_2)\big(\alpha_1(2\alpha_2(\gamma-1)(\alpha_2(\gamma-1)-\gamma)+1)+\alpha_2\big(-2\alpha_2(\gamma-1)^2+2\gamma(\gamma-1)+1\big)\big)}{(\alpha_2-1)\big(\alpha_1^2(\alpha_2(2\alpha_2-1)(\gamma-1)^2-\gamma)+\alpha_1\alpha_2((\gamma-2)\gamma-3\alpha_2(\gamma-1)^2)+\alpha_2^2(\gamma-1)\gamma\big)^2} > 0,$$

$$\frac{\partial\mu_1}{\partial\gamma} = \alpha_1\alpha_2\frac{f_5}{(\alpha_2-1)(\gamma-1)^2\big(\alpha_1^2(\gamma-\alpha_2(\gamma(\alpha_2+\gamma+1)-1))+\alpha_1\alpha_2(\alpha_2((\gamma-1)\gamma+1)+\gamma^2+\gamma)-\alpha_2^2(\gamma-1)\gamma\big)^2},$$

with

$$f_4 = \alpha_2(2\alpha_2(\gamma-1)-2\gamma+1)\alpha_1^2\left(\alpha_2^3(-(\gamma-1))\gamma - \alpha_2^2(\gamma-1)^2(2\gamma+1) + \alpha_2\left(\gamma\left(\gamma^2+\gamma-4\right)+1\right) - 2\gamma^2+\gamma\right)$$
$$+\alpha_2(2\alpha_2(\gamma-1)-2\gamma+1)(2\alpha_1\alpha_2^2(\gamma-1)\gamma^2 + \alpha_2^2(\gamma-1)\gamma(\alpha_2(\gamma-1)-2\gamma+1)),$$

and

$$f_5 = \gamma^4\left(-2(\alpha_1-1)(\alpha_2-1)\alpha_2\left(\alpha_1^2-2\alpha_1-(\alpha_2-2)\alpha_2\right)\right)+$$
$$\left(\alpha_1^3\left(\alpha_2-2\alpha_2^2\right)+\alpha_1^2(\alpha_2-1)^3(2\alpha_2+1)+\alpha_1\alpha_2\left(\alpha_2^2+\alpha_2-1\right)+\alpha_2^2\left(-2\alpha_2^2+3\alpha_2-1\right)\right)+$$
$$\gamma^3\left(2\alpha_2\left(\alpha_1^3(2\alpha_2-1)+\alpha_1^2\left(2\alpha_2^2-10\alpha_2+5\right)+\alpha_1\left(-4\alpha_2^3+7\alpha_2^2+4\alpha_2-4\right)+\alpha_2\left(4\alpha_2^2-4\alpha_2+4\right)\right)\right)+$$
$$\gamma^2\left(\alpha_1^3\left(-3\alpha_2^2+4\alpha_2-1\right)+\alpha_1^2\alpha_2\left(2\alpha_2^3-13\alpha_2^2+23\alpha_2-10\right)+\alpha_1\alpha_2\left(10\alpha_2^3-4\alpha_2^2-12\alpha_2+7\right)+\alpha_2^2\left(-12\alpha_2^2+21\alpha_2-7\right)\right)$$
$$\gamma\left(2(1-\alpha_2)(-2\alpha_1^3\alpha_2+\alpha_1^2(2\alpha_2^3-5\alpha_2^2+2\alpha_2+1)+2\alpha_1\alpha_2^2(\alpha_2+1)+2\alpha_2^2(1-2\alpha_2))\right).$$

In order to obtain the sign of derivative $\frac{\partial\mu_1}{\partial\gamma}$, note that the denominator is always negative. Then, the sign of the derivative will be given by the sign of polynomial f_5, which we write as $f_5(\gamma)$.

It can be shown that $f_5(\gamma)$ is increasing in γ, as $\frac{\partial f_5(\gamma)}{\partial\gamma} > 0$. Additionally, $f_5(\gamma=1)$ is always greater than zero, as $f_5(\gamma=1) = (\alpha_1-1)\alpha_1\left(\alpha_1\left(\alpha_2^2+\alpha_2-1\right)-\alpha_2(\alpha_2+2)\right) > 0$. Then, the sing of $f_5(\gamma)$ and consequently the sign of $\frac{\partial\mu_1}{\partial\gamma}$ depends on the sign of $f_5(\gamma=\frac{1}{2})$. We obtain

$f_5(\gamma=\frac{1}{2}) =$

$\frac{1}{4}\left(-2\alpha_1^3+\left(-4\alpha_1^2+\alpha_1+3\right)\alpha_2^3-(\alpha_1-3)\alpha_1^2\alpha_2+(\alpha_1-1)(4\alpha_1+1)\alpha_2^4+\alpha_1(4-3\alpha_1(\alpha_1+1))\alpha_2^2\right),$

with $f_5(\gamma=\frac{1}{2}) > 0$ if α_1 is smaller than the unique real root of polynomial $f_5(\gamma=\frac{1}{2}) = 0$ in α_1, which we denote by $\hat{\alpha}$.

Therefore, if $\alpha_1 < \hat{\alpha}$, then $f_5(\gamma=\frac{1}{2}) > 0$, which implies $f_5(\gamma) > 0$ and $\frac{\partial\mu_1}{\partial\gamma} < 0$. However, if $\alpha_1 > \hat{\alpha}$, then $f_5(\gamma=\frac{1}{2}) < 0$, which implies there will be a threshold of γ (let us called it $\hat{\gamma}$) such that if $\gamma < \hat{\gamma}$ then $\frac{\partial\mu_1}{\partial\gamma} > 0$ and if $\gamma > \hat{\gamma}$ then $\frac{\partial\mu_1}{\partial\gamma} < 0$. This concludes the proof of Corollary 2. ■

A.2 Part II: A strategic wise type expert

In this section we show that the honest strategy is always an equilibrium strategy for the wise type expert both under the APE and the RPE systems.

Prior to the proofs, let us denote by Λ_s^i the expected gain to the *wise* type expert i from taking action $\hat{r}_i$ rather than $\hat{l}_i$ after observing signal $s_i \in \{l_i, r_i\}$. Under the APE system we denote it by $\Lambda_s^{i,A}$, and we do it by $\Lambda_s^{i,R}$ under the RPE system.

A.2.1 APE system

Proposition 4. *Under APE and symmetric strategies, the honest strategy $\sigma_W^{i*} = 1$ is always an equilibrium strategy for the wise type expert, for all $i \in \{1,2\}$. In addition, if $\sigma_N^{i*} > 0$ for all $i \in \{1,2\}$, then the honest strategy is the unique equilibrium strategy for this type.*

Proof

Suppose expert $i \in \{1,2\}$ is a strategic wise type expert. Then $P(R \mid r_i, \hat{r}_j) = P(R \mid r_i, \hat{l}_j) = P(L \mid l_i, \hat{r}_j) = P(L \mid l_i, \hat{l}_j) = 1$ and $P(L \mid r_i, \hat{r}_j) = P(L \mid r_i, \hat{l}_j) = P(R \mid l_i, \hat{r}_j) = P(R \mid l_i, \hat{l}_j) = 0$. From expression (10) and the definition of $\Lambda_s^{i,A}$, the expressions for the expected gain of the wise type expert are given by:

$$\begin{aligned}
\Lambda_r^{i,A} \quad = \quad & P_W(\hat{r}_j \mid r_i)\Big(((1-\mu)\hat{\alpha}_i(\hat{r}_i,\hat{r}_j,\varnothing) + \mu\hat{\alpha}_i(\hat{r}_i,\hat{r}_j,R)) - ((1-\mu)\hat{\alpha}_i(\hat{l}_i,\hat{r}_j,\varnothing)\Big) + \\
& P_W(\hat{l}_j \mid r_i)\Big(((1-\mu)\hat{\alpha}_i(\hat{r}_i,\hat{l}_j,\varnothing) + \mu\hat{\alpha}_i(\hat{r}_i,\hat{l}_j,R)) - ((1-\mu)\hat{\alpha}_i(\hat{l}_i,\hat{l}_j,\varnothing)\Big), \qquad (15)
\end{aligned}$$

$$\begin{aligned}
\Lambda_l^{i,A} \quad = \quad & P_W(\hat{r}_j \mid l_i)\Big(((1-\mu)\hat{\alpha}_i(\hat{r}_i,\hat{r}_j,\varnothing)) - ((1-\mu)\hat{\alpha}_i(\hat{l}_i,\hat{r}_j,\varnothing) + \mu\hat{\alpha}_i(\hat{l}_i,\hat{r}_j,L))\Big) + \\
& P_W(\hat{l}_j \mid l_i)\Big(((1-\mu)\hat{\alpha}_i(\hat{r}_i,\hat{l}_j,\varnothing)) - ((1-\mu)\hat{\alpha}_i(\hat{l}_i,\hat{l}_j,\varnothing) + \mu\hat{\alpha}_i(\hat{l}_i,\hat{l}_j,L))\Big), \qquad (16)
\end{aligned}$$

for $i, j \in \{1,2\}$ and $i \neq j$, where

$$\begin{aligned}
P_W(\hat{l}_j \mid l_i) &= (\alpha_j + (1-\alpha_j)(\gamma\sigma_N^j(l) + (1-\gamma)(1-\sigma_N^j(r)))), && (17)\\
P_W(\hat{l}_j \mid r_i) &= (1-\alpha_j)(\gamma(1-\sigma_N^j(r)) + (1-\gamma)\sigma_N^j(l)), && (18)\\
P_W(\hat{r}_j \mid l_i) &= (1-\alpha_j)(\gamma(1-\sigma_N^j(l)) + (1-\gamma)\sigma_N^j(r)), && (19)\\
P_W(\hat{r}_j \mid r_i) &= (\alpha_j + (1-\alpha_j)(\gamma\sigma_N^j(r) + (1-\gamma)(1-\sigma_N^j(l))))\gamma. && (20)
\end{aligned}$$

Now, if we compare $\Delta_r^{i,A}$ with $\Lambda_r^{i,A}$, clearly $\Lambda_r^{i,A} > \Delta_r^{i,A}$. Similarly, $\Lambda_l^{i,A} < \Delta_l^{i,A}$. This implies that a wise type expert never lies more than a normal type expert.

Therefore, for any $i, j \in \{1,2\}$ with $i \neq j$, if in equilibrium the normal type expert optimally chooses $\sigma_N^{i*} > 0$, then $\Delta_r^{i,A} \geq 0$ and $\Delta_l^{i,A} \leq 0$, which further implies $\Lambda_r^{i,A} > 0$ and $\Lambda_l^{i,A} < 0$, and hence $\sigma_W^{i*} = 1$. Then, in this case, the unique equilibrium strategy of the wise type expert is the honest strategy.

On the other hand, if $\sigma_N^{i*} = 0$ for at least one normal type expert, we can show that $\sigma_W^{i*} = 1$, for $i \in \{1,2\}$, is an equilibrium strategy for the wise type expert. To this aim, let us assume $\sigma_W^{i*} = 1$ for $i \in \{1,2\}$ and, without loss of generality, consider $\sigma_N^{i*} = 0$ and $\sigma_N^{j*} \geq 0$ for $i, j \in \{1,2\}$ and $i \neq j$. Beliefs (2)-(8) apply in this case.

From (15) and for $i, j \in \{1,2\}$, after some algebra we obtain:

$$\begin{aligned}
\Lambda_r^i = & \tfrac{\alpha_i}{(2(\alpha_i-1))\gamma+1)^2} + (\alpha_i - 1)\gamma \\
& \Big(\tfrac{1-\mu}{2(\alpha_i-1)(\alpha_j-1)\gamma^2(2\sigma_N^j-1)-2(\alpha_i-2)(\alpha_j-1)\gamma\sigma_N^j-\gamma(\alpha_i+\alpha_j-2)-\alpha_j\sigma_N^j+\sigma_N^j-1} + \\
& \tfrac{\mu-1}{-2(\alpha_i-1)(\alpha_j-1)\gamma^2+(\alpha_j-1)(2\gamma-1)\sigma_N^j(2(\alpha_i-1)\gamma+1)-\gamma(\alpha_i+\alpha_j-2)} + 4\Big),
\end{aligned}$$

an expression that can be shown is always positive. Since $\Lambda_r^i > 0$, then $\Lambda_l^i < 0$ (by the same rationale used in Part I of the Appendix), and hence $\sigma_W^{i*} = 1$, for $i \in \{1,2\}$. ■

A.2.2 RPE system

Proposition 5. *Under RPE and symmetric strategies, the honest strategy $\sigma_W^{i*} = 1$ is always the unique equilibrium strategy for the wise type expert if $\sigma_N^{i*} > 0$ for all $i \in \{1,2\}$.*

Proof

The proof is analogous to the proof of Proposition 4.

First, we derive the expressions for $\Lambda_r^{i,R}$ and $\Lambda_l^{i,R}$:

$$
\begin{aligned}
\Lambda_r^{i,R} \quad = \quad & P_W(\hat{r}_j \mid r_i)\Big(((1-\mu)\hat{\alpha}_i^R(\hat{r}_i,\hat{r}_j,\varnothing)+\mu\hat{\alpha}_i^R(\hat{r}_i,\hat{r}_j,R)) - (1-\mu)\hat{\alpha}_i^R(\hat{l}_i,\hat{r}_j,\varnothing)\Big) + \\
& P_W(\hat{l}_j \mid r_i)\Big(((1-\mu)\hat{\alpha}_i^R(\hat{r}_i,\hat{l}_j,\varnothing)+\mu) - (1-\mu)\hat{\alpha}_i^R(\hat{l}_i,\hat{l}_j,\varnothing)\Big),
\end{aligned}
$$

$$
\begin{aligned}
\Lambda_l^{i,R} \quad = \quad & P_W(\hat{r}_j \mid l_i)\Big(((1-\mu)\hat{\alpha}_i^R(\hat{r}_i,\hat{r}_j,\varnothing)) - ((1-\mu)\hat{\alpha}_i^R(\hat{l}_i,\hat{r}_j,\varnothing)+\mu)\Big) + \\
& P_W(\hat{l}_j \mid l_i)\Big(((1-\mu)\hat{\alpha}_i^R(\hat{r}_i,\hat{l}_j,\varnothing)) - ((1-\mu)\hat{\alpha}_i^R(\hat{l}_i,\hat{l}_j,\varnothing)+\mu\hat{\alpha}_i^R(\hat{l}_i,\hat{l}_j,L))\Big),
\end{aligned}
$$

where $P_W(\hat{l}_j \mid l_i)$, $P_W(\hat{l}_j \mid r_i)$, $P_W(\hat{r}_j \mid l_i)$ and $P_W(\hat{r}_j \mid r_i)$ are defined by (17)-(20).

Second, the same argument used in the proof of Proposition 4 shows $\Lambda_r^{i,R} > \Delta_r^{i,R}$ and $\Lambda_l^{i,R} < \Delta_l^{i,R}$.

Third, for any $i, j \in \{1,2\}$ with $i \neq j$, if in equilibrium for the normal type expert optimally chooses $\sigma_N^{i*} > 0$, then $\Delta_r^{i,R} \geq 0$ and $\Lambda_l^{i} \leq 0$, which further implies $\Delta_r^{i,R} > 0$ and $\Delta_l^{i,R} < 0$, and hence $\sigma_W^{i*} = 1$. Then, in this case, the unique equilibrium strategy of the wise type expert is the honest strategy. ■

References

Andina-Díaz, Ascensión (2015), 'Competition and uncertainty in a paper's news desk', *Journal of Economics* **116**(1), 77–93.

Andina-Díaz, Ascensión and José A. García-Martínez (2020), 'Reputation and news suppression in the media industry', *Games and Economic Behavior* **123**, 240–271.

Andina-Díaz, Ascensión and José A. García-Martínez (2021), Garbling an evaluation to retain an advantage. Working paper.

Banerjee, Sanjay (2020), 'Does competition improve analysts' forecast informativeness?', *Management Science, forthcoming* .

Battaglini, Marco (2002), 'Multiple referrals and multidimensional cheap talk', *Econometrica* **70**(4), 1379–1401.

Bourjade, Sylvain and Bruno Jullien (2011), 'The roles of reputation and transparency on the behavior of biased experts', *RAND Journal of Economics* **42**(3), 575–594.

Canes-Wrone, Brandice, Michael C. Herron and Kenneth W. Shotts (2001), 'Leardhip and pandering: A theory of executive policymaking', *American Journal of Political Science* **45**(3), 532–550.

Clark, Andrew E., Paul Frijters and Michael A. Shields (2008), 'Relative income, happiness, and utility: An explanation for the easterlin paradox and other puzzles', *Journal of Economic Literature* **46**(1), 95–144.

Crawford, Vicent P. and Joel Sobel (1982), 'Strategic information transmission', *Econometrica* **50**(6), 1431–1452.

Cummins, Jason G. and Ingmar Nyman (2005), 'The dark side of competitive pressure', *RAND Journal of Economics* **36**(2), 361–377.

Effinger, Matthias R. and Mattias K. Polborn (2001), 'Herding and anti-herding: A model of reputational differentiation', *European Economic Review* **45**(3), 385–403.

Fershtman, Chaim, Hans K. Hvide and Yoram Weiss (2003), 'A behavioral explanation of the relative performance evaluation puzzle', *Annales d'Economie et de Statistique* **71-72**, 349–361.

Fershtman, Chaim, Hans K. Hvide and Yoram Weiss (2006), *Research in Labor Economics: The Economics of Immigration and Social Diversity*, Vol. 24, Emeral Group, chapter Cultural Diversity, Status Concerns and the Organization of Work, pp. 361–396.

Fox, Justin and Richard Van Weelden (2012), 'Costly transparency', *Journal of Public Economics* **96**(1), 142–150.

Gentzkow, Matthew and Jesse M. Shapiro (2006), 'Media bias and reputation', *Journal of Political Economy* **114**(2), 280–316.

Harris, Milton and Bengt Holmström (1982), 'A theory of wage dynamics', *Review of Economic Studies* **49**(3), 315–333.

Heidhues, Paul and Johan Lagerlöf (2003), 'Hiding information in electoral competition', *Games and Economic Behavior* **42**(1), 48–74.

Holmström, Bengt (1982), 'Moral hazard in teams', *The Bell Journal of Economics* **13**(2), 324–340.

Holmström, Bengt (1999), 'Managerial incentive problems: A dynamic perspective', *Review of Economic Studies* **66**(1), 169–182.

Kartik, Navin and Richard van Weelden (2019*a*), 'Informative cheap talk in elections', *Review of Economic Studies* **86**(2), 755–784.

Kartik, Navin and Richard van Weelden (2019*b*), 'Reputation effects and incumbency (dis)advantage', *Quarterly Journal of Political Science* **14**(2), 131–157.

Kreps, David M, Paul Milgrom, John Roberts and Robert Wilson (1982), 'Rational cooperation in the finitely repeated prisoners' dilemma', *Journal of Economic Theory* **27**(2), 245–252.

Krishna, Vijay and John Morgan (2001), 'A model of expertise', *Quarterly Journal of Economics* **116**(2), 747–775.

Levy, Gilat (2004), 'Anti-herding and strategic consultation', *European Economic Review* **48**(3), 503–525.

Levy, Gilat (2007), 'Decision making in committees: Transparency, reputation, and voting rules', *American Economic Review* **97**(1), 150–168.

Li, Ming and Kristóf Madarász (2008), 'When mandatory disclosure hurts: Expert advice and conflicting interests', *Journal of Economic Theory* **139**(1), 47–74.

Lichtendahl, Kenneth C., Yael Grushka-Cockayne and Phillip E. Pfeifer (2013), 'The wisdom of competitive crowds', *Operations Research* **61**(6), 1383–1398.

Lin, Desen (2015), A literature review of career concerns: Theories and empirics. WP.

López-Pintado, Dunia and Miguel A. Meléndez-Jiménez (2019), Far above others. WP.

Luttmer, Erzo F.P. (2005), 'Neighbors as negatives: Relative earnings and well-being', *The Quarterly Journal of Economics* **120**(3), 963–1002.

Meyer, Margaret A. and John Vickers (1997), 'Performance comparisons and dynamic incentives', *Journal of Political Economy* **105**(3), 547–581.

Morris, Stephen (2001), 'Political correctness', *Journal of Political Economy* **109**(2), 231–265.

Ottaviani, Marco and Peter N. Sørensen (2001), 'Information aggregation in debate: Who should speak first?', *Journal of Public Economics* **81**(3), 393–421.

Ottaviani, Marco and Peter N. Sørensen (2006*a*), 'Professional advice', *Journal of Economic Theory* **126**(1), 120–142.

Ottaviani, Marco and Peter N. Sørensen (2006*b*), 'Reputational cheap talk', *RAND Journal of Economics* **37**(1), 155–175.

Ottaviani, Marco and Peter N. Sørensen (2006*c*), 'The strategy of professional forecasting', *Journal of Financial Economics* **81**(2), 441–466.

Panova, Elena (2010), Confirmatory news. Working paper.

Prat, Andrea (2005), 'The wrong kind of transparency', *American Economic Review* **95**(3), 862–877.

Prendergast, Canice and Lars Stole (1996), 'Youngsters and jaded old-timers: Acquiring a reputation for learning', *Journal of Political Economy* **104**(6), 1105–1134.

Roels, Guillaume and Xuanming Su (2014), 'Optimal design of social comparison effects: Setting reference groups and reference points', *Management Science* **60**(3), 606–627.

Scharfstein, David S. and Jeremy C. Stein (1990), 'Herd behavior and investment', *American Economic Review* **80**(3), 465–479.