Migration and Growth in a Schumpeterian Growth Model with Creative Destruction

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Abstract: This paper incorporates endogenous migration into a second-generation Schumpeterian growth model to study how migration, innovation and growth interact one another. I find that migration always enhances the rates of innovation and growth of the receiving economy, but also that the other way round is not true when the gap in technical knowledge between country is fixed over time. However, when the technology gap is allowed to adjusts endogenously, I find that implementing pro-innovation policies in the receiving economy shrinks immigration flows and reduces the across-country technology.

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1 Introduction

International migration has risen significantly in recent years. According to the UN Migration Data Portal (UN DESA, 2020), about two-thirds of global migrants (around 205 million individuals) were migrant workers in 2020, 53 percent of which were located in North America and Europe.[1] These regions are the most R&D-intensive economies of the planet, with the US alone accounting for approximately 28% of the world's R&D funding in 2018. UN DESA (2020) also reveals that the number of working age migrants aging 20-64 years old located in the developed regions increased by a factor of 2.6 from 1990 to 2020, while the same statistics applied to less- and least- developed regions does not go beyond 1.3. A natural question then arises: given that innovation is the main growth driver for most developed countries, what are the effects of an increase in migration on R&D spending and economic growth? And what are the effects of increasing R&D spending on immigration flows of the innovating countries? And Also: Is it possible to state that implementing R&D-enhancing policies in big innovating countries such as the US increases immigration flows?

This paper aims at answering these questions by presenting a lab-equipment variant of the Schumpeterian growth models of Grossman and Helpman (1991) and Aghion and Howitt (1992) where the size of the workforce of the economy is endogenous and determined by immigration. At the aggregate level, there are at least two channels through which immigration can affect R&D investment and economic growth. Firstly, immigration can relax the resource constraint of the receiving economy due to an increase in the size of the workforce that it generates. This effect is commonly referred to as the "scale" effect and tends to increase the incentive of firms to invest in R&D. Secondly, the lower wage rate resulting from immigration reduces production costs, thereby creating a positive "cost-saving" effect that can potentially enhance R&D spending and innovation in the receiving economies.

The paper focuses on the first transmission channel and investigates to what extent the "scale" effect can serve it well to explain how migration affects innovation and growth in the receiving economies. As is known, the first-wave of Schumpeterian growth models predicts that the long-run growth rate of the economy is a positive function of its scale, and thus that larger economies grow faster than smaller economies. However, in a very influential paper, Jones (1995a) empirically rejected the scale effect prediction of most first-generation endogenous growth models (including the R&D based ones), and showed that in the presence of growing populations these models predict explosive growth in the long run. This critique gave rise to the emergence of a new generation of Schumpeterian models where: (i) R&D spending is still endogenous and determined by the investment decisions of profit-maximizing firms; (ii) Growth is scale-free, meaning that it no longer depends upon the size of the population.[2]

[1]Data are from the UN Migration Data Portal (last retrive, March 24, 2021), available at https://migrationdataportal.org/?i=stock_abs_&t=2020.

[2]This literature offers three main approaches to sterilize the scale effect: (i) Diminishing technological opportunities and increasing complexity of R&D [Jones (1995b); Kortum (1997); Segerstrom (1998)]; (ii)

The main objective of the paper is to show that extending a standard second-wave Schumpeterian growth model to migration can rehabilitate the "scale" effect to make it usable to shape the market effect without incurring in a situation of explosive growth. To do this, the paper focuses on the perspective of the receiving country of migration (henceforth, the *domestic* country) and assumes that the flows of migrant workers are governed by a migration function, similar to that recently used by Lecca et al. (2013), in which the net rate of migration of the domestic economy is postulated to depend upon two different components: (1) An exogenous component capturing the non-economic motivations that can make foreign workers wish to move permanently to the domestic country; (2) An endogenous component capturing all of the economic motivations that can make foreign workers wish to move temporarily abroad to exploit international wage differentials.

Since cross-country wage asymmetries can be due to the presence of differences in labor institutions as well as to the existence of international gaps in labor productivity and growth performance, the paper carries out the analysis of the interplay between migration, innovation and growth at two different levels. Initially, I focus on a benchmark scenario with complete technology transfer where the cross-country technology gap is exogenous and fixed by initial conditions. In such a scenario, immigration is a positive function of the differential between the wage rate paid to the immigrant worker in the domestic country and some external reference wage currently paid in their home country (henceforth, the *foreign* country). Next, the paper extends the baseline model to the case in which technology transmission is incomplete, meaning that all of the foreign countries involved in the process of migration present local barriers preventing their economies from fully absorbing the last-invented technology of the domestic economy. In this further scenario, international technology gaps can gradually bridge over time, with the result that changes in cross-country differences in productivity growth can contribute to the formation of cross-country wage differentials.

When productivity growth is assumed to be the same for both the sending and the receiving economy, and then the technology gap is exogenous and determined by initial conditions, I find that the Schumpeterian economy with migration tends to grow faster than its counterpart without migration. Under these circumstances, I find that permanent increases in the immigration ratio of the domestic economy lead to permanent increases in R&D spending and economic growth. However, when the effects of a pro-innovation policy in the form of an increase in patent breadth are introduced, I find that stronger patent protection, though still beneficial for innovation and growth, has no effects on immigration.

To investigate whether the latter result can be seen as a side effect of the fact that new technological knowledge completely spreads among countries when new innovations are developed, in the second part of the paper I consider an extension of the benchmark model in which the technology gap between the receiving and the sending economy can narrow only

Variety expansion [Dinopoulos and Thompson (1998); Peretto (1998); Young (1998); Howitt (1999)]; (*iii*) Rent Protection Activities [Dinopoulos and Syropoulos (2007)]. Though in these models R&D spending is endogenous and driven by firms' innovation efforts, the determinants of the long-run growth growth rates differ markedly across these approaches.

gradually over time. Surprisingly enough, I find that strengthening patent protection in the domestic country increases the rates of innovation and growth of the domestic economy, but decreases immigration. This result is due to the fact that pro-innovation policies tend to compress international technology gaps and can be explained by recalling Gerschenkron's (1986) theory of the "Advantage of Backwardness", according to which the countries that lags behind the technology frontier tends to grow faster than those laying close to it.

This paper relates to the literature that studies the dynamic effects of migration on macroeconomic stability and economic growth. This literature can be split into two main strands of investigation. The first strand aims at studying the macroeconomic effects of international migration through either one- or two-country neoclassical growth models of the Ramsey type. This literature is far from being homogeneous and comprises, among others, studies focusing on: (*i*) the interplay between business cycle and migration [e.g., Mandelman and Zlate (2012), Furlanetto and Robstad (2019), and Smith and Thoenissen (2019)]; (*ii*) The short- and long-run impact of migration on labor market performance and unemployment [e.g., Kiguchi and Mountford (2019), Lozej (2019) and Ikhenaode and Parello (2020)]; (*iii*) The welfare consequences of illegal migration [e.g., Hazari and Sgro (2003), Palivos (2009), Palivos and Yip (2010) and Liu (2010)]; (*iv*) The long-run impacts of migration policy on capital accumulation, macroeconomic stability and social welfare [Ben-Gad (2004), Klein and Ventura (2007, 2009), Khraiche (2015) and Parello (2019, 2021)].

With respect to this literature, this paper presents the following distinctive features. Firstly, the paper focuses on legal migration and endogenous productivity growth. Consequently, one of its contributions to the literature is to propose a tractable model in which the interplay between migration and growth can be jointly analyzed. Secondly, whereas this literature does not pay attention to the role that international technology gap may play in shaping international labor mobility, in this paper this topic is extensively addressed through an extension of the baseline model in which cross-country differences in technical knowledge is the result of the combined effect of past immigration and past R&D efforts exerted by the receiving economies. In this vein, the main result of the paper consists in showing how changes in international technology gaps might shape future migration flows. To the best of my knowledge, these features and results of the paper are new for the literature.

The second strand of literature this paper is related to is that on migration and endogenous growth. Far from being thick, this literature includes models in which the growth process follows an AK-type scheme as in Romer (1986, 1987) [e.g., Faini, (1996) Reichlin and Rustichini (1998), Kemnitz (2001), Larramona and Sanso (2001), Ben-Gad (2008)], and models in which TFP growth is the result of some form of R&D investment [e.g., Bretschger (2001), Lundborg and Segerstrom (2000, 2002), Drinkwater et al. (2007), Mondal and Gupta (2008) and Brunnschweiler et al. (2021)]. In particular, this paper is close to the studies of Lundborg and Segerstrom (2000, 2002), who propose two-country Schumpeterian models of growth with international trade to study the steady-state implications of migration and R&D policy on innovation, consumer welfare and growth; and Brunnschweiler et al. (2021), who develop an R&D-based model of growth with endogenous population to study the interactions between fertility, innovation-led productivity and wealth distribution.

With respect to Lundborg and Segerstrom (2000, 2002), this study differs for the following three reasons. First, the theoretical setting presented in this paper abstracts from international trade and focuses on the perspective of the receiving country of migration. Second, whereas Lundborg and Segerstrom consider immigrant workers as perfects substitutes for native workers, my paper focuses on the more empirically-relevant case in which native and immigrant workers are imperfect substitutes in production. Third, while in Lundborg and Segerstrom's models migration incentives are determined completely by international differences in worker's utility levels, which, in turn, implies that the type of migration considered by their paper is only permanent migration, my model admits both permanent and temporary migration and focuses on international wage differences as the major motivation for migrating. Here, the value added of incorporating temporary migration into a Schumpeterian growth are that: (i) the economy with migration grows faster than that without migration; (ii) strengthening of intellectual property rights (IPR) in the domestic country reduces the productivity-adjusted level of consumption of the native population.

With respect to Brunnschweiler et al. (2021), this paper presents the following distinctive features. First, whereas in Brunnschweiler et al. (2021) innovation is a result of *intra-muros* R&D in the spirit of Peretto (1998), in my framework it is the result of *extra-muros* R&D as in Grossman and Helpman (1991) and Aghion and Howitt (1992). Second, while Brunnschweiler et al. (2021) present a model with heterogeneous agents and endogenous fertility, in this paper agents are homogenous and the growth rate of the population is exogenous over time. Finally, whereas in this paper migration is affected by wages and technology, in Brunnschweiler et al. (2021) it is treated as a pure exogenous parameter to be used to shock the population size.

The paper is organized as follows. Sections 2 presents the baseline model with complete technology transfer and exogenous technology gap. Section 3 discusses the conditions under which the unique steady-state equilibrium of the model is asymptotically stable. Section 4 uses the model to assess the long-run effects that permanent increases in immigrants' reference wage and domestic patent protection might generate on international migration, R&D and growth. Section 5 extends the baseline model to the case of endogenous technology gap and then it checks whether the results of Section 4 are robust to such a change in the pattern of technology transmission. Finally, Section 6 concludes.

2 The model with complete technology transfer

2.1 An overview of the model

I consider an innovation-driven economy populated by a continuum $L_n(t)$ of infinitely-lived native consumers/workers and a continuum $L_m(t)$ of infinitely-lived immigrant consumers/workers, such that $m(t) := L_m(t)/L_n(t)$ is the immigration ratio of the host economy at time t. The growth rate of the native population is exogenous and equal to $\nu > 0$, while the growth rate of the immigrant population is assumed to be endogenously determined by the model through a migration function in the spirit of Harris and Todaro (1970). In particular, to endogenize migration I follow Layard et al. (1991) and Lecca et al. (2013), and assume that net migration flows are positively related to the difference between the log of real wages currently offered in national labor markets.

In the model, final output improves as R&D causes the average quality of technology to increase over time. I begin by presenting a baseline model with complete technology transfer in which technological knowledge is assumed to transfer from one country to another instantaneously. Then, in Section 5, I shall focus on an alternate version of the model with incomplete technology transfer in which technological knowledge is assumed to transfer only gradually between countries.

2.2 Consumption

2.2.1 Preferences

Let the subscript $\ell = \{n, m\}$ denote the type of the consumer, where n identifies native individuals and m identifies non-native individuals (hereinafter immigrants). Individuals are endowed with one unit of labor each. I assume that labor supply is inelastically supplied and that all workers of the same type earn the same wage rate w_ℓ. Regardless of whether individuals are natives or immigrants, they are assumed to have the same lifetime utility function

$$\mathcal{U}_\ell(t) = \int_0^\infty e^{-\rho t} \ln c_\ell(t) \, \mathrm{d}t, \quad \rho > \nu, \tag{1}$$

where ρ is the subjective discount rate of consumers.

Given (1), in the next two subsections, I shall consider the utility maximization problem of each consumer type separately. For expositional convenience, I shall drop the time dependency of the endogenous variables whenever notation is not confusing.

2.2.2 Natives

Natives own firms in equal shares and receive profits as dividends. Their goal is to choose the time path of consumption, $\{c_n(t)\}_{t\in[0,\infty)}$, that maximizes (1) subject to a sequence of budget constraints given by

$$\dot{a}_n = (r - \nu) a_n + w_n - c_n, \quad a_n(0) \text{ given},$$

where a_n is the stock of financial assets owned by the representative native at time t, r is the real rate of return on these assets and w_n is the wage rate paid to native workers. The current-value Hamiltonian associated with this problem writes

$$\mathcal{H} = \log c_n + \xi \left[(r - \nu) a_n + w_n - c_n \right],$$

where ξ is the shadow price of wealth which is the costate variable of the problem. The first-order conditions are $\partial\mathcal{H}/\partial c_n = 0$ and $\partial\mathcal{H}/\partial a_n = \rho\xi - \dot{\xi}$, while the transversality condition is $\lim_{t\to\infty}\left\{e^{-\int_0^t[r(s)-\nu]\mathrm{d}t}\xi a_n\right\}=0$. Therefore, combining these conditions I can write

$$\dot{c}_n = c_n\left(r-\nu-\rho\right) \tag{2}$$

$$\lim_{t\to\infty}\left\{e^{-\int_0^t(r(s)-\nu)\mathrm{d}s}\frac{a_n}{c_n}\right\}=0. \tag{3}$$

Equation (2) is the familiar Euler equation for consumption, according to which only when the real rate of return of assets r exceeds the rate of time preference ρ it is possible to see the per capita consumption of natives grow over time. Equation (3) is instead the standard transversality condition that prevents native consumers from jumping onto explosive paths.

2.2.3 Immigrants

At each time t, migration is supposed to depend upon two different components: a first, long-lasting component not directly linked to the relative economic conditions of the countries involved in the migration process; a second, temporary component in which people's decision to migrate is based on the need of exploiting, even for very short time frames, all of the employment opportunities appearing around the world.

In order to get analytical results, in the remainder of the paper I shall assume that the whole migration process towards the domestic economy is governed by the following migration function (see Layard et al., 1991; Treyz et al., 1993; Lecca et al., 2013)

$$\frac{\dot{L}_m}{L_m}=\varphi\left(\ln w_m-\ln w_f\right)+\zeta,\ \ \zeta\in[0,\nu),\ \ \varphi>0, \tag{4}$$

where $\dot{L}_m\ /L_m$ is the rate of net migration, w_f is the reference wage of immigrants, φ is an elasticity parameter measuring how a change in the wage gap may affect the rate of net migration and ζ is an exogenous parameter that captures all of the other motivations for migrating not related to the relative conditions of local labor markets. From (4), it follows that the immigration ratio of the domestic economy, m, changes over time according to the following differential equation

$$\dot{m}=\left[\varphi\left(\ln w_m-\ln w_f\right)-(\nu-\zeta)\right]m. \tag{5}$$

Given an initial condition $m(0)$, (5) governs the intertemporal evolution of the immigration ratio of the domestic economy. However, to solve for the steady state, I make the following assumption:

Assumption 1 *The reference wage of immigrants is given by*

$$w_f := \bar{w}_f\mathcal{A}_f,$$

where $\mathcal{A}_f$ denotes the level of technology of the foreign economy and $\bar{w}_f>0$ is the productivity-adjusted reference wage of immigrants.

Once in the domestic country, the goal of each immigrant consumer is to choose a path for consumption, $\{c_m(t)\}_{t\in[0,\infty)}$, that maximize (1). To simplify the model, I assume that immigrants are credit-constrained agents, not allowed to smooth consumption through capital markets. This implies that immigrants are hand-to-mouth consumers bounded to consume according to the individual consumption function

$$c_m = w_m. \tag{6}$$

Equation (6) completes the description of the demand side of the domestic economy.

2.3 Production

The production side of the domestic economy consists of a lab-equipment version of the Schumpeterian growth models of Grossman and Helpman (1991) and Aghion and Howitt (1992) extended to include immigrant labor.

2.3.1 The final output

Final output Y is produced by a unit continuum of competitive firms using the following Cobb-Douglas production function

$$Y = Z^{1-\alpha}L^{\alpha}, \ \ \alpha \in (0,1), \tag{7}$$

where Z and L are two composite inputs taking the following forms

$$Z := \exp\left[\int_0^1 \ln z(i)\,\mathrm{d}i\right] \tag{8}$$

$$L := \left[(1-\theta)L_n^{1-1/\phi} + \theta L_m^{1-1/\phi}\right]^{1/(1-1/\phi)}, \ \ \theta \in [0,1), \ \ \phi > 1. \tag{9}$$

The intermediate composite (8) is a Cobb-Douglas aggregator over a unit continuum of differentiated intermediate goods, where $z(i)$ denotes the quantity of the intermediate i used for the production of the final good at time t. The labor composite (9) is instead a CES aggregator over labor types, where θ is the distribution parameter of the CES and ϕ is the elasticity of substitution between labor types. This specification of the composite input L enables me to consider all degrees of substitutability between labor types that lie between the extreme of $\phi = 0$ (perfect complementarity) and $\phi = \infty$ (perfect substitutability).[3] However, since two influential contributions by Manacorda et al. (2012) and Ottaviano and Peri (2012) find significant values for ϕ ranging from 7 to 20 (imperfect substitutability), in

[3] Indeed, when $\phi = 0$, domestic and foreign workers are used in fixed proportions and (9) turns into the "Leontief-like" composite: $L = [\min\{L_n, L_m\}]^{\alpha}$. Likewise, when $\phi = \infty$, domestic and foreign workers are perfect substitutes in production and L becomes linear in L_n and L_m: $L = (1-\theta)L_n + \theta L_m$. Finally, when $\phi = 1$, domestic and foreign workers are complements in production with unitary elasticity of substitution, and (9) boils down to the Cobb-Douglas aggregator: $L = L_n^{1-\theta}L_m^{\theta}$.

the remainder of the paper I shall focus on $\phi > 1$ and restrict the scope of the analysis to the special case in which natives and immigrants are imperfect substitutes in production.

Given (7), (8) and (9), profit maximization leads to the following necessary and sufficient conditions

$$p(i) = \frac{(1-\alpha) Y}{z(i)}, \quad \text{for all } i \in [0,1], \tag{10}$$

$$w_n = \frac{\alpha Y (1-\theta) L_n^{-1/\phi}}{L^{1-1/\phi}} \tag{11}$$

$$w_m = \frac{\alpha Y \theta L_m^{-1/\phi}}{L^{1-1/\phi}}, \tag{12}$$

where $p(i)$ is the price of intermediate i at time t.

Equation (10) implicitly defines the conditional demand schedule of each intermediate i. As is easy to verify, such a demand function has a constant price elasticity equal to -1. Equations (11) and (10) are instead the conditional demand functions for native and immigrant workers respectively. From these conditional demands, it follows that:

Lemma 1 *The relative wage of natives is an increasing function of the immigration ratio of the domestic economy, $m = L_m/L_n$, and is given by*

$$\frac{w_n}{w_m} = \frac{(1-\theta) m^{1/\phi}}{\theta}.$$

Proof. The result follows straightforwardly by dividing (11) by (10). ■

2.3.2 Intermediates

The intermediate sector consists of a continuum of intermediate industries indexed on the unit interval $i \in [0,1]$. Each industry is dominated by a local leader who has been given fully-enforced patent rights for the use of the most advanced technological knowledge lastly introduced in the industry. The production function of the leader is

$$z(i) = A(i) \mathcal{Z}(i), \tag{13}$$

where $\mathcal{Z}(i)$ is the flow of final output used for the production of the ith intermediate and $A(i)$ is the quality-level of technology used by the leader of industry i at time t.

To improve technology, firms compete in innovation races. Following Grossman and Helpman (1991) and Aghion and Howitt (1992), I focus on a step-size quality index of the form $A(i) := \gamma^{j(i)}$, where $\gamma > 1$ is the exogenous step size of the quality improvement and $j(i)$ is a counting for the number of quality improvements that have occurred in industry i as of time t. Since the final good serves as the *numéraire* of the model, the above specification for $A(i)$ implies that the marginal cost of production of the ith industry leader can be written as $MC(i) = 1/A(i)$. Consequently, each time that an innovation occurs and the quality index jumps from level $A_j(i) := \gamma^{j(i)}$ to level $A_{j+1}(i) := \gamma^{j(i)+1}$, the marginal cost drops accordingly from $1/A_j(i)$ to $1/A_{j+1}(i)$.

As the demand curve (10) is unit-elastic, each industry leader will charge the maximum price that avoids competition by the next-best efficient producer. In Grossman and Helpman (1991) and Aghion and Howitt (1992), the optimal pricing consists in charging a markup over time marginal cost equal to the quality step size γ. In this paper, I follow Chu (2011) and assume that the markup depends on the the degree of protection of IPR; this leads to the limit price

$$p(i) = \frac{\mu}{A(i)}, \quad \mu \in (1, \gamma], \tag{14}$$

where μ is the markup over the marginal cost capturing the breadth of patent protection of the domestic country at each time t; i.e., the minimum size of improvements that the next innovating firm has to make in order to obtain a non-infringing patent.[4]

Substituting from (14) into the conditional demand for intermediates (10) and the firm' profit function $\pi(i) = [p(i) - 1] z(i)$, it follows that the flows of sales and profits of each industry leader i can be written as

$$z(i) = \frac{A(i)(1-\alpha)Y}{\mu} \tag{15}$$

$$\pi(i) = \frac{(\mu-1)(1-\alpha)Y}{\mu}. \tag{16}$$

In (15) and (16), the presence of Y causes $z(i)$ and $\pi(i)$ to be dependant upon the size of the domestic economy. However, while (16) turns out to be technology-free and identical across industries, (15) is clearly firm-specific and dependant upon the level of the productivity index $A(i)$. Yet, combining (13) and (15), it can be shown that all industry leaders employ the same amount of final output equal to $\mathcal{Z} = (1-\alpha)Y/\mu$. In Section 2.5, I shall show that a permanent increase in m generates a permanent increase in both $\mathcal{Z}$ and π due to an increase in final output, Y. In the reminder of the paper, I shall refer to this effect as the "scale" effect of migration.

2.4 R&D races

Innovations in any intermediate industry are assumed to follow a Poisson process with arrival rate

$$\iota(i) = \frac{R(i)}{\eta \mathcal{A} L_n}, \quad \eta > 0, \tag{17}$$

[4]The explanation for optimal pricing (14) is the following. When $\mu = \gamma$, patent protection covers the whole step size of innovation, implying that innovators are fully protected against imitation. However, when $\mu < \gamma$, a fraction $\gamma - \mu$ of the improvement in technological knowledge is not covered by the patent and can therefore be imitated by rivals.

where $R(i)$ is R&D spending expressed in terms of final output, η is a technology parameter and $\mathcal{A}$ is the aggregate level of technology of the domestic economy at time t. The presence of $\mathcal{A}$ at the denominator of (17) captures the fact that the inventions that are easier to discover tend to be discovered earlier in time, while the presence of L_n captures the idea that introducing successfully new intermediate products and to replace old ones is more difficult in a larger market. Both terms are necessary for the attainment of non-explosive growth rates in the steady state.[5]

To fund the up-front cost of R&D, firms sell equity shares to domestic households. Let $v(i)$ be the the value of a patented innovation in industry i at time t. Shares from innovating firms pay dividends at rate $\pi(i)\mathrm{d}t$, earn capital gain at rate $[\dot{v}(i)/v(i)]\mathrm{d}t$ and suffer capital loss $v(i)$ with probability $\iota(i)\mathrm{d}t$. This gives the following no-arbitrage condition for R&D investment

$$rv(i) = \pi(i) - \iota(i)\, v(i) + \dot{v}(i). \tag{18}$$

Due to perfect competition in R&D, patents are priced at marginal cost. This requires

$$v(i) = \eta \mathcal{A} L_n. \tag{19}$$

Equation (19) is the free-entry condition to R&D. Notice that when (19) holds, firms are indifferent between R&D projects in different industries and R&D spending can be chosen such that each industry has the same flow rate $\iota(i) = \iota$.

2.5 Aggregate final output and productivity growth

In Section 2.3.2, I have shown that all intermediate producers employ final output at the same intensity. Consequently, combining (7), (9) and (15), it follows that the aggregate final output is an increasing function of the immigration ratio

$$Y = \mathcal{A}\left(1 - \theta + \theta m^{1-1/\phi}\right)^{1/(1-1/\phi)} L_n, \tag{20}$$

where $\mathcal{A} := \exp\left[\int_0^1 \ln A(i)\, \mathrm{d}i\right] = \exp\left[\int_0^1 \ln \gamma^{j(i)} \mathrm{d}i\right]$ is an aggregate productivity index capturing the overall level of technological knowledge of the domestic economy at time t. Applying the law of large numbers, it follows that the growth rate of aggregate productivity can be written as

$$\frac{\dot{\mathcal{A}}}{\mathcal{A}} = \iota \ln \gamma. \tag{21}$$

From (21) it is easy to see that an increase in immigration does not affect productivity growth directly. However, as (20) clearly shows, migration can affect R&D spending and

[5] The approach used in this paper to get non-explosive growth rates is similar to that introduced by Li (2003). It can be shown that using $L_n + L_m$ to proxy the scale of the domestic economy does not change the main results of the paper while it complicates the analytical structure. The formal demonstration of this result is available upon request.

innovation indirectly through changes in the size of final output. To see this more succinctly, I combine (16), (18), (19), (20) and (21) to obtain the following expression for the rate of return on R&D

$$r = \frac{(1-1/\mu)(1-\alpha)\left(1-\theta+\theta m^{1-1/\phi}\right)^{1/(1-1/\phi)}}{\eta} + \nu - \iota\,(1-\ln\gamma)\,. \tag{22}$$

As is easy to verify from (22), increases in the immigration ratio m lead to increases in the real rate of return, r. This result is due to the "scale" effect of migration introduced in Section 2.3.2 and represents a distinguishing feature of this model with respect of the standard Schumpeterian framework without migration.

2.6 Resource constraints

The domestic economy has three production inputs: final good, native labor and immigrant labor. Given (20), the final-good market clearing condition requires the sum of aggregate consumption, $c_nL_n + c_mL_m$, intermediates production, $\int_0^1 \mathcal{Z}(i)\mathrm{d}i$, and R&D investment, $\int_0^1 R(i)\mathrm{d}i$, to equate aggregate final good, Y; that is, it must be that $Y = c_nL_n + c_mL_m + \int_0^1 \mathcal{Z}(i)\mathrm{d}i + \int_0^1 R(i)\mathrm{d}i$ is satisfied at each time t. Therefore, combining (6), (12) (13), (15), (17) and (20), and recalling that in the symmetric equilibrium $\mathcal{Z}(i) = \mathcal{Z}$ and $\iota(i) = \iota$, the final-good market clearing condition reads

$$1 = \frac{c_n/\mathcal{A}+\eta\iota}{\left(1-\theta+\theta m^{1-1/\phi}\right)^{1/(1-1/\phi)}} + \frac{\alpha\theta m^{1-1/\phi}}{1-\theta+\theta m^{1-1/\phi}} + \frac{1-\alpha}{\mu}. \tag{23}$$

In the labor market, conditional demands for native and immigrant labor are given by (11) and (10). Using (9) and (20) to substitute for L and Y in (10) and (11) yields

$$w_n = \alpha(1-\theta)\mathcal{A}\left(1-\theta+\theta m^{1-1/\phi}\right)^{1/(\phi-1)} \tag{24}$$

$$w_m = \alpha\theta\mathcal{A}\left(1-\theta+\theta m^{1-1/\phi}\right)^{1/(\phi-1)} m^{-1/\phi}. \tag{25}$$

The wage rates from (24) and (25) both grow at same rate as productivity (21). To ensure balanced growth, it is therefore necessary to make the following assumption:

Assumption 2 *The level of technology of the foreign economy, $\mathcal{A}_f$, grows over time at the same rate as the level of technology of the domestic economy, $\mathcal{A}$.*

Implicitly, Assumption 2 postulates that when a new innovation is introduced in the domestic economy, it is instantaneously transferred to (and applied by) the foreign country. This assumption is made just to make migration only dependant upon the relative conditions of national labor markets and can be justified in many ways. For instance, it can be justified by saying that no institutional barriers to the adoption of the innovations introduced by domestic firms exist in the foreign country, and that all of the newest technologies invented by the domestic country are always appropriate for the production system of the foreign country. In Section 5, I shall remove this assumption and shall show that if domestic innovations are only partially implementable abroad, many findings of the baseline model may change substantially.

From Assumption 2, it follows that the cross-country technology gap $\mathcal{A}/\mathcal{A}_f$ is always stationary over time and is fixed by initial conditions $\mathcal{A}(0)/\mathcal{A}_f(0)$.[6] Therefore, plugging (25) into (5), it can be shown that the rate of net migration towards the domestic country is independent of technical change and is given by

$$\frac{\dot{m}}{m} = \varphi \ln \left\{ \frac{\alpha\theta \left[(1-\theta) m^{1/\phi - 1} + \theta \right]^{1/(\phi-1)}}{\omega_f(0)} \right\} - (\nu - \zeta), \tag{26}$$

where $\omega_f(0) := \bar{w}_f \mathcal{A}_f(0)/\mathcal{A}(0)$ is the productivity-adjusted reference wage of immigrants.

3 The perfect-foresight dynamic equilibrium

3.1 Characterization of the equilibrium

Definition 1 *For any initial levels of aggregate technology, $\mathcal{A}(0)$, and immigration ratio, $m(0)$, a perfect-foresight dynamic equilibrium consists of time paths $\{c_n,\ c_m,\ \mathcal{A},\ m,\ \mathcal{Z},\ R\}_{t\in[0,\infty)}$ and $\{r,\ w_n,\ w_m,\ [p(i)]_{i\in[0,1]}\}_{t\in[0,\infty)}$ that: (i) satisfy the system of equations (2), (6), (14), (15), (17), (21), (24), (25), (26) and (22); (ii) fulfill the inequality constraints $c_n \geq 0$, $c_m \geq 0$, $\mathcal{A} \geq 0$, $m \geq 0$, $\mathcal{Z} \geq 0$, $R \geq 0$; (iii) satisfy the transversality conditions (3); (iv) clear the resource constraint (23).*

The intertemporal behavior of the model can be studied through a differential-algebraic system given by (2), (21), (22), (23) and (26). In the system, the differential equations (2), (21) and (26) determine the time paths of c_n, $\mathcal{A}$ and m, and the algebraic equations (23) and (22) determine the equilibrium values of ι and r. For a long-run equilibrium to exist, it must be that natives' consumption and productivity grow boundlessly at a common (and constant) rate $g > 0$. When this happens, the immigration ratio m and the consumption-to-technology ratio $c_n/\mathcal{A}$ appearing on the right-hand side of (23) are both stationary over time.

Therefore, defining the productivity-adjusted level of natives' consumption as $x_n := c_n/\mathcal{A}$, differentiating it with respect to time and then using (2), (21), (22) and (23) to get rid of $\dot{c}_n$, $\dot{\mathcal{A}}$, ι and r from the resulting expression, the study of the dynamic properties of

[6] Notice that assuming exogenous and everlasting differences in TFP between the receiving and sending economies is the custom in many dynamic two-country macro-models with endogenous migration and exogenous growth. Examples are the models of Klein and Ventura (2007, 2009), Mandelman and Zlate (2012) and Ikhenaode and Parello (2020).

the model reduces to study the time behavior of the following two-dimensional system of differential equations in x_n and m

$$\frac{\dot{x}_n}{x_n}=\frac{x_n-\alpha\left(1-\theta\right)\left(1-\theta+\theta m^{1-1/\phi}\right)^{1/(\phi-1)}}{\eta}-\rho. \tag{27}$$

$$\frac{\dot{m}}{m}=\varphi\left\{\ln\left(\frac{\alpha\theta}{\omega_f\left(0\right)}\right)+\frac{\ln\left[\left(1-\theta\right)m^{1/\phi-1}+\theta\right]}{\phi-1}\right\}-\left(\nu-\zeta\right), \tag{28}$$

where (28) comes directly from rearranging (26).

In system (27)-(28), x_n acts as a "pseudo" jump variable and m acts as a state variable, whose value at time $t=0$ is predetermined by history. A steady-state equilibrium for the domestic economy is reached if $\dot{x}_n=\dot{m}=0$ holds over time. When this happens, the long-run ratio of immigrants over natives is constant over time and equal to m^*, while natives' per capita consumption, c_n, and technology, $\mathcal{A}$, are pure exponential functions of time in the form

$$c_n\left(t\right)=c_n\left(0\right)e^{gt}\ \text{ and }\ \mathcal{A}\left(t\right)=\mathcal{A}\left(0\right)e^{gt}. \tag{29}$$

The following proposition shows under what conditions a steady-state equilibrium for the domestic economy exists and is unique

Proposition 1 *If*

$$\omega_f\left(0\right)<\alpha\theta^{\phi/(\phi-1)}e^{(\zeta-\nu)/\varphi} \tag{30}$$

$$\rho<\frac{\left(1-1/\mu\right)\left(1-\alpha\right)\left(1-\theta\right)^{1/(1-1/\phi)}}{\eta} \tag{31}$$

are satisfied, then, for any given initial condition $m\left(0\right)$, there exists a unique steady-state equilibrium for the dynamic system (27)-(28) where: (i) The levels of the immigration ratio m and productivity-adjusted per capita consumption x_n are given by

$$m^*=\left\{\left[\frac{\omega_f\left(0\right)e^{(\nu-\zeta)/\varphi}}{\alpha\theta\left(1-\theta\right)^{1/(\phi-1)}}\right]^{\phi-1}-\frac{\theta}{1-\theta}\right\}^{\phi/(1-\phi)} \tag{32}$$

$$x_n^*=\rho\eta+\alpha\left(1-\theta\right)\left[1-\theta+\theta\left(m^*\right)^{1-1/\phi}\right]^{1/(\phi-1)}; \tag{33}$$

(ii) The rates of innovation and growth are both positive and equal to

$$\iota^*=\frac{\left(1-1/\mu\right)\left(1-\alpha\right)\left[1-\theta+\theta\left(m^*\right)^{1-1/\phi}\right]^{1/(1-1/\phi)}}{\eta}-\rho \tag{34}$$

$$g^*=\left\{\frac{\left(1-1/\mu\right)\left(1-\alpha\right)\left[1-\theta+\theta\left(m^*\right)^{1-1/\phi}\right]^{1/(1-1/\phi)}}{\eta}-\rho\right\}\ln\gamma; \tag{35}$$

(iii) The unique equilibrium trajectory converging to the steady state (32)-(33) is saddle-path stable.

Proof. See appendix A. ■

Based on Proposition 1, if (30) and (31) are both satisfied, a unique steady state exists where the domestic economy grows smoothly over time at rate (35). The first restriction on $\omega_f(0)$ identifies an upper bound for the level of the productivity-adjusted reference wage of immigrants beyond which emigration is no longer convenient for non-natives. In fact, when (30) does not hold, foreign individuals do not find it convenient to leave their home country and work abroad. Consequently, it is only when (30) is satisfied that migration turns out to be positive at the steady-state equilibrium. The second restriction on the rate of time preference ρ guarantees that the rates of innovation and economic growth are both positive in the long run, even in the absence of migration. Indeed, when (31) is binding, (34) and (35) are both positive even in the case in which labor cannot move across countries (i.e. when $m^* = 0$).

Glancing at (35), it can be easily seen that one of the results of the paper is that the Schumpeterian economy with migration grows faster than that with labor immobility. Intuitively, this result is due to the presence of m^* on the right-hand side of (35), which, in turn, is due to the positive "scale" effect that migration on firms' profits. Indeed, when $\theta = m^* = 0$ holds, the rates of innovation and growth of the domestic economy reduce to $\tilde{\iota} = (1 - 1/\mu)(1 - \alpha)/\eta - \rho$ and $\tilde{g} = \tilde{\iota} \ln \gamma$, which are the rates of innovation and growth of the lab-equipment version of a standard quality-ladder Schumpeterian growth model without labor migration. This finding can be summarized by the following proposition

Proposition 2 *Suppose that restrictions (30) and (31) are satisfied. Then, opening-up a Schumpeterian economy to migration is beneficial for innovation and economic growth.*

Proof. See the text. ■

Propositions 1 and 2 complete the characterization of the dynamic properties of the perfect-foresight equilibrium of the model. In the next section, I shall apply the model to assess the macroeconomic effects of increasing immigration and patent protection on domestic R&D spending and growth.

4 Assessing the effects of an increase in migration and patent protection

4.1 The effects of an increase in migration

Consider the domestic economy in its own steady-state equilibrium and suppose that, for any given ι^*, the productivity-adjusted level of the reference wage of immigrants $\bar{w}_f$ decreases permanently. The following proposition summarizes the main findings of this section:

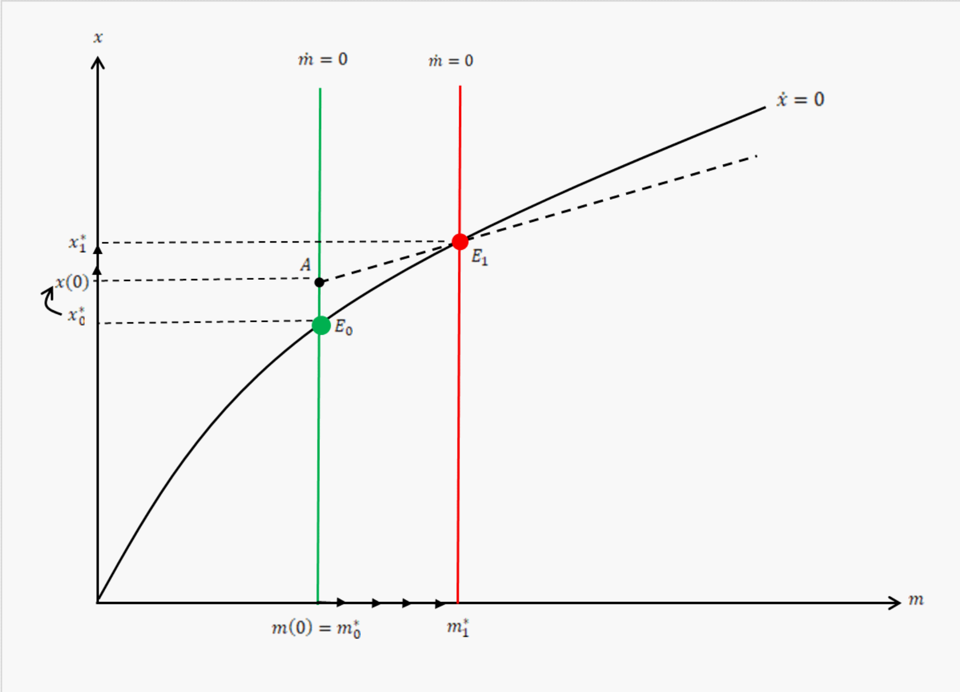

Figure 1: Transitional dynamics. The Figure assumes a permament fall in $\omega_f(0)$.

Proposition 3 *A permanent fall in the productivity-adjusted reference wage of immigrants $\bar{w}_f$ leads to: (i) a permanent increase in the steady-state immigration ratio, m^*; (ii) a permanent increase in the steady-state level of consumption per native, x_n^*; (iii) a permanent increase in the steady-state rates of innovation, ι^*, and economic growth, g^*.*

Proof. See Appendix B. ■

According to Proposition 3, an increase in migration is beneficial for the growth performance of the domestic economy. To see why this is so, suppose that the domestic economy in its own steady-state equilibrium - see point E_0 in Figure 1 - and assume that, at $t = 0$, a negative shock makes $\omega_f(0)$ fall permanently. Before the shock, the long-run equilibrium of the economy was characterized by the pair $\{m_0^*, x_{n,0}^*\}$ and for a time path of natives' consumption equal to $c_n(t) = x_{n,0}^* \mathcal{A}(0) e^{g_0^* t}$, where g_0^* is the equilibrium growth rate of the domestic economy when the equilibrium is at E_0.

The fall in $\omega_f(0)$ causes the economy to deviate from the steady state $\{m_0^*, x_{n,0}^*\}$. In response, natives modify their consumption/saving decision by making their consumption-to-productivity ratio jump from $x_{n,0}^*$ to $x_n(0)$. To do this, natives have to revise their initial level of consumption and choose $c_n(0) = x_{n,1}^* \mathcal{A}(0)$ so as to allow the economy to jump onto the unique stable arm (the dashed line in Figure 1) that monotonically converges to the after-shock steady-state equilibrium $\{m_1^*, x_{n,1}^*\}$ - see point E_1 of Figure 1. Along the converging path, both x_n and m grow smoothly over time, implying that, for a temporary time frame, consumption per capita, c_n, has to grow faster than domestic technology $\mathcal{A}$.

It is worth noticing that while the long-run effects on innovation and growth are clearly positive, the short-run effects on R&D spending and growth are negative because of the short-run increase in consumption. Indeed, while it is incontestable that the increase in m is beneficial for the economic growth in the long run, in the short run the initial jump of natives' per capita consumption $c_n(0)$ slows down innovation and growth because it reduces the amount of resources that can be devoted to research.

4.2 The effects of rising patent breadth

Consider now a further comparative statics exercise in which, starting from an initial steady-state equilibrium $\{m_0^*, x_{n,0}^*\}$, the government of the domestic economy decide to strengthening IPR protection by increasing patent breadth, μ. Overall, the steady-state effects of increasing μ can be summarized as follows

Proposition 4 *A permanent increase in patent breadth, μ: (i) has no effects on the steady-state immigration ratio, m^*; (ii) has no effects on the steady-state level of the technology-adjusted consumption of natives, x_n^*; (iii) increases both the steady-state rate of innovation, ι^* and economic growth, g^*.*

Proof. See Appendix B. ■

Whereas result (iii) of Proposition 4 lines up with the Schumpeterian growth literature, results (i) and (ii) are new for literature and deserve to be discussed. Result (i) is quite intuitive and can be easily explained by recalling Assumption 2 of the previous section. Indeed, when the foreign country is always able to bridge the technology gap with the domestic economy completely, each time that a new innovation occurs the wage ratio between countries remains unchanged and migration is not affected by domestic R&D. Therefore, in such a scenario it is the local conditions of labor markets that matters in determining relative wages and migration flows.

The explanation for result (ii) of Proposition 4 is instead a bit more involving and relies on the fact that (33) is independent of μ. This, in turn, can be explained by the need of the domestic economy to stay at the steady state when migration is unchanged, a need that the model can only satisfy if and only if the increase in the quality-level of technology $\mathcal{A}$ induced by the change in patent policy is exactly compensated by an increase in natives' consumption c_n.

To see this point more closely, consider again the initial steady-state equilibrium E_0 depicted in Figure 1. When μ increases, firms find it convenient to increase their R&D spending to speed-up the invention of new technologies. However, to accommodate the increase in R&D spending, natives' consumption must instantaneously adjust to compensate the increase in technology and thus let the domestic economy remain stuck at $\{m_0^*, x_{n,0}^*\}$.[7]

[7]Graphically, this result follows from the fact that the two isoclines $\dot{x}_n = 0$ and $\dot{m} = 0$ in Figure 1 do not move because of the increase in μ, and hence from the fact that that the initial steady-state equilibrium

Evidently, this result is quite unrealistic, especially when the process of migration involves least developed and emerging economies. Consequently, in the next section I shall consider an alternate scenario in which the technology gap between countries is endogenously determined by the model.

5 Endogenous technology gap

In this section, I shall remove Assumption 2 and postulate that: (*i*) international technology transfer is incomplete, so that only a share of domestic R&D is useful to improve productivity in the foreign country; (*ii*) the knowledge spillover that domestic innovations generate on foreign productivity tend to fade away over time because of technology obsolescence.

5.1 The model with incomplete technology transfer

I consider the same innovation-driven economy with migration of Section 2, in which agents' preferences and firms' technologies are still given by (1), (7), (8), (9), (13) and (17). In this economy, immigration still evolves according to (5), where the reference wage of immigrants is still supposed to be proportional to the technology level of the foreign country; i.e. $w_f = \bar{w}_f \mathcal{A}_f$. However, in contrast with the baseline model of Section 2, in this section I assume that the foreign economy can update her technology to that of the domestic country only gradually, so that $\mathcal{A}_f$ always differs from $\mathcal{A}$, with $\mathcal{A}_f < \mathcal{A}$.

More specifically, throughout this section I shall assume the following

Assumption 3 *At each time t, the level of technology of the foreign economy is given by*

$$\mathcal{A}_f(t) = \varepsilon \int_0^t \left[\frac{R(s)}{L_n(s)} \right] e^{\psi(s-t)} ds, \quad \varepsilon \in [0,1), \quad \psi \geq 0. \tag{36}$$

In (36), the stock of technical knowledge of the foreign economy is described as a declining weighted average of the past R&D investment of the domestic economy, $R(t)/L_n(t)$, where ε is an externality parameter capturing how much of the domestic R&D spending contributes in the formation of $\mathcal{A}_f$ and ψ is a persistence parameter measuring how long an increase in R/L_n contributes to the formation of $\mathcal{A}_f$.[8] Again, division by L_n of R&D spending R serves to avoid the generation of explosive paths for innovation and growth rates.

obtained for system (28) and (27) is not affected by the policy change of the domestic economy.

[8]The presence of the weighting function $e^{\psi(s-t)}$ implies that new R&D spending contributes more to foreign technology than those further back in time. Indeed, when $s = t$, only the share ε of the domestic country's R&D spending to improve the state of the art of technology of the foreign country. However, when $s < t$, the contribution of the domestic country's R&D spending to the formation of $\mathcal{A}_f(t)$ is smaller the bigger is the distance of s from t. This feature of (36) is intended to introduce some form of obsolescence induced by creative destruction even in the foreign country.

Assumption 3 does not modify the main optimality conditions of the baseline model, implying that natives' consumption and domestic technology still evolve over time according to the same dynamic equations (2) and (21). Nor it modifies the resource constraint of the domestic economy, which can be still written as in (23). What it does change because of Assumption 3 is (26) due to the fact that now the technology gap between countries plays a key role in shaping the migration flows towards the domestic country. Indeed, plugging (25) into (5), it is easy to verify that the time path of the immigration ratio m now depends on the technology gap $\mathcal{A}/\mathcal{A}_f$ according to

$$\frac{\dot{m}}{m} = \varphi \left\{ \ln\left(\frac{\alpha\theta}{\bar{w}_f}\right) + \ln\left(\frac{\mathcal{A}}{\mathcal{A}_f}\right) + \frac{\ln\left[(1-\theta)\, m^{1/\phi-1} + \theta\right]}{\phi - 1} \right\} - (\nu - \zeta), \tag{37}$$

where, upon differentiation of (36), it can be easily shown that the level of technology of the foreign country is expected to evolve over time according to the following differential equation

$$\dot{\mathcal{A}}_f = \frac{\varepsilon R}{L_n} - \psi \mathcal{A}_f. \tag{38}$$

Notice that, in contrast to (26), in (37) cross-country R&D spillovers play an active role in shaping migration flows. In fact, permanent changes in R&D spending per domestic worker, R/L_n, have permanent effects on the long-run immigration ratio m through (38).

5.2 Characterization of the dynamic equilibrium

A perfect-foresight dynamic equilibrium for the model with incomplete technology transfer can be defined as follows:

Definition 2 *For any initial conditions $\mathcal{A}(0)$, $\mathcal{A}_f(0)$ and $m(0)$, a perfect-foresight dynamic equilibrium for the extended model consists of time paths $\{c_n,\ c_m,\ \mathcal{A},\ m,\ \mathcal{Z},\ R\}_{t\in[0,\infty)}$ and $\{r,\ w_n,\ w_m,\ [p(i)]_{i\in[0,1]}\}_{t\in[0,\infty)}$ that: (i) satisfy the system of equations (2), (6), (14), (15), (17), (21), (24), (25), (22), (37) and (38); (ii) fulfill the inequality constraints $c_n \geq 0$, $c_m \geq 0$, $\mathcal{A} \geq 0$, $\mathcal{A}_f \geq 0$, $m \geq 0$, $\mathcal{Z} \geq 0$, $R \geq 0$; (iii) satisfy the transversality conditions (3); (iv) clear the resource constraint (23).*

From Definition 2, it follows that the dynamics of the extended model can be studied through a new differential-algebraic system formed by the differential of equations (2), (21), (37) and (38), which establish the equilibrium paths of c_n, $\mathcal{A}$, m and $\mathcal{A}_f$, and by the algebraic equations (22) and (23), which determine the values of ι and r. Thus, by defining the productivity-adjusted level of consumption of natives by $x_n = c_n/\mathcal{A}$ and the relative technology level of the domestic economy by $\Omega := \mathcal{A}/\mathcal{A}_f$, the dimension of the dynamic system of the model can be reduced to only three nonlinear differential equations in x_n, m and Ω. In particular, combining (2), (21), (23) and (22), it can be seen that the dynamic equation governing the time path of x_n does not change because of Assumption 3 and is still given by (27). Next, to yield the law of motion of m, it suffices to replace $\mathcal{A}/\mathcal{A}_f$ with Ω in (37) to obtain

$$\frac{\dot{m}}{m} = \varphi \left\{ \ln \left(\frac{\alpha\theta}{\bar{w}_f} \right) + \ln \Omega + \frac{\ln \left[(1-\theta)\, m^{1/\phi - 1} + \theta \right]}{\phi - 1} \right\} - (\nu - \zeta) \,. \tag{39}$$

Finally, to determine the differential equation governing the dynamics of the relative productivity of the domestic economy Ω, I log-differentiate Ω with respect to time and then use (17), (21), (23) and (36) to get rid to $\dot{\mathcal{A}}$, ι and $\dot{\mathcal{A}}_f$; this gives the following expression

$$\frac{\dot{\Omega}}{\Omega} = \psi - \left[\frac{(\mu + \alpha - 1)(1-\theta) + (\mu - 1)(1-\alpha)\,\theta m^{1-1/\phi}}{\eta\mu \left(1 - \theta + \theta m^{1-1/\phi}\right)^{1/(1-\phi)}} - \frac{x_n}{\eta} \right] (\varepsilon\eta\Omega - \ln\gamma) \,. \tag{40}$$

A steady state equilibrium for the domestic economy is given by the triple $\{m^*, \Omega^*, x_n^*\}$ solving (27), (39) and (40) when $\dot{x}_n = \dot{m} = \dot{\Omega} = 0$ holds. In the steady state, the immigration ratio m and the relative technology of the domestic country are both constant over time and equal to m^* and Ω^*, while natives' per capita consumption, c_n, and technology, $\mathcal{A}$, are pure exponential functions as in (29) growing smoothly over time at the common rate $g > 0$.

Analytically, the steady-state equilibrium of the model can be obtained by solving the following algebraic system[9]

$$x_n = \eta\rho + \alpha(1-\theta)\left(1 - \theta + \theta m^{1-1/\phi}\right)^{1/(\phi-1)} \tag{41}$$

$$\Omega \left(\frac{\alpha\theta}{\bar{w}_f} \right) \left[(1-\theta)\, m^{1/\phi-1} + \theta \right]^{1/(\phi-1)} = e^{(\nu-\zeta)/\varphi} \tag{42}$$

$$\frac{(1 - 1/\mu)(1-\alpha)\left(1 - \theta + \theta m^{1-1/\phi}\right)^{1/(1-1/\phi)}}{\eta} = \rho + \frac{\psi}{\varepsilon\eta\Omega - \ln\gamma}. \tag{43}$$

The following proposition indicates under which restrictions a steady-state equilibrium for the domestic economy exists and is unique.

Proposition 5 *If*

$$\varepsilon > \frac{\alpha\theta^{\phi/(\phi-1)} \ln\gamma}{\eta\bar{w}_f e^{(\nu-\zeta)/\varphi}} \tag{44}$$

$$\rho < \frac{(1-\alpha)(1-1/\mu)(1-\theta)^{1/(1-1/\phi)}}{\eta} \tag{45}$$

are satisfied, then, for any given pair of initial conditions $\langle \Omega(0), m(0) \rangle$, there exists a unique steady-state equilibrium where: (i) $m^* > 0$, $\Omega^* > 0$ and $x_n^* > 0$ are the solution of system (41)-(43); (ii) the rates of innovation and economic growth are both positive and still given by (34) and (35).

[9] To obtain the steady-state system (41)-(43), it suffices to set $\dot{x}_n = \dot{m} = \dot{\Omega} = 0$ in (27), (39) and (40), and then rearrange the resulting equations.

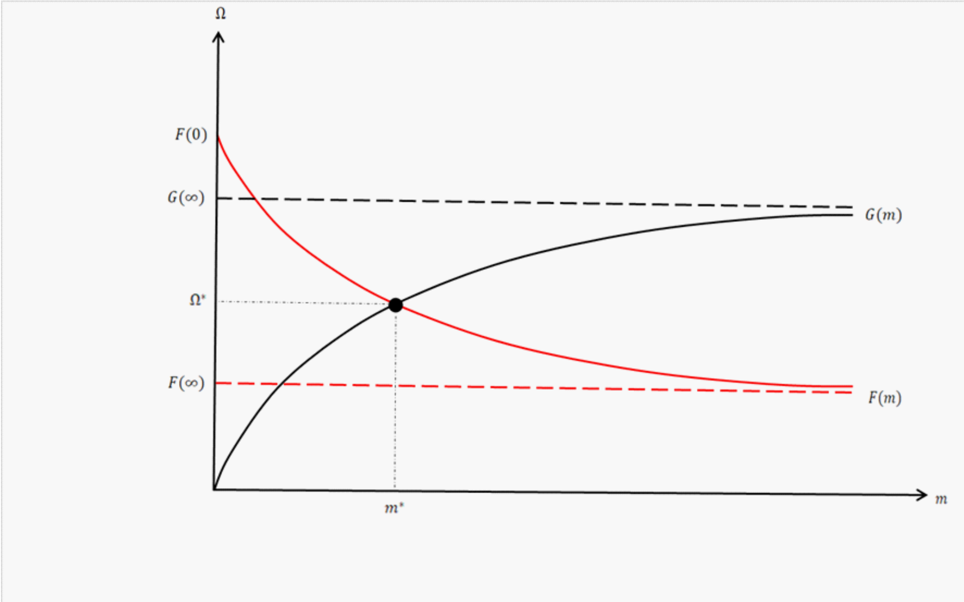

Figure 2: The ABGP of the extended model.

Proof. See Appendix C. ■

Figure 2 can help to explain the results summarized by Proposition 5. In the figure, the convex curve $F(m)$ indicates all of the combinations of m and Ω that cause the productivity-adjusted level of per capita consumption of natives, x_n, and the relative technology of the domestic country, Ω, to be stationary over time.[10] The concave curve $G(m)$ is instead the isocline of the migration function (39), obtained upon setting $\dot{m} = 0$.

The equilibrium condition $F(m)$ presents a positive vertical intercept $F(0)$ and a positive horizontal asymptote at $F(\infty)$, where restriction (45) on the rate of time preference ρ makes $F(0)$ to be larger than $F(\infty)$. The isocline $G(m)$ has instead a zero vertical intercept and a positive horizontal asymptote at $G(\infty)$. For $F(m)$ and $G(m)$ to intersect it must be that $F(\infty) < G(\infty)$ is satisfied, and then that the horizontal asymptote of $G(m)$ lays above that of $F(m)$ as portrayed in Figure 2. However, for this to happen, ε must satisfy restriction (44); i.e. the cross-country R&D spillover must be sufficiently larger to make the foreign economy to benefit from domestic innovation. Once that m^* and Ω^* have been determined through (42) and (43), (41) can be used to determine x_n^*.

Notice that when Assumption 3 holds, the upper bound for the reference wage of immigrants (see restriction (30) of Proposition 1) disappears from the model. This is due to the fact that, when the technology gap is endogenous, the reference wage of immigrants presents an additional (and ever-growing) component given by the variable Ω that always affects the foreign workers' decision to either migrate or not migrate to the domestic economy. However, in the next section I shall show that whereas all of the comparative statics

[10] To obtain the function $F(m)$, it suffices to set $\dot{x}_n = \dot{m} = 0$ to equations (27) and (39), and then combining the two resulting expressions so as to cancel the variable x_n. For the analytical details about the derivation of the two curves appearing in Figure 2 see Appendix C.

properties of the baseline model with respect to changes in migration are not affected by the presence of incomplete technology transfer[11], those related to changes in the R&D policy of the domestic economy change dramatically.

5.3 Increasing migration and patent breadth policy

In this section, I shall perform two comparative statics exercises to assess (i) how migration can effect innovation and growth and (ii) to what extent an increase in patent protection in the domestic economy can serve as a propeller for new immigration.[12] I begin by considering the effects of an increase in migration on innovation and growth. To do this, I keep assuming that the domestic economy is in its own steady-state equilibrium and that, at a given point in time, a negative shock hits the reference wage of immigrants causing $\bar{w}_f$ to fall permanently.

The following proposition summarizes how the fall in $\bar{w}_f$ impacts on the domestic economy

Proposition 6 *A permanent decrease in the productivity-adjusted reference wage of immigrants $\bar{w}_f$: (i) increases the steady-state immigration ratio, m^*; (ii) shrinks the technology gap between countries, Ω^*; (iii) increases the steady-state level of consumption per native, x_n^*; (iv) increases the steady-state rates of innovation, ι^* and economic growth, g^*.*

Proof. See Appendix D ■

The steady-state results of Proposition 6 can be explained through of the two vertically-integrated Cartesian diagrams of Figure 3. The upper diagram shows the values of m and Ω that solves (42) and (43) simultaneously, while the lower diagram reports the isocline of (41), showing all of the combinations of m and x_n that makes the productivity-adjusted value of natives' consumption be stable over time.

The permanent fall in $\bar{w}_f$ worsens the economic conditions of the foreign labor market, thereby giving foreign workers more incentives to migrate. Graphically, this effect can be seen through the downward shift in the horizontal asymptote of the concave function $G(m)$, which moves from $G_0(\infty)$ to $G_1(\infty)$, with $G_0(\infty) > G_1(\infty)$ - see the upper diagram

[11] Notice that since the steady-state rates of innovation and growth are identical to that shown by Proposition 1, all of the results summarized by Proposition 2 are still valid in the presence of incomplete technology transfer.

[12] To be viable, any comparative statics exercise involving dynamic models should require long-run equilibria to be either globally or locally (i.e. saddle-path) stable. Unfortunately, due to the mathematical complexity of the extended model, the dynamic properties of the steady-state equilibrium cannot be studied analytically. Nevertheless, in a supplementary appendix of this paper (available upon request), I show that a calibrated version of the extended model exhibits saddle-path stability of the unique steady state established by Proposition 5, meaning that the comparative-statics exercises presented on the next section are acceptable.

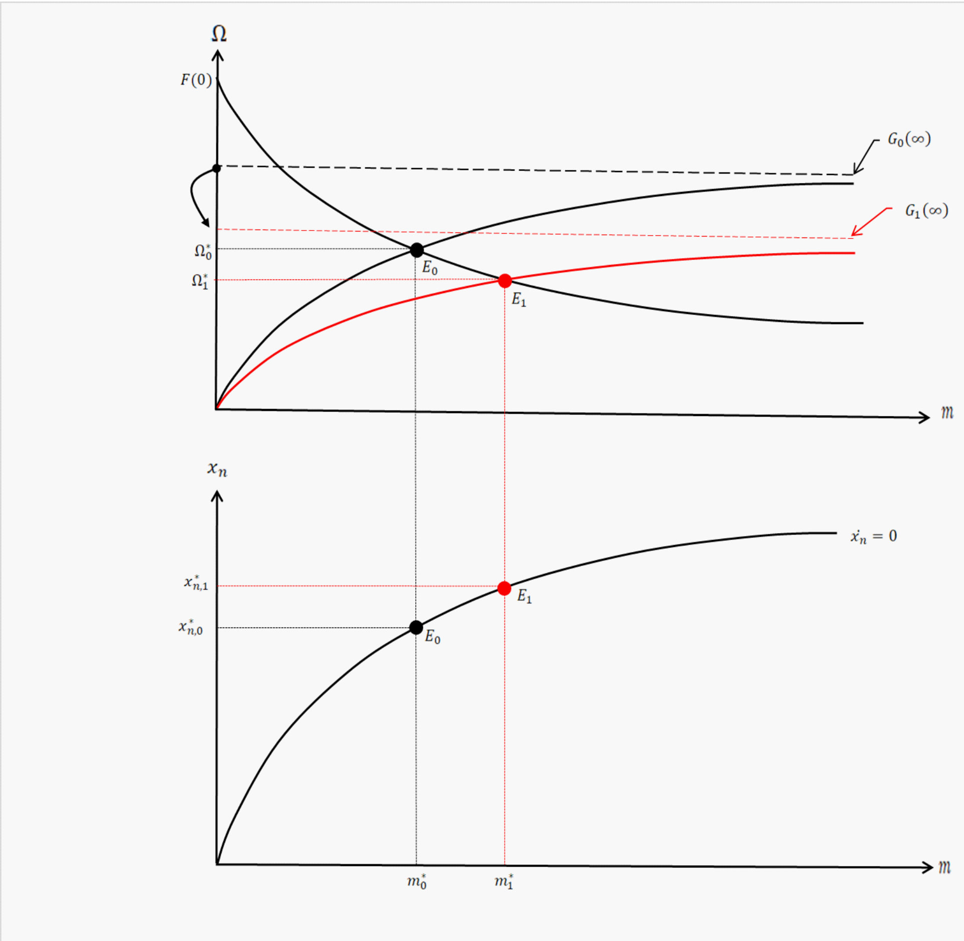

Figure 3: The steady-state effects of a fall in the reference wage of immigrants, $\bar{w}_f$.

of Figure 3. The steady-state equilibrium thus moves from E_0 to E_1, implying that the immigration rate of the domestic economy shifts upward from m_0^* to m_1^*, and the relative technology index downward from Ω_0^* to Ω_1^*. Despite of the reduction in Ω, the steady-state consumption-to-technology ratio of natives x_n^* increases in the new steady-state, meaning that - during the transition from E_0 to E_1 - natives' per capita consumption c_n grew faster than productivity $\mathcal{A}$.

I now consider the effects of a permanent increase in patent breadth, μ. The steady-state impacts of such a increase in the patent protection policy of the domestic government are the following.

Proposition 7 *A permanent increase in patent breadth μ: (i) decreases the steady-state immigration ratio, m^*; (ii) decreases the technology gap between countries, Ω^*; (iii) decreases the steady-state level of consumption per native, x_n^*; (iv) increases the steady-state rates of innovation, ι^* and economic growth, g^*.*

Proof. See Appendix D. ■

Surprisingly enough, strengthening patent policy in the domestic economy is detrimental for natives' consumption and immigration, despite it turns out to be effective in enhancing innovation and growth. To explain the economic logic of these results, I rely again on two vertically-integrated charts as those portrayed in Figure 4.

A permanent increase in patent breadth μ allows intermediate producers to charge an higher markup over their marginal cost and makes innovation more profitable. However, the rise in R&D that follows the improvement in patent protection against imitation enhances the transmission of technology towards the foreign economy, which, eventually, tends to increase the reference wage of immigrants and therefore discourage emigration. In Figure 4, this *technology transfer* effect is captured by the downward shift of the vertical intercept of the convex function $F(m)$, which moves from $F_0(0)$ to $F_1(0)$. The steady-state equilibrium of the domestic economy thus moves from E_0 to E_1, implying that both the immigration ratio of the domestic economy m_1^* and the cross-country technology gap Ω_1^* decrease because of the rise in firms' markup μ.

The fall in Ω can be explained by recalling Gerschenkron's "advantage of backwardness" (see Gerschenkron, 1962), according to which the further a country lags behind the world technology frontier, the bigger the productivity improvement it will get if it implements the frontier technology. However, the fact that Ω^* decreases because of the rise in patent breadth might lead to conclude that permanent increases in μ had negative impacts on the long-run innovation rate, ι^*. Yet, even though Ω^* clearly decreases in the new steady state, this result is not at odds with a scenario in which a possible increase in ι^* might cause domestic productivity $\mathcal{A}$ to grow less that foreign productivity $\mathcal{A}_f$. To see this point more

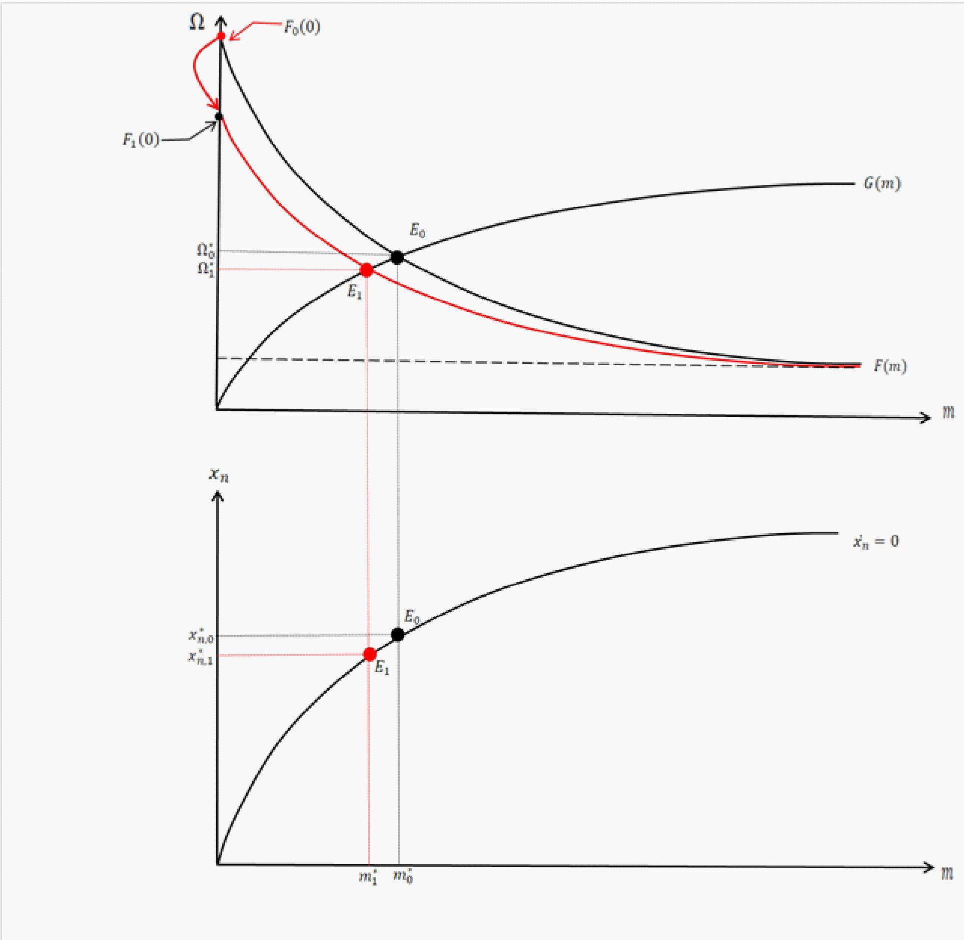

Figure 4: The steady-state effects of an increase in patent breath, μ.

clearly, I differentiate (34) with respect to μ to obtain (after heavy simplification)

$$\frac{\mathrm{d}\iota^*}{\mathrm{d}\mu} = \underbrace{\frac{\iota^*\mathcal{M}(\mu)}{\mu^2\left[(1-1/\mu)\mathcal{M}(\mu)-\rho\right]}}_{+} + \underbrace{\frac{\iota^*\mathcal{M}(\mu)(1-1/\mu)\theta}{m^*\left[(1-1/\mu)\mathcal{M}(\mu)-\rho\right]\left[(1-\theta)(m^*)^{1/\phi-1}+\theta\right]}\frac{\mathrm{d}m^*}{\mathrm{d}\mu}}_{-}, \tag{46}$$

where ι^* is given by (34) and, for simplicity, I set

$$\mathcal{M}(\mu) := \frac{(1-\alpha)\left[1-\theta+\theta(m^*)^{1-1/\phi}\right]^{1/(1-1/\phi)}}{\eta}.$$

The first term of the right-hand side of (46) is positive and includes the standard "profit-margin" effect captured by the term $1-1/\mu$ in (34). The second term on the right-hand side of (46) is instead negative and stems from the indirect effect that a higher value of μ has on innovation through the fall in the net rate of migration ($\mathrm{d}m/\mathrm{d}\mu<0$). Indeed, the fall in the immigration ratio that follows the rise in μ generates a negative "scale" effect on firms' profits that tends to discourage innovation. In Appendix D, I show that the "profit-margin" effect always overrides the "scale" effect, implying that, on balance, the long-run effects of stronger IPR protection on the steady-state innovation rate are always positive ($\mathrm{d}\iota^*/\mathrm{d}\mu>0$).

Notice that even though ι^* increases in the new steady-state equilibrium E_1, the fall in the immigration rate has a negative impact on the productivity-adjusted level of final output, $Y/\mathcal{A}=\left(1-\theta+\theta m^{1-1/\phi}\right)^{1/(1-1/\phi)}L_n$, because of the permanent fall in labor supply due to the reduction in immigration. Moreover, as the lower diagram of Figure 4 makes it clear, the ultimate impacts on the productivity-adjusted level of consumption of natives x_n^* is also negative, meaning that, along the adjustment path, natives' consumption tend to grow at a lower rate than productivity. This result enriches the literature in that it gives a instance in which the strengthening of patent protection is not effective in improving the productivity-adjusted level of consumption of the population in the long run.

6 Concluding remarks

In this paper, I have presented a lab-equipment model of Schumpeterian growth with endogenous migration to analyze the interlink between migration, innovation and long-run growth. To do this, the paper focused on the perspective of the innovating country - which also acts as the receiving country of migration - and analyzed two different scenarios in which: (i) the technology transfer between countries is complete, meaning that all the productivity improvements occurring in the innovating country are always fully and instantaneously transferred to the sending countries of migration; (ii) the technology transfer between countries is incomplete, meaning that each time that the innovating country is able to introduce a productivity improvement in her economy, this is only partially transferred to the sending countries of migration.

When technology transfer is complete, I find that an increase in migration has a positive impact on the innovation activity of the receiving country. In particular, compared with the baseline Schumpeterian model without international labor mobility, the paper finds that migration tends to improve the long-run rates of innovation and economic growth of the innovating economy. However, whereas the effects of pro-innovation policies on migration are analyzed, the paper finds that the resulting increase in the innovation rate of the receiving economy has no effect on migration.

While the positive effects of migration on innovation and growth are confirmed even when technology transmission turns into incomplete, the neutrality result of innovation on migration is not and becomes negative. More specifically, I find that implementing innovation-enhancing policies in the innovating economy reduces the flow of migration towards the domestic economy when the cross-country productivity gap is allowed to narrow only gradually over time. The economic reason underlying this result stands in the fact that a rise in innovation in advanced economies tends to compress the long-run technology gap between countries, thereby causing international wage differentials to reduce accordingly. That, in turn, makes emigration less attractive for migrant workers, with the result that, as Gerschenkron (1986) pointed out, countries lagging behind the technology frontier may enjoy an advantage in backwardness that causes emigration to decrease because of technology adoption.

Appendix

A Proof of Proposition 1

I start demonstrating part (i) of the proposition. Rewrite system (27)-(28) as follows

$$\dot{x}_n = \frac{x_n^2}{\eta} - \left[\frac{\alpha (1-\theta) \left(1-\theta+\theta m^{1-1/\phi}\right)^{1/(\phi-1)}}{\eta} - \rho \right] x_n \tag{A.1}$$

$$\dot{m} = \varphi \left\{ \ln \left(\frac{\alpha\theta}{\omega_f (0)} \right) + \frac{1}{\phi - 1} \ln \left[(1-\theta) m^{1/\phi - 1} + \theta \right] \right\} m - (\nu - \zeta) m. \tag{A.2}$$

In the system, x_n acts as a "pseudo" jump variable and m as a state variable, whose value at time $t = 0$ is predetermined and equal to $m(0)$. Let "*" denote stationary values for the endogenous variables of the model. A steady-state equilibrium for system (A.1)-(A.2) is attained when $\dot{x}_n = \dot{m} = 0$. This gives the following system of two algebraic equations in x_n and m

$$x_n - \alpha (1-\theta) \left[1-\theta+\theta m^{1-1/\phi}\right]^{1/(\phi-1)} = \eta\rho \tag{A.3}$$

$$\frac{\alpha\theta \left[(1-\theta) m^{1/\phi-1} + \theta\right]^{1/(\phi-1)}}{\omega_f (0)} = e^{(\nu-\zeta)/\varphi}, \tag{A.4}$$

whose solution are given by (32)-(33). The following lemma provides the restriction of the parameter space that makes both x_n and m to be positive in the steady state.

Lemma 2 *If*

$$\omega_f (0) < \alpha\theta^{1/(\phi-1)+1} e^{\zeta/\varphi}$$

occurs, then it follows that $x_n^* > 0$ *and* $m^* > 0$.

Proof. From (33) it follows that for m^* to present nonnegative values, it must be that

$$\left\{ \left[\frac{\omega_f (0) e^{(\nu-\zeta)/\varphi}}{(1-\theta)^{1/(\phi-1)} \alpha\theta} \right]^{\phi-1} - \frac{\theta}{1-\theta} \right\}^{\phi/(1-\phi)} > 0.$$

On rearranging:

$$\left\{ \left[\frac{\omega_f (0) e^{(\nu-\zeta)/\varphi}}{(1-\theta)^{1/(\phi-1)} \alpha\theta} \right]^{\phi-1} - \frac{\theta}{1-\theta} \right\}^{\phi/(\phi-1)} < 0 \quad \rightarrow \quad \omega_f (0) < \alpha\theta^{1/(\phi-1)+1} e^{(\zeta-\nu)/\varphi}.$$

When $\omega_f (0) < \alpha\theta^{1/(\phi-1)+1} e^{(\zeta-\nu)/\varphi}$ is satisfied, from (A.3) and (A.4) it follows that $x_n^* > 0$ and $m^* > 0$, and this completes the proof of the lemma. ■

I now turn to parts (ii) and (iii) of the proposition. Suppose that $\omega_f(0) < \alpha\theta^{1/(\phi-1)+1}e^{(\zeta-\nu)/\varphi}$ holds. Plugging (32) and (33) into (21) and (23) gives (34) and (35) of Proposition 1. From (34), it follows that the steady-state innovation rate of the economy is positive iff

$$\frac{(1-1/\mu)(1-\alpha)\left[1-\theta+\theta(m^*)^{1-1/\phi}\right]^{1/(1-1/\phi)}}{\eta} > \rho.$$

If $\theta \to 0$, from the above expression it follows that $\iota^* > 0$ requires $(1-1/\mu)(1-\alpha)/\eta > \rho$; i.e., when migration is not allowed, the rate of time preference must not exceed the threshold $(1-1/\mu)(1-\alpha)/\eta$. When $(1-1/\mu)(1-\alpha)/\eta > \rho$ is satisfied, both (34) and (35) are positive and this concludes the demonstration of part (ii) of Proposition 1.

Finally, to demonstrate part (iii) of the proposition, I Taylor-expand system (A.1)-(A.2) about the steady state (32)-(33) to obtain

$$\begin{pmatrix} \dot{x}_n \\ \dot{m} \end{pmatrix} = \begin{pmatrix} z_{11} & z_{12} \\ 0 & z_{22} \end{pmatrix} \begin{pmatrix} x_n - x_n^* \\ m - m^* \end{pmatrix}, \tag{A.7}$$

where

$$z_{11} = \frac{x_n^*}{\eta} > 0$$

$$z_{12} = -\frac{\alpha(1-\theta)\theta\left[1-\theta+\theta(m^*)^{1-1/\phi}\right]^{1/(\phi-1)-1}(m^*)^{-1/\phi}x_n^*}{\eta\phi} < 0$$

$$z_{22} = -\frac{(1-\theta)(m^*)^{1/\phi-1}}{\phi\left[(1-\theta)(m^*)^{1/\phi-1}+\theta\right]} < 0.$$

The model exhibits saddle-path stability iff the determinant of the 2×2 coefficient matrix in (A.7) is negative. When this happens, the coefficient matrix of the linearized system presents two real eigenvalues with opposite sign. Straightforward computations show that

$$\det = z_{11}z_{22} = -\frac{(1-\theta)(m^*)^{1/\phi-1}}{\phi\left[(1-\theta)(m^*)^{1/\phi-1}+\theta\right]}\frac{x_n^*}{\eta} < 0, \text{ for } m^* > 0 \text{ and } x_n^* > 0. \tag{A.8}$$

The steady state (32)-(33) is thus locally saddle-path stable and the demonstration of the part (iii) of Proposition 1 is done.

B Proof of Propositions 3 and 4.

In this appendix, I provide the formal demonstrations of Propositions 3 and 4. I begin by studying the steady-state effects of a decrease in $\bar{w}_f$ on m^*, ι^* and g^* (Proposition 3) Let the foreign and domestic economies be in their own steady-state equilibria and suppose that a fall in $\bar{w}_f$ makes $\omega_f(0)$ decrease permanently. Differentiating (32)-(35) with respect to $\omega_f(0)$ yields

$$\frac{\mathrm{d}m^*}{\mathrm{d}\omega_f(0)} = -\left\{\left[\frac{\omega_f(0)\,e^{(\nu-\zeta)/\varphi}}{(1-\theta)^{1/(\phi-1)}\,\alpha\theta}\right]^{\phi-1} - \frac{\theta}{1-\theta}\right\}^{\phi/(1-\phi)-1} \frac{\phi\left[\omega_f(0)\,e^{(\nu-\zeta)/\varphi}\right]^{\phi-1}\omega_f(0)^{\phi-2}}{(1-\theta)(\alpha\theta)^{\phi-1}} < 0$$

$$\frac{\mathrm{d}x_n^*}{\mathrm{d}\omega_f(0)} = \frac{\alpha(1-\theta)\left[1-\theta+\theta(m^*)^{1-1/\phi}\right]^{1/(\phi-1)-1}\theta(m^*)^{-1/\phi}}{\phi}\frac{\mathrm{d}m^*}{\mathrm{d}\omega_f(0)} < 0$$

$$\frac{\mathrm{d}\iota^*}{\mathrm{d}\omega_f(0)} = \frac{(\mu-1)(1-\alpha)\left[1-\theta+\theta(m^*)^{1-1/\phi}\right]^{1/(1-1/\phi)}\theta(m^*)^{-1/\phi}}{\mu\eta}\frac{\mathrm{d}m^*}{\mathrm{d}\omega_f(0)} < 0$$

$$\frac{\mathrm{d}g^*}{\mathrm{d}\omega_f(0)} = \left(\frac{\mathrm{d}\iota^*}{\mathrm{d}\omega_f(0)}\right)\ln\gamma < 0.$$

Since $\mathrm{d}\omega_f(0) < 0$, the sign of the derivatives are reversed and all the statements of Proposition 3 hold accordingly.

I now turn to Proposition 4 and analyze the steady-state effects on m^*, ι^* and g^* of a permanent increase in μ. Differentiation of (32)-(35) with respect to μ gives

$$\frac{\mathrm{d}m^*}{\mathrm{d}\mu} = 0$$

$$\frac{\mathrm{d}m^*}{\mathrm{d}\mu} = 0$$

$$\frac{\mathrm{d}\iota^*}{\mathrm{d}\mu} = \frac{(1-\alpha)\left[1-\theta+\theta(m^*)^{1-1/\phi}\right]^{1/(1-1/\phi)}}{\eta\mu^2} > 0$$

$$\frac{\mathrm{d}g^*}{\mathrm{d}\mu} = \left(\frac{\mathrm{d}\iota^*}{\mathrm{d}\mu}\right)\ln\gamma > 0.$$

Since an increase in patent breadth requires $\mathrm{d}\mu > 0$, the sign of the derivatives are preserved, implying that all the statements of Proposition 4 hold accordingly.

C Proof of Proposition 5

I start demonstrating part (*i*) of the proposition. Rewrite system (27), (39) and (40), as follows

$$\dot{x}_n = \frac{x_n^2}{\eta} - \left[\frac{\alpha(1-\theta)\left(1-\theta+\theta m^{1-1/\phi}\right)^{1/(\phi-1)}}{\eta} + \rho\right]x_n \tag{C.1}$$

$$\dot{m} = \varphi\left\{\ln\left(\frac{\alpha\theta}{\bar{w}_f}\right) + \ln\Omega + \frac{\ln\left[(1-\theta)\,m^{1/\phi-1}+\theta\right]}{\phi-1}\right\}m - (\nu-\zeta)\,m. \tag{C.2}$$

$$\dot{\Omega} = \psi\Omega - \left[\frac{(\mu+\alpha-1)(1-\theta)+(\mu-1)(1-\alpha)\,\theta m^{1-1/\phi}}{\eta\mu\left(1-\theta+\theta m^{1-1/\phi}\right)^{1/(1-\phi)}} - \frac{x_n}{\eta}\right](\varepsilon\eta\Omega-\ln\gamma)\,\Omega \tag{C.3}$$

In the above system, x_n acts as a "pseudo" jump variable and m and Ω as a state variables, with predetermined values time $t = 0$ given by $m(0) \geq 0$ and $\Omega(0) > 0$. A steady-state equilibrium for system (C.1)-(C.3) is attained when $\dot{x}_n = \dot{m} = \dot{\Omega} = 0$. This gives the following system of three algebraic equations in x_n, m and Ω:

$$x_n = \eta\rho + \alpha(1-\theta)\left(1-\theta+\theta m^{1-1/\phi}\right)^{1/(\phi-1)} \tag{C.4}$$

$$\frac{\alpha\theta\left[(1-\theta)m^{1/\phi-1}+\theta\right]^{1/(\phi-1)}\Omega}{\bar{w}_f} = e^{(\nu-\zeta)/\varphi} \tag{C.5}$$

$$\frac{(1-1/\mu)(1-\alpha)\left(1-\theta+\theta m^{1-1/\phi}\right)^{1/(1-1/\phi)}}{\eta} = \rho + \frac{\psi}{\varepsilon\eta\Omega - \ln\gamma} \tag{C.6}$$

The following lemma provides the conditions under which a steady-state equilibrium for system (C.1)-(C.3) with $x_n^* > 0$, $m^* > 0$ and $\Omega^* > 0$ exists

Lemma 3 *Let*

$$\tilde{\rho} := \frac{(1-\alpha)(1-1/\mu)(1-\theta)^{1/(1-1/\phi)}}{\eta}$$

$$\tilde{\omega}_f := \frac{(\rho\ln\gamma - \psi)\alpha\theta^{(\phi-2)/(\phi-1)}}{\varepsilon\mu\rho e^{(\nu-\zeta)/\varphi}}.$$

denote two threshold levels for the rate of time preference ρ and the reference wage of immigrants $\bar{w}_f$. If $\rho > \tilde{\rho}$ and $\bar{w}_f > \tilde{\omega}_f$ are both satisfied, then the system (C.4)-(C.6) admits a unique solution with $x_n^* > 0$, $m^* > 0$ and $\Omega^* > 0$.

Proof. Rewrite equations (C.5) and (C.6) as follows

$$\Omega = \frac{\bar{w}_f e^{(\nu-\zeta)/\varphi}}{\alpha\theta\left[(1-\theta)m^{1/\phi-1}+\theta\right]^{1/(\phi-1)}} := G(m) \tag{C.7}$$

$$\Omega = \frac{\ln\gamma}{\varepsilon\eta} + \frac{\psi/\varepsilon}{(1-1/\mu)(1-\alpha)\left(1-\theta+\theta m^{1-1/\phi}\right)^{1/(1-1/\phi)} - \eta\rho} := F(m) \tag{C.8}$$

Consider first (C.7). Because $\phi > 1$, for $m \to 0$, it follows that $G(0) = 0$; for $m \to \infty$, (C.7) approaches the horizontal asymptote:

$$G(\infty) = \frac{\bar{w}_f e^{(\nu-\zeta)/\varphi}}{\alpha\theta^{\phi/(\phi-1)}} > 0.$$

Moreover, differentiation of (C.7) with respect to t gives

$$G'(m) = \frac{(1-\theta)\bar{w}_f e^{(\nu-\zeta)/\varphi} m^{1/\phi-2}}{\alpha\theta\phi\left[(1-\theta)m^{1/\phi-1}+\theta\right]^{1/(1-1/\phi)}} > 0$$

$$G''(m) = -\frac{(1-\theta)\bar{w}_f e^{(\nu-\zeta)/\varphi}\left[(\phi-1)\left(1-\theta+\theta m^{1-1/\phi}\right)+\phi\theta m^{1-1/\phi}\right]m^{1/\phi-2}}{\alpha\theta\phi^2\left(1-\theta+\theta m^{1-1/\phi}\right)^{1-1/(1-1/\phi)}} < 0,$$

when $m \geq 0$. It can thus be established that (C.7) is concave and monotonically increasing in the positive orthant of $\langle m, \Omega \rangle$ space.

Consider next (C.8). Because $\phi > 1$, when $m \to 0$ it follows that

$$F(0) = \frac{\ln \gamma}{\varepsilon \eta} + \frac{\psi/\varepsilon}{(1-1/\mu)(1-\alpha)(1-\theta)^{1/(1-1/\phi)} - \eta\rho}.$$

Instead, when $m \to \infty$ it follows that

$$F(\infty) = \frac{\ln \gamma}{\varepsilon \eta},$$

where $F(0) > F(\infty)$ holds iff the following restriction is satisfied

$$\rho < \frac{(1-\alpha)(1-1/\mu)(1-\theta)^{1/(1-1/\phi)}}{\eta} := \tilde{\rho}.$$

Hence, differentiating (C.8) with respect to m yields

$$F'(m) = -\frac{\psi(1-\alpha)(\mu-1)\theta\left[(1-\theta)m^{1/\phi-1} + \theta\right]^{1/(\phi-1)}}{\varepsilon\left[(\mu-1)(1-\alpha)(1-\theta+\theta m^{1-1/\phi})^{1/(1-1/\phi)} - \mu\eta\rho\right]^2} < 0$$

$$F''(m) = \frac{\psi(1-\alpha)(\mu-1)\theta\left[(1-\theta)m^{1/\phi-1} + \theta\right]^{1/(\phi-1)} m^{-1/\phi}}{\varepsilon\left[(\mu-1)(1-\alpha)(1-\theta+\theta m^{1-1/\phi})^{1/(1-1/\phi)} - \mu\eta\rho\right]^2 (1-\theta+\theta m^{1-1/\phi})}$$
$$\times \left[\frac{(1-\theta)m^{1/\phi-1}}{\phi} + \frac{2(\mu-1)(1-\alpha)\theta}{(\mu-1)(1-\alpha) + \rho\eta\mu(1-\theta+\theta m^{1-1/\phi})^{1/(1-1/\phi)}}\right] > 0,$$

for $m \geq 0$. Thus, it can be established that (C.8) is convex and monotonically decreasing in $\langle m, \Omega \rangle$ space.

For a steady state to exists, it must be that (C.7) and (C.8) intersects at least once in the positive orthant of $\langle m, \Omega \rangle$ space. This requires $F(\infty) < G(\infty)$; i.e., it must be that

$$\varepsilon > \frac{\alpha\theta^{\phi/(\phi-1)} \ln \gamma}{\eta \bar{w}_f e^{(\nu-\zeta)/\varphi}} := \tilde{\varepsilon}.$$

When $\varepsilon > \tilde{\varepsilon}$ holds, the horizontal asymptote of $G(m)$ lays above of the horizontal asymptote of $F(m)$, implying that (C.7) and (C.8) cross only once in the positive orthant of $\langle m, \Omega \rangle$ space. This determines the steady-state values of $m^* > 0$ and $\Omega^* > 0$. To determine x^*, it suffices to substitute m^* in (C.4) and the proof of Lemma 3 is done. ■

D. Proofs of Propositions 6 and 7

This appendix provides the formal proof of all of the comparative statics results of Propositions 6 and 7. Rewrite steady-state conditions (41)-(43) as follows

$$x_n = \eta\rho + \alpha\left(1-\theta\right)\left(1-\theta+\theta m^{1-1/\phi}\right)^{1/(\phi-1)} := \chi\left(m\right) \tag{D.1}$$

$$\Omega = \frac{\bar{w}_f e^{(\nu-\zeta)/\varphi}}{\alpha\theta\left[\left(1-\theta\right)m^{1/\phi-1}+\theta\right]^{1/(\phi-1)}} := \mathcal{M}\left(m,\ \bar{w}_f\right) \tag{D.2}$$

$$\Omega = \frac{\ln\gamma}{\varepsilon\eta} + \frac{\psi/\varepsilon}{\left(1-1/\mu\right)\left(1-\alpha\right)\left(1-\theta+\theta m^{1-1/\phi}\right)^{1/(1-1/\phi)}-\eta\rho} := \Gamma\left(m,\ \mu\right). \tag{D.3}$$

System (D.1)-(D.3) is bloc recursive; after solving (D.2)-(D.3) for m and Ω, one can proceed to solve (D.1) and obtain x_n. Hence, total differentiating (D.2)-(D.3) with respect to Ω, μ and $\bar{w}_f$. I obtain the following 2×2 system

$$\mathrm{d}\Omega = \frac{\partial\mathcal{M}\left(m,\ \bar{w}_f\right)}{\partial m}\mathrm{d}m + \frac{\partial\mathcal{M}\left(m,\ \bar{w}_f\right)}{\partial\bar{w}_f}\mathrm{d}\bar{w}_f \tag{D.4}$$

$$\mathrm{d}\Omega = \frac{\partial\Gamma\left(m,\ \mu\right)}{\partial m}\mathrm{d}m + \frac{\partial\Gamma\left(m,\ \mu\right)}{\partial\mu}\mathrm{d}\mu, \tag{D.5}$$

where:

$$\chi'\left(m\right) = \frac{\alpha\left(1-\theta\right)\left(1-\theta+\theta m^{1-1/\phi}\right)^{1/(\phi-1)-1}\theta m^{-1/\phi}}{\phi} > 0 \tag{D.6}$$

$$\frac{\partial\mathcal{M}\left(m,\ \bar{w}_f\right)}{\partial m} = \frac{\bar{w}_f e^{(\nu-\zeta)/\varphi}\left(1-\theta\right)m^{1/\phi-2}}{\alpha\theta\phi\left[\left(1-\theta\right)m^{1/\phi-1}+\theta\right]^{1/(1-1/\phi)}} > 0 \tag{D.7}$$

$$\frac{\partial\mathcal{M}\left(m,\ \bar{w}_f\right)}{\partial\bar{w}_f} = \frac{e^{(\nu-\zeta)/\varphi}}{\alpha\theta\left[\left(1-\theta\right)m^{1/\phi-1}+\theta\right]^{1/(\phi-1)}} > 0 \tag{D.8}$$

$$\frac{\partial\Gamma\left(m,\ \mu\right)}{\partial m} = -\frac{\left(\psi/\varepsilon\right)\left(1-1/\mu\right)\left(1-\alpha\right)\left(1-\theta+\theta m^{1-1/\phi}\right)^{1/(\phi-1)}\theta m^{-1/\phi}}{\left[\left(1-1/\mu\right)\left(1-\alpha\right)\left(1-\theta+\theta m^{1-1/\phi}\right)^{1/(1-1/\phi)}-\eta\rho\right]^2} < 0 \tag{D.9}$$

$$\frac{\partial\Gamma\left(m,\ \mu\right)}{\partial\mu} = -\frac{\left(\psi/\varepsilon\right)\left(1/\mu^2\right)\left(1-\alpha\right)\left(1-\theta+\theta m^{1-1/\phi}\right)^{1/(1-1/\phi)}}{\left[\left(1-1/\mu\right)\left(1-\alpha\right)\left(1-\theta+\theta m^{1-1/\phi}\right)^{1/(1-1/\phi)}-\eta\rho\right]^2} < 0. \tag{D.10}$$

Proof of Proposition 6

Set $\mathrm{d}\mu = 0$, so that the system (D.4)-(D.5) can be written as:

$$\begin{bmatrix} 1 & -\frac{\partial\mathcal{M}\left(m,\ \bar{w}_f\right)}{\partial m} \\ 1 & -\frac{\partial\Gamma\left(m,\ \mu\right)}{\partial m} \end{bmatrix}\begin{bmatrix} \frac{\mathrm{d}\Omega}{\mathrm{d}\bar{w}_f} \\ \frac{\mathrm{d}m}{\mathrm{d}\bar{w}_f} \end{bmatrix} = \begin{bmatrix} \frac{\partial\mathcal{M}\left(m,\ \bar{w}_f\right)}{\partial\bar{w}_f} \\ 0 \end{bmatrix}$$

The demonstration of points $(i)-(ii)$ of Proposition 6 is a straightforward application of the Cramer's Rule, which gives:

$$\frac{\mathrm{d}\Omega}{\mathrm{d}\bar{w}_f} = \frac{\begin{bmatrix} \frac{\partial\mathcal{M}\left(m,\ \bar{w}_f\right)}{\partial\bar{w}_f} & -\frac{\partial\mathcal{M}\left(m,\ \bar{w}_f\right)}{\partial m} \\ 0 & -\frac{\partial\Gamma\left(m,\ \mu\right)}{\partial m} \end{bmatrix}}{\begin{bmatrix} 1 & -\frac{\partial\mathcal{M}\left(m,\ \bar{w}_f\right)}{\partial m} \\ 1 & -\frac{\partial\Gamma\left(m,\ \mu\right)}{\partial m} \end{bmatrix}} = \frac{-\frac{\partial\Gamma\left(m,\ \mu\right)}{\partial m}\frac{\partial\mathcal{M}\left(m,\ \bar{w}_f\right)}{\partial\bar{w}_f}}{\frac{\partial\mathcal{M}\left(m,\ \bar{w}_f\right)}{\partial m}-\frac{\partial\Gamma\left(m,\ \mu\right)}{\partial m}} > 0 \tag{D.11}$$

$$
\frac{\mathrm{d}m}{\mathrm{d}\bar{w}_f} = \frac{\begin{bmatrix} 1 & \frac{\partial \mathcal{M}\left(m,\ \bar{w}_f\right)}{\partial \bar{w}_f} \\ 1 & 0 \end{bmatrix}}{\begin{bmatrix} 1 & -\frac{\partial \mathcal{M}\left(m,\ \bar{w}_f\right)}{\partial m} \\ 1 & -\frac{\partial \Gamma(m,\ \mu)}{\partial m} \end{bmatrix}} = \frac{-\frac{\partial \mathcal{M}\left(m,\ \bar{w}_f\right)}{\partial \bar{w}_f}}{\frac{\partial \mathcal{M}\left(m,\ \bar{w}_f\right)}{\partial m} - \frac{\partial \Gamma(m,\ \mu)}{\partial m}} < 0 \tag{D.12}
$$

From (D.11)-(D.12), it can be shown that when $\mathrm{d}\bar{w}_f < 0$ holds, the following results follow

$$
\mathrm{d}\Omega = \underbrace{\frac{-\frac{\partial \Gamma(m,\ \mu)}{\partial m}\frac{\partial \mathcal{M}\left(m,\ \bar{w}_f\right)}{\partial \bar{w}_f}}{\frac{\partial \mathcal{M}\left(m,\ \bar{w}_f\right)}{\partial m} - \frac{\partial \Gamma(m,\ \mu)}{\partial m}}}_{+}\mathrm{d}\bar{w}_f < 0, \quad \frac{\mathrm{d}m}{\mathrm{d}\bar{w}_f} = \underbrace{\frac{-\frac{\partial \mathcal{M}\left(m,\ \bar{w}_f\right)}{\partial \bar{w}_f}}{\frac{\partial \mathcal{M}\left(m,\ \bar{w}_f\right)}{\partial m} - \frac{\partial \Gamma(m,\ \mu)}{\partial m}}}_{-}\mathrm{d}\bar{w}_f > 0,
$$

so the demonstration of $(i)-(ii)$ of Proposition 6 is done. To demonstrate points $(iii)-(iv)$ of the proposition, I differentiate (D.1), (34) and (35) with respect to $\bar{w}_f$ and then use resulting expressions in (D.11)-(D.12) to obtain

$$
\mathrm{d}x_n = \left[\underbrace{\chi'(m)}_{+}\underbrace{\frac{\mathrm{d}m}{\mathrm{d}\bar{w}_f}}_{-}\right]\underbrace{\mathrm{d}\bar{w}_f}_{-} > 0,
$$

$$
\mathrm{d}\iota = \left[\underbrace{\frac{(1-1/\mu)(1-\alpha)\left(1-\theta+\theta m^{1-1/\phi}\right)^{1/(1-1/\phi)-1}\theta m^{-1/\phi}}{\eta}}_{+}\underbrace{\frac{\mathrm{d}m}{\mathrm{d}\bar{w}_f}}_{-}\right]\underbrace{\mathrm{d}\bar{w}_f}_{-} > 0
$$

$$
\mathrm{d}g^* = \left\{\left[\underbrace{\frac{(1-1/\mu)(1-\alpha)\left(1-\theta+\theta m^{1-1/\phi}\right)^{1/(1-1/\phi)-1}\theta m^{-1/\phi}}{\eta}}_{+}\underbrace{\frac{\mathrm{d}m}{\mathrm{d}\bar{w}_f}}_{-}\right]\underbrace{\mathrm{d}\bar{w}_f}_{-}\right\}\ln\gamma > 0
$$

which completes the proof of Proposition 6.

Proof of Proposition 7

Set $\mathrm{d}\bar{w}_f = 0$, so that system (D.4)-(D.5) can be rewritten as

$$
\begin{bmatrix} 1 & -\frac{\partial \mathcal{M}\left(m,\ \bar{w}_f\right)}{\partial m} \\ 1 & -\frac{\partial \Gamma(m,\ \mu)}{\partial m} \end{bmatrix}\begin{bmatrix} \frac{\mathrm{d}\Omega}{\mathrm{d}\mu} \\ \frac{\mathrm{d}m}{\mathrm{d}\mu} \end{bmatrix} = \begin{bmatrix} 0 \\ \frac{\partial \Gamma(m,\ \mu)}{\partial \mu} \end{bmatrix}.
$$

Likewise the proof of Proposition 6, the proof of the first two points of Proposition 7 is another straightforward application of the Cramer's rule:

$$
\frac{\mathrm{d}\Omega}{\mathrm{d}\mu} = \frac{\begin{bmatrix} 0 & -\frac{\partial \mathcal{M}\left(m,\ \bar{w}_f\right)}{\partial m} \\ \frac{\partial \Gamma(m,\ \mu)}{\partial \mu} & -\frac{\partial \Gamma(m,\ \mu)}{\partial m} \end{bmatrix}}{\begin{bmatrix} 1 & -\frac{\partial \mathcal{M}\left(m,\ \bar{w}_f\right)}{\partial m} \\ 1 & -\frac{\partial \Gamma(m,\ \mu)}{\partial m} \end{bmatrix}} = \frac{\frac{\partial \mathcal{M}\left(m,\ \bar{w}_f\right)}{\partial m}\frac{\partial \Gamma(m,\ \mu)}{\partial \mu}}{\frac{\partial \mathcal{M}\left(m,\ \bar{w}_f\right)}{\partial m} - \frac{\partial \Gamma(m,\ \mu)}{\partial m}} < 0 \tag{D.13}
$$

$$\frac{\mathrm{d}m}{\mathrm{d}\mu}=\frac{\begin{bmatrix}1 & 0\\ 1 & \frac{\partial\Gamma(m,\ \mu)}{\partial\mu}\end{bmatrix}}{\begin{bmatrix}1 & -\frac{\partial\mathcal{M}\left(m,\ \bar{w}_f\right)}{\partial m}\\ 1 & -\frac{\partial\Gamma(m,\ \mu)}{\partial m}\end{bmatrix}}=\frac{\frac{\partial\Gamma(m,\ \mu)}{\partial\mu}}{\frac{\partial\mathcal{M}\left(m,\ \bar{w}_f\right)}{\partial m}-\frac{\partial\Gamma(m,\ \mu)}{\partial m}}<0, \tag{D.14}$$

from which I can conclude that a rise in μ increases both Ω and m.

To demonstrate point (*iii*) of the proposition, I differentiate (D.1) and then use (D.14) to get

$$\frac{\mathrm{d}x_n}{\mathrm{d}\mu}=\underbrace{\chi'(m)}_{+}\underbrace{\frac{\mathrm{d}m}{\mathrm{d}\mu}}_{-}<0.$$

Finally, to demonstrate point (*iv*) of the proposition, consider the following reduced version of (34)

$$\iota^*(\mu)=(1-1/\mu)\,\mathcal{M}(\mu)-\rho,\ \text{ where: }\mathcal{M}(\mu):=\frac{(1-\alpha)\left[1-\theta+\theta m(\mu)^{1-1/\phi}\right]^{1/(1-1/\phi)}}{\eta}. \tag{D.15}$$

Log-differentiation of (D.15) with respect to μ gives

$$\frac{\mathrm{d}\iota^*(\mu)/\mathrm{d}\mu}{\iota^*(\mu)}=\frac{\mathcal{M}(\mu)+\mu^2(1-1/\mu)\,\mathcal{M}'(\mu)}{\mu^2\left[(1-1/\mu)\,\mathcal{M}(\mu)-\rho\right]}, \tag{D.16}$$

from which it follows that $\mathrm{d}\iota^*(\mu)/\mathrm{d}\mu>0$ is satisfied iff the following inequality holds true:

$$\frac{\mu\mathcal{M}'(\mu)}{\mathcal{M}(\mu)}>-\frac{1}{\mu-1}. \tag{D.17}$$

Upon differentiation of $\mathcal{M}(\mu)$ with respect to μ, it can be shown that

$$\frac{\mu\mathcal{M}'(\mu)}{\mathcal{M}(\mu)}=\frac{\theta}{(1-\theta)\,m^{1/\phi-1}+\theta}\frac{\mu m'(\mu)}{m(\mu)}.$$

Thus, substituting from $\mu\mathcal{M}'(\mu)/\mathcal{M}(\mu)$ into (D.17), I can establish that

$$\frac{\mathrm{d}\iota^*(\mu)}{\mathrm{d}\mu}>0\quad\text{iff}\quad\frac{\mathrm{d}m}{\mathrm{d}\mu}\frac{\mu}{m}>-\frac{(1-\theta)\,m^{1/\phi-1}+\theta}{\mu(1-1/\mu)\,\theta}. \tag{D.18}$$

Plugging (D.7), (D.9) and (D.10) into (D.14), I can write

$$\frac{\mathrm{d}m}{\mathrm{d}\mu}=-\frac{\frac{\left(1/\mu^2\right)\left[(1-\theta)m^{1/\phi-1}+\theta\right]m}{(1-1/\mu)\theta}}{\frac{\left[(1-1/\mu)(1-\alpha)\left(1-\theta+\theta m^{1-1/\phi}\right)^{1/(1-1/\phi)}-\eta\rho\right]^2\bar{w}_f e^{(\nu-\zeta)/\varphi}(1-\theta)m^{1/\phi-2}}{(\psi/\varepsilon)(1-1/\mu)\alpha(1-\alpha)\theta^2\phi\left[(1-\theta)m^{1/\phi-1}+\theta\right]}+1}.$$

Therefore, substituting from $\mathrm{d}m/\mathrm{d}\mu$ into (D.18), it follows that for $\mathrm{d}\iota^*(\mu)/\mathrm{d}\mu>0$ to hold, the following inequality must satisfied

$$-\frac{\frac{\left(1/\mu^2\right)\left[(1-\theta)m^{1/\phi-1}+\theta\right]m}{(1-1/\mu)\theta}}{\frac{\left[(1-1/\mu)(1-\alpha)\left(1-\theta+\theta m^{1-1/\phi}\right)^{1/(1-1/\phi)}-\eta\rho\right]^2\bar{w}_f e^{(\nu-\zeta)/\varphi}(1-\theta)m^{1/\phi-2}}{(\psi/\varepsilon)(1-1/\mu)\alpha(1-\alpha)\theta^2\phi\left[(1-\theta)m^{1/\phi-1}+\theta\right]}+1}\frac{\mu}{m}>-\frac{(1-\theta)\,m^{1/\phi-1}+\theta}{\mu(1-1/\mu)\,\theta}.$$

After heavy simplification, the previous expression boils down to:

$$\frac{\left[\left(1-1/\mu\right)\left(1-\alpha\right)\left(1-\theta+\theta m^{1-1/\phi}\right)^{1/(1-1/\phi)}-\eta\rho\right]^{2}\bar{w}_{f}e^{(\nu-\zeta)/\varphi}\left(1-\theta\right)m^{1/\phi-2}}{\left(\psi/\varepsilon\right)\left(1-1/\mu\right)\alpha\left(1-\alpha\right)\theta^{2}\phi\left[\left(1-\theta\right)m^{1/\phi-1}+\theta\right]}>0,$$

which always holds true for whatever constellation of the exogenous parameters. I can therefore conclude that $\mathrm{d}\iota^{*}\left(\mu\right)/\mathrm{d}\mu>0$ always holds true when $m^{*}>0$. With this result in hand, $\mathrm{d}g^{*}/\mathrm{d}\mu>0$ follows directly from differentiation of (35), and this completes the Proof of Proposition 7.

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Online Appendix

(Not to be published)

E. Stability Analysis of the Extended Model

In this appendix, I shall simulate the model to study the dynamic properties of the steady state described by Proposition 5. My goal is to show that the comparative statics exercises of Section 5.3 are meaningful when the values of the exogenous parameters are close to those of the real economies. Therefore, the material collected in this section cannot be taken as an alternate demonstration of the saddle-path stability of the steady state equilibrium of the extended model.

To simulate the model, I proceed as follows. First, I calibrate the model to US data and choose $\bar{w}_f$ and ρ such that the restrictions (44) and (45) are satisfied. Next, I simulate a linearized version of the extended model and analyze the eigenspace of the coefficient matrix.

Calibration

Table 1 shows the benchmark values for all the calibrated parameters.

Parameter	Description	Value
ρ	Subjective discount rare	0.016
α	Labor share	0.67
γ	Step-size of innovation	1.25
ϕ	Elasticity of substitution between intermediates	20
$\bar{w}_f$	Initial reference wage of immigrants	0.53
η	R&D productivity	0.2
φ	Migration sensitivity	5
ζ	"Secular" component of migration	0.04
μ	Industry leaders' mark-up	1.12
ν	Growth rate of the native population	0.006
θ	CES share parameter	0.4321
ε	Intensisty parameter of technology transfer	0.95
ψ	Persistence parameter of technology transfer	0.01

Table 1: Calibrated parameter values.

Following Siegel (2002)[13], I set the subjective discount rate of individuals to $\rho = 0.016$, so

[13] Gollin, D. (2002). Getting income shares right. Journal of Political Economy 110, 458-474.

to obtain a steady-state rate of return on financial assets of roughly 3.8% and the growth rate of the native population to $\nu = 0.006$, so as to capture the tendency of the US population of the last three decades (see UN-DESA, 2019)[14]. Further, I choose the labor share parameter $\alpha = 0.67$ to match the empirical evidence of Gollin (2002) and a value for the labor-input elasticity parameter ϕ of 20 which is the value of the elasticity of substitution between native and immigrant workers with similar levels of education that Ottaviano and Peri (2012) find for the US. The share parameter $\theta = 0.4321$ is thus chosen to match a wage ratio between native and migrant workers of about 1.2, which lines up with Bureau of Labor Statistics data. Indeed, according to the Bureau of Labor Statistics annual release (BLS, 2020), in 2019 the level of earnings of the foreign-born workers was only 85% of the earnings of their native-born counterparts. This leads to a value of the relative wage of natives of $w_n/w_m \approx 1.18$, which is not far away from the calibrated value of 1.2.[15]

As for immigration, according to UN-DESA (2019), in 2019 the stock of immigrants of the US was 50.7 million from 23.3 million in 1990. On average, the yearly growth rate of the immigrants is 4%, so in the simulation I set $\zeta = 0.04$. As for the wages elasticity parameter φ and the productivity-adjusted reference wage $\bar{w}_f$, my calibration strategy consists of setting them so to obtain a long-run value for the migrant stock as a percentage of the total population of 15.4%. Since in the model the ratio between the stock of immigrants and the total population writes $L_m / (L_n + L_m) = m/(1+m)$, a value of the ratio of 0.154 implies $m \approx 0.182$. To match this value, in the simulation I set $\varphi = 5$ and $\bar{w}_f = 0.553$, which allow me to get a steady-state immigration rate of 18.2%.

Finally, to calibrate the R&D parameters of the model, I proceed as follows. First, I choose the productivity level of R&D firms $\eta = 0.2$ because it is in line with a waiting time for innovation of 10 years; i.e. $1/\iota^* \approx 10$. Patent breadth μ is set to 1.12, which implies a mark-up of 12% and a R&D share of GDP of US of about 3%, while the step size of productivity improvement γ to 1.25 so as to get a steady-state growth rate of 2.2% per year. Last but not least, I set $\varepsilon = 0.95$ and $\psi = 0.01$ so as to get an equilibrium technology gap between the domestic and the foreign country Ω^* of 1.712, meaning that the technology level of the domestic economy is almost twice as much as that of the foreign economy.

Determination of the steady state and eigenspace analysis

Table 2 displays the steady-state equilibrium generated by the model, while Table 3 shows the values of the three eigenvalues (denoted by λ) and associated eigenvectors of the linearized version of the dynamic system. Since m and Ω are predetermined/state variables while x_n is a non-predetermined/"pseudo" jump variable, the steady state shown by Table 2 is determinate (i.e. it is saddle-path stable) if two eigenvalues out of three present negative real part. Based on Table 3, it follows that, for any initial values of $m(0)$ and $\Omega(0)$,

[14] Data for immigration and population of the US economy are from the UN "Migration Data Portal" 2019, available at: https://migrationdataportal.org/about

[15] BLS (2020) Foreign-Born Workers: Labor Force Characteristics - 2019 (News Release: May 15, 2020). Bureau of Labor Statistics, U.S. Department of Labor (visited: January 6, 2021)

there exists a unique equilibrium trajectory that makes the economy converge to the unique steady-state equilibrium of Table 2.

	ABGP values.
Productivity-adjusted natives' consumption, x_n^*	0.375
Immigrantion ratio, m^*	0.182
Relative technology, Ω^*	1.717
Relative wage of native workers, w_n^*/w_m^*	1.206
BGP rate of return, r^*	3.8%
Innovation's waiting time , $1/\iota^*$	10.3 years
BGP growth rate, g	2.2%
R&D share	3.0%

Table 2: The steady-state equilibrium

	λ_1	λ_2	λ_3
Eigenvalues	1.873	−0.147	−0.098
Eigenvectors	$\begin{pmatrix} -0.902 \\ -0.106 \\ -0.418 \end{pmatrix}$	$\begin{pmatrix} -0.012 \\ -0.991 \\ -0.131 \end{pmatrix}$	$\begin{pmatrix} 0.012 \\ 0.976 \\ 0.219 \end{pmatrix}$

Table 3: Stability properties of the ABGP equilibria.

F. Robustness check: Using $L_n + L_m$ to remove the scale effect

In this Appendix, I present a robustness check of the model in which the level of immigration of the domestic economy enters in the determination of the Poisson arrival rate. In doing so, I replace the Poisson arrival rate (17) with the following

$$\iota\left(i\right)=\frac{R\left(i\right)}{\eta\mathcal{A}\left(L_n+L_m\right)},\quad \eta>0, \tag{F.1}$$

where (17) differs from (F.1) because of the presence of L_m at the denominator of (F.1).

Due to (F.1), only the rate of return on R&D investment (22) and the resource constraint (23) change to become

$$r=\frac{\left(1-1/\mu\right)\left(1-\alpha\right)\left(1-\theta+\theta m^{1-1/\phi}\right)^{1/(1-1/\phi)}+\eta\dot{m}}{\eta\left(1+m\right)}+\nu-\iota\left(1-\ln\gamma\right) \tag{F.2}$$

$$1=\frac{c_n/\mathcal{A}+\eta\left(1+m\right)\iota}{\left(1-\theta+\theta m^{1-1/\phi}\right)^{1/(1-1/\phi)}}+\frac{\alpha\theta m^{1-1/\phi}}{1-\theta+\theta m^{1-1/\phi}}+\frac{1-\alpha}{\mu},\tag{F.3}$$

whereas all of the other key equations of the model remain unchanged. Consequently, the dynamic equilibrium of the model can be written as

$$\begin{aligned}\frac{\dot{x}_n}{x_n} &= \frac{x_n-\alpha\left(1-\theta\right)\left(1-\theta+\theta m^{1-1/\phi}\right)^{1/(\phi-1)}+\eta\left(\nu+m\zeta\right)}{\eta\left(1+m\right)}-\rho+\\ &\quad+\frac{m\varphi}{1+m}\left\{\ln\left(\frac{\alpha\theta}{\omega_f\left(0\right)}\right)+\frac{\ln\left[\left(1-\theta\right)m^{1/\phi-1}+\theta\right]}{\phi-1}\right\}\end{aligned}\tag{F.6}$$

$$\frac{\dot{m}}{m}=\varphi\left\{\ln\left(\frac{\alpha\theta}{\omega_f\left(0\right)}\right)+\frac{\ln\left[\left(1-\theta\right)m^{1/\phi-1}+\theta\right]}{\phi-1}\right\}-\left(\nu-\zeta\right),\tag{F.7}$$

where $\omega_f\left(0\right):=\bar{w}_f\mathcal{A}_f\left(0\right)/\mathcal{A}\left(0\right)$ as in Section (2.6) of the paper.

A steady-state equilibrium for the domestic economy is attained when $\dot{x}_n=\dot{m}=0$. When this happens, the long-run ratio of immigrants over natives is constant over time and equal to m^*, while natives' per capita consumption, c_n, and technology, $\mathcal{A}$, are pure exponential functions of time as in (29).

I can therefore establish the following alternative version of Proposition 1:

Proposition 8 *Suppose restrictions (30) and (31) of Proposition 1 are satisfied. Then, for any given initial condition* $m\left(0\right)$*, there exists a unique steady-state equilibrium for the dynamic system (F.6)-(F.7) where:* (i) *The levels of the immigration ratio* m *and productivity-adjusted per capita consumption* x_n *are given by*

$$m^*=\left\{\left[\frac{\omega_f\left(0\right)e^{(\nu-\zeta)/\varphi}}{\alpha\theta\left(1-\theta\right)^{1/(\phi-1)}}\right]^{\phi-1}-\frac{\theta}{1-\theta}\right\}^{\phi/(1-\phi)}\tag{F.8}$$

$$x_n^*=\rho\eta\left(1+m^*\right)+\alpha\left(1-\theta\right)\left[1-\theta+\theta\left(m^*\right)^{1-1/\phi}\right]^{1/(\phi-1)};\tag{F.9}$$

(ii) *The rates of innovation and growth are both positive and equal to*

$$\iota^*=\frac{\left(1-1/\mu\right)\left(1-\alpha\right)\left[1-\theta+\theta\left(m^*\right)^{1-1/\phi}\right]^{1/(1-1/\phi)}}{\eta\left(1+m^*\right)}-\rho\tag{F.10}$$

$$g^*=\left\{\frac{\left(1-1/\mu\right)\left(1-\alpha\right)\left[1-\theta+\theta\left(m^*\right)^{1-1/\phi}\right]^{1/(1-1/\phi)}}{\eta\left(1+m^*\right)}-\rho\right\}\ln\gamma;\tag{F.11}$$

(iii) *The unique equilibrium trajectory converging to the steady state (F.8)-(F.9) is saddle-path stable.*

Proof. Rewrite system (F.6)-(F.7) as follows

$$\dot{x}_n = \frac{x_n^2 - \left[\alpha(1-\theta)\left(1-\theta+\theta m^{1-1/\phi}\right)^{1/(\phi-1)} - \eta(\nu+m\zeta)\right]x_n}{\eta(1+m)} - \rho x_n + \\ + \frac{m\varphi x_n}{1+m}\left\{\ln\left(\frac{\alpha\theta}{\omega_f(0)}\right) + \frac{\ln\left[(1-\theta)m^{1/\phi-1}+\theta\right]}{\phi-1}\right\} \tag{F.12}$$

$$\dot{m} = \varphi\left\{\ln\left(\frac{\alpha\theta}{\omega_f(0)}\right) + \frac{1}{\phi-1}\ln\left[(1-\theta)m^{1/\phi-1}+\theta\right]\right\}m - (\nu-\zeta)m. \tag{F.13}$$

■

A steady-state equilibrium for system (A.1)-(A.2) is attained when $\dot{x}_n = \dot{m} = 0$. This gives the following system of two algebraic equations in x_n and m

$$\frac{x_n}{1+m} - \frac{\alpha(1-\theta)\left(1-\theta+\theta m^{1-1/\phi}\right)^{1/(\phi-1)}}{1+m} = \rho\eta \tag{F.14}$$

$$\frac{\alpha\theta\left[(1-\theta)m^{1/\phi-1}+\theta\right]^{1/(\phi-1)}}{\omega_f(0)} = e^{(\nu-\zeta)/\varphi}, \tag{F.15}$$

whose solution are given by (32)-(33). Since Lemma 2 of Appendix A is still valid in establishing the conditions under which x_n and m are positive in the steady state, the proof of parts (i) and (ii) of the proposition is done.

To demonstrate part (iii) of the proposition, I Taylor-expand system (A.1)-(A.2) about the steady state (32)-(33) to obtain

$$\begin{pmatrix} \dot{x}_n \\ \dot{m} \end{pmatrix} = \begin{pmatrix} z_{11} & z_{12} \\ 0 & z_{22} \end{pmatrix} \begin{pmatrix} x_n - x_n^* \\ m - m^* \end{pmatrix}, \tag{F.16}$$

where

$$z_{11} = \frac{x_n^*}{\eta(1+m^*)} + \nu > 0$$

$$z_{22} = -\frac{(1-\theta)(m^*)^{1/\phi-1}}{\phi\left[(1-\theta)(m^*)^{1/\phi-1}+\theta\right]} < 0,$$

and where the sign of z_{12} is irrelevant for determining the sign of the determinant of Jacobian matrix in (F.16). Again, the steady state is saddle-path stable iff the determinant of the 2×2 coefficient matrix in (F.16) is negative; which is the case since:

$$\det = z_{11}z_{22} = -\frac{(1-\theta)(m^*)^{1/\phi-1}}{\phi\left[(1-\theta)(m^*)^{1/\phi-1}+\theta\right]}\left[\frac{x_n^*}{\eta(1+m^*)}+\nu\right] < 0, \text{ for } m^* > 0 \text{ and } x_n^* > 0.$$

That demonstrates that the steady state (F.14)-(F.15) is locally saddle-path stable and the proof of the part (iii) of Proposition 8 is done.