Optimal taxation with positional considerations

Abstract

This paper analyzes the optimal commodity tax policy, in a generalized vertical differentiation model in which consumers have positional considerations. Consumers enjoy having a product which is better than that owned by others, and feel envy when others own a better product than them. We examine the impact of these positional considerations on the optimal tax and welfare when a monopoly produces one or two variants of such good. The standard result that the government should subsidize the product, can be reversed in our setting. In the presence of positional concerns, the optimal tax rate can be positive, when the status and envy effects are strong enough. For the multiproduct monopoly: the positional effects determine the level of the tax pass-through on prices. Moreover, because of the presence of these effects, the tax levied on the high quality variant affects the price of the low quality variant and vice versa.

Keywords: Commodity tax, vertically differentiated market, positional effects.

1 Introduction

Consumers' utility often depends not only on the characteristics of the product they buy but also on the knowledge that they own a better product than others. Higher quality products such as designer clothing and handbags, latest versions of smartphones and tablets, drones, luxury cars, etc are few of the many products that consumers buy not only for their quality but also for status reasons. On the other hand, the purchase of a product may affect negatively those who have not made a similar purchase.

We analyze the optimal taxation of a monopoly that produces a positional good. It is well known that in the absence of positional considerations the government should subsidize the product to correct for the monopoly pricing. This would induce more consumers to buy it. However, when positional considerations are present, the increase in the number of consumers who afford to buy the product at the subsidized price creates additional effects in consumers' utility. How these concerns affect government's choice for the optimal tax or subsidy and welfare? Does a price change of a luxury product due to the tax increase the price of less positional products?

We model positional considerations as network externalities. Each consumer's utility is affected positively by the number of people who have bought a lower quality model of the good or have made no purchase at all (status effect). In a similar way, each consumer's utility is negatively affected by the number of consumers who have bought a higher quality model (envy effect). In this paper, we introduce commodity taxation in a monopoly framework in which the market is vertically differentiated and consumers have positional considerations. We derive the optimal commodity tax formula and examine the effect of commodity taxation on prices and welfare when a monopoly produces either one or two variants of such good.

The standard result that the government should subsidize the product to correct for the monopoly pricing, can be reversed in our setting. When positional considerations are present, the increase in utility of those who initially could not afford to buy the good but now buy it at the lower (subsidized) price is not the only effect. As more consumers afford to buy the product, the status effect reduces the utility of those who initially purchase it. Finally, as more people buy the product, the envy effect makes those who still do not buy the product worse off. The sign and the level of the optimal tax depend on how strong these effects are. When these are strong enough, the government taxes the good.

When the monopoly is multi-product and the status and envy effects are of the same magnitude, the government switches from a subsidy to a tax as the positional effects become stronger. We show that the optimal tax policy is to tax less the high quality variant.

We also examine the pass-through of taxation, that is, the extent at which taxes are passed through to consumer prices. This is a classic public policy concern and has not been explored in frameworks with positional effects. We find that the pass through of a single-product monopoly is not affected by the status and envy effects. However, positional considerations affect the level of the tax pass-through for a multiproduct monopoly.

Moreover, because of the presence of positional considerations, there is cross-tax pass-through. A tax levied in one variant of the product is passed through to the price of the other variant. A tax levied on the high quality variant reduces the mass of consumers who can afford to buy this variant.

This leads to an increase in utility of the consumers who buy the low quality variant as fewer people own a better quality variant than them. As a result, they are willing to pay a higher price. These effects have important tax policy implications, because a tax on the high quality variant, which is expected to affect rich consumers only, increases the price of the low quality variant which is chosen by the poorer consumers.

On the other hand, in the presence of status and envy effects, a tax on the low quality variant decreases the price of the high quality variant. This happens because a tax increase in the low quality variant raises the mass of consumers who buy the high quality variant and therefore, reduces their status. The lower status effect reduces the price of the high quality variant.

Veblen (1899) was the first who pointed out the importance of social factors on consumption. He used the term conspicuous consumption to describe consumption that is used to signal high income or wealth.[1] Such conspicuous consumption affects consumption and financial decisions and may create negative positional externalities (Frank (1985) and Hopkins and Kornienko (2004)).

The idea that consumers value owning a good that is better than that of other consumers has find support in empirical research. Solnick and Hemenway (2005) provide evidence that individuals care about relative position, for both public and private goods and bads. Carlsson, et al. (2007) find that income and cars are highly positional, on average, in contrast to leisure and car safety in Sweden. More recently, Bursztyn et al. (2018), in a field experiment show that demand for the platinum credit card (which signals high income) exceeds demand for a nondescript credit card with identical benefits, suggesting demand for the status aspect of the card. Banuri and Nguyen (2020) in a lab experiment find that consumption increases when it is observable and signals status.

Status refers to the satisfaction of possessing a good. On the other hand, envy refers to the decrease in utility of not having a good. Both effects are present in our model. Concerning the idea that conspicuous consumption generates envy, Winkelmann (2012) finds that living in a municipality with a higher number of Ferrari and Porsche cars has a negative impact on own income satisfaction. Also, Bellet (2019) shows that new constructions of houses at the top of the house size distribution in a suburb, lower the satisfaction that neighbours derive from their own house size.

Such social factors are particularly present in vertically differentiated markets. However, in studies that examine the effects of taxation in such markets, they are completely ignored.[2] Cremer and Thisse (1994) introduce commodity taxation into a duopoly model of vertical product differentiation. They show that a uniform ad valorem tax lowers the qualities and the consumer prices of both variants. Arakawa (2017) analyzes the effects of commodity taxation with tax brackets under a vertically differentiated multiproduct monopoly. Constantatos and Sartzetakis (1999) examine the impact of commodity taxation on market structure in vertically differentiated product markets. They show that an ad valorem tax may induce the entry of a large number of firms in what was previously a natural monopoly. In these models consumers care only about the characteristics of the product they buy. In contrast to this literature, we assume that consumers derive utility both from the characteristics of the product and from the status the product offers.

[1] See also Duesenberry (1949), and Bagwell and Bernheim (1996).

[2] Commodity tax policy implications in models with status considerations have been explored in signalling models (Ireland (1994), Truyts (2012)).

The implications of positional considerations on monopolist's behaviour have received some attention in the literature. Lambertini and Orsini (2002) compare a single-product monopoly equilibrium and the first best optimum when consumers' utility depends, not only on the intrinsic characteristics of the product, but also on the social status the purchase of it confers to them. They find that the monopolist's quantity decreases with the level of the positional effect only when the taste parameter is high enough while the first best optimum quantity always increases as the positional concern becomes stronger.

Deltas and Zacharias (2018) explore the implications of positional considerations for product pricing, firm profit and the number of products a monopolist offers to consumers. These effects can induce the monopoly to offer products of different quality. In particular the high quality product becomes more exclusive as the status effect strengthens. Friedrichsen (2018) analyzes quality provision, prices and optimal product lines both in the monopoly and perfect competition settings, when consumers differ not only in their valuation of a product's quality but also in their desire for the social image attached to it. Unlike these contributions, the focus of our paper is on optimal taxes.

This paper is also related to the literature on tax incidence under imperfect competition. The pass-through of taxation has not been explored in frameworks with positional effects. The pass-through of a tax imposed on a single-product monopoly with constant marginal cost, when the demand is linear, is 50% (Amir, Maret and Troege (2004), Genakos and Pagliero (2019)). We show that, for a multiproduct monopoly, positional considerations determine the level of the tax pass-through.

The paper is structured as follows. Section 2 describes the model. Section 3, examines the optimal tax when the monopoly produces one variant of the positional good. Section 4 analyzes optimal taxation when the monopoly produces two variants of the positional good and Section 5 concludes.

2 The model

There is a continuum of consumers who differ in their willingness to pay θ for quality. We assume that θ is distributed according to $F(\cdot)$ on the interval $[0, 1]$. There is a monopoly in the market that can produce one or two variants of the product. The quality of product j is denoted by S_j, with higher values of j indicating products of higher quality. That is, if the monopoly offers two variants of the product, the product 2 is the high quality variant ($S_2 > S_1$). Consumers buy at most one unit of the vertically differentiated good.

Furthermore, a_i, where $i = 1, 2$, are non-negative parameters that show the intensity of the positional considerations. These are modeled as network effects as follows: a_1 denotes the status effect from the purchase of a variant of higher quality. The utility of those who purchase increases, when the mass of those who buy less advanced variants and those who have made no purchase at all increases (note that as the demand remains constant, this is equivalent to less consumers buying a variant of the same or higher quality). Similarly, a_2 denotes the envy effect from either not purchasing at all or purchasing a variant of lower quality. The utility of these consumers increases the less consumers buy a product of higher quality than theirs.

Assume there are k variants, with $k = 1, 2$. The indirect utility that a consumer of type θ_i derives from the consumption of product j, at price P_j is given by:

$$U_{i,j} = \theta_i S_j + a_1 \left(\sum_{k<j} Q_k + Q_0 \right) - a_2 \sum_{k>j} Q_k - P_j. \tag{1}$$

where Q_k is the mass of consumers who purchase product k and Q_0 is the mass of consumers that do not buy any variant of the product. Consumers' utility who buy variant j rises as the mass of those who buy variants of lower quality than theirs and those who do not buy the product at all increases. However, it decreases as more consumers buy a product of higher quality than theirs.

A consumer who does not buy the product incurs positional disutility due to the envy effect. Her indirect utility is affected negatively as the number of those who buy any variant of the product increases:

$$U_{i,0} = -a_2 \sum_k Q_k. \tag{2}$$

The monopoly produces each variant of quality S_i with $i = 1, 2$ at a constant marginal cost c_i with

$$c_i < S_i. \tag{3}$$

The government imposes a tax t_i per unit of the product i. The profits π of the monopoly are:

$$\pi = \sum_{i=1}^{K} \left[(P_i - t_i - c_i) Q_i \right]. \tag{4}$$

Tax revenues are the sum of revenues from taxing the variants of the product

$$R = \sum_{i=1}^{K} t_i Q_i. \tag{5}$$

The welfare of the country is given by the sum of the consumer surplus, the profits and the government's tax revenues[3]

$$W = CS + \pi + R. \tag{6}$$

In the next sections we characterize the optimal tax policy when the monopoly produces one or two variants of the positional good.

3 The monopoly produces only one variant of the positional good

In this section we assume that the monopoly produces only one variant of the good of quality S. As a result, consumers can either buy the positional good, or not buy at all. Let the consumer of type θ_1 be indifferent between buying the good and not buying at all. As a result, there is a mass of $1 - F(\theta_1)$ consumers who buy the product and a mass of $F(\theta_1)$ consumers who do not buy

[3] We assume that the tax revenues are returned to the residents of the country in a lump sum fashion.

at all. To simplify the analysis, in what follows we assume that $F(\cdot)$ is the uniform distribution. Therefore, the demand function is defined by

$$Q_1 = 1 - \theta_1. \tag{7}$$

From (1), the indirect utility that a consumer of type θ_i derives from the consumption of the product with quality S at price P is given by

$$U_i = \theta_i S + a_1\theta_1 - P. \tag{8}$$

From (2), the indirect utility of a consumer who does not buy the product is

$$U_0 = -a_2(1 - \theta_1). \tag{9}$$

The consumer who is indifferent between buying the product or not is of theta

$$\theta_1 = \frac{P - a_2}{S + a_1 - a_2}, \tag{10}$$

and can be obtained by setting $U_0 = U_i$. Solving the demand function $Q = 1 - \theta_1$ for P, after using (10), yields the inverse demand function

$$P = S + a_1 - (S + a_1 - a_2)\, Q. \tag{11}$$

The consumer surplus is the sum in the surplus of the consumers that buy the product and of those that do not make a purchase, and is given by

$$CS = \int_0^{\theta_1} (-a_2(1 - \theta_1))\, d\theta + \int_{\theta_1}^{1} (\theta S + a_1\theta_1 - P)\, d\theta. \tag{12}$$

For simplicity, we normalize to zero the marginal cost of the good.[4] The profits of the monopoly are:

$$\pi = (P - t)(1 - \theta_1). \tag{13}$$

Furthermore, to guarantee that the profit and welfare functions are concave, we make the assumption that the envy effect is sufficiently low. We have the following Assumption:

- **Assumption 1**: We have $a_2 < S/2 + a_1$.

The monopoly maximizes its profits with respect to the price, given the tax rate. After using (10) and (13), we have the price and quantity the monopoly sets:[5]

$$P = \frac{a_1 + S + t}{2}, Q = \frac{1}{2}\frac{S + a_1 - t}{S + a_1 - a_2}. \tag{14}$$

[4] Qualitatively, the results do not change if the marginal cost is positive but the model becomes much more complicated without necessarily being more insightful.

[5] By Assumption 1 we have $a_2 < \frac{S}{2} + a_1$. We know that $\frac{S}{2} + a_1 < S + a_1$. Thus $a_2 < S + a_1$ and therefore, the second order conditions for profit maximization $\frac{\partial^2\pi}{\partial P^2} = -\frac{2}{S+\alpha_1-\alpha_2} < 0$ are satisfied.

Positional considerations lead to an increase in the price, as the product offers a status to those who purchase it.

We now investigate the tax pass-through rate, which is the rate at which the consumer price rises when a tax is imposed on the monopoly. Does the price increase by the full amount of the tax? From (14) it is straightforward to show that

$$\frac{\partial P}{\partial t} = \frac{1}{2}.$$

The tax is not passed completely to the consumers. The price rises by half the amount of the tax. Moreover, the status and envy effects do not affect the level of the tax pass-through. This result in a vertically differentiated market with positional concerns, is in line with the literature on the tax pass-through of a monopoly. In particular, the pass-through of a tax imposed on a monopoly firm with constant marginal cost, when the demand is linear, is 0.5 (Amir, Maret and Troege (2004), Genakos and Pagliero (2019)).

The welfare of the country is given by (see the Appendix for a derivation)

$$W = \frac{(S - t + a_1)\left[t\left(S + 2(a_1 - a_2)\right) + S(5a_1 + 3S - 6a_2) + 2(a_2 - a_1)(2a_2 + a_1)\right]}{8(S + a_1 - a_2)^2}. \tag{15}$$

The government chooses the tax rate that maximizes country's welfare. Maximizing (15) with respect to t yields the optimal tax rate

$$t^* = \frac{2a_2\left(S + a_1 - a_2\right) - S\left(S + a_1\right)}{S + 2\left(a_1 - a_2\right)}. \tag{16}$$

Using (15) we find that $\frac{\partial^2 W}{\partial t^2} = -\frac{S+2(a_1-a_2)}{4(S+a_1-a_2)^2}$. The second order conditions for welfare maximization are satisfied by Assumption 1.

Note that we have the constraint $\theta_1 \geq 0$, which after using (14) in (10) becomes[6]

$$t \geq -(S - 2a_2 + a_1). \tag{17}$$

It is easy to show that the optimal tax in (16) satisfies the constraint in (17) when $a_1 \geq a_2$. That is, the solution is interior when $a_1 \geq a_2$. More specifically, the market is not covered when $a_1 > a_2$. When $a_1 = a_2$ the market is covered and all consumers buy.

However, when $a_1 < a_2$ the optimal tax does not satisfy this constraint as it is lower than $-\left(S + a_1 - 2a_2\right)$. Therefore we have a corner solution, $\theta_1 = 0$ and the market is covered. The tax is such that all consumers buy. The monopoly maximizes its profits by setting price equal to $P = a_2$ (from (10) we can see that this price makes the market fully covered, that is, $\theta_1 = 0$). Therefore, when $a_1 < a_2$, for $t \in \left[\frac{2a_2(S+a_1-a_2)-S(S+a_1)}{S+2(a_1-a_2)}, -(S - 2a_2 + a_1)\right]$ we have a corner solution i.e., $\theta = 0$ where the monopoly sets price $P = a_2$ and sells to the whole market. For this price the welfare is $W = \frac{S}{2}$.

When the magnitude of the status and envy effects are the same i.e., $a_1 = a_2 = a$, the optimal tax is given by $t^* = a - S$.

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[6] The constraint $\theta_1 \geq 0$, after using (14) in (10) becomes $\frac{1}{2}\frac{S-2a_2+a_1+t}{S+a_1-a_2} \geq 0$. From Assumption 1 the denominator is always positive. Thus the constraint can be rewritten as $t \geq -(S - 2a_2 + a_1)$.

In what follows, we examine the effect of the positional considerations on the tax and welfare when there is an interior solution, that is when $a_1 \geq a_2$.

The standard result in the optimal taxation literature for imperfectly competitive markets is that the optimal policy of the government would be to impose a subsidy, to reduce the deadweight loss from imperfect competition (see Guesnerie and Laffont (1978)). The problem with monopoly pricing is that it induces consumers to consume too little of the good. To achieve an efficient allocation of resources, the government induces them to consume more by subsidizing the good.

This result holds here as well, when there are no positional considerations. The optimal tax in the case in which there are no status and envy effects can be easily found by setting $a_1 = a_2 = 0$ in (16). The optimal tax is negative and equal to $t_{a_2=a_1=0} = -S$. The optimal strategy for the government is to impose a negative tax (subsidy) to reduce the price the monopoly sets. Therefore, in line with the optimal commodity taxation literature for a monopoly, in the vertically differentiated market without positional concerns, the government corrects the distortion that is created from monopoly pricing by imposing a subsidy. In contrast, in the presence of positional concerns, the optimal tax can be positive, as we show in the following Proposition.

Proposition 1 *When the monopoly produces one variant of the positional good, the optimal tax rate is positive when* $a_1 > S$ *and* $a_2 \in \left(\frac{1}{2}\left(S + a_1 - \sqrt{a_1^2 - S^2}\right), a_1\right)$.

The proof of the Proposition is in the Appendix. Intuitively, this result is driven mainly by the presence of the envy and status effects. A subsidy, corrects the distortion from monopoly pricing by making the good available to consumers that couldn't afford buying it. However, when consumers have positional considerations, the subsidy has two negative effects.

First, recall that the utility of the consumers that do not purchase the product decreases the more people buy it, due to the envy effect. As a result, the subsidy reduces the utility of the consumers that do not buy the product. Second, when consumers have status concerns, the utility of those who could without the subsidy buy the product decreases with the subsidy, because the number of people who do not buy the product is smaller. Thus, when the status the good offers exceeds the intrinsic value of the good and the envy effect is high enough, it is optimal to tax the product.

Substituting the optimal tax when both positional effects exist (given by (16)) in (13) and (14), we obtain the price and quantity that the monopoly sets

$$P = \frac{(a_1 - a_2)(a_2 + a_1) + Sa_1}{S + 2(a_1 - a_2)}, Q = \frac{S + a_1 - a_2}{S + 2(a_1 - a_2)} \tag{18}$$

and its profits

$$\pi = \frac{(S + a_1 - a_2)(S + a_1 - a_2)^2}{[S + 2(a_1 - a_2)]^2}. \tag{19}$$

The critical value of θ below which the consumers do not buy the product becomes

$$\theta_1 = \frac{a_1 - a_2}{S + 2(a_1 - a_2)}. \tag{20}$$

Substituting the optimal tax in (15), gives the welfare

$$W = \frac{1}{2}\frac{(S + a_1 - a_2)^2}{S + 2(a_1 - a_2)}. \tag{21}$$

3.1 Effects of social factors on welfare and tax

The explicit solution of the tax equilibrium allows us to get insights about comparative statics with respect to the envy and status parameters. We now examine the effects of the two positional considerations on welfare. We have the following Proposition.

Proposition 2 *The effect of the two positional parameters on welfare is of equal but opposite magnitude. Furthermore, the welfare is higher, the stronger is the status effect and the weaker is the envy effect.*

We can easily prove Proposition (2) by using (21), which gives

$$\frac{\partial W}{\partial a_1} = -\frac{\partial W}{\partial a_2} = \frac{(a_1 - a_2)(S + a_1 - a_2)}{(S + 2(a_1 - a_2))^2} = \theta_1 Q > 0.$$

Intuitively, an increase in the status parameter, increases welfare as those who buy the product enjoy a higher utility. Similarly, as the envy parameter increases, the welfare decreases, as those who do not buy suffer more from not buying.

Another interesting aspect is how the optimal tax changes when social issues start being important.

Proposition 3 *The optimal tax is a) decreasing in the quality of the product, b) increasing in the status and the envy parameter.*

The proof of the proposition is in the Appendix. Intuitively, an increase in a_2 raises the disutility that consumers suffer when they do not own the product. A tax increase lowers the purchases, and thus decreases the disutility of the consumers who do not buy. This beneficial effect of a tax increase is higher when a_2 raises. A rise in a_1 increases the utility of the consumers who buy the product. A tax increase decreases the utility of those who can not manage to buy the product but increases the utility of the consumers who still buy. As a_1 raises the latter effect exceeds the former and the welfare increases.

In addition, the higher the quality of the product is, the lower the optimal tax. Intuitively, an increase in the product quality raises consumers' utility. As a result, the government has an incentive to impose a lower tax so as to reduce the price and make the good available to more people.

4 The monopoly produces two variants of the positional good

In the previous section, we have specified the equilibrium and the optimal tax when the monopoly produces one variant of the positional good. In this section, we turn our attention to the case of a multiproduct monopoly. In particular, we assume that the monopoly produces two variants of different quality.

Let the consumer of type θ_2 be indifferent between consuming the high and the low quality version of the product. Also, let the consumer of type θ_1 be indifferent between consuming the low quality version and not buying the good. As a result, there is a mass of $1 - F(\theta_2)$ consumers who buy the high end product, a mass of $F(\theta_2) - F(\theta_1)$ consumers who buy the less advanced product and a mass of $F(\theta_1)$ consumers who do not buy at all. $F(\theta_2)$ denotes the mass of consumers who either buy the low quality variant or do not buy at all.

Furthermore, the government imposes different tax rates in each variant. As in the case in which the monopoly produces only one variant of the product, we assume that $F(\cdot)$ is the uniform distribution. From (1), the indirect utility that consumer i derives from purchasing the high quality product, at price P_2, is

$$U_{i,2} = \theta_i S_2 + a_1\theta_2 - P_2 \tag{22}$$

Consumers' utility who buy the high quality variant rises as θ_2 increases. From (1), the indirect utility that consumer i derives from purchasing the low quality product becomes

$$U_{i,1} = \theta_i S_1 + a_1\theta_1 - a_2(1-\theta_2) - P_1. \tag{23}$$

Finally, the indirect utility of a consumer who does not buy the product, given by (2), becomes

$$U_{i,0} = -a_2(1-\theta_1). \tag{24}$$

Consumer surplus is the sum in the surplus of the consumers that consume the product and of those that do not make a purchase, and is given by

$$CS = \int_0^{\theta_1} (-a_2(1-\theta_1)\, d\theta + \int_{\theta_1}^{\theta_2} (\theta S_1 + a_1\theta_1 - a_2(1-\theta_2) - P_1)\, d\theta + \\ + \int_{\theta_2}^{1} (\theta S_2 + a_1\theta_2 - P_2)\, d\theta. \tag{25}$$

Due to the complexity of the results, our attention is focused on the case in which the magnitude of the status and envy effects are the same that is, $a_1 = a_2 = a$. The value of θ_i for the consumer who is indifferent between buying the low-quality version of the product and not buying the good at all, is denoted by θ_1 and is obtained by setting $U_{i,0} = U_{i,1}$. In addition, θ_2 (which is the value of θ_i for the consumer who is indifferent between consuming the high and the low-quality version of the product) is obtained by setting $U_{i,2} = U_{i,1}$. Solving simultaneously these two equations, after using (22), (23) and (24) yields

$$\theta_1 = \frac{a^2 + a(P_1 - P_2) + P_1(S_2 - S_1)}{a^2 + S_1(S_2 - S_1)}, \tag{26}$$

$$\theta_2 = \frac{a(P_1 - S_1) + S_1(P_2 - P_1)}{a^2 + S_1(S_2 - S_1)}. \quad (27)$$

Consumers with strong preferences for quality, that is, with $\theta_i > \theta_2$, prefer the product 2, while consumers with $\theta_1 < \theta_i < \theta_2$ prefer product 1. The demand functions are given by

$$Q_1 = \theta_2 - \theta_1 = \frac{P_2 S_1 - P_1 S_2 - aS_1 - a^2 + aP_2}{a^2 + S_1(S_2 - S_1)} \quad \text{and} \quad (28)$$

$$Q_2 = 1 - \theta_2 = \frac{S_1(S_2 - S_1) - P_2 S_1 + a^2 + a\,(S_1 - P_1) + P_1 S_1}{a^2 + S_1(S_2 - S_1)}. \quad (29)$$

Now consider the monopoly's problem. For simplicity, we normalize to zero the marginal cost of S_1.[7] The monopoly maximizes its profits by the appropriate choice of the two prices P_1 and P_2, given the tax rates. Substituting (28) and (29) in monopoly profits, given by (4), and differentiating with respect to P_1 and P_2 we get the following first order conditions for profit maximization[8]

$$\begin{aligned} P_1(P_2) &= \frac{S_1}{S_2} P_2 + \frac{S_2 t_1 + (a - S_1)\,t_2 + a\,(c_2 - S_1 - a) - S_1 c_2}{2S_2}, \\ P_2(P_1) &= P_1 + \frac{S_1 t_2 - (a + S_1)\,t_1 + a\,(S_1 + a) + S_1\,(c_2 + S_2 - S_1)}{2S_1}. \end{aligned} \quad (30)$$

The system of first order conditions for the prices gives

$$\begin{aligned} P_1 &= \frac{a\,(c_2 + t_2 - t_1) + (S_2 - S_1)\,(S_1 + t_1)}{2\,(S_2 - S_1)}, \\ P_2 &= \frac{a^2\,(S_2 - S_1) - a t_1 S_2 + (a + S_2)\,(S_2 - S_1 + c_2 + t_2)\,S_1 - (t_2 + c_2)\,S_1^2}{2\,(S_2 - S_1)\,S_1}. \end{aligned} \quad (31)$$

Using (31) in the demand functions (28) and (29), we get

$$\begin{aligned} Q_1 &= \frac{a\,(S_2 - S_1) + S_1\,(c_2 + t_2) - S_2 t_1}{2S_1\,(S_2 - S_1)} \\ Q_2 &= \frac{S_2 - S_1 + t_1 - t_2 + c_2}{2\,(S_2 - S_1)}. \end{aligned} \quad (32)$$

We now examine the extent to which taxes are passed to the consumer prices. The question that naturally arises is whether positional considerations determine the level of the tax pass-through. Unlike the case of the one variant, pass-through depends on positional considerations when we have two variants. Using (31), we see that, an increase in the tax levied on the high quality variant, for given t_1, raises its price:

$$\frac{\partial P_2}{\partial t_2} = \frac{1}{2} + \frac{a}{2\,(S_2 - S_1)} > 0. \quad (33)$$

Without the positional effect, that is, when $a = 0$, an increase in the tax of the high quality variant rises its price by half the amount of the tax increase. With positional concerns, however, the size of the tax pass-through depends on the value of a. In particular, the tax pass-through rate becomes

[7] As in the case of one variant, the results do not change qualitatively when the marginal cost of the low quality variant is positive.

[8] The proof of the second order conditions for profit maximization is in the Appendix.

greater as the positional considerations rise. Consumers are willing to pay a higher price as an increase in a corresponds to an increase in the status effect (consumers who buy the high quality product do not suffer from the envy effect). Thus, the monopoly can pass a larger part of the tax to the price when a raises.

In addition, because of the presence of positional considerations, the cross-tax pass-through is positive

$$\frac{\partial P_1}{\partial t_2} = \frac{1}{2} \frac{a}{S_2 - S_1} > 0, \tag{34}$$

that is, an increase in the tax levied on the high quality variant raises the price of the low quality variant. A tax levied on the high quality variant reduces the mass of consumers who can afford to buy this variant. This leads to an increase in utility of the consumers who buy the low quality variant as fewer people own a better quality variant than theirs. Thus, they are willing to pay a higher price. The higher the positional considerations, the greater the cross-tax pass-through.

To have a better understanding of these results, we look at Figure 1, which shows graphically the monopoly's first order conditions for profit maximization given by (30). The first condition, $P_1(P_2)$, yields the optimal price for the low quality product 1 that should be set for any given price P_2 of the high quality product. The second one, $P_2(P_1)$, shows the optimal price of the high quality variant P_2 that the monopoly sets for any given price of the low quality variant. They are upward sloping since monopoly increases P_2 after an increase in P_1 and vice versa. They are represented by the solid lines in Figures 1 and 2.

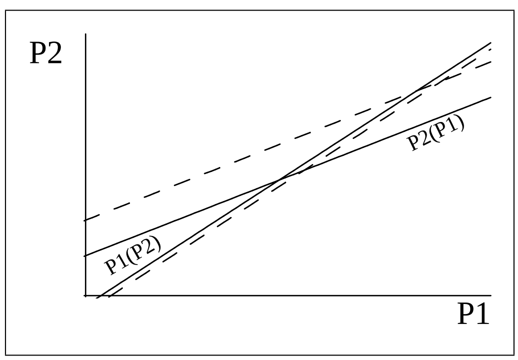

Figure 1: The effect of a rise in t_2 when $a > S_1$

Using (30), we see that an increase in t_2 shifts up $P2(P1)$, (note that $\frac{\partial P_2(P_1)}{\partial t_2} = \frac{1}{2}$), leading to a new equilibrium with higher prices. However, at the same time an increase in t_2 shifts $P1(P2)$ (note that in (30) $\frac{\partial P_1(P_2)}{\partial t_2} = \frac{1}{2}\frac{a-S_1}{S_2}$). In the absence of positional effects this curve would be shifted up. As a rises, this shift becomes smaller and for high enough values of a the curve will move down, leading to an increase in the equilibrium prices. Thus, the presence of positional considerations leads to higher prices after an increase in t_2.

Consider now the impact of an increase in the tax levied in the low quality variant for given t_2. Using (31), we find the change in the price of the low quality variant from an increase in the tax levied on it:

$$\frac{\partial P_1}{\partial t_1} = \frac{1}{2} - \frac{a}{2(S_2 - S_1)}.$$

In the absence of positional considerations, an increase in the tax of the low quality variant t_1 rises its price by half the amount of the tax increase. However, as the positional considerations rise the tax pass-through decreases.

For high enough values of a, the tax pass-through can be negative, and therefore a tax increase can lead to a reduction in the price of the product.[9]

Moreover

$$\frac{\partial P_2}{\partial t_1} = -\frac{1}{2}\frac{a}{S_1}\frac{S_2}{S_2 - S_1} < 0. \tag{35}$$

Equation (35) reveals that an increase in the tax on the low quality variant reduces the price of the high quality product. As t_1 increases, Q_2 increases (see (32)). In the absence of positional effects this happens while P_2 remains the same. However now, the price of P_2 decreases (thus we have the cross pass through) as an increase in Q_2 reduces the status of those who buy S_2. The higher the positional considerations, the greater the price reduction from the tax increase.

In Figure 2, the effect of an increase in t_1 is to shift outwards $P_1(P_2)$ (from (30), we can see that $\frac{\partial P_1(P_2)}{\partial t_1} = \frac{1}{2}$). The price of good 1, for any given price P_2, is higher than it was before the increase in t_1. If the change in t_1 would not affect $P_2(P_1)$, in the new equilibrium both prices would be higher. But an increase in t_1 shifts downwards $P_2(P_1)$, ($\frac{\partial P_2(P_1)}{\partial t_1} = -\frac{1}{2}\frac{a+S_1}{S_1}$). This tends to reduce the prices. Moreover, the presence of a makes the monopoly to react in the increase in t_1 by choosing an even lower price for product 2 for any given price P_1. This shifts more the curve, leading to even lower prices. Thus, because of the presence of positional considerations, there is a stronger downward pressure on the prices after an increase in t_1. The tax is shifted to consumer prices less than it would be without positional considerations.

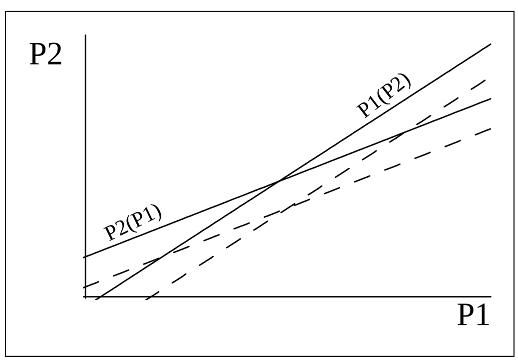

Figure 2: The effect of a rise in t_1

Summarizing

Proposition 4 *In a vertically differentiated market in which consumers have positional concerns of the same magnitude and the monopoly produces two variants of a positional good*

[9] The following numerical example shows that the tax pass-through for the low quality variant can be negative. Suppose that the positional effect parameter is $a = 5$ and the other parameters are $c_2 = 3$, $c_1 = 0$, $S_2 = 5$, and $S_1 = 1$. In this case, the tax pass-through is negative: $\frac{\partial P_1}{\partial t_1} = -0.13$. Therefore, an increase in the tax decreases the price of the low quality variant. If however $a = 3$ while the other parameters are the same as in the previous example, then the tax pass-through is positive: $\frac{\partial P_1}{\partial t_1} = 0.13$.

a) the pass-through of a tax levied on the high quality variant on its price is always positive and becomes higher as the positional considerations rise
b) the pass-through of a tax levied on the low quality variant on its price is positive for small values of the positional considerations and negative for high values of the positional considerations. As the positional considerations rise the tax pass-through on the price of the low quality product decreases.
c) an increase in the tax levied on the high quality variant raises the price of the low quality variant. However, an increase in the tax on the low quality variant reduces the price of the high quality variant.

Proposition 4 shows that positional considerations affect the level of the tax pass-through. Moreover, the cross-tax pass-through exists due to the presence of the positional considerations. In this case, a tax levied in the high quality variant (which targets consumers who buy this variant) affects the price of the low quality one and vice versa. These results have important implications for tax policymakers as the impact of taxing the high quality product extends to the low quality one.

We now turn to the characterization of the optimal tax policy. Substituting the Eqs. (4)–(5) in (6) and then Eqs. (26), (27), (31) and (32), the welfare becomes

$$\begin{aligned} W &= \frac{1}{8S_1}\left[-a^2+2a\left(S_1+t_1\right)\right] \\ &+\frac{1}{8S_1\left(S_2-S_1\right)}[t_2S_1\left(2c_2+2S_1-2S_2+2t_1-t_2\right)+t_1\left(-S_2t_1-2S_1c_2\right) \\ &+3S_2S_1\left(S_2-S_1\right)+3S_1c_2^2-6S_1c_2\left(S_2-S_1\right)]. \end{aligned} \tag{36}$$

Maximizing the welfare in (36) with respect to t_1 and t_2 gives the following first order conditions[10]

$$\begin{aligned} t_1 &= \frac{-c_2S_1+\left(S_2-S_1\right)a+t_2S_1}{S_2}, \\ t_2 &= S_1-S_2+t_1+c_2. \end{aligned} \tag{37}$$

Solving simultaneously the first-order conditions gives the optimal tax rates.

Proposition 5 *When the status and envy effects are of the same magnitude, the optimal taxes are given by:*

$$\begin{aligned} t_1^* &= -S_1+a, \\ t_2^* &= -S_2+a+c_2. \end{aligned} \tag{38}$$

Although in the absence of positional concerns the optimal choice of the government is to subsidize the products, when consumers have positional concerns, it is optimal for the government to impose a tax when the positional effect exceed the positional parameter and the cost.

[10]The proof of the second order conditions for welfare maximization is in the Appendix.

To interpret the optimal tax rates, we need to observe the different forces that are at work. First, the government has an incentive to impose a subsidy on both variants of the product to reduce the deadweight loss from the monopoly pricing. The subsidy would lead to a lower price and would make the goods available to more consumers. However, this would not only reduce the utility of those who cannot buy the product due to the envy effect but also of those that buy the low quality version of the product (since their utility is negatively affected by the number of consumers who buy the high quality product). In addition, the increased number of purchases would reduce the utility of those who buy the high and the low quality variants due to the status effect. For high enough values of the status and envy effects, these negative effects overcome the efficiency gain from reducing the monopoly price. As a result the government finds it optimal to impose a tax.

By substituting the optimal taxes in (31) and (32), we obtain the equilibrium prices and quantities

$$\begin{aligned} P_1^{t_1,t_2^{\cdot\cdot}} &= \frac{ac_2}{(S_2 - S_1)}, \\ P_2^{t_1,t_2} &= c_2 + \frac{a\left(S_1\left(S_2 - S_1\right) + S_1 c_2\right)}{S_1\left(S_2 - S_1\right)}, \end{aligned} \tag{39}$$

$$Q_1^{t_1,t_2} = \frac{S_1 c_2}{\left(S_2 - S_1\right) S_1}, Q_2^{t_1,t_2} = \frac{S_2 - S_1 - c_2}{\left(S_2 - S_1\right)}. \tag{40}$$

For the monopoly to produce the high quality variant it is required $S_2 - c_2 > S_1$, that is, the difference between the quality and the cost needs to be greater for the high quality variant than for the low quality one. The difference between the two taxes, after using (38) and (40), is

$$t_1^* - t_2^* = \left(S_2 - S_1\right) Q_2^{t_1,t_2} > 0. \tag{41}$$

From (41) we can see that $t_1^* > t_2^*$ if $Q_2^{t_1,t_2} > 0$, that is, if the high quality variant is produced, the optimal tax policy is to tax more the low quality variant.[11]

Corollary 1 *When the monopoly produces two qualities, the optimal tax policy is to tax less the high quality variant.*

We now investigate whether the welfare is higher when two variants of the good are produced rather than one. To see this, we use the values of the optimal taxes in (36) to get

$$W^{t_1,t_2} = \frac{1}{2} \frac{-S_2 S_1\left(S_1 + 2c_2 - S_2\right) - S_1\left(-2S_1 c_2 - c_2^2\right)}{S_1\left(S_2 - S_1\right)}, \tag{42}$$

and comparing it with (21) we have

$$W^{t_1,t_2} - W^1 = \frac{1}{2} \frac{\left(-S_2 + S_1 + c_2\right)^2}{S_2 - S_1}. \tag{43}$$

[11] Myles and Uyduranoglu (2004) show that a fiscal intervention which includes a balanced-budget tax policy with a tax on the basic variant of the product combined with a subsidy to the new one, can induce a monopoly to produce the new variant of the product (for which an investment in new technology is required). Although they abstract from optimal taxation issues, they find that there are circumstances where this policy raises social welfare.

Therefore, the welfare achieved when the monopoly produces two variants is higher than when it produces one variant. Note that consumers in this model differ in their marginal willingness to pay for quality. A consumer with higher θ_i is willing to pay more for higher quality. When the monopoly produces two variants of different quality, consumers can choose the variant that is closer to their preference. As a result, the consumer surplus is higher than in the case of one variant only.

5 Concluding Remarks

Consumers often buy a product both for its characteristics and the knowledge that they own a better product than others. In the paper we examine the effects of imposing a tax on such goods. Government imposes negative taxes to correct for monopoly inefficiencies. However, this result holds when there are no positional considerations. In an extended vertically differentiation model which incorporates status and envy concerns, we find that these should be taken into account when the government imposes a tax to correct for the inefficiencies due to the monopolistic structure of the market. We show that the optimal tax rate is related to the intensity of the status and envy effects. Moreover, we show that the optimal tax rate can be positive for high enough values of these effects.

Because taxes are a widely used policy tool, it is important to understand how positional considerations affect the level of the tax pass-through to prices. We find that, for a multiproduct monopoly, the intensity of the positional considerations determine the level of the tax pass-through on prices. Furthermore, as the intensity of the positional effects depends on the mass of consumers who buy any of the two variants, a tax on the high quality variant affects not only its price, but also the price of the low quality variant, and vice versa. For example, a tax rate increase in real estate in an expensive suburb will, according to our analysis, also increase the prices in less expensive districts.

The findings of this paper have significant policy implications for tax policymakers. Our analysis shows that positional concerns are important for the tax policy outcome and should not be ignored by the government.

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6 Data Availability Statement

Data sharing not applicable to this article as no datasets were generated or analysed during the current study.

7 Appendix

Proof of Equation (15): Consumer surplus and profits in (12) and (13), and revenues $R = tQ$, after using (10) and (14), become

$$CS = \frac{\left[4a_2\left(a_2 - a_1\right) + S\left(a_1 - 4a_2 + S - t\right)\right]\left(S - t + a_1\right)}{8\left[S + a_1 - a_2\right]^2}, \tag{44}$$

$$\pi = \frac{\left(S + a_1 - t\right)^2}{4\left[S + a_1 - a_2\right]}, \tag{45}$$

$$R = \frac{t(S + a_1 - t)}{2\left[S + a_1 - a_2\right]}. \tag{46}$$

After substituting the above equations in (6), we can find country's welfare in (15).

Proof of Proposition 1: The denominator of the optimal tax in (16) is always positive, by Assumption 1. The numerator has two roots: $a_2 = \frac{1}{2}\left[S + a_1 \pm \sqrt{a_1^2 - S^2}\right]$.

If the discriminant of the quadratic equation in the numerator of the optimal tax is negative or zero (that is, when $a_1 < S$), the numerator is always negative and consequently, the optimal tax is negative.

If, on the other hand, the discriminant is positive (which occurs when $a_1 > S$), the optimal tax can take both signs. As $a_1 > a_2$, the optimal tax is negative for $a_2 \in \left(0, \frac{1}{2}\left(S + a_1 - \sqrt{a_1^2 - S^2}\right)\right)$ and positive, for $a_2 \in \left(\frac{1}{2}\left(S + a_1 - \sqrt{a_1^2 - S^2}\right), a_1\right)$.

Proof of Proposition 3: The partial derivative of the equilibrium tax in (16) with respect to S is given by

$$\frac{\partial t}{\partial S} = -\frac{2(a_1 - a_2)(S + a_1 - a_2) + S(S + 2(a_1 - a_2))}{(S + 2(a_1 - a_2))^2} \tag{47}$$

The numerator has two roots: $a_2 = S + a_1 \pm \frac{1}{2}S\sqrt{2}$. Note that a_2 cannot be greater than the lower root. To see that, assume, for a proof by contradiction, that $a_2 > \left(S + a_1 - \frac{1}{2}S\sqrt{2}\right)$. Multiplying by 2 and rearranging yields $S\sqrt{2} > (2S + 2a_1 - 2a_2)$. We square both sides (we know from Assumption 1 that $S + 2a_1 - 2a_2 > 0$) and we get $2S^2 > (2S + 2a_1 - 2a_2)^2$. This can be rewritten as $-2S(a_1 - a_2) - 2(S + a_1 - a_2)(S + 2a_1 - 2a_2) > 0$ which is a contradiction since we know from Assumption 1 that $S + 2a_1 - 2a_2 > 0$ and that $a_2 < a_1$ for the market to be non covered.

Therefore

$$a_2 < \left(S + a_1 - \frac{1}{2}\sqrt{2}S\right). \tag{48}$$

Thus the numerator of (47) is always negative, which implies that $\frac{\partial t}{\partial S} < 0$.

Using (16), we can see that the optimal tax increases with the status parameter a_1 as:

$$\frac{\partial t}{\partial a_1} = \frac{S^2}{(S + 2(a_1 - a_2))^2}. \tag{49}$$

Finally, the change in the optimal tax when the envy effect parameter a_2 increases is

$$\frac{\partial t}{\partial a_2} = \frac{2(2(a_1 - 2a_2)a_1 + 2a_2^2 + 2S(a_1 - a_2))}{(S + 2(a_1 - a_2))^2}. \tag{50}$$

The numerator has two roots: a_1 and $a_1 + S$. As $a_2 < a_1$,the numerator is positive and therefore $\frac{\partial t}{\partial a_2}$ is positive.

Second order conditions for profit and welfare maximization when the monopoly produces two variants of the positional good: The second order conditions for profit maximization are satisfied:

$$\frac{\partial^2 \pi}{\partial P_1^2} = \frac{2S_2}{-a^2 + S_1(S_1 - S_2)} < 0, \frac{\partial^2 \pi}{\partial P_2^2} = \frac{2S_1}{-a^2 + S_1(S_1 - S_2)} < 0,$$

and

$$\frac{\partial^2 \pi}{\partial P_1^2}\frac{\partial^2 \pi}{\partial P_2^2} - \frac{\partial^2 \pi}{\partial P_1 \partial P_2}\frac{\partial^2 \pi}{\partial P_2 \partial P_1} = \frac{4S_1(S_2 - S_1)}{[-a^2 + S_1(S_1 - S_2)]^2} > 0.$$

The second order conditions for welfare maximization are satisfied:

$$\frac{\partial^2 W}{\partial t_1^2} = \frac{S_2}{4S_1(S_1 - S_2)} < 0, \frac{\partial^2 W}{\partial t_2^2} = \frac{1}{4(S_1 - S_2)} < 0$$

and

$$\frac{\partial^2 \pi}{\partial t_1^2}\frac{\partial^2 \pi}{\partial t_2^2} - \frac{\partial^2 \pi}{\partial t_1 \partial t_2}\frac{\partial^2 \pi}{\partial t_2 \partial t_1} = \frac{1}{16S_1(S_2 - S_1)} > 0.$$