

Advantageous Symmetric Cross-Ownership

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Abstract

We model an industry where a subset of firms is interlinked via a mutual and symmetric share exchange agreement. A merger aiming at acquisition of market power can be reproduced by the same firms under a symmetric cross-ownership scheme. Both concentration and market power indices increase due to cross-ownership. Under linear demand, a non-controlling symmetric cross-ownership scheme is always advantageous to each members if at least $(2 - \sqrt{2})(1 + n)$ firms in an n -firm industry participate. The threshold drops to $(1 + n)/2$ for relatively low levels of cross-ownership. Last, we show that cross-ownership schemes require fewer participants than mergers to be advantageous and can always be more profitable than mergers, unless a merger involves more than 88 per cent of industry firms.

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1 Introduction

Minority stake acquisitions in rival firms is considered an alternative option to full integration. The empirical works of Ouimet (2013) and Allen and Phillips (2000) attribute minority acquisitions in the US economy to the preservation of managerial incentives of the target firm, relief from financial constraints at the target, and access to information before taking a majority stake. Minority acquisitions may also serve as a remedy to incomplete contracts, provide management with effective monitoring of the target and help induce more R&D cooperation between acquirer and target. Alongside these positive effects, the literature on common ownership¹ has demonstrated that minority stake acquisitions may increase market power and harm competition. Recently, Nain and Wang (2018) used a sample of 774 acquisitions among rival firms in which less than 50% of the target's equity is acquired and showed that the real producer price index (RPPI) and price-cost margins increased significantly between 1980 and 2010 in US manufacturing industries.

The objective of this work is to show that market power may not only be an effect but also a cause of common-ownership. Inspired by merger theory, which has established that market power alone is an incentive for merger, we focus on market power as an incentive for minority stake acquisitions. In Cournot oligopolies with identical firms, mergers are profitable provided a certain number of firms collude, Salant et al. (1983), Cheung (1992), Faulí-Oller (1997). We extend this result to a particular type of symmetric cross-ownership, where two or more firms exchange an equal number of their shares. Although symmetry is a simplifying assumption, it allows us to derive explicit conditions on the profitability of cross-ownership. Moreover, our results can be extended to asymmetric cross-holdings provided a specific ownership structure of an industry is given. Examples of cross-ownership are frequent in the automobile, financial, energy and steel industry.²

We rely on the theoretical model of Reynolds and Snapp (1986) to show that a wide range of symmetric cross-ownership schemes can be profitable at equilibrium and have adverse effects on competition. Our model encompasses their monopoly equivalence result as a limit case. They have shown that if all n firms in an industry are involved in a symmetric cross-ownership scheme (COS henceforth) where each firm owns $1/n$ of the shares of its rival, the equilibrium outcome is monopoly.

We extend the result of Reynolds and Snapp (1986) to the case where a number $k \in [2, n]$ of firms participate in a COS where each firm owns $1/k$ of the shares of its rival and show that equilibrium coincides with the k -firm merger equilibrium. Thus, in an industry with symmetric firms that face constant marginal cost and a general demand function, cross-ownership can indeed serve as a substitute to full integration aiming at market power. This finding is relevant from the organizational viewpoint but also antitrust. Moreover, we show that HHI and Lerner indices are positively related to higher levels of cross-ownership, expressed either in terms of firm participation or percentage of shares being exchanged.

¹See Anton et al. (2018), Azar et al. (2018), Backus et al. (2019), Azar et al. (2019) and Schmalz (2021) for a recent review.

²In 2018, Renault owned 43% of Nissan, while Nissan had a 15% share in Renault. From 2011 to 2015, VW and Suzuki held shares in each other, 19.89% and 2.5% respectively. More examples are reported in Seldeslachts et al. (2017), Dietzenbacher et al. (2000), Brito et al. (2014), Nain and Wang (2018).

In order to quantify the effects of symmetric cross-ownership, we retreat to a linear demand function. Salant et al. (1983) have established that in the symmetric Cournot model with linear demand and cost functions, a full merger is profitable if the number of merging firms represent at least 80% of industry size. Similarly, we prove that the minimal participation ratio that renders a COS profitable drops to 58.758% for any COS that preserves the initial status-quo of control within a firm, or even to 50% for certain low cross-ownership levels that we explicitly identify. Surprisingly, the 50% rule appears in various distinct models that study coalitional profitability, as in Levin (1990), Kamien and Zang (1990), Cheung (1992) and Benchekroun et al. (2020). We explain the differences with our model in the next section.

Our model predicts that, compared to mergers, cross ownership schemes are more likely to appear, unless mergers involve more than 88% of the firms in the industry. If the participation ratio ranges from 58.785% to 88%, firms can always arrange a COS that outperforms a merger. Bearing in mind the 80% lower bound for profitability of Salant et al. (1983), it is evident that there exists a non-trivial range in which a merger is loss-making while a COS is profitable. To give an example, in an industry with 5 firms, a merger of 4 firms is equally profitable to a COS with 4 firms, each firm owning $1/4$ in each of the other 3 participating firms. While a merger of 3 firms is unprofitable, a COS with 3 firms is profitable if each participating firm owns a percentage $s \in (0, 1/3)$ in the other 2 firms or $s \in (0, 1/4]$ if we consider only silent financial interests.

The results in the paper are driven by the incentive of firms to relax competition and reduce output once they participate in a COS. Any sales foregone by lower output are offset by revenues from dividends in rival firms. We identify conditions on the number of participating firms, industry size and distribution of ownership that make the dividend effect stronger and render a COS advantageous. Outsiders, i.e. the firms that do not participate in a COS, are always favoured by the formation of a COS, because due to strategic substitutability they produce more as a reaction to the contraction of output by insiders.

The paper proceeds as follows. Section 2 reviews the related literature. Section 3 presents the model with a general demand function, demonstrates the equivalence between a COS and a merger and derives the formulas of concentration and market power indices. In Section 4, we specify a linear demand function and derive explicit conditions on the profitability of symmetric cross-ownership schemes and their payoff dominance over mergers. Section 5 concludes. All proofs are in the appendix.

2 Related Literature

This work relates to the theory of mergers and acquisitions and common ownership. Early theory developed in Rubinstein and Yaari (1983) and Rotemberg (1984) showed that common ownership relaxes competition or leads to monopoly.

Reynolds and Snapp (1986) constructed a general model of asymmetric equity linkages across firms and have shown that under Cournot competition, if firms increase their shareholdings in other rival firms, then the equilibrium market output declines. Our model can be considered as a special case of Reynolds and Snapp (1986) where a subset of firms has zero percentage

interlinks with other firms while participants in a COS have symmetric and mutual interlinks. However, further results in their paper rely exclusively on symmetric equity holdings among *all* n industry firms, thus not allowing for any outsiders. Their assumption of "widespread linkages" across all firms neutralizes the output expansion effect that a subset k of insider firms causes on $n - k$ outsiders. The output expansion effect, which is well known from merger theory, seriously limits the profitability of a COS that does not involve all firms in the industry. This is exactly the point that our comparative statics analysis stresses by varying the number of firms k that participate in a COS and thus showing explicitly which sizes of insiders' group are profitable.

Bresnahan and Salop (1986) extend the work of Reynolds and Snapp (1986) and devise a modified HHI (MHHI) index to account for various financial interest and control mechanisms concerning common ownership. Although they quantify the competitive effects of common ownership, they do not link the industry ownership structure with its profitability. MHHI was generalized by Salop and O'Brien (2000) to include also other forms of common ownership, e.g. minority shareholdings by institutional investors.

Perry and Porter (1985) consider a model with multiple oligopolists behaving as Stackelberg leaders and a competitive fringe in an industry with a fixed supply of an asset which is essential to production. They examine the incentives of fringe firms to merge putting together their assets. The merger increases the number of oligopolists in the market while decreases the fringe size. This leads to a higher industry price and profit of the merged entity despite its output contraction. The profitability of mergers then depends on the conjecture of each oligopolist on how rivals will respond to its output changes.

Kwoka (1989) drops the assumption that a group of merged firms behaves as a Cournot player and attributes merger profitability to different types of conjectural variations that players may have. Similarly to Perry and Porter (1985) he shows that a merger is likely to be more profitable when it absorbs firms with particularly rivalrous conjectures.

Farrell and Shapiro (1990) analyze the effects of one firm increasing its stake in a rival on price, profits, industry performance, and measured concentration. They identify market structure conditions under which common ownership increases both concentration and welfare.

Kamien and Zang (1990) develop a multistage game where symmetric Cournot firms may acquire their rivals in their entirety and then decide how many of the acquired production plants to operate. Under constant marginal cost and general demand function, they show that an owner finds it optimal to operate more than one firm and have the owned firms compete with each other. They also show that mergers cannot arise at equilibrium unless an owner can possess $(n + 1)/2$ or more firms. Similarly, in our model cross-ownership is not advantageous, if less than $(n + 1)/2$ firms in the industry participate in a COS. Despite the similarity, the result is derived from very distinct models. Kamien and Zang (1990) consider full and controlling acquisitions while we consider only partial and non-controlling ones. Moreover, Kamien and Zang (1990) endogenize the acquisition decision, while we compare equilibria of exogenously given industry ownership structures.

Levin (1990) studies the consequences of an horizontal merger by a subset of possibly asymmetric firms that face constant marginal cost and general demand. Following a merger of asymmetric firms, the new merged entity will operate only the most efficient firm, thus achiev-

ing production efficiencies and increase its profitability. In our model, the shut-down decision is not an option, because all stakes are non-controlling. Levin (1990) shows even in the absence of production efficiencies, if a group of firms with less than 50 percent of premerger market output considers a horizontal merger, then any contraction of output cannot be profitable. Outsiders always remain Cournot players after a merger, but insiders can become a Stackelberg leader, a conjectural variation player or remain Cournot. Moreover, he identifies conditions under which a merger can be both profitable and total welfare increasing.

Flath (1991) builds a two stage model where firms acquire shares in rivals and then produce. He shows that under Cournot competition, if the stock market is efficient, any possible gain from minority acquisitions will be reflected on share prices and hence be neutralized. However under Bertrand competition, firms do have incentives to acquire silent interests in rivals.

Cheung (1992) show that the minimal market share for a merger to be profitable is 50%, if one extends the model of Salant et al. (1983) to demand functions that result in decreasing marginal revenue of the industry. Faulí-Oller (1997) explains the difference between Cheung (1992) and Salant et al. (1983) by showing that the profitability of a merger is inversely related to the degree of concavity of the demand function.

Reitman (1994) considers asymmetric cross holdings in a 2-stage game where firms buy and sell shares in the first stage and choose output levels in the second stage. He provides conditions under which a COS is pairwise profitable when firms have possibly non-Cournot conjectures about the choices of their opponents. The model of Reitman (1994) differs from ours in many respects. First, he allows for cross shareholdings that exceed 50% of the firm shares, but give no control rights to the majority owner. Second, he shows that when the number of firms in the industry is greater than three, there can be no profitable single COS, while profitable multiple COS may co-exist only if firms behave more competitively than Cournot. We show exactly the opposite, that is, a single COS with sufficient size among Cournot players can be profitable. The difference in our results lie on the fact that Reitman (1994) considers COS between two firms only, possibly spanning the whole industry through chain rings, while we consider multi-firm COS which cause a much greater output contraction at equilibrium.

Benchekroun et al. (2020) consider static and dynamic versions of a symmetric cross-ownership model with an application to the renewable resources industry. They derive a lower bound on profitability of cross-ownership which is identical to ours albeit different assumptions. In particular, we assume that firm managers take into account direct shareholdings on rivals while they assume indirect or ultimate shareholdings. Direct and ultimate shareholdings generically never coincide and lead to different firm objectives. Moreover, they give rise to different sets of permissible cross-ownership schemes that guaranty the existence of a majority shareholder.³

Last, the bright side of common-ownership, that is, the facilitation of transfer of knowledge, R&D investments and cost reducing innovation can be found in Shelegia and Spiegel (2015), Ghosh and Morita (2017), Bayona and López (2018), Anton et al. (2018), Papadopoulos et al. (2019), López and Vives (2019), who identify conditions under which common ownership is

³For a discussion on indirect shareholdings see Dietzenbacher et al. (2000), Dorofeenko et al. (2008) and the models of Bolle and Güth (1992), Flath (1992a,b), Brito et al. (2018).

welfare improving. However, according to the empirical findings of Nain and Wang (2018), innovation and technology sharing do not explain the increase in price cost margins observed after minority stake acquisitions.

3 The Model

Let n be the number of firms in an industry, and i be a firm. We treat the number of firms as a continuous variable. Firms are symmetric facing a cost function $C_i(q_i) = cq_i$, where q_i is the quantity produced by firm i and c is the marginal cost. Market demand is given by an inverse demand function $p(Q)$ which is twice-continuously differentiable and $p'(Q) < 0$. Moreover $p(0) > c \geq 0$. Initially, we assume that each firm is owned by a single owner who is also the manager. Firms are assumed to play a Cournot game, choosing simultaneously the quantities to be offered for sale to the market. The total quantity is $Q = \sum_i q_i$. Each firm maximizes profit $\Pi_i = (p(Q) - c)q_i$ taking as given $Q_{-i} = \sum_j q_j, j \neq i$.

Now suppose that a k number of firms, $2 \leq k \leq n$, engage in a mutual symmetric share exchange agreement of $s\%$, that is, each firm offers an ownership share s of itself to the rest $k - 1$ firms in exchange for a share s in each one of the $k - 1$ firms.⁴

Definition 1 A (k, s) symmetric common ownership scheme is a group of k firms, $2 \leq k \leq n$, such that each firm in the group owns a percentage $0 \leq s \leq 1/k$ of the stock of the rest $k - 1$ firms.

A firm that participates in a (k, s) symmetric cross-ownership scheme is entitled to $s\%$ of the product market profits of each one of the rest $k - 1$ firms, but no control over their output decisions. In order to guaranty that no control issue will arise and that the objective function of the firm is well defined, we make the following assumptions on the set of permissible cross-ownership schemes.

Assumption 1 A (k, s) COS satisfies $0 \leq s < \frac{1}{2(k-1)}$.

The assumption guarantees that each firm retains at least 50% of its initial shares, the absolute majority, under any reciprocal and symmetric share exchange agreement. If we permit symmetric cross-ownership structures without a dominant majority shareholder, there are issues of corporate control which contradict the notion of a silent interest. For example, if 4 firms exchange 1/4 of their shares among them, they will all end up having a minority share, so the production decision will not rest on one firm only, unless a specific decision rule is specified. For this case we will have to use the following assumption.

Assumption 2 For any (k, s) COS such that $\frac{1}{2(k-1)} \leq s \leq \frac{1}{k}$, the initial owner firm will preserve corporate control after the formation of a symmetric COS.

⁴For instance, such a multilateral exchange of shares could occur simultaneously at any uniform non-negative price, nevertheless we will not examine the endogenous formation of a COS, but consider it as exogenous throughout the paper.

Assumption 2 ensures that after a COS is formed, the initial firm owner will remain a dominant shareholder despite owning a minority share in its firm. Assumptions 1 and 2 are necessary if we wish to refer to silent financial interests and allow us to further assume that the initial firm owner-manager maximizes the value of its own portfolio, disregarding the preferences of any other direct or indirect minority owners.⁵ This last assumption is also made in Reynolds and Snapp (1986), where firms do not consider dividend profits from indirect shareholdings when they maximize profit.

A firm i that takes part in a (k, s) COS or an *insider* solves

$$\max_{q_i^I} \Pi_i^I(k, s) = [1 - (k - 1)s][p(Q) - c]q_i^I + \sum_{j \neq i}^I s[p(Q) - c]q_j^I \quad (1)$$

where q_j^I is the quantity of another member firm $j \neq i$ and I denotes the set of insiders with $|I| = k$.

Let $n - k$ be the number of independent firms. An independent firm or an *outsider* solves

$$\max_{q_i^O} \Pi_i^O(k, s) = [p(Q) - c]q_i^O \quad (2)$$

where q_i^O is the quantity chosen by an outsider firm with $|O| = (n - k)$. A Cournot equilibrium is a solution to the following system of k and $(n - k)$ first order conditions of problems (1) and (2) respectively

$$[1 - (k - 1)s] \left(\frac{\partial p}{\partial q_i^I} q_i^I + p(Q) - c \right) + s \frac{\partial p}{\partial q_i^I} \sum_{j \neq i}^I q_j^I = 0, \quad (3)$$

$$\frac{\partial p}{\partial q_i^O} q_i^O + p(Q) - c = 0 \quad (4)$$

and solving for q_i^I and q_i^O we obtain

$$q_i^I = \frac{c - p(Q)}{\partial p / \partial q_i^I} - \frac{s}{1 - (k - 1)s} \sum_{j \neq i}^I q_j^I, \quad (5)$$

$$q_i^O = \frac{c - p(Q)}{\partial p / \partial q_i^O}. \quad (6)$$

All insiders are symmetric so at a Nash equilibrium $q_i^I = q_j^I$ and $\sum_{j \neq i}^I q_j^I = (k - 1)q_i^I$, hence (5) reduces to

$$q_i^I = [1 - (k - 1)s] \frac{c - P(Q)}{\partial p / \partial q_i^I} \quad (7)$$

Given that $\partial p / \partial q_i^I = \partial p / \partial q_i^O$, from (7) and (4) we may deduce that $q_i^I = [1 - (k - 1)s]q_i^O$, hence $q_i^I < q_i^O$. When k , the number of firms that participate in a COS, or s , the level of cross-ownership, increases, the reaction functions of insiders shift towards the origin, $\partial q_i^I / \partial s < 0$, $\partial q_i^I / \partial k < 0$, while those of the outsiders remain unchanged $\partial q_i^O / \partial s = \partial q_i^O / \partial k = 0$. Once a

⁵If firm A owns 30% of firm B and firm B owns 30% of firm C, then firm A owns an indirect share of 9% in firm C. We assume that the 9% share of A in C, does not affect the output decision of A. Also if firm D owns 30% of firm B, firm A will not assign any positive control weight on the 9% indirect share of D in A.

(k, s) COS is formed, given the output of outsiders, it is no longer a Nash equilibrium strategy for an insider firm to keep its output unchanged. Insiders wish to contract their output, the higher the cross-ownership level s or the number of participating firms k . Given that quantities are strategic substitutes in the Cournot model, outsiders benefit from a COS by expanding their output.

Reynolds and Snapp (1986) have shown that when *all* n firms in an industry are interlinked symmetrically with a share $s = 1/n$, then the Cournot equilibrium coincides with the monopoly equilibrium. We may generalize their result for the case of symmetric cross-ownership among a *subset* of firms in the industry, $k \leq n$.

Lemma 1 *In an industry with n firms, let $k \leq n$ be the number of firms that merge. Then, if Assumption 2 holds, the k -firm merger Cournot equilibrium is equivalent to an equilibrium with a (k, s) COS where $s = 1/k$.*

To give an example, for any $n \geq 3$, the managers of three independent firms will find it equally profitable either to fully merge their firms or split their respective firms in three equal pieces and share the pieces evenly among them. The equivalence rests on the fact that both a $(k, 1/k)$ COS and a k -firm merger induce exactly the same contraction of insiders' total output. Hence outsiders will expand output equally under a COS or a merger.

Conclusively, our model contains the k -firm merger equilibrium as an equilibrium with a $(k, 1/k)$ COS, the standard Cournot equilibrium as an equilibrium with a degenerate $(k, 0)$ COS and the monopoly equilibrium as an equilibrium with a $(n, 1/n)$ COS.

We may next derive concentration and market power indices for an industry where a subset of firms participates in a (k, s) COS. The Herfindahl-Hirschman index (HHI) is

$$HHI(k, s) = \frac{n - (k - 1)ks[2 - (k - 1)s]}{[n - (k - 1)ks]^2}. \quad (8)$$

The more interlinked the firms become, the higher the concentration in the industry because $dHHI/ds > 0$. As s increases, total output decreases because insiders contract output more than outsiders expand theirs. Consequently the market shares of insiders (outsiders) decrease (resp. increase), leading overall to a higher concentration level. The effect of the number of insiders on HHI, $dHHI/ds$ is ambiguous. The market share of outsiders grows larger as the number of insiders k increases, but at the same time outsiders $n - k$ are getting fewer. As more firms participate in a COS, their market share may either increase or decrease depending on the level of cross-ownership s .

Let m be the number of firms with the higher market share in an industry, then the C_m concentration index is,

$$C_m(k, s) = \begin{cases} \frac{m}{n - (k - 1)ks} & \text{if } m \leq n - k, \\ \frac{m - (k - 1)(k + m - n)s}{n - (k - 1)ks} & \text{if } m > n - k. \end{cases} \quad (9)$$

which increases with s and k .

The Lerner index of an industry where firms participate in a (k, s) COS is

$$L(k, s) = -\frac{1}{\varepsilon_{Q,p}} \left(\frac{1}{n - (k-1)ks} \right), \quad (10)$$

where $\varepsilon_{Q,p}$ is the elasticity of demand. A higher level of cross-ownership or larger participation in a COS increase the price-cost margin because $dL(k, s)/ds > 0$ and $dL(k, s)/dk > 0$.⁶

Notice that the formulas of concentration and market power indices in an industry with a (k, s) COS reduce to the respective formulas of the standard Cournot model, once we set $s = 0$, i.e. $HHI(k, 0) = 1/n$, $C_m(k, 0) = m/n$ and $L(k, 0) = -1/(\varepsilon_{Q,p}(n - k + 1))$. However, in line with Bresnahan and Salop (1986) and Salop and O'Brien (2000), they do not reduce to the respective formulas for the k -firm merger when $s = 1/k$ as Lemma 1 suggests. The $HHI(k, 1/k) = [(n - k) + 1/k]/(n - k + 1)^2$ is lower than the HHI at an equilibrium with k -firm merger, $HHI_{k-merger} = 1/(n - k + 1)$. The difference $HHI(k, 1/k) - HHI_{k-merger} = (1 - k)/[k(n - k + 1)^2] < 0$ is due to the fact that the total number of firms n in the industry is reduced by $k - 1$ after a merger takes places, whereas n is not affected by the formation of a COS, so there are more squared market shares in the $HHI(k, 1/k)$. For instance, in an industry with ten firms, a merger of six firms is equivalent to a $(6, 1/6)$ COS. Nonetheless, $HHI_{6-merger} = 0.20$, while $HHI(6, 1/6) = 0.166667$.

Similarly, for $s = 1/k$ the C_m concentration index reduces to

$$C_m(k, 1/k) = \begin{cases} \frac{m}{n-k+1}, & \text{if } m \leq n - k, \\ \frac{m+(k-1)(n-k)}{k(n-k+1)}, & \text{if } m > n - k. \end{cases} \quad (11)$$

Again, if $m > n - k$, then $C_m(k, 1/k)$ is lower than $C_{m,k-merger} = m/(n - k + 1)$, despite that a k -firm merger and a $(k, 1/k)$ COS result in the same equilibrium. For example, in an industry with ten firms, $C_5 = 1$ after a merger of six firms, while $C_5(6, 1/6) = 0.833$.

On the other hand, consistently with Lemma 1, the Lerner index $L(k, 1/k)$ reduces the k -firm merger formula for $s = 1/k$.

4 The Linear Case

For the purpose of quantifying the profitability of cross-ownership, we rely to the tractable case where firms face a demand function of the type $p(Q) = a - Q$.

We may solve the system of (3) and (4) to obtain the reaction functions of insiders and outsiders respectively,

$$q_i^I(Q_{-i}^I, Q^O) = \frac{1}{2} \left[a - c - Q^O - \left(1 + \frac{s}{1 - (k-1)s} \right) Q_{-i}^I \right], \quad (12)$$

$$q_i^O(Q^I, Q_{-i}^O) = \frac{1}{2} (a - c - Q_{-i}^O - Q^I), \quad (13)$$

where $Q^O = \sum_i q_i^O$, $Q_{-i}^O = \sum_{j \neq i} q_j^O$, $Q^I = \sum_i q_i^I$, $Q_{-i}^I = \sum_{j \neq i} q_j^I$. Notice that for $s > 0$, an insider (outsider) firm reacts asymmetrically (resp. symmetrically) to the quantities chosen by

⁶The calculation of $HHI(k, s)$, $C_m(k, s)$, $L(k, s)$ and the signs of relevant derivatives can be found in the appendix.

outsiders and insiders. In particular, $|dq_i^I/dQ_{-i}^I| > |dq_i^I/dQ^O|$, that is an insider firm prefers to play less aggressively against insiders than outsiders. Also the higher the number of insiders or the level of cross-ownership, the less aggressive insiders become. Let $Z = 1/[1 + n - (k - 1)ks]$, then the Cournot equilibrium quantities under a (k, s) COS are,

$$q_i^I(k, s) = (a - c)[1 - (k - 1)s]Z, \quad (14)$$

$$q_i^O(k, s) = (a - c)Z. \quad (15)$$

The total quantity is $Q = kq_i^I + (n - k)q_i^O$ or

$$Q(k, s) = (a - c)[n - (k - 1)ks]Z. \quad (16)$$

Then under a (k, s) COS the Cournot equilibrium price, and profits are respectively

$$\begin{aligned} p(k, s) &= c + (a - c)Z, \\ \Pi_i^I(k, s) &= [1 - (k - 1)s][(a - c)Z]^2, \\ \Pi_i^O(k, s) &= [(a - c)Z]^2, \\ \Pi_{Industry}(k, s) &= (n - (k - 1)s)[(a - c)Z]. \end{aligned}$$

Allowing for symmetric cross-ownership among an explicitly given *subset* of firms, enables us to characterize the profitability of a COS with respect to the number of participating firms (insiders), in the spirit of Salant et al. (1983), where profitability is analysed as a function of relative merger size.

4.1 Profitability of Cross-Ownership Schemes

By comparing the equilibrium profits at a Cournot equilibrium with a (k, s) COS to the profits at the standard Cournot equilibrium with n firms we obtain the following proposition.

Proposition 1 *Under Assumption 1, for any $k \geq 2$, $n \geq k$, at a Cournot equilibrium a (k, s) COS is*

$$\begin{aligned} &i) \text{ unprofitable if } \frac{k}{1+n} \leq \frac{1}{2}, \\ &ii) \text{ profitable if } \frac{1}{2} < \frac{k}{1+n} < 2 - \sqrt{2} \text{ and } 0 < s < \frac{(2k - n - 1)(1 + n)}{(k - 1)k^2}, \\ &iii) \text{ unprofitable if } \frac{1}{2} < \frac{k}{1+n} < 2 - \sqrt{2} \text{ and } s > \frac{(2k - n - 1)(1 + n)}{(k - 1)k^2}, \\ &iv) \text{ profitable if } \frac{k}{1+n} \geq 2 - \sqrt{2}. \end{aligned}$$

Any non-controlling COS is profitable, when at least $(2 - \sqrt{2})(1 + n)$ firms in the industry participate in it. Also, any COS with at most $(1 + n)/2$ firms is unprofitable. When $(1 + n)/2 < k < (2 - \sqrt{2})(1 + n)$, a COS maybe profitable for relatively low levels of cross-ownership.

It is insightful to express the results of Proposition 1 in terms of the percentage of firms that participate in a COS relative to the industry size so that they are comparable to Salant

et al. (1983).⁷

Corollary 1 *Let $\hat{k} = k/n$ be the percentage of firms that participate in a COS. Then, for all industry sizes it is sufficient for a COS to be unprofitable that $\hat{k} \leq 50\%$.*

While a merger is loss-making when less than 80% of firms in an industry participate, the respective participation percentage is greater than 50% and drops to 50% for a COS when $n \rightarrow \infty$. In other words, the smaller the industry, the higher the participation ratio that renders a COS unprofitable. To illustrate, for $n \leq 5$, a COS is unprofitable if $\hat{k} \leq 60\%$, while for $n \leq 10$, $\hat{k} \leq 55\%$. From condition *iv*) in Proposition 1, it follows that a COS is profitable if $k/n \geq (2 - \sqrt{2})(1 + n^{-1})$ where the r.h.s decreases as the industry grows larger, hitting a minimum at 0.58578 when $n \rightarrow \infty$. From condition *ii*), it follows that for low enough cross-ownership levels, the minimum percentage of firms that can render a COS profitable drops to 50%.

Corollary 2 *The minimum percentage of firms that is sufficient for a (k, s) COS to be profitable is 58.758% for all s and 50% for $0 < s < \frac{(2k-n-1)(1+n)}{(k-1)k^2}$. The minimum percentage is increasing as the industry size becomes smaller.*

For $n \geq 4$, it is sufficient for a COS to be profitable that $\hat{k} \geq 73.22\%$ while for $n \geq 7$, $\hat{k} \geq 66.94\%$. As n increases, competition in the symmetric Cournot model intensifies thus driving down the equilibrium profits. Therefore, the more competitive the market, the potential gains from arranging a COS are much more profound compared to a market where few firms already enjoy market power.

To illustrate the comparison of profits between the standard Cournot equilibrium and a (k, s) COS equilibrium, let's assume that each firm owner that participates in a (k, s) COS, holds 90% of its own stock and the rest 10% is distributed to the remaining $(k - 1)$ firms. For any k , such a COS is realized when $s = \frac{1}{10(k-1)}$. The following graph depicts the ratio $\Pi_i^I(k, s)/\Pi_i(n)$ which is the profit of an insider at a (k, s) COS equilibrium over the standard Cournot equilibrium profit, where $k = \hat{k}n$ and $s = \frac{1}{10(\hat{k}n-1)}$. The ratio is calculated as a function of industry size for values of \hat{k} equal to 50%, 60%, 70% and 80%.

[INSERT FIGURE01 HERE]

In an industry with 10 firms for example, the following (k, s) COS are profitable $(6, \frac{1}{50})$, $(7, \frac{1}{60})$, $(8, \frac{1}{70})$. Notice also that as the participation rate k/n increases, each firm holds a decreasing quantity of stock in an increasing number of its rivals, because by construction $s = \frac{1}{10(k-1)}$. Hence, higher participation in a COS, induces higher profits for insiders.

4.2 Cross-ownership vs Merger

In Lemma 1 we have shown that any k -firm merger is equivalent to a $(k, 1/k)$ COS. So if we restrict the set of possible cross-ownership schemes to non-controlling ones as described by Assumption 1, we question whether silent financial interests can be more advantageous than a k -firm merger.

⁷The result *i*) of Proposition 1 and Corollary 1 hold also under the assumption of indirect shareholdings in Benchenkroun et al. (2020).

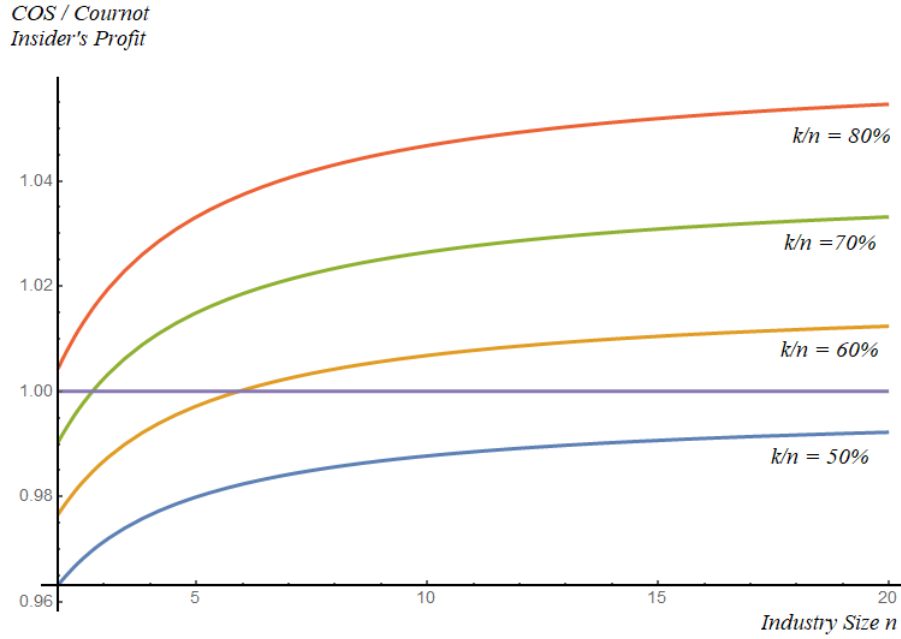


Figure 1: Profit of an Insider Firm under a COS over the standard Cournot profit

Proposition 2 *Let $n > 3$ and Assumption 1 hold, then for any Cournot merger (profitable or not) of \bar{k} firms, there exists a (\bar{k}, s) COS that is more advantageous to its members if and only if $\bar{k} < \frac{1}{4}(4n + 5 - \sqrt{8n + 9})$. A merger is more profitable than a (k, s) COS, if at least 88% of the firms in the industry merge.*

Proposition 2 provides an incentive for firms to engage in a COS instead of a merger. Not only a COS can be profitable when a merger is loss-making, but a COS may outperform a profitable merger with the same member firms. In Figure 1, the shaded area shows all positive differences between an insider's profit in a COS and in a merger when the industry size is twelve. We may observe that ten out of twelve firms in the industry achieve higher profits if they engage in a (10,4%) COS rather than a 10-firm merger. Each firm owner keeps 64% of the shares of its own firm and exchanges the rest 36% for 4% stakes in nine of its rivals. Besides the profit motive, managers may chose a COS over a merger for market power because the former is far less alarming to competition authorities as it does not change the HHI index as significantly as a merger or require any legal permission.

[INSERT FIGURE 2 HERE]

5 Conclusions

Symmetric cross-ownership schemes align the strategic incentives of competing firms and soften competition. They may serve as a alternative to a merger, or even be more advantageous. They certainly provide a means to cartelize an industry. On the other hand, symmetric cross-ownership schemes are hardly profitable unless at least more than half of industry firms participate. Conclusively, when we observe overlapping ownership between two or three competitors in

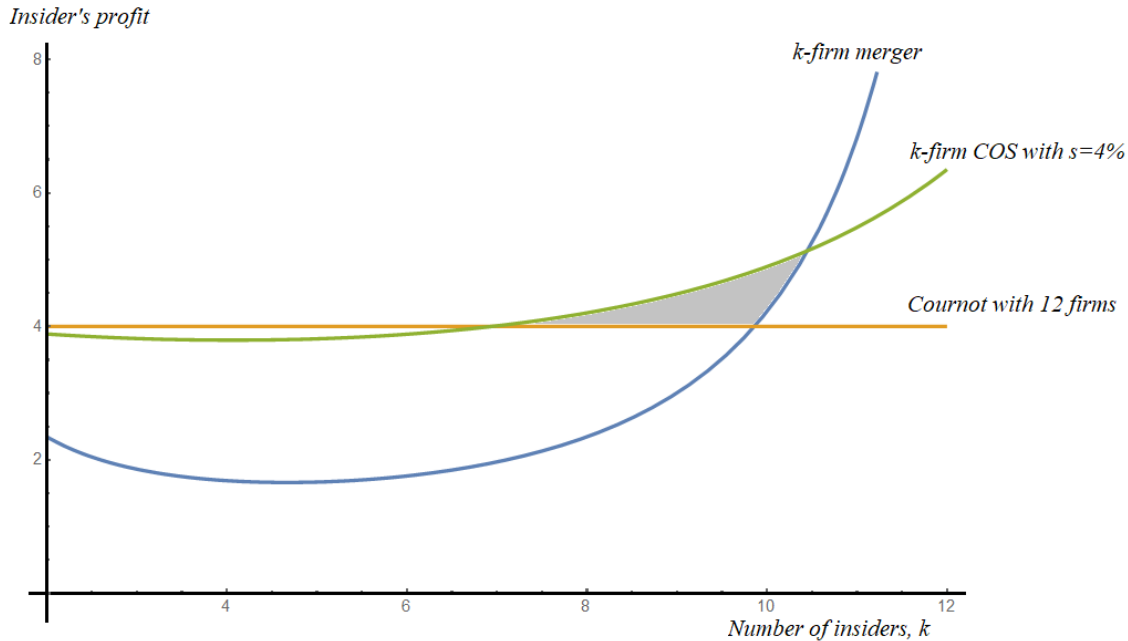


Figure 2: Profit of an Insider Firm, $n=12$, $a=32$, $c=6$

real markets, it is rather not driven by the acquisition of market power. Competition authorities should not be concerned, unless the market share of interlinked firms is substantial.

Symmetry of cross-ownership, cost symmetry and linearity of market demand were necessary simplifications in our model. If one drops these assumptions, it is not possible to obtain specific profitability thresholds for generic ownership structures. We expect firms with higher cross-holdings in other firms to contract their individual output relatively more. Similarly, firms with higher production cost are expected to produce even less if they happen to have shares in their rivals. Also, due to Faulí-Oller (1997), the degree of demand concavity will certainly affect negatively the profitability of a COS. Of course, if we specify demand and supply parameters that satisfy Cournot stability conditions, it is possible to derive explicit profitability thresholds for given asymmetric cross-ownership schemes.

Our works links profitability with the degree of cross-ownership s or the number k of participating firms in a COS (insiders). This link can be supported by the empirical findings of Banal-Estañol et al. (2020), if we interpret insiders as passive investors who diversify in a subset of industry firms, and outsiders as active investors who remain completely undiversified. Banal-Estañol et al. (2020) show that in the department stores and publishers industries, when the holdings of the passive investors increase, common ownership incentives increase. The latter can be related to higher markups, which are empirically demonstrated in De Loecker et al. (2020).

Considering welfare, it has been shown by Reynolds and Snapp (1986) that at a Cournot equilibrium total market output will drop, if ownership interlinks between firms increase. Therefore, cross-ownership schemes are welfare decreasing, exactly as mergers aiming at the acquisition of market power. This work highlights an apparent contradiction that should be considered by antitrust authorities. On the one hand, mergers for market power are banned, even though

they are less likely to occur, because they require a large market size to be profitable. On the other hand, cross-ownership schemes in the form of silent financial interests may be freely arranged, despite requiring a smaller market size to be profitable and being more advantageous than mergers.

6 Appendix

Proof of Lemma 1. Take a Cournot equilibrium in an industry with n symmetric firms where k firms merge. The manager of the merged entity maximizes the joint profit a k -plants, i.e. $\sum_{i \in I} [p(Q) - c]q_i^I$, w.r.to q_i^I , for all $i \in I$, where I is the set of insiders with $|I| = k$. The system of FOCs is given by

$$\frac{\partial p}{\partial q_i} \sum_i q_i^I + p(Q) - c = 0, \text{ for every } i \in I \quad (17)$$

The manager could separate the merged k -plant firm into k independent firms and engage them into a (k, s) COS with $s = 1/k$. Then each firm $i \in I$ would maximize its profit w.r.to q_i , which is

$$\begin{aligned} & \left(1 - \sum_{j \neq i} \frac{1}{k}\right) [p(Q) - c]q_i^I + \sum_{j \neq i} \frac{1}{k} [p(Q) - c]q_j^I \\ &= \left(1 - (k-1)\frac{1}{k}\right) [p(Q) - c]q_i^I + \frac{1}{k} \sum_{j \neq i} [p(Q) - c]q_j^I \\ &= \frac{1}{k} \sum_{i \in I} [p(Q) - c]q_i^I. \end{aligned} \quad (18)$$

The system of FOCs given by the maximization of profit in (18) for each firm $i \in I$ reduces to (17). The FOCs of the outsider firm remain the same under a k -firm merger and a $(k, 1/k)$ COS, so the Cournot equilibria of these market structures coincide. ■

Calculation of HHI, C_m , Lerner index and relevant derivatives. At a Cournot equilibrium where firms participate in a (k, s) COS, total output can be written as $Q = kq_i^I + (n-k)q_i^O$. From (7) and (4) we know that $q_i^I = [1 - (k-1)s]q_i^O$, hence total output is $Q = k[1 - (k-1)s]q_i^O + (n-k)q_i^O$ or $Q = q_i^O[n - (k-1)ks]$. Then the equilibrium market share of an outsider is $s_i^O = q_i^O/Q$ or $s_i^O = 1/[n - (k-1)ks]$. Similarly, the market share of an insider is $s_i^I = [1 - (k-1)s]/[n - (k-1)ks]$. Then at a Cournot equilibrium $HHI(k, s) = k(s_i^I)^2 + (n-k)(s_i^O)^2$ or

$$\begin{aligned} HHI(k, s) &= k(s_i^I)^2 + (n-k)(s_i^O)^2 \\ &= k \left(\frac{1 - (k-1)s}{n - (k-1)ks} \right)^2 + (n-k) \left(\frac{1}{n - (k-1)ks} \right)^2 \\ &= \frac{n - (k-1)ks[2 - (k-1)s]}{[n - (k-1)ks]^2}. \end{aligned}$$

Differentiating $HHI(k, s)$ we obtain

$$\frac{dHHI(k, s)}{ds} = \frac{2(k-1)^2ks(n-k)}{(n - (k-1)ks)^3} > 0,$$

$$\frac{dHHI(k, s)}{dk} = \frac{(k-1)s^2 [k((k-1)^2s - 4k + 3n + 2) - n]}{[n - (k-1)ks]^3} \leq 0.$$

In particular $dHHI(k, s)/ds > 0$ if either $n > \frac{2k(2k-1)}{3k-1}$ or $\frac{1-3k^2}{1-3k} < n \leq \frac{2k(2k-1)}{3k-1}$ and $s > \frac{k(4k-3n-2)+n}{(k-1)^2k}$. Otherwise $dHHI(k, s)/ds < 0$ if either $n \leq \frac{1-3k^2}{1-3k}$ or $\frac{1-3k^2}{1-3k} < n < \frac{2k(2k-1)}{3k-1}$ and $s < \frac{k(4k-3n-2)+n}{(k-1)^2k}$.

For the calculation of the Lerner index we rearrange the terms in (3) to obtain

$$p - c = -\frac{s}{1 - (k-1)s} \frac{\partial p}{\partial q_i^I} \sum_{j \neq i}^I q_j^I - \frac{\partial p}{\partial q_i^I} q_i^I,$$

and then we divide both sides by p and multiply the right hand side by Q/Q ,

$$\frac{p - c}{p} = -\frac{s}{1 - (k-1)s} \frac{\partial p}{\partial q_i^I} \frac{Q}{p} \frac{1}{Q} \sum_{j \neq i}^I q_j^I - \frac{\partial p}{\partial q_i^I} \frac{q_i^I}{p} \frac{Q}{Q},$$

by symmetry $\sum_{j \neq i}^I q_j^I = (k-1)q_i^I$ so

$$\begin{aligned} \frac{p - c}{p} &= -\frac{s}{1 - (k-1)s} \frac{1}{\varepsilon_{Q,P}} \frac{1}{Q} (k-1)q_i^I - \frac{1}{\varepsilon_{Q,P}} \frac{q_i^I}{Q}, \\ &= -\frac{1}{\varepsilon_{Q,P}} s_i^I \left[\frac{(k-1)s}{1 - (k-1)s} + 1 \right]. \end{aligned} \quad (19)$$

Similarly (4) becomes

$$\begin{aligned} \frac{p - c}{p} &= -\frac{\partial p}{\partial q_i^O} \frac{q_i^O}{p} \frac{Q}{Q}, \\ &= -\frac{1}{\varepsilon_{Q,P}} s_i^O. \end{aligned} \quad (20)$$

So multiplying both sides of (19) and (20) with s_i^I and s_i^O respectively we obtain

$$\begin{aligned} s_i^I \frac{p - c}{p} &= -\frac{1}{\varepsilon_{Q,P}} (s_i^I)^2 \left[\frac{(k-1)s}{1 - (k-1)s} + 1 \right] \\ s_i^O \frac{p - c}{p} &= -\frac{1}{\varepsilon_{Q,P}} (s_i^O)^2 \end{aligned}$$

and summing up for all insiders and outsiders respectively

$$\begin{aligned} \sum_{i=1}^k s_i^I \frac{p - c}{p} + \sum_{i=k+1}^n s_i^O \frac{p - c}{p} &= -\frac{1}{\varepsilon_{Q,P}} \left[\left(\frac{(k-1)s}{1 - (k-1)s} + 1 \right) \sum_{i=1}^k (s_i^I)^2 + \sum_{i=k+1}^n (s_i^O)^2 \right] \Rightarrow \\ \frac{p - c}{p} \left(\sum_{i=1}^k s_i^I + \sum_{i=k+1}^n s_i^O \right) &= -\frac{1}{\varepsilon_{Q,P}} \left[\frac{1}{1 - (k-1)s} \sum_{i=1}^k \left(\frac{1 - (k-1)s}{n - (k-1)ks} \right)^2 + \sum_{i=k+1}^n \left(\frac{1}{n - (k-1)ks} \right)^2 \right] \Rightarrow \\ \frac{p - c}{p} &= -\frac{1}{\varepsilon_{Q,P}} \left[\frac{1}{1 - (k-1)s} k \left(\frac{1 - (k-1)s}{n - (k-1)ks} \right)^2 + (n - k) \left(\frac{1}{n - (k-1)ks} \right)^2 \right] \Rightarrow \\ L(k, s) &= -\frac{1}{\varepsilon_{Q,P}} \left[\frac{k[1 - (k-1)s]}{[n - (k-1)ks]^2} + \frac{(n - k)}{[n - (k-1)ks]^2} \right] \end{aligned}$$

$$= -\frac{1}{\varepsilon_{Q,P}} \left(\frac{1}{n - (k-1)ks} \right).$$

Differentiating $L(k, s)$ we obtain

$$\begin{aligned} \frac{dL(k, s)}{dk} &= -\frac{1}{\varepsilon_{Q,P}} \frac{(2k-1)s}{(n - (k-1)ks)^2} > 0, \\ \frac{dL(k, s)}{ds} &= -\frac{1}{\varepsilon_{Q,P}} \frac{(k-1)k}{(n - (k-1)ks)^2} > 0. \end{aligned}$$

Regarding the calculation of C_m index, from (7) and (4) we have shown that $q_i^I = [1 - (k-1)s]q_i^O$, hence $q_i^I < q_i^O$. So when $m \leq n - k$, only outsider firms have the higher market shares, so $C_m(k, s) = ms_i^O$ where $s_i^O = 1/[n - (k-1)ks]$. When $m > n - k$, all $n - k$ outsiders and $m - n + k$ insiders have the higher market shares, so $C_m(k, s) = (n - k)s_i^O + (m - n + k)s_i^I$ where $s_i^I = [1 - (k-1)s]/[n - (k-1)ks]$. ■

Proof of Proposition 1. The standard n -firm Cournot equilibrium is given by

$$q_i(n) = \frac{a-c}{1+n}, Q(n) = n \frac{a-c}{1+n}, p(n) = c + \frac{a-c}{1+n}, \Pi_i(n) = \left(\frac{a-c}{1+n} \right)^2, \Pi(n) = n \left(\frac{a-c}{1+n} \right)^2 \quad (21)$$

where $\Pi(n) = \sum_i \Pi_i(n)$ is the industry profit. Given Assumption 1, if $\frac{k}{1+n} \leq \frac{1}{2}$, then the profit of a firm which participates in a (k, s) COS is lower compared to that in the standard Cournot model, or

$$\Pi_i^I(k, s) - \Pi_i(n) = (a-c)^2 \left(\frac{1+s-ks}{(1+n-(k-1)ks)^2} - \frac{1}{(1+n)^2} \right) < 0,$$

when either $k = 2, n = 3$ or $k \geq 2, n > 3$. If $\frac{1}{2} < \frac{k}{1+n} < 2 - \sqrt{2}$ then $\Pi_i^I(k, s) - \Pi_i(n) > 0$, for any $0 < s < \frac{(2k-n-1)(1+n)}{(k-1)k^2}$. Furthermore, the condition $s < \frac{(2k-n-1)(1+n)}{(k-1)k^2}$ does not violate assumption 1, that is for all $k \geq 2, n \geq k$ that satisfy $\frac{1}{2} < \frac{k}{1+n} < 2 - \sqrt{2}$, it is true that

$$0 < \frac{(2k-n-1)(1+n)}{(k-1)k^2} < \frac{1}{2(k-1)}.$$

It can be checked that if $\frac{1}{2} < \frac{k}{1+n} < 2 - \sqrt{2}$ and $\frac{(2k-n-1)(1+n)}{(k-1)k^2} < s < \frac{1}{2(k-1)}$, then $\Pi_i^I(k, s) - \Pi_i(n) < 0$. Last, if $\frac{k}{1+n} \geq 2 - \sqrt{2}$, then $\Pi_i^I(k, s) - \Pi_i(n) > 0$ for any $0 < s < \frac{1}{2(k-1)}$, $k \geq 2, n \geq k$. ■

Proof of Corollary 1. From proposition 1 a COS is unprofitable if $\frac{k}{1+n} \leq \frac{1}{2}$. By rearranging we obtain $k/n \leq \frac{1+n}{2n}$ with $\lim_{n \rightarrow \infty} \frac{1+n}{2n} = 1/2$. ■

Proof of Corollary 2. From condition *iv*) in proposition 1, it follows that

$$\frac{k}{n} \geq (2 - \sqrt{2}) \frac{(1+n)}{n},$$

where the r.h.s is decreasing in n with $\lim_{n \rightarrow \infty} (2 - \sqrt{2})(1+n)/n = 0.58578$. Similarly for $0 < s < \frac{(2k-n-1)(1+n)}{(k-1)k^2}$,

$$\frac{1+n}{2n} < \frac{k}{n} < (2 - \sqrt{2}) \frac{(1+n)}{n}. \quad \blacksquare$$

Proof of Proposition 2. Following a profitable merger, the profit of an outsider is given if we substitute n in $\Pi_i(n)$ in (21) with $n - k + 1$, so that

$$\Pi_i^O(k, n) = \left(\frac{a - c}{2 + n - k} \right)^2,$$

while the profit of an insider I , assuming equal distribution of profits among the merged firm is

$$\Pi_i^I(k, n) = \frac{1}{k} \left(\frac{a - c}{2 + n - k} \right)^2.$$

If instead of merging, k firms had formed a COS the profit of a participant firm would have been

$$\Pi_i^I(k, s) = (1 - s(k - 1)) \left(\frac{a - c}{1 + n - (k - 1)ks} \right)^2$$

Then

$$\Pi_i^I(k, s) - \Pi_i^I(k, n) = (a - c)^2 \left(\frac{1 - s(k - 1)}{[1 + n - (k - 1)ks]^2} - \frac{1}{k(2 + n - k)^2} \right) \geq 0$$

if

$$\frac{1 - 3k + k^2 + 2n - 2kn + n^2}{k - k^2} \leq s < \frac{1}{2(k - 1)}. \quad (22)$$

The above interval is non empty when $n > 3$ and $k < (4n + 5 - \sqrt{8n + 9})/4$. Hence, given a \bar{k} -firm merger, there exists an s that satisfies (22) and can give rise to a more profitable (\bar{k}, s) COS. Salant et al. (1983) have shown that for a simple Cournot merger to be profitable, it is required that $k > (2n + 3 - \sqrt{4n + 5})/2$, which leads to the so-called the 80% rule. Given that $(4n + 5 - \sqrt{8n + 9})/4 > (2n + 3 - \sqrt{4n + 5})/2$, then for all k such that

$$\frac{1}{2} (2n + 3 - \sqrt{4n + 5}) < k < \frac{1}{4} (4n + 5 - \sqrt{8n + 9}), \quad (23)$$

a (\bar{k}, s) COS outperforms a profitable \bar{k} -firm merger. Otherwise, if $k > \frac{1}{4} (4n + 5 - \sqrt{8n + 9})$, then $\Pi_i^I(k, s) - \Pi_i^I(k, n) < 0$, for $n > k \geq 2$, or in terms of relative group size,

$$\frac{k}{n} > \frac{1}{4n} (4n + 5 - \sqrt{8n + 9}). \quad (24)$$

The r.h.s. of the inequality is minimized at $n = 9$. Substituting the minimizer in (24), we obtain that 88.88% is the minimum size of a \bar{k} -firm merger which is more advantageous to a (\bar{k}, s) COS. Furthermore, given *iv*) in Proposition 1, for all k such that

$$(2 - \sqrt{2})(1 + n) < k < \frac{1}{2} (2n + 3 - \sqrt{4n + 5}) \quad (25)$$

a (\bar{k}, s) COS is profitable while a \bar{k} -firm merger is loss making. ■

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