Licensing of a process innovation with heterogeneous firms

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Abstract

A significant strand of literature has analyzed the problem of an external innovator who licenses a superior technology to one or many firms competing in the market. This article contributes to that debate by analyzing licensing to firms with different efficiency levels. Novel questions arise when one takes heterogeneity in the ex-ante firms' characteristics into consideration. Moreover, the traditional modeling technique of process innovation as a linear cost-reducing parameter ceases to have economic soundness. This paper aims to find an alternative modeling solution that accounts for diversity in firms' efficiency. Assuming a per-unit linear price licensing scheme (royalty), I show that the external innovator is subject to two distinct forces: a price effect (PE) and a market share effect (MSE). The former describes the inefficient firms' willingness to pay for a cost-reducing technology. The latter describes the market penetration of the new technology - i.e., the level of output produced with the innovative process. On the one hand, the innovator wants to set the largest royalties possible. On the other hand, she wants to achieve the most significant market penetration, which implies many royalties. When PE dominates MSE, the least efficient firms get the innovation. Vice-versa, the most efficient firms get the license. If the innovator cannot exclude firms from purchasing the innovation, then, starting from the less efficient firms, the number of licensees increases as the MSE gets more intense.

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1 Introduction

A patent represents an Intellectual Property (IP) protection strategy that enables an innovator to appropriate the public good she creates (knowledge) while guaranteeing the complete disclosure of information in order for the innovative process to proceed. The patent's value from the innovator's perspective does not only consist of the quasi-rent granted by the monopoly over the new creation but also of the revenues derived by the possibility of licensing it to other firms. Focusing on licensing, several scholars have investigated which selling mechanism is the most profitable to the innovators. Licensing under imperfect competition was first analysed by Kamien and Tauman (1986), Kamien et al. (1992), and Katz and Shapiro (1985). These early contributions seek to identify the most efficient licensing scheme. They suggest that upfront fees dominate royalties from the innovator's perspective, while the auction is the most efficient licensing scheme for an outside innovator. In the light of these results, the dominance of royalties in empirical evidence has been considered puzzling.[1] An explanation for this has been advanced, among others, by Gallini and Wright (1990), who suggest that asymmetry of information can explain the dominance of royalties in empirical evidence. In contrast, Sen (2005) focuses on the technical constraints on the number of adopting firms. A non-exhaustive list of references on licensing under imperfect competition includes Lapan and Moschini (2000), Kamien et al. (2002), Erutku and Richelle (2007), Sen and Tauman (2007), Sen and Stamatopoulos (2016), Sen and Tauman (2018), Marshall and Parra (2019), and Parra (2019). In particular, Lapan and Moschini (2000) provide the conditions that ensure incomplete adoption of a superior technology when a licensing contract is a per-unit linear price (royalties) and firms are ex-ante homogeneous in terms of marginal costs of production. According to the author, if the production of a good requires more than one input, and if the innovation affects the productivity of only one of the inputs employed, then some firms may prefer not to adopt the new technology. This happens if the demands of the other inputs and their prices vary due to the variation in the demand of the more efficient input.[2]

The majority of these studies analyze the outcomes of a cost-reducing innovation when firms share a homogeneous technology that can be improved under a licensing agreement. As long as heterogeneity is considered, it is mainly intended as an output heterogeneity - i.e., product differentiation - not a technological one.

By analyzing the licensing problem in a context where candidate adopting firms have heterogeneous efficiency levels, novel questions arise. Moreover, the traditional modeling technique of process innovation as a linear cost-reducing parameter ceases to have economic soundness. In fact, despite being mathematically appealing, it is doubtful that one innovation would have the same

[1] Rostoker (1984) shows that royalties (39%) and a combination of royalties and fees (49%) are largely more common than upfront fees (13%) in corporate licensing transactions.

[2] Sandrini (2020) shows that incomplete adoption of a superior technology under royalties may be an equilibrium also in the case of a single input production function. If the innovation is k-drastic, the patent holder may decide to sell the technology at a discounted price to a subset of firms.

effect, in absolute terms, on different existing technologies. This paper aims to find an alternative way to model this problem. To simplify it at most, suppose there are two firms, say i and j, whose costs are $c_i < c_j$. Assume an innovator licenses a new technology that allows the licensees to produce the final good at a cost $\underline{c} < c_i$. Will the innovator sell the innovation to both firms? If not, should she supply the least or the most efficient firm? What are the main drivers of this choice? Intuitively, each outcome implies a different effect on social welfare. Providing an already efficient firm with even better technology, given the rival's production costs, is not the same as improving the least efficient firm's technology. Which outcome would the policy-maker prefer? Are the social incentives aligned with the private ones? If so, under which conditions?

Assuming a per-unit linear price licensing scheme (royalty), I disentangle two main forces that affect the innovator's choice: a price effect (PE) and a market share effect (MSE).

On the one hand, the less efficient firms would benefit the most in comparative terms from adopting the technology. For this reason, their willingness to pay is higher than their more efficient rivals' one. Under the linear cost assumption, the gross benefit from licensing are $c_i - \underline{c}$ for firm i and $c_j - \underline{c}$ for firm j. As $c_i < c_j$, it is apparent that firm j would benefit the most from the adoption. However, in absolute terms, both firms would end up producing at $\underline{c}$. Considering that no firms would ever adopt the new technology if the net benefit turns out to be a cost - i.e., if the price of the technology exceeds its benefits - it is possible to state that $r_i \leq \bar{r}_i \equiv c_i - \underline{c}$ and $r_j \leq \bar{r}_j \equiv c_j - \underline{c}$. One can notice that the maximum price is higher for the less efficient firm - i.e., $\bar{r}_j > \bar{r}_i$. The price effect identifies the above-mentioned higher willingness to pay of the less efficient firm. Everything else being equal, the innovator can charge a higher price for the same innovation if she decides to sell it to comparatively inefficient firms.

On the other hand, if the less efficient firm gets the new technology, she has to compete against an efficient rival. Instead, the efficient firm would be competing against a rival who is, in comparative terms, even less efficient. Thus, the market penetration of the new technology - i.e., the amount of output produced employing the innovative process - is the highest if the ex-ante efficient firm gets the innovation, as her market share would be the largest. In other words, $q_i(C_i) > q_j(C_j)$, with $C_i = \underline{c} + r_i \leq \underline{c} + r_j = C_j$. As revenues from linear per-unit price licensing depend on both the price and the amount of output produced with the technology, the dominance of either the price effect or the market share effect determines the innovator's choice on whom to sell the innovation to and to foreclose.

Under the assumption of ex-ante symmetric firms, PE and MSE do not play a role. The innovator does not face any trade-off, as all firms are willing to pay the same and share an ex-ante identical market share. Instead, a more general approach that accounts for firms' heterogeneity in the ex-ante efficiency shows that PE and MSE are, in fact, among the main driver of the innovator's choice. As a result, the innovator may prefer limiting the diffusion of her superior technology to just a subset of firms, even when the licensing scheme is a per-unit linear price, traditionally associated with complete adoption of innovation.

2 The model

There is a continuous of firms. Each firm i produces the same good with a technology that displays linear and constant marginal cost c_i. The firms are heterogeneous and the technologies are uniformly distributed between $\underline{c}$ and $\bar{c}$, with $\underline{c} < \bar{c}$. The total production level is $Q = \int_{\underline{c}}^{\bar{c}} q(c)\,\mathrm{d}c$. Moreover, $\partial q(c)/\partial c < 0$ and $q(\bar{c}) > 0$. In other words, the firm with the lowest cost (the most efficient technology) has the largest share of the market, while the firm with the highest cost (the least efficient) has the smallest market share. The total production Q is decreasing in c_i $\forall i$, while the individual production level $q_i(c_i, c_j)$ is increasing in the costs of rivals and decreasing in own costs:

$$\frac{\partial q_i(c_i, c_j)}{\partial c_i} < 0 \qquad\qquad \frac{\partial q_i(c_i, c_j)}{\partial c_j} > 0$$

Let us assume there is an outside innovator who owns a patent for a technology that allows the firms to produce at a cost $c^* < \underline{c}$. For the sake of simplicity, we assume that technology adoption by any of the firms in the market will not lead the least efficient out of business. The licensing scheme is the per-unit price (royalties).

3 Results and analysis

First, let us consider the case in which the innovator sells the technology to one firm only, which means that the licensing contract includes an exclusivity agreement.

The innovator has to decide which firm is most profitable to sell the technology to and its price. Intuitively, the maximum price she can charge a firm i is $r_i \leq c_i - c^*$, such that the net cost of production of the licensees is $c^* + r_i \leq c_i$. Otherwise, the adoption represents a pure cost and the firm i would not accept the offer. The price set by the innovator is $r = r_i(c_i)$, with $r'_{c_i} > 0$.

The choice of the innovator is:

$$\max_{c_i} \quad R^{ph} = r_i(c_i)\, q_i\left(c_i, r_i(c_i)\right)$$

where the superscript ph stands for *patent holder*. From the f.o.c.

$$\frac{\partial R}{\partial c_i} = \frac{d\, r_i}{d\, c_i}\left(q_i + r_i\, \frac{\partial q_i}{\partial ri}\right) + r_i \left(\frac{\partial q_i}{\partial c_i}\right) = 0 \tag{1}$$

The first term of eq. 1 (price effect, PE hereafter) can be either positive or negative, while the second one (market share effect, MSE hereafter) is negative. The PE identifies the increase in the maximum price that less efficient firms are willing to pay. As the price r is upward bound by the differential between the original cost and the new one, an inefficient firm is willing to pay more for the new technology, as her gains are comparatively high. However, if the royalty rate increases, the net benefit of innovation shrinks. Whether one effect dominates the other, the PE can be positive or negative. Instead, MSE implies that the less efficient the firm selected by the innovator, the lower the technology penetration in the market. Since the revenues from licensing depend on how many goods the licensee sells, the less efficient the adopting firm, the lower the market penetration of the new technology. We should notice that, although the licensee would technically be the most efficient firm in the market, the net costs of production are weighed by the royalty rate, which is increasing in c_i.

The innovator chooses to supply the firm whose c_i solves eq. 1. Intuitively, there are two corner solutions to consider. First, if the MSE dominates PE (or if PE is negative) for all $c_i \in [\underline{c}, \bar{c}]$, the innovator sells the technology to the most efficient firm. Instead, if PE is positive and dominates MSE for all c_i, the innovator selects the least efficient firm. In every other case, there exists a firm i such that $\underline{c} < c_i < \bar{c}$ which is selected by the innovator as the only licensee. Let us assume $c^* \approx \underline{c}$ and that $q(\bar{c}) \approx 0$. Intuitively, the innovator's revenues from selling the technology to either $\underline{c}$ and $\bar{c}$ are meager: in the first case, the maximum royalty rate is very close to zero ($\underline{c} - c^* \approx 0$); similarly, in the second case, the revenues are driven downward by the small market share of the firm $\bar{c}$. The innovator can make the firm gain market shares from the rivals, but it has to lower the price r_i. Under these conditions, the revenues from licensing are bell-shaped in c_i.

Remark 1. *The stronger MSE, the more likely that an efficient firm gets the license. The stronger the PE, the more likely that an inefficient firm gets the license.*

Intuitively, the innovator prefers an inefficient firm if the benefits from a higher royalty rate outweigh the opportunity cost - i.e., a small market share. Instead, if the demand function is such that firms' output is exceptionally responsive to reducing costs, then efficient firms are more likely to be selected.

Conjecture 1. *When the innovation is "drastic", meaning that some inefficient firms exit if an efficient one improves her technology, then MSE dominates PE.*

Let us now relax the assumption of exclusive licensing. The innovator can sell the technology to as many firms as she prefers. We assume that the royalty rate is public knowledge and that the innovator cannot negotiate the rate individually. In other words, the royalty rate is a spot price and that the innovator cannot prevent firms from purchasing the innovation.

Let us start from a case which is the opposite of the one analyzed above, i.e., when all firms get the license. In this case, the maximum price that the innovator can set is the price that the firms with a lower willingness to pay will accept. As the costs of the most efficient firm improve from $\underline{c}$ to c^*, and as $\underline{c}$ is the lower bound of costs distribution, the maximum price $r = \underline{c} - c^*$. Such a situation would be optimal from a welfare perspective. In fact, as a consequence of complete adoption, all firms' costs of production boil down to $\underline{c}$. As the aggregate costs of production collapse, the total output increases. At the same time, as the royalty rate is fixed, less efficient firms would be the ones that gain the most from adoption. Market shares become equally distributed among all firms. Therefore, in terms of payoffs, efficient firms lose from adoption.

It is easy to observe that the most efficient firm, whose original cost is $\underline{c}$, is indifferent between adopting the innovation or not. Indeed, it may strategically commit not to adopt it if, by doing so, she can induce the innovator to raise the royalty rate and sell the technology to all firms but her. Although the most efficient firm is still worse off, as her market share is downsized, she can keep a relatively larger share than the one kept by the rivals.

The innovator align her incentives to those of the efficient firm if it is profitable to her, i.e. if

$$r(\underline{c})\left(Q(\underline{c}, r(\underline{c})) - q\left(\underline{c}, r(\underline{c})\right)\right) < r(c_i) \int_{c_i}^{\bar{c}} q(c)\mathrm{d}c \equiv r(c_i)q\left(c_i, r(c_i)\right)(\bar{c} - c_i)$$

with $\underline{c} < c_i$. Let us define $Q^i(c_i, r(c_i)) \equiv q_i(c_i, r(c_i))(\bar{c} - c_i)$, with $Q^{i'}_{c_i} < 0$ and $Q^{i'}_{r_i} < 0$. Then, the problem of the innovator

$$\max_{c_i} \quad R^{ph} = r_i(c_i)Q^i(c_i, r(c_i)) \tag{2}$$

from the f.o.c.

$$\frac{\partial R}{\partial c_i} = \frac{d\, r_i}{d\, c_i}\left(Q^i + r_i\, \frac{\partial Q^i}{\partial ri}\right) + r_i\left(\frac{\partial Q^i}{\partial c_i}\right) = 0 \tag{3}$$

As before, the patent holder would choose to sell the technology to the subset of firms $(\bar{c} - c_i)$. Thus, she would chose c_i such that eq. 2 is satisfied. Intuitively, two corner solutions may emerge: first if eq. 2 is negative for all $c_i \in [\underline{c}, \bar{c}]$, then the price of the technology would be $r(\underline{c})$ so that all firms have access to it. Conversely, if eq. 2 is positive for all $c_i \in [\underline{c}, \bar{c}]$, then only the least efficient firm will receive the technology, which means that the price is $r(\bar{c})$. In the former case, the MSE always dominates the PE, while the opposite is true in the latter case. If eq. 2 is satisfied for some $c_i^* \in [\underline{c}, \bar{c}]$, then the two forces compensate each other and there is an interior solution for which the price of the technology is $r(c_i^*)$ and $(\bar{c} - c_i^*)$ firms adopt the technology.

Remark 2. *Assume that no exclusive dealing is allowed. The stronger MSE, the more firms get the license. The stronger the PE, the fewer firms get the license.*

3.1 Example: Duopoly with linear demand and linear marginal costs of production

The following cases are build upon the standard model of Competition à la Cournot. In order to ensure clarity of exposition, trivial mathematical steps are omitted and are available on request. Assume there exist two firms, labeled firm i and firm j. Without loss of generality, assume that firm i is endowed with a better technology, such that her costs of production are $c_i < c_j$. An outsider patent holder sells a technology that enables firms to produce at a cost $\underline{c} < c_i$. The firms compete in quantities for a homogeneous good. Their production levels coincide with the standard Cournot outcome:

$$q_i(c_i, c_j) = \frac{1 - 2c_i + c_j}{3} \qquad q_j(c_j, c_i) = \frac{1 - 2c_j + c_i}{3}$$

3.1.1 Exclusive licensing

First, let us consider the case in which the patent holder can only sign an exclusive contract with one firm.

Firm i gets the license. Assume the patent holder licenses the new technology to the most efficient firm, namely, firm i. Then, the corresponding output levels become:

$$q_i(c_i, r_i, c_j) = \frac{1 - 2(\underline{c} + r_i) + c_j}{3} \qquad q_j(c_j, c_i) = \frac{1 - 2c_j + (\underline{c} + r_i)}{3}$$

For the sake of simplicity, let us assume that $1-2c_j+(\underline{c}+r_i) > 0$, meaning that the less efficient firms can still stay active in the market. The choice of r_i follows from maximization of $R_i^{ph} = q(\underline{c}, r_i, c_j)r_i$, subject to $r_i \leq c_i - \underline{c}$. The constraint ensures that the technology does not represent a net cost for the adopting firms - i.e., it is the participation constraint.

$$r_i = \begin{cases} \frac{1-2\underline{c}+c_j}{4} & \text{if} \quad \underline{c} < \frac{4c_i-c_j-1}{2} \\ c_i - \underline{c} & \text{otherwise} \end{cases} \qquad R_i^{ph} = \begin{cases} \frac{(1-2\underline{c}+c_j)^2}{24} & \text{if} \quad \underline{c} < \frac{4c_i-c_j-1}{2} \\ \frac{(c_i-\underline{c})(1-2c_i+c_j)}{3} & \text{otherwise} \end{cases}$$

Firm j gets the license. Assume that the patent holder licenses the new technology to the less efficient firm, namely, firm j. Then, the corresponding output levels become:

$$q_i(c_i, c_j) = \frac{1 - 2c_i + (\underline{c} + r_j)}{3} \qquad q_j(\underline{c}, r_j, c_i) = \frac{1 - 2(\underline{c} + r_j) + c_i}{3}$$

The choice of r_j follows from maximization of $R_j^{ph} = q(\underline{c}, r_j, c_i)r_j$, subject to $r_j < c_j - \underline{c}$. The constraint ensures that the technology does not represent a net cost for the adopting firms - i.e., it is the participation constraint.

$$r_j = \begin{cases} \frac{1-2\underline{c}+c_i}{4} & \text{if} \quad \underline{c} < \frac{4c_j-c_i-1}{2} \\ c_j - \underline{c} & \text{otherwise} \end{cases} \qquad R_j^{ph} = \begin{cases} \frac{(1-2\underline{c}+c_i)^2}{24} & \text{if} \quad \underline{c} < \frac{4c_j-c_i-1}{2} \\ \frac{(c_j-\underline{c})(1-2c_j+c_i)}{3} & \text{otherwise} \end{cases}$$

By simple comparison, we can state that:

Proposition 1. *Assume* $(1-2\underline{c}+c_j)/4 < c_i - \underline{c}$ *- i.e., the participation constraint is never binding in both scenarios. Then the patent holder always supplies the most efficient firm.*

Proof. See mathematical appendix. □

If the constraint is not binding, the patent holder can freely set the price r^* that maximizes her revenues. As the market share of the most efficient firm is larger than that of the less efficient one, the new technology will have a more extensive market penetration if i is licensed. As the price is not bound, the net benefit from adoption must be positive. This means that the adopting firm becomes even more efficient. If the most efficient firm is the one which gets selected, she becomes even more efficient, with an even larger market share. In other words, as both firms are willing to pay the monopoly price, only the market share effect is in force. Thus, to sell the technology to firm i is the profit-maximizing choice from the patent holder's perspective.

Proposition 2. *Assume* $(1-2\underline{c}+c_j)/4 \geq c_i - \underline{c}$ *and* $(1-2\underline{c}+ci)/4 < c_j - \underline{c}$ *- i.e., the participation constraint is binding only if the most efficient firm is licensed. Then, there exists a threshold* $\bar{\bar{c}} < c_i$ *such that the most efficient firm is licensed if* $\underline{c} < \bar{\bar{c}}$*. Vice-versa, the less efficient firm gets the license.*

Proof. See mathematical appendix. □

To provide a numerical example, assume $c_i = 0.3$ and $c_j = 0.5$, such that the assumptions in proposition 2 are satisfied for all $\underline{c} < c_i$, as well as condition $1 - 2c_j + c_i > 0$. Then, one can see that the threshold exists and is $\bar{\bar{c}} = 0.174 < 0.3$. We should notice that for some values of c_i and c_j, the threshold $\bar{\bar{c}}$ can be negative, meaning that the price effect always dominates the market share effect. If she decides to sell the technology to a firm i, the patent holder cannot freely set the price. Thus, on the one hand, if the differential between c_i and $\underline{c}$ is not sufficiently large, the more diffuse market penetration is more than compensated by the negative price distortion. On the other hand, the patent holder can choose her profit-maximizing strategy if she deals with the less efficient firm. Therefore, provided that the price distortion is sufficiently large, the price effect dominates the market share effect.

Proposition 3. *Assume* $(1-2\underline{c}+c_j)/4 \geq c_i - \underline{c}$ *and* $(1-2\underline{c}+ci)/4 \geq c_j - \underline{c}$ *- i.e., the participation constraint is always binding. Then, there exists a threshold* $\tilde{\tilde{c}} < c_i$ *such that the most efficient firm is licensed if* $\underline{c} < \tilde{\tilde{c}}$*. Vice-versa, the less efficient firm gets the license.*

Proof. See mathematical appendix. □

To provide a numerical example, assume $c_i = 0.4$ and $c_j = 0.42$, such that the assumptions in proposition 3 are satisfied for all $\underline{c} < c_i$, as well as condition $1 - 2c_j + c_i > 0$. Then, one can see that the threshold exists and is $\tilde{\tilde{c}} = 0.213 < 0.4$. We should notice that for some value of c_i and c_j ,the threshold $\tilde{\tilde{c}}$ can be negative, meaning that the price effect always dominates the market share effect.

3.1.2 Non-exclusive dealing

Let us assume now that the innovator cannot prevent one or the other firm from obtaining the new technology if they are willing to pay for it. Depending on the price r^u set by the patent holder, one or more firms might decide to become licensees. More particularly, let us define m as the number of adopting firms, then $m = 2$ if $r^u < c_i - \underline{c}$, $m = 1$ if $c_i - \underline{c} \leq r^u < c_j - \underline{c}$, and $m = 0$ otherwise. With exclusive dealing, the patent holder could choose which firm to supply, preventing access to the other. Thus, even if one firm, namely the less efficient one, had been willing to pay the price r_i set for the most efficient one, she could have been foreclosed. Here, we assume such a restriction is not in place. The patent holder posts a price, and all the firms which are willing to pay for it will purchase the technology. Intuitively, as the least efficient firm has much more than the most efficient one to gain from becoming a licensing, she is willing to pay a high price for it. In other words, if at least one firm adopts the technology, that firm must be the least efficient one. If both firms adopt the technology, what used to be a market with heterogeneous firms becomes a market with symmetric technologies. The former most efficient firm loses the strategic advantage of being more productive, and the market share is split into two halves. Similarly, let us suppose that the least efficient firm only adopts the technology and that the price is $r^u \leq c_j - \underline{c}$. In that case, the productivity gap between the two firms shrinks, and the market share becomes less concentrated. This means that:

$$q_i(\underline{c}, r^u) = q_j(\underline{c}, r^u) = \frac{1 - \underline{c} - r^u}{3}$$

$$q_i(c_i, \underline{c}, r^u) = \frac{1 - 2c_i + \underline{c} + r^u}{3} \qquad q_j(\underline{c}, r^u, c_i) = \frac{1 - 2(\underline{c} + r^u) + c_i}{3}$$

Starting from here, there are three cases to be considered: i) the profit-maximizing price (r^{u*}) exceeds the participation constraint of the least efficient firm; ii) the profit-maximizing price exceeds the participation constraint of the most efficient firm, but not that of the least efficient one; iii) the profit-maximizing price is not constrained.

The first case is the simplest to analyze. The price of the technology boils down to $r_j^u = c_j - \underline{c}$ if the innovator sells the technology only to j, and $r_i^u = c_i - \underline{c}$, in case she wants both firms to join.

Proposition 4. *Assume $r^{u*} \geq c_j - \underline{c}$. Then the patent holder supplies both firms if $\frac{2(1-c_i)}{1-2c_j+c_i} > \frac{c_j - \underline{c}}{c_i - \underline{c}}$. Otherwise, she supplies only the least efficient firm.*

Proof. See mathematical appendix. □

We should notice that $\frac{2(1-c_i)}{1-2c_j+c_i}$ is the ratio between market penetration of the new technology when both firms are licensed (numerator) and when only j is licensed (denominator). The inequality in proposition 4 means that if the advantage in quantities of a full adoption outweighs the advantage in price of serving the least efficient firm only, then the market share effect dominates the price effect, and both firms are licensed. Otherwise, the opposite occurs. Let us notice that as $\underline{c} \to c_i$, the patent holder must reduce the price to convince the most efficient firm to join. Thus, the price effect increases, and so does the opportunity cost of full adoption.

Proposition 5. *Assume* $c_j - \underline{c} > r^{u*} \geq c_i - \underline{c}$. *Then the patent holder supplies both firms if* $\frac{2(1-c_i)}{1-2\underline{c}+c_i} > \frac{1-2\underline{c}+c_i}{8(c_i-\underline{c})}$. *Otherwise, she supplies only the less efficient firm.*

Proof. See mathematical appendix. □

As stated above, when $\underline{c} \to c_i$, the patent holder must reduce the price to convince the most efficient firm to join. Thus, the price effect increases, and so does the opportunity cost of full adoption.

Eventually:

Proposition 6. *Assume* $c_i - \underline{c} > r^{u*}$. *Then, it follows logically from Proposition 1 that both firms are supplied.*

Proof. See mathematical appendix. □

From proposition 1, we know that if the price is not bound, then the most efficient firm is supplied, as this is the best strategy for the patent holder. As firm j is willing to pay the profit-maximizing price, the revenues of the patent holder increase as the market penetration of her technology reaches the highest possible level.

4 Conclusion

The literature on innovation licensing has focused chiefly on which licensing scheme provides the innovator with the highest returns. To that extent, assuming firms in the market where technologically symmetric is a reasonable approximation allows to single out the value of the innovation as a cost-reducing parameter. Being the firms ex-ante homogeneous, also the effect of the innovation on each of them is symmetric. However, when firms have heterogeneous costs of productions, they also have different reservation prices for the new technology. The innovator faces a novel problem. She chooses the price of the license and which firms to supply. This article analyzes this problem under linear per unit price licensing (royalties) and disentangles two main forces: a price effect and a market share effect. The former identifies the higher willingness to pay for firms that are not efficient, as the cost reduction would be more significant in their case. Instead, the latter suggests that providing the most efficient firms with the innovation allows the patent holder to collect more royalties, as the market is filled with goods produced by means of innovative technology. The dominance of one of these two forces determines which firm (or subset of firms) get licensed in equilibrium.

Mathematical appendix

Proof of proposition 1

Proof. Assume $(1-2\underline{c}+c_j)/4 < c_i - \underline{c}$. Then, the revenues from licensing can be written as:

$$R^{ph} = \begin{cases} \frac{(1-2\underline{c}+c_j)^2}{24} & \text{if} \quad \text{i gets the license} \\ \frac{(1-2\underline{c}+c_i)^2}{24} & \text{if} \quad \text{j gets the license} \end{cases}$$

As $c_j > c_i$ by assumption, the innovator's rational choice is to license the most efficient firm. □

Proof of proposition 2

Proof. Assume $(1-2\underline{c}+c_j)/4 \geq c_i - \underline{c}$ and $(1-2\underline{c}+c_i)/4 < c_j - \underline{c}$. Then, the revenues from licensing can be written as:

$$R^{ph} = \begin{cases} \frac{(c_i-\underline{c})(1-2c_i+c_j)}{3} & \text{if} \quad \text{i gets the license} \\ \frac{(1-2\underline{c}+c_i)^2}{24} & \text{if} \quad \text{j gets the license} \end{cases}$$

It is easy to prove that there exists a threshold

$$\bar{\bar{c}} \equiv \sqrt{(c_i - c_j)(2c_i - c_j - 1)} + \frac{5c_i}{2} - c_j - \frac{1}{2}$$

such that

$$\begin{aligned} \frac{(c_i-\underline{c})(1-2c_i+c_j)}{3} &> \frac{(1-2\underline{c}+c_i)^2}{24} \quad \text{if} \quad c < \min\{\bar{\bar{c}}, c_i\} \\ \frac{(c_i-\underline{c})(1-2c_i+c_j)}{3} &< \frac{(1-2\underline{c}+c_i)^2}{24} \quad \text{otherwise} \end{aligned}$$

As a numerical example, assume $c_i = 0.3$ and $c_j = 0.5$. One can see that, under that assumption, conditions $(1-2\underline{c}+c_j)/4 \geq c_i - \underline{c}$ and $(1-2\underline{c}+c_i)/4 < c_j - \underline{c}$ become

$$(1.5 - 2\underline{c})/4 \geq 0.3 - \underline{c} \quad \text{and} \quad (1.3 - 2\underline{c})/4 < 0.5 - \underline{c}$$

which are mutually satisfied for $\underline{c} < c_i$. Moreover, $1 - 2c_j + c_i = 0.3 > 0$ (condition 1) is also satisfied. Then,

$$\bar{\bar{c}} \to \sqrt{(0.18)} + \frac{1.5}{2} - 1 \approx 0.174$$

□

Proof of proposition 3

Proof. Assume $(1 - 2\underline{c} + c_j)/4 \geq c_i - \underline{c}$ and $(1 - 2\underline{c} + c_i)/4 \geq c_j - \underline{c}$. Then, the revenues from licensing can be written as:

$$R^{ph} = \begin{cases} \frac{(c_i - \underline{c})(1-2c_i+c_j)}{3} & \text{if} \quad \text{i gets the license} \\ \frac{(c_j - \underline{c})(1-2c_j+c_i)}{3} & \text{if} \quad \text{j gets the license} \end{cases}$$

It is easy to prove that there exists a threshold

$$\tilde{\tilde{c}} \equiv \frac{2c_i + 2c_j - 1}{3}$$

such that

$$\frac{(c_i - \underline{c})(1 - 2c_i + c_j)}{3} > \frac{(c_j - \underline{c})(1 - 2c_j + c_i)}{3} \quad \text{if} \quad c < \min\{\tilde{\tilde{c}}, c_i\}$$
$$\frac{(c_i - \underline{c})(1 - 2c_i + c_j)}{3} < \frac{(c_j - \underline{c})(1 - 2c_j + c_i)}{3} \quad \text{otherwise}$$

As a numerical example, assume $c_i = 0.4$ and $c_j = 0.42$. One can see that, under that assumption, conditions $(1 - 2\underline{c} + c_j)/4 \geq c_i - \underline{c}$ and $(1 - 2\underline{c} + c_i)/4 \geq c_j - \underline{c}$ become

$$(1.42 - 2\underline{c})/4 \geq 0.4 - \underline{c} \quad \text{and} \quad (1.4 - 2\underline{c})/4 \geq 0.42 - \underline{c}$$

which are mutually satisfied for $0.14 < \underline{c} < c_i$. Moreover, $1 - 2c_j + c_i = 0.56 > 0$ (condition 1) is also satisfied. Then,

$$\tilde{\tilde{c}} \to \frac{0.64}{3} \approx 0.213$$

□

Non-exclusive dealing

Intuitively, if the patent holder cannot exclude any firm from purchasing the innovation, the possible outcomes of the licensing stage are the following: i) only the firm with the highest reservation price gets the license (i.e., firm j), or ii) both the firms get the license if the price is sufficiently low. For obvious reasons, we exclude the analysis of the case in which no firms get the license. Let r^u identify the royalty rate set by the innovator (i.e., the price of the technology). In the first case, the output levels in the Cournot game are:

$$q_i(c_i, \underline{c}, r^u) = \frac{1 - 2c_i + \underline{c} + r^u}{3} \quad \text{and} \quad q_j(\underline{c}, r^u, c_i) = \frac{1 - 2(\underline{c} + r^u) + c_i}{3}$$

Instead, in the second case, the output levels in the Cournot game are symmetric and equal to:

$$q_i(\underline{c}, r^u) = q_j(\underline{c}, r^u) = q(\underline{c}, r^u) \equiv \frac{1 - \underline{c} - r^u}{3}$$

From these levels, it is possible to derive the profit maximizing price r^{u*}, subject to the participation constraint $r^{u*} \leq c_k - \underline{c}$, where $k = i, j$.

Case i): only j gets the license. This scenario is equivalent to the case in which the patent holder decides to deal exclusively with firm j. The output level of the less efficient firm $q_j(\underline{c}, r^u, c_i)$ represents the demand faced by the innovator. The consequent profit maximizing royalty rate is:

$$r^{u*} = \min \left\{ c_j - \underline{c}; \frac{1 - 2\underline{c} + c_i}{4} \right\}$$

where $c_j - \underline{c}$ is the participation constraint.

Let assume that the profit maximizing price r^{u*} is not bound, i.e., $r^{u*} = \frac{1-2\underline{c}+c_i}{4}$. This happens when

$$\left(\frac{1}{4} < c_j < \frac{1}{3} \wedge ((0 < c_i \leq C_j \wedge 0 < \underline{c} < c_i) \vee (C_j < c_i < 2C_j \wedge 0 < \underline{c} < \chi)) \right)$$
$$\vee \left(\frac{1}{3} \leq c_j < 1 \wedge ((0 < c_i < C_j \wedge 0 < \underline{c} < c_i) \vee (C_j \leq c_i < c_j \wedge 0 < \underline{c} < \chi)) \right)$$

where $C_j = \frac{4c_j - 1}{2}$ and $\chi = \frac{-c_i + 4c_j - 1}{2}$. The profits of the patent holder can be written as:

$$R^u = \frac{(1 - 2\underline{c} + c_i)^2}{24}$$

Otherwise, the price is bound to $r^{u*} = c_j - \underline{c}$, which leads to:

$$R^u = \frac{(1 - 2c_j + c_i)(c_j - \underline{c})}{3}$$

Case ii): both i and j get the license. In this scenario, the total output leve $Q(\underline{c}, r^u) = 2\frac{1-\underline{c}-r^u}{3}$ represents the demand faced by the innovator. The consequent profit maximizing royalty rate is:

$$r^{u*} = \min \left\{ c_i - \underline{c}; \frac{1 - \underline{c}}{2} \right\}$$

where $c_i - \underline{c}$ is the participation constraint (namely, the maximum willingness to pay of the most efficient firm).

Let assume that the profit maximizing price r^{u*} is not bound, i.e., $r^{u*} = \frac{1-2\underline{c}+c_i}{4}$. This happens when:

$$\frac{1}{2} < c_i < 1 \wedge 0 < \underline{c} < 2c_i - 1 \wedge c_i < c_j < 1$$

The profits of the patent holder can be written as:

$$R^u = \frac{(1-\underline{c})^2}{6}$$

Otherwise, the price is bound to $r^{u*} = c_i - \underline{c}$, which leads to:

$$R^u = \frac{2(1-c_i)(c_i-\underline{c})}{3}$$

Proof of proposition 4

Proof. Assume that the optimal price r^{u*} is bound both in cases i) and ii). Then, the innovator sells the technology to both the firms if :

$$\frac{(1-2c_j+c_i)(c_j-\underline{c})}{3} < \frac{2(1-c_i)(c_i-\underline{c})}{3}$$

$$\frac{(c_j-\underline{c})}{(c_i-\underline{c})} < \frac{2(1-c_i)}{(1-2c_j+c_i)}$$

□

Proof of proposition 5

Proof. Assume that the innovator can set the optimal price r^{u*} in case i), while it is bound by the participation contraint in case ii). Then, the innovator sells the technology to both the firms if :

$$\frac{(1-2\underline{c}+c_i)^2}{24} < \frac{2(1-c_i)(c_i-\underline{c})}{3}$$

$$\frac{(1-2\underline{c}+c_i)}{8(c_i-\underline{c})} < \frac{2(1-c_i)}{(1-2\underline{c}+c_i)}$$

□

Proof of proposition 6

Proof. From proposition 1, if the innovator is able to set the price that maximizes her profits, then, the MSE dominates and the most efficient firm gets the license. Consequently, the innovator chooses to supply both the firms:

$$\frac{(1-\underline{c})^2}{6} > \frac{(1-2\underline{c}+c_i)^2}{24}$$

$$3(1-\underline{c})^2 > (1-2\underline{c}+c_i)^2$$

which is always satisfied for any feasible value of c_i and $\underline{c}$. □

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