Efficient audits by pooling independent projects: Separation vs. conglomeration

Anna Maria C. Menichini[1]
Università di Salerno and CSEF

Peter J. Simmons[2]
University of York

October 1, 2020

[1]Corresponding author: Dipartimento di Scienze Economiche e Statistiche, Via Giovanni Paolo II 132, 84084 Fisciano (SA). Tel.: +39 089 962174. E-mail: amenichini@unisa.it

[2]Department of Economics and Related Studies, University of York.

Abstract

In a costly state verification model with endogenous audit and commitment, the paper shows that jointly financing multiple uncorrelated projects mitigates credit rationing and reduces the deadweight loss of inefficient audits as compared with standalone finance. The results hold even when risk-contamination rather than coinsurance arises from conglomeration and work through the incentive effects brought about by optimally chosen variable intensity audits, in which the worst outcomes are audited intensively, while the intermediate ones residually. This implies that for certain parameters' regions, the optimal contract is a standard debt contract. The results hold for both simultaneous and sequential audit as, due to independence of project returns, there is no gain from conditioning the audit of one project on the result of prior audit of another.

Keywords: contracts, auditing, project finance, conglomerations.

JEL Nos: D82, D83, D86.

1 Introduction

In costly state verification (CSV, henceforth) models, costly audit is required to control borrower cheating in loan contracts. Borrowers must report their revenue outcomes and, under commitment, random audit is often sufficient to elicit truth-telling and cheaper than universal audit on all debtors who report default. However, if audit costs are high enough, loan financing may prove unfeasible although without costly audit the project has positive net present value (NPV). And even if audit costs are not so high, any audit, though random, still involves a deadweight loss, as it always just reveals a truthful report. An issue then arises of whether it is possible to guarantee financing and reduce the deadweight loss of audit through contract design. To this aim, we investigate the role of multiple project financing. In particular, we consider an entrepreneur who has two projects but no financial resources to carry them out. The entrepreneur can apply for a separate loan for each project, or instead combine the projects under one roof and apply for a single loan from a single lender to finance both projects. Each project can succeed or fail and the outcomes are uncorrelated. The outcome of each project is ex-post private information to the entrepreneur, who, upon its realisation, has to send an outcome report to the lender stating which projects have failed or succeeded. The truthfulness of the report can be verified by the lender at a cost, who, in the case of joint financing, has to decide also whether to audit one or both projects, and, in this latter case, whether to audit them sequentially or simultaneously.

We study when it is optimal to combine distinct projects under one roof, as opposed to setting them up as stand-alone projects. In doing this, we explore the relative merits of three systems, providing for each of them a detailed characterization of the optimal contract: 1) individual project finance and random audit of fail reported outcomes; 2) group finance with sequential random audit of individual fail reports, with the audit policy and repayment required on the second project conditioned on ex-post knowledge of the first project outcome; 3) group finance with simultaneous audit of fail reports on both the projects which are part of the joint loan.

We find that joint financing has lower deadweight loss than individual financing and may allow the realisation of otherwise unfeasible projects. Within such a setting, the audit frequency is weakly decreasing in the total reported income and may be deterministic or random, depending on the revenues generated from a single success. Last, under joint finance, simultaneous or sequential audit of projects are equivalent in efficiency.

To understand the drivers of these results, consider that both in individual and joint finance there must be some audit of each failed report to stop the borrower from always reporting the low revenue outcome. However, an audit is costly and the size of such cost determines whether a viable project, i.e., a positive NPV project, is feasible or not. When the expected net return on each project separately falls short of the expected audit cost, the projects cannot be carried out as stand-alones, despite each being viable. However, when undertaken jointly, the expected net return on the joint projects taken together may exceed the expected audit cost. This is because under joint finance, the dimension of the reporting and audit space increases allowing finer targeting of audit policy and cross-pledging the cash flows of various projects. In particular, with stand-alone finance there are only two reports for each project, fail or success. To maximise the reporting incentives (and minimise the frequency of audit), it is optimal to audit only a fail report stochastically and leave a rent to the borrower in case of a success report, pledging to the lender the entire returns from failure and as much as necessary of the remaining revenues to meet the lenders costs. Under joint finance, there are three actual and reported states: zero, one and two successes. Again, to maximise the reporting incentives, it is optimal to pledge the entire returns from zero successes to the lender. Since this amount is, by assumption, insufficient to let the lender break even, it is necessary to pledge part or all of the returns from one success. In this way, the borrower cross-pledges the return from one successful project and gives up the rent she could have obtained when this was financed as a stand-alone, thereby slackening the reporting constraints. It turns out that the form of the optimal joint finance contract depends on whether the amount so pledged is sufficient to meet the lender's participation constraint. If it is, audit can be concentrated only on reports of zero successes, neglecting audits of single (and two) success reports. This allows a pooling of the top states (and repayments) and a saving in audit cost relative to the single project financing, in which any fail report must be audited to preserve incentives for truthtelling. We refer to this subsequently as a *joint finance contract with pooling*. When this is feasible, the payment structure may actually resemble that of a standard debt contract, despite the simple two state distribution on each project.

If the expected return from reports of one and zero successes is insufficient to meet the lender's participation constraint, the borrower can pledge also part of the returns from two successes. We refer to this as a *joint finance contract with no pooling*. In this case, besides auditing deterministically the worst state report (zero successes), a report of a single success is also audited, but randomly. Also in this case, despite the intensive audit frequency of the no success report, there is a saving in audit cost relative to the single project financing, because audit is highly concentrated in states that are less likely to occur, two fails, and minimal in intermediate states.

We characterise conditions under which both financing regimes are feasible and the relation between these conditions. We compare the profitability of each regime, finding that not only joint financing always dominates stand-alone financing, but also that in some cases it is the only feasible financing regime.

The above results hold whether the borrower reports the project outcomes simultaneously or sequentially (the latter would allow the audit of the last project to be conditioned on the report of the first). The irrelevance in efficiency of the timing of audit arises because, due to independence of project returns, there is no gain from conditioning the audit of one project on the result of prior audit of another. Moreover, with sequential audit there is an indeterminacy in the audit combination for steps in the sequence. What matters is the overall net probability of the audit of a reported list of fails. In particular, with a report of two fails, a sequential audit strategy sets an audit probability for the first fail and then another audit probability (conditional on the audit outcome of the first fail) on the second reported fail. The overall probability of auditing the two fail reports is a combination of these two and only this combination matters in the deadweight loss and incentive problem. Hence, auditing the first fail intensively and then the second fail with a light touch is equivalent in welfare to auditing the first lightly and the second intensively.

The idea that misreporting incentives can be controlled by costly audits started in the costly state verification literature (Townsend (1979), Gale and Hellwig (1985)) in a world with deterministic audits and a single project with continuous revenue outcomes. Here the solution is a standard debt contract. The range of possible audit strategies was extended in Border and Sobel (1985) and Mookherjee and Png (1989), who allow stochastic audit and show that generally the audit probabilities are interior and fall with the profitability of the state, with the highest revenue state not audited. An alternative cost-efficient information acquisition system has been studied by Menichini and Simmons (2014), who, still within a single project setting, show that by adding a layer of ex-ante information acquisition correlated with future project returns, audits become deterministic and targeted on some signal-state combinations. In the present paper, pooling projects together is yet another reason that may make deterministic audit emerge as the optimal solution in a commitment scenario.

The paper is also related to Diamond (1984), who, with multiple lenders financing several independent projects, shows the optimality of delegating auditing to an intermediary so as to eliminate wasteful duplication of monitoring. However, delegation creates the problem of controlling the incentive of the intermediary to misreport to lenders. This is solved using a standard debt contract between the intermediary and lenders that pays a fixed repayment to lenders and punishes the intermediary any time he fails to deliver it. In such circumstances, the risk of the intermediary failing is minimised by financing several projects at once as the chance of them all failing falls as the number of projects rises. Thus, there is a pooling of risks across projects that drives the intermediary's default risk to zero as the number of projects rises. And it is partly the reduced risk of multiple fails coming from the grouping of projects together, which, within the CSV framework, implies a lower frequency with which audits occur, one of the drivers of our results. Such lowered audit frequency, although accompanied by a more intensive audit, has an overall net effect of a reduction in the expected audit cost as compared with single finance.

However, this reduced risk of bankruptcy is possible in Diamond only so long as pooling returns from one success and one failure covers the total debt (Diamond, 1996), i.e., when coinsurance is feasible. If this is infeasible, risk contamination may occur in the sense that a successful project may be driven bankrupt by a failing one. This possibility has been first uncovered by Winton (1999) and explored in Banal-Estañol, Ottaviani and Winton (2013).[1] Within a setting in which external financing is obtained through debt and default costs depend on total realised project returns, the authors show that losses from risk contamination may arise and separate financing dominates joint financing. One of our contributions is to show that, even with risk contamination, joint financing dominates single financing when optimal stochastic audit is used. There are two reasons for this. First, when audits are stochastic, generally debt is not the optimal contract and there may be a cost saving from optimising the audit policy, with the worst outcomes, less likely to occur, audited intensively, and the intermediate ones audited residually. In addition, the default cost structure allowing project specific audit implies that there are no losses from risk contamination of the type highlighted in Banal-Estañol, Ottaviani and Winton (2013).[2] It follows that in our setting joint financing always weakly dominates single financing.

The paper is also related to the literature on internal capital markets, showing that headquarters may add or destroy value, for example, by better assessing the relative ex-ante profitability of different projects (Stein, 1997), improving asset redeployability (Gertner, Scharfstein and Stein, 1994), or weakening managerial incentives (Stein, 2002). Within an optimal contracting approach between headquarters and outside investors, Inderst and Müller (2003) show that conglomeration may bring coinsurance benefits, but may tighten financing constraints ex ante relative to stand-alone firms due to a subsequent reduced capital market discipline. In a similar vein, Faure-Grimaud and Inderst (2005) highlight the trade-off between the pooling of cash flows across divisions and the allocation of funds to the most profitable divisions. In these papers conglomeration always brings a benefit of coinsurance and this is traded-off against costs

[1] The potential for risk contamination has also been analysed by Leland (2007), while Luciano and Nicodano (2014) have considered the possibility of mitigating such risk by introducing conditional guarantees which, preserving the guarantor's limited liability, do not trigger its default.

[2] We discuss the generality of this assumption in Section 6.2.

of weaker investor control. In our setting, rather than coinsurance, conglomeration may bring risk contamination. Notwithstanding, audit cost savings may be still achieved through the incentive effects brought about by optimally chosen variable intensity audits.

In the results of equivalence of simultaneous and sequential audit and indeterminacy in the audit combination under sequential audit, independence of the project returns also matters. If returns on different projects are correlated, then knowing the outcome on one project gives information about the expected outcomes of others, and hence the need to audit them. Within a no commitment setting, Phelan (2017) shows that, if there is correlation, sequential audit may dominate simultaneous audit. Our results imply that a non-zero correlation is necessary for this (not just sufficient). If the project returns are uncorrelated, knowing the outcome on one project gives no information about the outcome of the other. Hence, auditing both project outcomes simultaneously is as desirable as auditing them sequentially.

The remainder of the paper is organised as follows. Section 2 lays out the model assumption. Section 3 develops a standard CSV model in which two individual projects are financed as stand-alones in the competitive banking sector. Section 4 considers the case of two independent projects to illustrate the basic role of joint project financing in reducing the deadweight loss of audits, distinguishing between the case in which audits are sequential (Section 4.1) and simultaneous (Section 4.2), when pooling is feasible. Section 4.3 assumes that individual financing is feasible but pooling is not. Section 5 compares the individual and joint project financing setting, finding that joint finance dominates in all cases, and analyses the relation between the conditions determining whether pooling is feasible or not. Section 6 discusses some robustness issues. Section 7 concludes. All the proofs, unless otherwise specified, are in the Appendix.

2 The Model Assumptions

An entrepreneur/borrower has two investment projects with uncorrelated returns, each costing I, which can be funded from a risk neutral investor. Each project gives a random return s, $s \in \{H, L\}$, with $H > I >$

$L > 0$, with probabilities p and $1 - p$, respectively. Each project is socially profitable, i.e., the expected return covers the investment cost: $pH + (1-p)L - I > 0$. The return of the project is freely observable only to the entrepreneur and not to the investor. Once the return is realised, the borrower makes a report for each project to the investor. Because of output unobservability, the borrower has an incentive to report $s = L$ on each. But since $I > L$, the only way for the investor to recoup the investment cost on a single project is to carry out an audit. This has a cost $c > 0$ per project and its result is observable and verifiable.

The possible ex-post outcomes vary with how projects are grouped in their financing. With stand-alone projects there are only the two outcomes to the contract on each project, $s = \{L, H\}$, with probability $(p, 1-p)$. The reports can be $\sigma \in \{0, 1\}$, where $\sigma = 0$ denotes a report of zero successes, $\sigma = 1$ denotes a report of one success.

With the two projects jointly financed in a single contract there are more possible outcomes corresponding to the number of projects which succeed $(0, 1$ or $2)$. Four outcomes are then possible: two successes, with probability p^2, two failures, with probability $(1-p)^2$, one success and one failure, with probability $2p(1-p)$. Thus, $s \in \{LL, HL, LH, HH\}$. The reports can be $\sigma \in \{0, 1, 2\}$, where $\sigma = 0$ denotes a report of zero successes, $\sigma = 1$ denotes a report of one success (and one failure), and $\sigma = 2$ denotes a report of two successes. If a single success is reported, the borrower also reports which project has failed.[3]

Treated individually, to attract finance each project must have an ex-ante expected return which covers the investment cost on the project and the expected cost of auditing the reported ex-post outcome:[4]

Condition 1 *(Individual Feasibility) The NPV of each project net of its expected audit cost is strictly positive, i.e.,*

$$IF = pH + (1-p)L - I > (1-p)c.$$

Under the individual feasibility condition, each project as a stand-alone

[3]Subsequently, we relax this in the robustness section below.

[4]Conventionally the CSV literature uses a sufficient condition for this: the true expected revenue of the project(s) with certain audit on relevant failed reports covers the loan cost.

is ex-ante profitable even if enforcement involves auditing for sure a low state report. Whenever this holds the two projects are jointly ex-ante profitable even if all failed reports on either project are audited for sure, i.e., $2p^2H + 2p(1-p)(H+L-c) + 2(1-p)^2(L-c) > 2I$.[5] So, if two projects are viable as stand-alones, they are also viable jointly.

However, projects can also be financed jointly under an alternative ex-ante profitability assumption. This requires that:

Condition 2 *(Pooling Feasibility) Collecting at most $H+L$ revenue from projects with at least one success and $2L$ from two failed projects yields enough expected revenue to cover the investment cost and the certain audit cost of the two failed projects report, i.e.:*

$$PF = p(2-p)(H-L) - 2(I-L) > 2(1-p)^2 c.$$

The interpretation of Condition 2 is that joint finance can be attracted in a contract which pools together the top two outcomes (one or two successes) thus eliminating the need for audit of a one success report. A necessary condition for Condition 2 to hold is that $H + L > 2I$. Thus, it entails the possibility of coinsurance between the two projects. The loss from a failed project can be covered by the gains of a success and still meet the joint cost of the two loans.

3 Single project financing

Under single project financing, each project is funded as a stand-alone. A contract specifies repayments and probability with which an audit will occur. Let m_σ be the probability of auditing a report $\sigma \in \{0,1\}$. Because reports must be feasible, a report of success ($\sigma = 1$) must be truthful. So, there is no audit, $m_1 = 0$. Let R_1 be the repayment due following a report $\sigma = 1$, $R_{0|s}$ be the repayment due following a report $\sigma = 0$, and an audit which reveals that the state is $s \in \{L, H\}$, and $R_{0|\cdot}$ be the repayment with report $\sigma = 0$, but with no audit.

[5]This means that when a report $\sigma = 1$ occurs, the reported fail is audited for sure. Similarly, when a report $\sigma = 0$ occurs, the two reported fails are audited for sure. To deduce it, multiply Condition (1) by 2 and rearrange.

The contract has commitment so that in the play of the game an audit must actually occur even though the lender knows that a fail report must be truthful. This monitoring has a deadweight loss which must be paid. All repayments are non-negative and the agent has limited liability.

The sequence of events is as follows, with the corresponding game tree sketched in Fig. 1.

1. A financing contract is offered and, if accepted, the borrower is committed to the investment.
2. Nature (N) chooses the project outcome, $s = \{H, L\}$. This is only observed by the borrower (A), who makes a report σ to the investor (P).
3. If $\sigma = 1$ is reported, there is no audit. If $\sigma = 0$ is reported, the investor can audit with probability m_0 to discover the true project outcome, or not audit, with probability $1 - m_0$.
4. Conditional on the report and audit decisions, repayments are made as described.

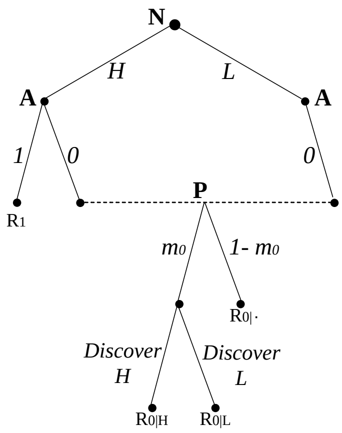

Fig. 1. Game tree with single project financing

The contract $\mathcal{P}^{\text{sin}}$ is to choose repayments $R_{0|H}$, R_1, $R_{0|\cdot}$, $R_{0|L}$, and monitoring probability m_0 to

$$\max E\Pi_S = p\left(H - R_1\right) + (1-p)\left[m_0\left(L - R_{0|L}\right) + (1-m_0)\left(L - R_{0|\cdot}\right)\right] \tag{1}$$

$$\text{st } pR_1 + (1-p)\left[(1-m_0)\,R_{0|\cdot} + m_0\left(R_{0|L} - c\right)\right] \geq I \tag{2}$$

$$R_1 \leq m_0 R_{0|H} + (1-m_0)\,R_{0|\cdot} \tag{3}$$

$$0 \leq R_1, R_{0|H} \leq H \text{ and } 0 \leq R_{0|\cdot}, R_{0|L} \leq L \tag{4}$$

where (1) is the objective function, (2) is the participation constraint, ensuring that the lender breaks even in expected terms on each project, (3) is the truth-telling constraint, ensuring that upon a high state the borrower prefers to report truthfully rather than cheating and be audited with probability m_0, and (4) the limited liability conditions.

The solution to programme P^{sin} is described in Proposition 1:

Proposition 1 *When the borrower applies for funding each project as a stand-alone, if Condition 1 holds, the optimal contract has:*

(i) *maximum punishment for false low state report:* $R_{0|H} = H$;

(ii) *zero low state return for the borrower:* $R_{0|L} = R_{0|\cdot} = L$;

(iii) *random audit of low state reports,* $m_0^{\text{sin}} \equiv m_0$:

$$m_0^{\text{sin}} = \frac{I - L}{p\,(H - L) - (1 - p)\,c} < 1; \tag{5}$$

(iv) *lender repayment following a high state report equal to* $R_1 = \frac{(H-L)I-(1-p)L(H-L+c)}{p(H-L)-(1-p)c} < H$, *and expected return to the borrower equal to the expected return net of the expected audit cost:*

$$E\Pi^{sin} = pH + (1 - p)\,L - I - \underbrace{\frac{(I - L)}{p\,(H - L) - (1 - p)\,c}}_{m_0^{\text{sin}}}(1 - p)\,c > 0. \tag{6}$$

If Condition 1 does not hold, $m_0^{\text{sin}} \geq 1$ *and the expected return to the borrower is negative. Thus, financing does not occur.*

The intuition behind these results is the following. When Condition 1 does not hold, the audit frequency exceeds one and the borrower makes losses (expected returns $E\Pi^{\text{sin}}$ (6) are negative). Thus, no contract is signed, despite the project having positive NPV. If Condition 1 does hold, instead, the frequency of audits is lower but positive (since if $m_0^{\text{sin}} = 0$, from (3), $R_1 = R_{0|\cdot} \leq L$ and there is insufficient revenue to meet the investment cost). The deadweight loss of monitoring is minimised by raising $R_{0|H}$ up to H and reducing the monitoring probability until the incentive constraint (3) holds with equality. In addition, low state repayments, whether monitored or not, are set to give zero surplus to the borrower: $R_{0|\cdot} = R_{0|L} = L$. However, since $R_1 < H$, the borrower gets a reward in the high state, thus making profits (expected returns $E\Pi^{\sin}$ (6) are positive). These results are analogous to those obtained by Khalil and Parigi (1998), except here the investment size is exogenous.

4 Joint project financing

We now introduce the possibility of jointly financing two projects with independent ex-post private returns costlessly observed by the borrower. This is relevant when single project financing proves infeasible, i.e., Condition 1 does not hold. But actually, even when stand-alone financing is feasible, there are advantages from joint financing. To explore the nature of such advantages, we consider two possible scenarios. One in which Condition 2 holds, and one in which it is violated, but Condition 1 holds.

Under joint financing, the two projects may give four possible outcomes: $s \in \{LL, HL, LH, HH\}$. We assume that the borrower makes a single report to the lender stating the number of successes, $\sigma \in \{0, 1, 2\}$, and which project has succeeded when reporting a single success. Any report the borrower makes must be feasible in that she has to have funds to make the appropriate repayment. Conditional on the report, the lender can audit. The contract has to list an audit strategy that overcomes the temptations to cheat in the report. In particular, in case of a report of zero successes, the lender can audit sequentially, i.e., first audit one project and then, possibly conditional on the outcome of the first audit, the other, or simultaneously, i.e., audit both projects reported to have failed. In case of a report of one success, only the project reported to have failed is audited. Both in the case of sequential or simultaneous audit we assume there is commitment in the contract, so the lender has to carry through the audit policy even knowing that it will never catch a cheat.

4.1 Joint finance with pooling and sequential audits

We first consider the case in which Condition 2 holds. This amounts to assuming that the total investment and maximum audit cost on a report of two fails can be met by never repaying more than the returns from one success and one fail, $H + L$.

Given the four possible outcomes $s \in \{LL, HL, LH, HH\}$, the borrower can report no success, one success, or two successes, $\sigma \in \{0, 1, 2\}$. Because reports must be feasible, a report of two successes ($\sigma = 2$) must be truthful. So, there is no audit: $m_2 = 0$. Let R_2 be the repayment due following a report $\sigma = 2$.

In case of a report of one success ($\sigma = 1$), because reports must be feasible, the lender knows that at least one project has been successful, while the other may have succeeded or failed. We assume that, along with the outcome, the borrower also reports which project has succeeded.[6] So, to minimise audit cost, the lender can target monitoring on the reported failed project only, with probability m_1, or not audit at all, with probability $1 - m_1$. In the case in which he does not audit, he demands a repayment $R_{1|\cdot}$ in total on the two projects, where the first subscript denotes the report, and the dot stands for no audit. In the case in which he does audit, instead, he demands a repayment $R_{1|i}$, where $i \in \{L, H\}$ is the outcome of the audit. So, upon an audit of a report $\sigma = 1$ with probability m_1, if the lender finds that the project has truly failed, he demands repayment $R_{1|L}$, while he demands repayment $R_{1|H}$ if he discovers a success on the reportedly failed project.

In case of a report of zero successes ($\sigma = 0$), knowing that the borrower claims that both projects have failed, the lender can randomly choose which one to audit, if any, with probability $1/2$ on each (by the principle of insufficient reason). Denote with m_0 the probability to audit one of the two projects, and with $1 - m_0$ the probability of auditing neither. In cases in which the lender does not audit, he demands a repayment $R_{0|\cdot}$ and the game ends. When the lender does audit and discovers the outcome for the selected project, he can decide to go further and audit the remaining project, with probability $m_{0,i}$, $i \in \{L, H\}$, where the second subscript denotes the outcome of the first audit, or to stop, with probability $1 - m_{0,i}$. Denote with $R_{0|ij}$, $i, j \in \{L, H\}$, the repayment the lender gets

[6] Given that audit costs are ultimately borne by the borrower, it is in his interest to provide this information. We discuss more thoroughly this possibility in the robustness section.

upon receiving a report of zero successes when he audits both projects and discovers the true state to be i on the first and j on the second, and with $R_{0|i\cdot}$, the repayments in case he audits just one project and discovers the true state to be i, but does not audit the other.

The sequence of events is as follows, with the corresponding game tree sketched in Fig. 2.

1. A financing contract is offered and, if accepted, the borrower is committed to the investment.
2. Nature (N) chooses the project outcome, $s = \{LL, HL, LH, HH\}$. This is only observed by the borrower (A), who makes a report σ to the investor (P).
3. If $\sigma = 2$ is reported, there is no audit. If $\sigma = 1$ is reported, the investor can audit the project reported to have failed with probability m_1 or not audit with probability $1 - m_1$. If $\sigma = 0$ is reported, the investor can audit the first project with probability m_0 or not audit with probability $1 - m_0$. Conditional on having audited the first project, the investor can audit the second with probability $m_{0,i}$, or not audit, with probability $1 - m_{0,i}$, $i \in \{L, H\}$.
4. Conditional on the report and audit decisions, repayments are made.

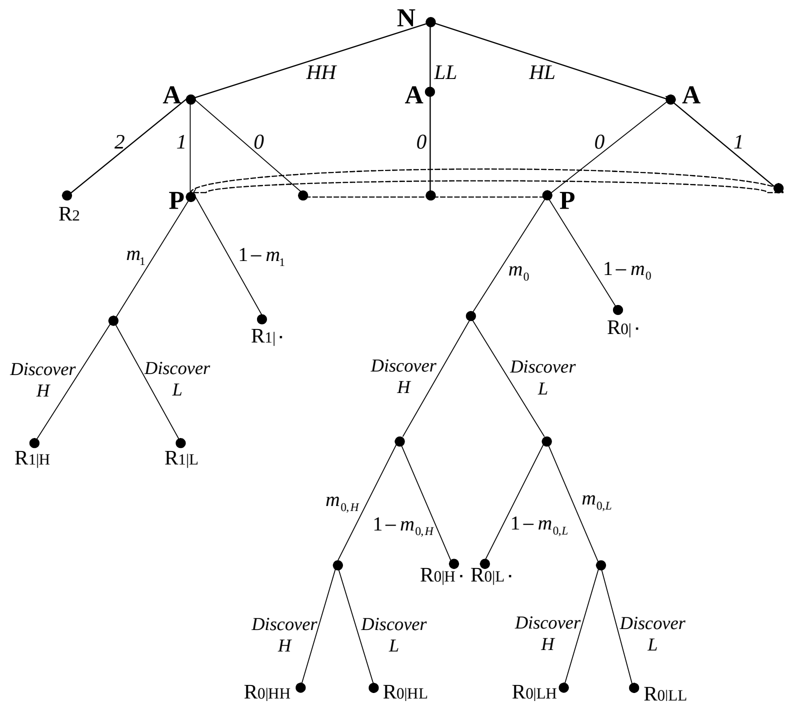

Fig. 2. Game tree with two projects and sequential audits

The borrower's joint payoff function with truthtelling is

$$E\Pi_J^{seq} = p^2 (2H - R_2) + 2p(1-p)\left\{H + L - m_1 R_{1|L} - (1-m_1) R_{1|\cdot}\right\} + (1-p)^2 \left\{2L - m_0\left[m_{0,L} R_{0|LL} + (1-m_{0,L}) R_{0|L\cdot}\right] - (1-m_0) R_{0|\cdot}\right\}. \quad (7)$$

The participation constraint requires the expected return to the lender from financing both projects cover the joint loan costs and the expected audit costs:

$$E\Pi_L^{seq} = p^2 R_2 + 2p(1-p)\left\{m_1\left(R_{1|L} - c\right) + (1-m_1) R_{1|\cdot}\right\} + \quad (8)$$
$$(1-p)^2\left\{m_0\left[m_{0,L}\left(R_{0|LL} - c\right) + (1-m_{0,L}) R_{0|L\cdot} - c\right] + (1-m_0) R_{0|\cdot}\right\} \geq 2I.$$

With two true successes, there are two ways of cheating. To declare zero successes, or to declare one. Thus, the incentive constraints that ensure that a borrower with two successes reports truthfully, i.e., prefers to make a truthful report $\sigma = 2$ rather than a false report $\sigma = 0$ (constraint 9) or $\sigma = 1$ (constraint 10) are:

$$R_2 \leq (1-m_0) R_{0|\cdot} + m_0\left[m_{0,H} R_{0|HH} + (1-m_{0,H}) R_{0|H\cdot}\right] \quad (9)$$
$$R_2 \leq (1-m_1) R_{1|\cdot} + m_1 R_{1|H} \quad (10)$$

With one true success, the only way of cheating is to declare zero successes. The incentive constraint ensuring that a borrower with one success prefers to truthfully report $\sigma = 1$ rather than falsely $\sigma = 0$ is:

$$(1-m_1) R_{1|\cdot} + m_1 R_{1|L} \leq (1-m_0) R_{0|\cdot} + \quad (11)$$
$$+ m_0\left[\frac{1}{2}\left(m_{0,H} R_{0|HL} + (1-m_{0,H}) R_{0|H\cdot}\right) + \frac{1}{2}\left(m_{0,L} R_{0|LH} + (1-m_{0,L}) R_{0|L\cdot}\right)\right].$$

Last, the limited liability conditions are

$$\begin{aligned} R_2, R_{1|H}, R_{0|HH} &\leq 2H, \quad (12) \\ R_{1|\cdot}, R_{1|L}, R_{0|H\cdot}, R_{0|HL}, R_{0|LH} &\leq H + L, \\ R_{0|\cdot}, R_{0|L\cdot}, R_{0|LL} &\leq 2L. \end{aligned}$$

The contract problem $\mathcal{P}^{\text{seq}}$ is to choose R_2, $R_{1|\cdot}$, $R_{1|H}$, $R_{1|L}$, $R_{0|H\cdot}$,

$R_{0|HL}$, $R_{0|LH}$, $R_{0|\cdot\cdot}$, $R_{0|L\cdot}$, $R_{0|LL}$, $R_{0|HH}$, and monitoring probabilities $m_0, m_{0,H}, m_{0,L}, m_1 \in [0,1]$ to maximise the objective function (7), subject to the participation constraint (8) being non-negative, to the incentive constraints (9), (10), and (11), and to the limited liability conditions (12). The solution to the above programme gives the joint finance contract with pooling described in the following proposition.

Proposition 2 *Under Assumption 2, with joint contracting and sequential monitoring the optimal contract has:*

(i) *maximum punishment for detected false reporting:* $R_{0|HH} = R_{1|H} = 2H$; $R_{0|H\cdot} = R_{0|HL} = R_{0|LH} = H + L$;

(ii) *zero rent to the borrower in the lowest true state (both projects fail):* $R_{0|L\cdot} = R_{0|\cdot\cdot} = R_{0|LL} = 2L$;

(iii) *probability of monitoring reports of zero successes at first stage,* $m_0^{\text{seq}} \equiv m_0$, *and at second stage having discovered a truthful report by first stage audit,* $m_{0,L}^{\text{seq}} \equiv m_{0,L}$, *not uniquely defined but* $0 < m_0^{\text{seq}}\left(1 + m_{0,L}^{\text{seq}}\right) \leq 2$, *with* $m_0^{\text{seq}} \in (0,1]$, *and* $m_{0,L}^{\text{seq}} \in [0,1]$. *In particular, any combination* $m_0^{\text{seq}}, m_{0,L}^{\text{seq}}$ *along the curve satisfying*

$$m_0^{\text{seq}}\left(1 + m_{0,L}^{\text{seq}}\right) = \frac{4\left(I - L\right)}{p\left(2 - p\right)\left(H - L\right) - 2\left(1 - p\right)^2 c} \tag{13}$$

is optimal;

(iv) *probability of monitoring at the second stage having discovered a cheat by first stage audit set at its highest value,* $m_{0,H} = 1$;

(v) *repayments pooled in the top two states, so that there is a common repayment after a report of two successes or a single success. This gives a reward to truthfully reporting one or two successes no higher than* $H + L$, *and* $R_{1|\cdot} = R_2 = 2L + \frac{2(I-L)(H-L)}{p(2-p)(H-L)-2(1-p)^2 c}$;

(vi) *probability of monitoring reports of one success set at its lowest value,* $m_1 = 0$. *Because of this,* $R_{1|L}$ *is never paid and can be set to any value between zero and* $H + L$;

(vii) the borrower with at least one success indifferent between truthfully reporting one or two successes, getting a positive expected return:

$$2\left[pH+(1-p)L-I\right]-\frac{4(1-p)^2(I-L)c}{p(2-p)(H-L)-2(1-p)^2c}. \tag{14}$$

The intuition behind these results is the following. Maximum punishment and zero rent to the borrower in the lowest truthfully reported states maximises the incentives for truth-telling whilst also keeping the observation cost of low reports as small as possible.

Setting a strictly positive probability of auditing a report of two fails is required since otherwise the borrower could always report no success and get away with cheating, leading to repayments which do not cover the investment cost.

The incentive to cheat between a report of one success or zero successes is controlled by a binding incentive constraint in which $R_{1|\cdot}$ is set at the lowest possible level compatible with the lender's participation constraint. Thus, repayments and audit probabilities are set so as ensure that the borrower is indifferent between declaring one success and zero successes.

The incentive to cheat between a report of one or two successes is controlled by a binding incentive constraint again, which involves pooling the repayments due after reporting one or two successes, $R_{1|\cdot}=R_2$. These repayments must be above $2L$, since otherwise there would be insufficient revenue to the lender to recoup the loans cost. With flat repayments for one or two successes, audit of one fail report is unnecessary, so $m_1=0$.

Some of the properties of the solution mirror those of the fundamental literature, i.e., maximum punishment on detected cheats, zero rent for the borrower in the lowest state. But there are many novelties.

In particular, if the projects taken together are sufficiently profitable, then optimally there is a common repayment from reports of one or two successes, and thus a pooling across the top two states.

Moreover, truthful reporting of zero successes is ensured by many equivalent combinations of first and second stage audit probabilities and they lie along the curve satisfying Eq. (13). One extreme possibility has zero monitoring at the second stage, $m_{0,L}^{\text{seq}}=0$, and $m_0^{\text{seq}}=$

$\frac{4(I-L)}{p(2-p)(H-L)-2(1-p)^2c}$. The other extreme possibility has $m_{0,L}^{\text{seq}} = 1$, and $m_0^{\text{seq}} = \frac{2(I-L)}{p(2-p)(H-L)-2(1-p)^2c}$. At the extremes, monitoring only the first stage but with twice the intensity is equivalent in deadweight loss to monitoring both stages, the second stage with probability one. The intuition for the irrelevance of the timing of monitoring the projects after a zero report is that the total repayment due is $2L$, however the audit is divided between the first or second stage. So, every first and second stage audit combination that satisfies (13) has the same incentive effect.

In Fig. 3, the shape of the locus highlights that m_0^{seq} is a more powerful instrument than $m_{0,L}^{\text{seq}}$: a fall in m_0^{seq} requires a proportionally larger increase in $m_{0,L}^{\text{seq}}$. This is because m_0^{seq} affects incentives directly, at the first stage monitoring following a zero report, and indirectly, through the effect that it has at the second stage monitoring, after first stage monitoring has occurred upon a zero report. However, the combinations have the same expected audit cost.

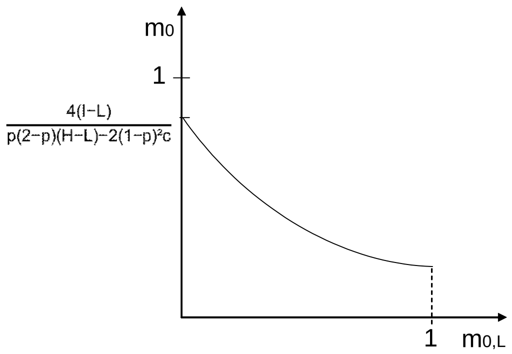

Fig. 3. Locus of indifferent sequential audit probabilities upon a zero success report

4.2 Joint finance with pooling and simultaneous audits

We now consider the case of simultaneous audits. In case of a report of one or two successes, the audit and repayment structure is analogous to the sequential one. In particular, there is no audit following a report of two successes ($\sigma = 2$), while it is only necessary to audit the project reported to have failed in case of a report of just one success ($\sigma = 1$).[7] The case in which no success is reported ($\sigma = 0$) is different. Here the lender will audit both projects simultaneously with probability m_0, and neither of them with probability $1-m_0$. As for the sequential case, in case of no audit, the lender demands a repayment $R_{0|\cdot}$ and the game ends. If he does audit, instead, he can discover that both projects have failed, that only one has failed, or that none has, getting respectively $R_{0|LL}, R_{0|LH}, R_{0|HH}$.

The sequence of events is as the one described under sequential audits, except for stage 3, in which, in case of a report $\sigma = 0$, either both projects are audited simultaneously with probability m_0 or neither with probability $1 - m_0$.

A game tree is sketched in Fig. 4. Note that if we set $m_{0,H} = m_{0,L} = m_0$ in the sequential audit game tree, we have the simultaneous audit game tree. Hence optimally sequential audit must weakly dominate simultaneous audit.

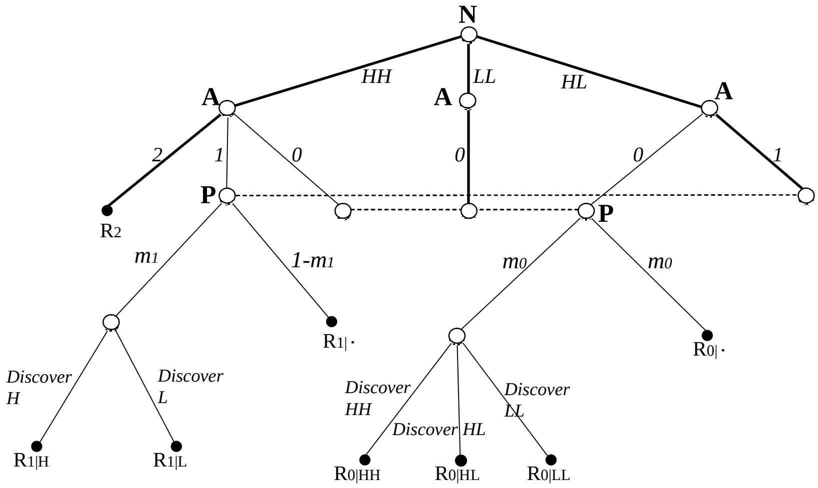

Fig. 4. Game tree with two projects and simultaneous audits

The objective function is:

$$\begin{aligned} E\Pi_J^{\text{sim}} &= p^2 (2H - R_2) + 2p(1-p)\left[H + L - m_1 R_{1|L} - (1-m_1) R_{1|\cdot}\right] \\ &\quad + (1-p)^2 \left[2L - m_0 R_{0|LL} - (1-m_0) R_{0|\cdot}\right], \end{aligned} \tag{15}$$

[7]As in the case with sequential audit, we assume that the borrower reveals which project has failed.

while the participation constraint is:

$$E\Pi_L^{\text{sim}} = p^2 R_2 + 2p(1-p)\left[m_1\left(R_{1|L} - c\right) + (1-m_1) R_{1|\cdot}\right] \qquad (16)$$
$$+ (1-p)^2\left[m_0\left(R_{0|LL} - 2c\right) + (1-m_0) R_{0|\cdot}\right] \geq 2I.$$

Last, the relevant limited liability conditions (12) hold.

As for the sequential case, there are two ways of cheating: to declare zero successes, or to declare one. Thus, the incentive constraints are:

$$R_2 \leq (1-m_0) R_{0|\cdot} + m_0 R_{0|HH} \qquad (17)$$

$$R_2 \leq (1-m_1) R_{1|\cdot} + m_1 R_{1|H} \qquad (18)$$

$$m_1 R_{1|L} + (1-m_1) R_{1|\cdot} \leq (1-m_0) R_{0|\cdot} + m_0 R_{0|HL} \qquad (19)$$

where constraints (17) and (18) ensure that a borrower with two successful projects prefers to make a report $\sigma = 2$ rather than $\sigma = 0$ or $\sigma = 1$. Similarly, constraint (19) ensures that a borrower with one successful and one failed project prefers to report $\sigma = 1$ rather than $\sigma = 0$.

The contract problem is to choose R_2, $R_{1|\cdot}$, $R_{1|L}$, $R_{1|H}$, $R_{0|\cdot}$, $R_{0|LL}$, $R_{0|HL}$, $R_{0|HH}$, m_0, m_1 to maximise the objective function (15), subject to the participation constraint (16), to the incentive constraints (17), (18), and (19), and to the limited liability conditions (12).

In the following we show that the optimal simultaneous contract is closely related to the optimal sequential one, as stated in Proposition 3.

Proposition 3 *(Irrelevance of the timing of monitoring) Defining $m_0^{sim} \equiv m_0$ in the simultaneous case, if we set $m_0^{\text{seq}}\left(1 + m_{0,L}^{\text{seq}}\right) = 2m_0^{sim}$, the two contract problems are identical.*

Since sequential audit must be at least as good as simultaneous audit, if there are choices of variables under simultaneous audit which attain the best sequential audit payoff, these must be the simultaneous audit optimal values. Inspecting the two contract problems they are identical (and we know that only the combined value $m_0^{\text{seq}}\left(1 + m_{0,L}^{\text{seq}}\right)$ matters in the sequential problem), so the optimal audit strategies in both problems must coincide, with $m_0^{\text{seq}}\left(1 + m_{0,L}^{\text{seq}}\right) = 2m_0^{sim}$.

Corollary 1 *Optimal sequential and simultaneous audits have identical ex-ante welfare.*

Choosing $m_0^{sim} = \frac{2(I-L)}{p(2-p)(H-L)-2(1-p)^2 c} = \frac{1}{2} m_0^{\text{seq}} (1 + m_{0,L}) \leq 1$, the optimal ex-ante welfare coincides in the two contracts. The equivalence arises because the returns on the two projects are independent. So, knowing the outcome on one project is not informative about the distribution of returns on the other project. By contrast, if the returns were correlated, there would be a potential information gain in sequential audit (Phelan, 2017).

4.3 Joint finance with no pooling and sequential audits

In the previous section we have seen that when Condition 2 holds, only reports of two fails are audited randomly, while reports of only one fail are never audited ($m_1 = 0$). The intuition is that there are enough revenues from the bottom and intermediate state to cover the investment and audit cost, so that there is no need to collect extra resources over $H + L$ from the top state.

In the present section we consider what happens if Condition 2 does not hold, i.e., $p(2-p)(H-L) - 2(I-L) - 2(1-p)^2 c < 0$, but Condition 1 does hold.[8] Thus, while each project can be carried out as a stand-alone, it is not possible to carry them out jointly by pooling the top two returns and auditing, even if deterministically, only reports of two fails, as the returns so obtained are insufficient to meet the lender's participation constraint. Thus, a report of a single fail must also be audited and the top repayment cannot be pooled with the intermediate one. These results are stated in Proposition 4**.**

Proposition 4 *If Condition 1 is satisfied but Condition 2 is not, with joint contracting and simultaneous or sequential audits, in addition to the properties reported in points (i)-(ii) of Proposition 2, the optimal contract has:*

[8] This implies that $p(H-L) - 2(1-p)c > 0$.

1. *Deterministic monitoring for reports of two fails:* $m_0 = m_{0,L} = 1$;

2. *Zero rent to the borrower in the intermediate true state (one project fails):* $R_{1|L} = R_{1|\cdot} = H + L$;

3. *Random monitoring for single fail reports:*[9]

$$m_1 = \frac{2(I-L) - p(2-p)(H-L) + 2(1-p)^2 c}{p^2(H-L) - 2p(1-p)c} < 1; \tag{20}$$

4. *Repayment after a report of two successes higher than* $H + L$: $R_2 = 2H - 2(H-L)\frac{[pH+(1-p)L-I-(1-p)c]}{p^2(H-L)-2p(1-p)c}$;

5. *Expected return to the borrower*

$$2[pH + (1-p)L - I] - \frac{2(I-L) - p(H-L)}{p(H-L) - 2(1-p)c} 2p(1-p)c. \tag{21}$$

The intuition behind these results is that, since the revenues from just one success or no successes do not cover the investment plus audit cost of the two projects, additional revenues in excess of $H+L$ must be raised from the report of two successes. But to ensure truthful reports of two successes and prevent cheating by declaring one success, reports of only one success must sometimes be audited: $m_1 > 0$. Thus, in the absence of pooling the one success report must be audited but only stochastically.

5 Comparisons

We now compare the relative efficiency of the various contract problems.

Under single project financing, the expected profits obtainable from two stand-alone projects are

$$2\left[pH + (1-p)L - I - (1-p)m_0^{\sin}c\right], \tag{22}$$

where $m_0^{\sin}$ is the probability of auditing a fail report under single project financing, as defined in (5).

[9]Because Condition 1 holds but Condition 2 does not, it follows that the denominator of m_1 is positive. Similarly $m_1 > 0$ when Condition 2 does not hold. Last, $m_1 < 1$ follows from Condition 1 holding.

Under joint financing and sequential audit, expected profits are

$$2\left[pH+(1-p)L-I\right]-2p(1-p)m_1c-(1-p)^2m_0^{\text{seq}}\left(1+m_{0,L}^{\text{seq}}\right)c, \quad (23)$$

where $m_0^{\text{seq}}\left(1+m_{0,L}^{\text{seq}}\right)$ is the combination of first and second stage audit probabilities upon a report of zero successes, as defined in (13), and m_1 is the probability of auditing a report of one success (20).

For viable stand-alone finance Condition 1 must always hold. The detailed comparison of joint and single project finance depends on whether Condition 2 is satisfied, i.e., whether a pooling contract is feasible.

Case 1: Condition 2 is satisfied. In this case, reports of one success are never audited, $m_1=0$, and only reports of zero successes are audited with probability $m_0^{\text{seq}}\left(1+m_{0,L}^{\text{seq}}\right)<2$. By comparing the expected audit cost under single and joint project finance (Eqs. 22 and 23), we see that joint project financing with pooling dominates single project financing iff:

$$(1-p)\,c\underbrace{\frac{2(I-L)}{p(H-L)-(1-p)c}}_{2m_0^{sin}}>(1-p)^2c\underbrace{\frac{4(I-L)}{p(2-p)(H-L)-2(1-p)^2c}}_{m_0^{\text{seq}}\left(1+m_{0,L}^{\text{seq}}\right)}.$$

The difference reduces to $\frac{2p^2(1-p)(H-L)(I-L)}{[p(H-L)-(1-p)c]\left[p(2-p)(H-L)-2(1-p)^2c\right]}$, which is always positive. Thus, joint project financing with pooling has lower expected audit cost and dominates single project financing.

To determine the driver of this result, we compare the audit probabilities under joint and single financing, $m_0^{\text{seq}}\left(1+m_{0,L}^{\text{seq}}\right)-2m_0^{sin}$:

$$\frac{2p(I-L)\left[p(H-L)-2(1-p)\right]c}{\left[p(H-L)-(1-p)c\right]\left[p(2-p)(H-L)-2(1-p)^2c\right]},$$

which is positive. Thus, there is more intensive audit under joint financing. It follows that the dominance of joint pooling finance over stand-alone finance is ascribed to the lower probability with which default occurs ($(1-p)^2$ under joint financing rather than $1-p$ under separate financing), and thus the lower frequency with which audit is applied, along with the pooling of returns implied by Condition 2, that allows concentration of audit only on reports of two fails. This result in which an intensive audit is applied with a low frequency is reminiscent of Becker (1968) in which maximum deterrence is obtained at minimal cost by inflicting a high punishment with a sufficiently low probability.

Case 2: Condition 2 is not satisfied. When it is not possible to meet the lender's participation constraint by pooling the top two returns and auditing only reports of zero successes, even with the highest possible intensity ($m_0^{\text{seq}} = m_{0,L}^{\text{seq}} = 1$), extra-resources must be raised from the two success outcome, which implies that also reports of one success and one fail must be audited, although minimally: $m_1 > 0$. Overall, by comparing the expected audit cost under single (22) and joint project financing (23), we get:

$$\underbrace{(1-p)\,2m_0^{\text{sin}}\,c}_{\text{single}} - \underbrace{\left[(1-p)^2\,m_0^{\text{seq}}\left(1+m_{0,L}^{\text{seq}}\right) + 2p\,(1-p)\,m_1\right]c}_{\text{joint}},$$

which reduces to $\frac{p(H-L)[pH+(1-p)L-I-(1-p)c]}{[p(H-L)-2(1-p)c][p(H-L)-(1-p)c]}$, strictly positive under Condition 1. Thus, even in the case in which Condition 2 is violated and a pooling contract is infeasible, joint financing dominates single financing.

These results contrast with Banal-Estañol, Ottaviani and Winton (2013) who show that when the required repayment obligation cannot be met in the intermediate outcome of one success and one failure risk contamination losses arise and single financing dominates joint financing. In our setting, although risk contamination may arise (in the sense that, despite having the successful project, the failing one drags the firm into bankruptcy), still it involves no extra loss as the default costs of joint financing never exceed those of single financing.

To disentangle the determinants of such lower default costs, notice that, as in Banal-Estañol, Ottaviani and Winton (2013), and unlike Case 1 above, the probability of default is actually higher under joint than single financing ($(1-p)^2+2p\,(1-p) = 1-p^2 > 1-p$). Given that a report of zero successes is audited with probability one ($m_0^{\text{seq}} = m_{0,L}^{\text{seq}} = 1$), it turns out that in our setting the saving in expected default costs relative to single financing may be ascribed to the random (and minimal) audit of reports of one success and one fail ($m_1 < 1$). So long as audit of a single fail report is stochastic, joint financing always dominates single financing. In the extreme case in which the audit of a single success report is deterministic ($m_1 = 1$), the borrower is indifferent between financing the projects separately or jointly.

It follows that requiring deterministic audit is not a sufficient condition for Banal-Estañol, Ottaviani and Winton (2013) to find the strict dominance of single finance over joint finance. An additional factor is the difference between the default cost structures assumed. In Banal-Estañol, Ottaviani and Winton (2013), bankruptcy costs are proportional to total realised returns,[10] so that when the entrepreneur is unable to meet its repayment obligation under one success and one failure a fraction of the high state returns are lost. This loss never occurs if each project is financed separately, and so neither does the loss from risk contamination and suboptimality of joint financing highlighted by the authors. In our setting, the possibility of disentangling the successful project from the failing one, targeting audits only on fail reports, rules out the extra inefficiency from investigating the successful project (and thus the monetary loss associated with risk contamination). Along with the cost saving due to the endogenous (and stochastic) audit probability, this reduces bankruptcy/default costs. We can thus state Proposition 5.

Proposition 5 *Joint project financing dominates single project financing.*

5.1 Graphical analysis

From the above we have seen that the form of the optimal contract depends on whether the Individual Feasibility condition 1 (IF) and the Pooling Feasibility condition 2 (PF) hold and how they relate to each other. Indeed, they are not nested and projects could satisfy both, one or neither of them, depending on project return distribution, investment and audit cost. In particular, projects that are unviable alone can be viable jointly, while projects that are viable alone are also viable jointly. In the present section we explore how the interaction of these conditions determines whether stand-alone or either form of joint finance are possible. To this aim, let

[10] However, their results also hold with a more general structure of default costs, provided there are not too extreme diseconomies of scale in default (Banal-Estanol and Ottaviani, 2013).

us take the difference between twice the individual feasibility ($2IF$) and the pooling feasibility (PF) conditions:

$$\Delta F \equiv 2IF - PF = p\left[p\left(H-L\right) - 2\left(1-p\right)c\right].$$

The ex-ante revenue in individual feasibility on each project will tend to be higher than that in pooling feasibility when the spread $H - L$ is large and/or p is high and/or c is low. Pooling feasibility has a lower expected audit cost but also reduces the potential amount of repayment the lender can secure from the two success outcome.

In the diagrams below the lines $2IF$ and PF show how expected revenue net of audit cost varies with c. The slopes of these lines depend only on p. The pooling feasibility condition PF falls more slowly with c. At $c = 0$ the intercept of individual feasibility, $2[pH + (1-p)L]$, is higher than that of pooling feasibility, $p(2-p)(H-L) + 2L$. Projects will be able to attract finance with auditing when $2IF > 2I$ for stand-alone finance or when $PF > 2I$ for joint finance. The locus of $2I$ is horizontal. The vertical intercept of the line $2IF$ must be above $2I$ since, by assumption, each project has positive NPV. So, depending on the position of the locus $2I$ in relation to the intersection of the pooling and individual feasibility lines, there are areas in which neither, one or both of the individual and pooling feasibility conditions (1) and (2) hold. When all three types of finance are possible, the one that minimises audit cost is pooling with joint finance. This dominates joint finance with no pooling, which in turn dominates stand-alone finance.

Fig. 5 shows the case when the common value of PF and $2IF$ at their intersection is above $2I$. As c rises from zero, initially both stand-alone and pooling joint are possible. But as c rises further, stand-alone is not possible but pooling joint is for a range of audit costs, after which pooling joint also becomes infeasible. So, in this case, whenever the projects can be financed at all, joint financing with pooling is the best form of finance.

For sufficiently high c there is no way of financing the projects.

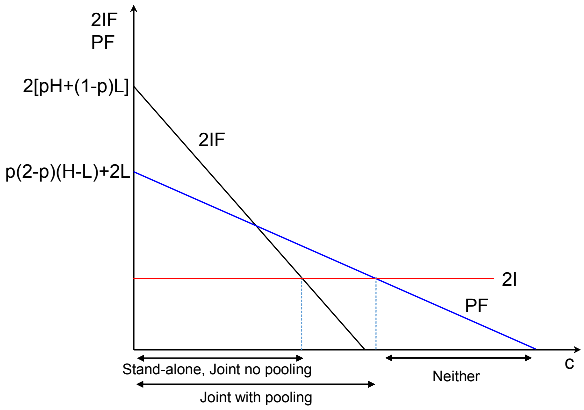

Fig. 5. Only joint finance with pooling is optimal

In Fig. 6 the common value of PF and $2IF$ at their intersection is below $2I$, which in turn is below the value of PF at $c = 0$.

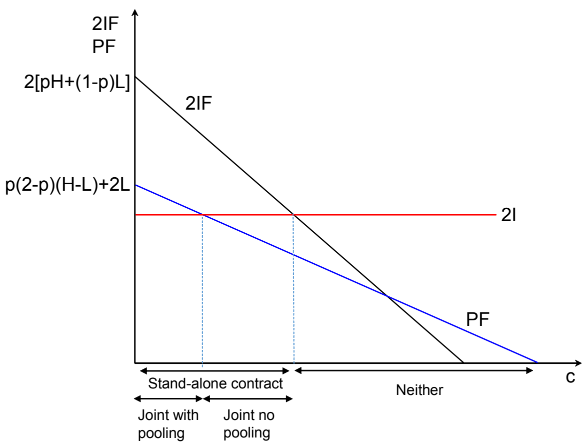

Fig. 6. Joint finance with pooling and no pooling both optimal, depending on audit cost

Here as c rises from zero, initially both stand-alone and pooling joint are possible, but as c continues to rise pooling joint gets infeasible, becoming feasible with no pooling, along with stand-alone finance. Thus, in this case, for sufficiently low audit cost, the optimum is joint finance with pooling, but once this becomes infeasible as c rises the best contract is joint finance with no pooling. For sufficiently high c there is no way of financing the projects.

In Fig. 7 the common value of PF and $2IF$ at their intersection is below $2I$, which in turn is above the value of PF at $c = 0$ and below the value of $2IF$ at $c = 0$. Then pooling joint is never feasible although as c rises from zero, joint with no pooling becomes feasible along with stand-alone until for higher c that gets infeasible too.

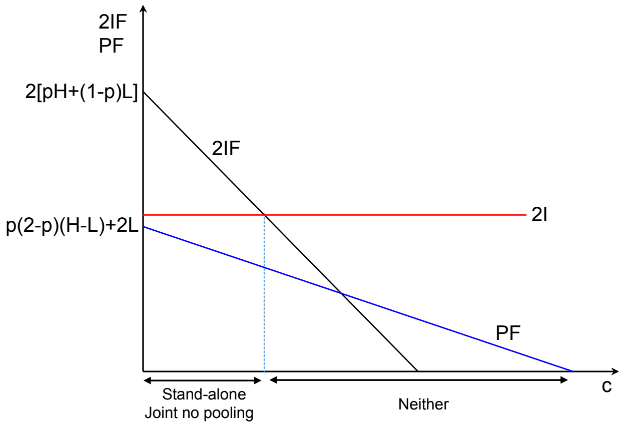

Fig. 7. Only joint finance with no pooling is optimal

In this case joint finance with pooling is never possible and the optimal contract, when projects can be financed, is joint finance with no pooling.

6 Robustness

In this section we consider the relevance of the arguments to more general settings.

6.1 More than two projects

We have assumed only two projects and a single investor with sufficient funds to finance both of them. We found that joint finance increases the number of states allowing more precisely targeted audit. With more than two projects, the number of states will further increase, the monotonicity of audit probability and state should continue, and the superiority of joint over single project finance magnified. We conjecture some of the details of this. With n projects, if each is financially viable in the sense that Condition 1 holds, then any subset of the n projects can be jointly financed. The types of pooling of repayments across states is much more diverse with n projects. Choosing any k^* $(1 \leq k^* \leq n-1)$ the pooling of repayments for reports of $k \geq k^*$ at a level equal to the cash flows at k^*,taking all the reported revenue from audited states with $k < k^*$, may cover the joint investment cost nI and the certain audit cost of k^*-1 projects. In the sense of Condition 2 above, for each such k^* we have a k^* pooling condition $PF(k^*)$ which should allow pooling of repayments above k^*. Then it should follow that if projects are k^*-feasible in this sense they are also k^*+1-feasible.

The audit cost of such a joint finance contract is $\Pr(k < k^*)c$ and hence the lowest deadweight loss pooling contract will involve the lowest possible k^* allowing the maximum degree of pooling. This will be characterised by $R_k = kH + (n-k)L$ for $k \leq k^*-1$ and to be incentive compatible must have positive audit chance below k^*. And indeed, from the description above, we expect $m_k = 1, k < k^*$. With n projects the chance of extreme number of fails/succeses falls, eg., the chance of k fails is $(1-p)^k$, which falls with k. So one expects that reports of fewer successes will be audited more intensively. For $k > k^*$, $m_k = 0$ and $R_k = k^*H + (n-k^*)L$. But if no pooling contract is possible, i.e., if the equivalent of the pooling feasibility (PF) condition fails to hold, but the individual feasibility (IF) condition still holds, the optimal joint finance contract cannot involve pooling.

This sketch extends the idea of pooling states and lowering expected audit cost to the multiproject case $(n > 2)$. The close relation to a standard debt contract is clear.

6.2 Revealing the project outcomes

We have assumed that the borrower knows the realised profit of each project and, in the event of just one project succeeding, reports this to the lender, specifying which project has failed. The possibility of disentangling the successful project from the failing one allows the lender to target audit only on fail reports. By contrast, in Banal-Estañol, Ottaviani and Winton (2013) the bankruptcy cost is incurred on each project and proportional to total realised returns, which is important in generating the risk contamination loss. An issue arises here, namely, why should the borrower reveal which project is the fail to the lender? In the following we show that it is in the borrower's interest to reveal this truthfully to the lender. This is easiest to see in the case of deterministic audit. Indeed, if he does declare which project failed, then the lender just has to audit it at cost c. If he does not, the lender randomises which project to audit after a report $\sigma = 1$. With probability 1/2 he picks the true fail to audit and does not have to go on to audit the other. With probability 1/2 he audits the successful project and then goes on to audit the other. So the expected audit cost overall is $c + 0.5c = 1.5c > c$. So the borrower chooses to reveal which is the fail truthfully. Similar forces arise in the case of endogenous random audit of projects. So long as the borrower can truly identify the realised profit on each project, it minimises his expected deadweight loss to report which project failed truthfully to the lender. Thus, our assumption that the borrower does this is without loss of generality.

6.3 No commitment

We have assumed commitment, i.e., the lender carries out the audit strategy announced in the contract even though he knows that there is always truthtelling. If the lender is an intermediary in turn financed by shareholders, then they can hold the lender to account to ensure audits are fulfilled. In a repeated contract setting, the lender could get away once with not carrying out his announced audit strategy. But in the next round the borrower should start anticipating that maybe if he cheats the lender will not monitor as stated in the contract. An alternative, even in a one shot contract, is that there is no commitment. After writing the contract, the lender can readjust his audit probability simultaneously with the borrower deciding what report to make. Typically this leads to a Nash equilibrium outcome of a non-cooperative game (Menichini and Simmons, 2006).

An alternative approach to ensuring commitment is to add a renegotiation proof constraint on the lender. Here the ex-ante contract is restricted to satisfy a renegotiation-proof constraint which removes any incentive for the ex-ante uninformed principal to change his action from that contracted once he has learnt the agent's action. For example, in our loan setting the ex-ante contract induces truthful borrower reports via the audit strategy, the lender knows the reports are truthful and so after receiving a low report has no incentive to audit. Generally in a loan contracting scenario this gives motivation for pooling repayments across true project outcomes in the ex-ante contract to prevent information revelation to the lender. This tends to favour aspects of a standard debt contract (Krasa & Villamil, 2000).

7 Conclusion

The CSV literature established some general principles in the context of a single risky borrower and lender where only the borrower knows her ex-post outcome. Since then, many studies have examined whether these principles hold up under different settings. One part of this has looked at some issues arising with many lenders and one borrower (Winton, 1995) or with many borrowers and lenders (Bond, 2004) still imposing a deterministic audit setting. In this paper we add to these contributions by seeing how some of these principles extend to a single borrower seeking finance from a single lender for two risky projects when the audit policy is optimally chosen. Should such a borrower seek a single loan for all the projects collectively or instead distinct loans for each project? And how should the borrowers incentive to cheat be controlled, namely, what is the cheapest incentive compatible audit strategy?

We find that some of the detailed principles of the optimal contract stand up, e.g., monotonicity of the audit probabilities or zero monitoring of the top state. But there are also some new forces, namely, joint finance is preferable to multiple single loans because it allows truthtelling at lower expected audit costs. This is obtained through the enlargement of the reporting space, with an intensive audit of the collective worst outcomes, less likely to occur, and a lower or no intensive audit of the intermediate outcomes. As a result, for certain regions of the parameter space, the resulting optimal joint finance contract is a standard debt contract.

In some more general settings, for example with several independent projects, the forces we identify should remain and the type of mechanism in which the reduced probability of a joint failure goes together with the highest audit frequency should lead to deterministic audit of the worst outcomes and no audit of the remaining ones with a debt contract for the conglomerate emerging endogenously for a sufficiently large number of projects. Another interesting avenue of analysis concerns the impact of correlation between project returns. We conjecture that while on one side positive correlation may reduce the benefits of joint financing, on the other side it may allow an audit cost saving to sequential audit as knowing the outcome on one project provides information about the outcome of the other. We leave the development of these extensions to future research.

References

[1] Banal-Estañol, A., Ottaviani, M., Winton, A.: The flip side of financial synergies: Coinsurance versus risk contamination, Rev. Fin. Stud. **26,** 3142–3181 (2013)

[2] Banal-Estañol, A., Ottaviani, M.: Separate or joint financing? Optimal conglomeration with bankruptcy costs. Working Paper, Universitat Pompeu Fabra and Bocconi University (2013)

[3] Becker, G. S.: Crime and punishment: An economic approach, J. Pol. Econ. **76**, 169-217 (1968)

[4] Bond, P.: Bank and nonbank financial intermediation, J. Finance, **59**, 2489–2529 (2004)

[5] Border, K. C., Sobel, J.: Samurai accounting: A theory of auditing and plunder. Rev. Econ. Stud. **54**, 525–540 (1987)

[6] Diamond, D. W.: Financial Intermediation and Delegated Monitoring, Rev. Econ. Stud. **51**, 393–414 (1984)

[7] Diamond, D.W.: Financial intermediation as delegated monitoring: A simple example, Econ. Quart. **82,** 51–66 (1996)

[8] Faure-Grimaud, A., Inderst, R.: Conglomerate entrenchment under optimal financial contracting. Am. Econ Rev. **95**, 850–61 (2005).

[9] Gale, D., Hellwig, M.: Incentive-compatible debt contracts: the one period problem. Rev. Econ. Stud. **52**, 647–663 (1985)

[10] Gertner, R. H., Scharfstein, D. S., Stein J. C.: Internal versus External Capital Markets. Quart. J. Econ. **109**, 1211-1230 (1994)

[11] Inderst, R., Müller, H.M.: Internal versus external financing: An optimal contracting approach. J. Finance **58**, 1033–1062 (2003)

[12] Khalil, F., Parigi, B.: Loan size as a commitment device. Int. Econ. Rev. **39**, 135–150 (1998)

[13] Krasa, S., Villamil, A. P.: Optimal contracts when enforcement is a decision variable. Econometrica **68**, 119–134 (2000)

[14] Leland, H.: Purely financial synergies and the optimal scope of the firm: Implications for mergers, spinoffs, and structured finance. J. Finance **62**, 765–807 (2007)

[15] Luciano, E., Nicodano, G.: Guarantees, leverage, and taxes, Rev. Fin. Stud. **27**, 2736–2772 (2014)

[16] Menichini, A. M. C., Simmons, P. J.: Liars and inspectors: optimal financial contracts when monitoring is non-observable. Contrib. Theor. Econ. **6**, 1–19 (2006)

[17] Menichini, A. M. C., Simmons, P. J.: Sorting the good guys from bad: on the optimal audit structure with ex-ante information acquisition. Econ. Theory **57**, 339–376 (2014)

[18] Mookherjee, D., Png, I.: Optimal auditing, insurance and redistribution. Q. J. Econ. **104**, 399–415 (1989)

[19] Phelan, G.: Correlated default and financial intermediation. J. Finance **72**, 1253–1284 (2017)

[20] Stein, J. C.: Internal capital markets and the competition for corporate resources. J. Finance, **52**, 111-133 (1997)

[21] Stein, J. C.: Information production and capital allocation: Decentralized vs. hierarchical firms, J. Finance **57**, 1891–1921 (2002).

[22] Townsend, R.: Optimal contracts and competitive markets with costly state verification. J. Econ. Theory **21**, 265–293 (1979)

[23] Winton, A.: Costly state verification and multiple investors: The role of seniority. Rev. Fin. Stud. **8**, 91–123 (1995)

[24] Winton, A.: Don't put all your eggs in one basket? Diversification and specialization in lending. Working Paper, U. Minnesota (1999)

A Appendix

Proof of Proposition 1. Using maximum punishment ($R_{0|H} = H$) in the optimisation problem $\mathcal{P}^{\text{sin}}$ and forming a Lagrangian with multiplier λ and μ, the FOC's wrt R_1, $R_{0|\cdot}$, $R_{0|L}$ and m_0 are

$$\begin{aligned}
\frac{\partial\mathcal{L}}{\partial R_1} &: (\lambda-1)\,p-\mu \geq 0, R_1 \leq H \\
\frac{\partial\mathcal{L}}{\partial m_0} &: (1-p)\left(R_{0|\cdot}-R_{0|L}\right)(1-\lambda)-\lambda\,(1-p)\,c+\mu\left(H-R_{0|\cdot}\right) \geq 0, m_0 \leq 1 \\
\frac{\partial\mathcal{L}}{\partial R_{0|L}} &: (\lambda-1)\,m_0\,(1-p) \geq 0, R_{0|L} \leq L \\
\frac{\partial\mathcal{L}}{\partial R_{0|\cdot}} &: (1-m_0)\,[(\lambda-1)\,(1-p)+\mu] \geq 0, R_{0|\cdot} \leq L
\end{aligned}$$

1. $\lambda > 1$.
   Suppose $\lambda = 1$. Then by $\frac{\partial\mathcal{L}}{\partial R_1}$, $\mu = 0$. By $\frac{\partial\mathcal{L}}{\partial m_0}$, this implies $-\lambda\,(1-p)\,c \leq 0$, a contradiction, since $\frac{\partial\mathcal{L}}{\partial m_0} \geq 0$.

2. $R_{0|L} = R_{0|\cdot} = f_L$.
   By $\lambda > 1$, $\frac{\partial\mathcal{L}}{\partial R_{0|L}}, \frac{\partial\mathcal{L}}{\partial R_{0|\cdot}} > 0$ and $R_{0|L} = R_{0|\cdot} = f_L$.

3. $R_1 < H$
   Using $R_{0|L} = R_{0|\cdot} = L$, $R_{0|H} = H$ and $m_0 = \frac{R_1-L}{H-L}$ from the incentive constraint, the contract problem becomes to choose R_1 to max $p\,(H-R_1)\,|\text{ st } pR_1+(1-p)\left(L-\frac{R_1-L}{H-L}c\right) = I$. The objective function is decreasing in R_1, while the participation constraint is increasing in it, provided Condition 1 holds ($\frac{\partial PC}{\partial R_1} = \frac{1}{H-L}\,[p\,(H-L)-(1-p)\,c]$). R_1 is then obtained by solving the participation constraint, giving $R_1 = \frac{(H-L)I-(1-p)L(H-L+c)}{p(H-L)-(1-p)c}$. Substituting out in m_0, gives m^{sin} (5). For $m^{\text{sin}} < 1$, $pH+(1-p)\,L-I-(1-p)\,c > 0$, which certainly holds under Assumption 1. This in turn implies from (3) that $R_1 < H$. The expected return to the borrower (6) is obtained using the solutions to the programme set out above in the objective function.

■

Proof of Proposition 2.

1. Maximum punishment for false reports
   From programme $\mathcal{P}^{\text{seq}}$ we see that the punishment repayments $R_{0|HH}, R_{1|H}, R_{0|H\cdot}, R_{0|HL}, R_{0|LH}$ only enter the incentive constraints. So by setting maximum punishment the right hand side of these increases and so either m_0 or m_1 or both can be reduced. For example if $R_{0|HH} < 2H$ then we can increase $R_{0|HH}$ and reduce m_0 keeping $m_0 m_{0,H} R_{0|HH}$ constant. This raises the right hand side of (9) because it raises $(1-m_0)\,R_{0|\cdot}$ and slackens (8) due to the decreased frequency of the audit cost $m_0 c$. In turn this allows

a reduction in R_2. Similar arguments apply to variations keeping $(1-m_{0,H})R_{0|H\cdot}$ constant and variations keeping $m_1 R_{1|H}$ constant (which has the added benefit of *ceteris paribus* reducing the left hand side and so slackening (11)). And finally variations keeping successively each of $m_0 m_{0,H} R_{0|HL}, m_0(1-m_{0,H})R_{0|H\cdot}$ constant. Thus $R_{0|HH} = R_{1|H} = 2H$, $R_{0|H\cdot} = R_{0|HL} = R_{0|LH} = H+L$. The right hand side of (9) and (11) is increasing in $m_{0,H}$, but this is neither in the objective nor in the participation constraint. So we can set $m_{0,H} = 1$, to slacken (9) and (11).

We can subsequently write the contract problem as:

$$\max\ p^2(2H-R_2) + 2p(1-p)\left\{H+L-m_1R_{1|L}-(1-m_1)R_{1|\cdot}\right\}$$
$$+(1-p)^2\left\{2L-m_0\left[m_{0,L}R_{0|LL}+(1-m_{0,L})R_{0|L\cdot}\right]-(1-m_0)R_{0|\cdot}\right\}$$

$$\text{st } p^2R_2+2p(1-p)\left\{m_1\left(R_{1|L}-c\right)+(1-m_1)R_{1|\cdot}\right\}+$$
$$(1-p)^2\left\{m_0\left[m_{0,L}\left(R_{0|LL}-c\right)+(1-m_{0,L})R_{0|L\cdot}-c\right]+(1-m_0)R_{0|\cdot}\right\}=2I$$

$$\begin{aligned} R_2 &\leq m_0 2H + (1-m_0)R_{0|\cdot} \\ R_2 &\leq m_1 2H + (1-m_1)R_{1|\cdot} \end{aligned}$$

$$m_1R_{1|L}+(1-m_1)R_{1|\cdot}\leq$$
$$m_0\left[\frac{1}{2}(1+m_{0,L})(H+L)+\frac{1}{2}(1-m_{0,L})R_{0|L\cdot}\right]+(1-m_0)R$$

2. $R_{0|L\cdot} = R_{0|\cdot} = R_{0|LL} = 2L$.
If $R_{0|L\cdot} < 2L$ and $R_2 > 0$ we can reduce R_2 and raise $R_{0|L\cdot}$ so that $p^2R_2 + (1-p)^2 m_0(1-m_{0,L})R_{0|L\cdot}$ stays constant, leaving both the objective function and the participation constraint unchanged. This slackens the first two incentive constraints and the right hand side of the third incentive constraint, again allowing a reduction in m_0. Similarly, we can reduce R_2 and raise $R_{0|\cdot}$ so that $p^2R_2 + (1-p)^2(1-m_0)R_{0|\cdot}$ stays constant, leaving both the objective function and the participation constraint unchanged, while slackening the three incentive constraints. We know $R_2 > 2L > 0$ since if $R_2 \leq 2L$ there is insufficient revenue to recoup the investment cost. Hence such reductions in R_2 are always possible. The result is $R_{0|L\cdot} = R_{0|\cdot} = 2L$. $R_{0|LL}$ only appears in the objective function and the participation constraint. Using $R_{0|\cdot} = R_{0|L\cdot} = 2L$, we have that lowering R_2 and raising $R_{0|LL}$ so as to keep $p^2R_2 + (1-p)^2 m_0 m_{0,L} R_{0|LL}$ constant

leaves both the objective and the participation constraint unchanged, while slackening the first and second incentive constraint. So, also $R_{0|LL} = 2L$.

3. $m_0 > 0$ and constraint (26) binding
With $R_{0|\cdot} = R_{0|L\cdot} = R_{0|LL} = 2L$, the contract problem becomes $(\mathcal{P}^{\text{seq}'})$:

$$\max p^2 (2H - R_2) + 2p(1-p)\left[H + L - m_1 R_{1|L} - (1-m_1) R_{1|\cdot}\right]$$

$$\text{st } p^2 R_2 + 2p(1-p)\left[m_1 \left(R_{1|L} - c\right) + (1-m_1) R_{1|\cdot}\right] + \\ + (1-p)^2 \left[2L - m_0 (1 + m_{0,L}) c\right] = 2I$$

$$R_2 \leq 2m_0 H + 2(1-m_0) L \tag{24}$$
$$R_2 \leq 2m_1 H + (1-m_1) R_{1|\cdot} \tag{25}$$

$$m_1 R_{1|L} + (1-m_1) R_{1|\cdot} \leq \frac{1}{2} m_0 (1 + m_{0,L})(H - L) + 2L \tag{26}$$

If $m_0 = 0$, the first and third incentive constraints would give $R_2, R_{1|L}, R_{1|\cdot} \leq 2L$. Then the available revenue is no higher than $2L - 2p(1-p) m_1 c$, which is less than $2I$. So we must have $m_0 > 0$. Moreover, constraint (26) must be binding. If not, it would be possible to lower m_0 slackening the participation constraint, thus allowing a reduction in R_2.

4. $m_1 = 0$
The variables are $R_2, R_{1|\cdot}, R_{1|L}, m_0, m_{0,L}, m_1$. We know that $m_0 > 0, R_2 > 2L$ to provide sufficient expected revenue to repay the debt. So we can eliminate these two variables from the binding participation constraint and the binding incentive constraint (26), obtaining $m_0 = 2\frac{(1-m_1)R_{1|\cdot} + m_1 R_{1|L} - 2L}{\left(1+m_{0,L}\right)(H-L)}$, and $R_2 = \frac{2(1-p)\left\{(1-p)\left[m_1 R_{1|L} + (1-m_1) R_{1|\cdot} - 2L\right] + m_1 p (H-L)\right\} c}{p^2 (H-L)}$. Substituting them out in the objective function (obj) and in the incentive constraints 24 and 25 (IC_1, IC_2) leaves the variables $R_{1|\cdot}, R_{1|L}, m_{0,L}, m_1$. Starting from any feasible position in the variables, we can locally vary all the variables except $m_{0,L}$ in ways which keep each constraint unchanged (thus requiring $dIC_1 = dIC_2 = 0$) and see which directions of change will improve the objective function $(dobj)$. This requires the variations to

satisfy

$$
\begin{aligned}
dIC_1 &= \frac{\partial IC_1}{\partial R_{1|\cdot}} dR_{1|\cdot} + \frac{\partial IC_1}{\partial R_{1|L}} dR_{1|L} + \frac{\partial IC_1}{\partial m_1} dm_1 = 0 \\
dIC_2 &= \frac{\partial IC_2}{\partial R_{1|\cdot}} dR_{1|\cdot} + \frac{\partial IC_2}{\partial R_{1|L}} dR_{1|L} + \frac{\partial IC_2}{\partial m_1} dm_1 = 0.
\end{aligned}
$$

We use this to express local variations in $R_{1|\cdot}, R_{1|L}$ in terms of the variations in m_1, holding $m_{0,L}$ constant. Finally, we see the effect on the objective function:

$$
dobj = \frac{\partial obj}{\partial R_{1|L}} dR_{1|L} + \frac{\partial obj}{\partial R_{1|\cdot}} dR_{1|\cdot} + \frac{\partial obj}{\partial m_1} dm_1.
$$

Substituting in the variations in $dR_{1|\cdot}$ and $dR_{1|L}$ which ensure that IC_1 and IC_2 hold, we get:

$$
\frac{dobj}{dm_1} = \frac{-2p^2(1-p)c\left[1+m_{0,L}+p\left(1-m_{0,L}\right)\right](H-L)}{p\left[1+m_{0,L}+p\left(1-m_{0,L}\right)\right](H-L)-\left(1+m_{0,L}\right)(1-p)^2 c}.
$$

The above expression is negative. To see this, notice that the numerator is always negative, while the denominator term is positive. This can be seen by noticing that the derivative of the denominator with respect to $m_{0,L}$, $(1-p)\left[p(H-L)-(1-p)c\right]$, is always positive, and at $m_{0,L}=0$ the value of the denominator is positive. Thus, the objective function can be increased by reducing m_1 as far as possible, whilst preserving feasibility. A solution will then have $m_1 = 0$ so long as the implied $R_{1|\cdot}, R_{1|L}, R_2 \geq 0, R_2 < 2H, R_{1|L}, R_{1|\cdot} \leq H+L, 0 < m_0 < 1$, and there are sufficient revenues to repay the debt cost.[11] This amounts to Condition 2. Setting $m_1 = 0$ allows pooling the repayments in the top two states, thus removing any cheating incentives. So audit costs can be minimised.

5. $R_2 = R_{1|\cdot}$
With $m_1 = 0$, from the incentive constraint (25), because of monotonicity of repayments, we deduce that $R_2 = R_{1|\cdot}$. Also, because $m_1 = 0$, $R_{1|L}$ is never paid and it can be set to any value between 0 and $H+L$. Thus, the incentive constraint (24) becomes $R_{1|\cdot} \leq 2m_0(H-L)+2L$. By comparing constraints (24) and (26) we see that if constraint (26) is satisfied, then certainly (24) is. So, we can

[11] Notice this holds for any non-negative fixed value of $m_{0,L}$.

ignore constraint (24) and the contract problem becomes:

$$\max\ p^2 (2H - R_2) + 2p(1-p)\left(H + L - R_{1|\cdot}\right) \tag{27}$$

$$\text{st } p^2 R_2 + 2p(1-p) R_{1|\cdot} + (1-p)^2 \{2L - m_0 (1 + m_{0,L}) c\} = 2I$$

$$R_{1|\cdot} = \frac{1}{2} m_0 (1 + m_{0,L}) (H - L) + 2L$$

6. Determination of $R_{1|\cdot}$ and $m_0 (1 + m_{0,L})$
   The remaining monitoring probabilities only enter the constraints and only through the composite variable $m_0 (1 + m_{0,L})$. There is then a redundancy of instruments. In the participation constraint $R_{1|\cdot}$ needs to be high enough to cover the investment plus audit cost, but in the incentive constraint low enough to make cheating unprofitable between one and zero successes. Similarly, the audit probability $m_0 (1 + m_{0,L})$ has to be high enough to control the cheating incentive but low to minimise the audit cost. The balance between the two comes from solving the remaining binding constraints for $m_0 (1 + m_{0,L})$ and $R_{1|\cdot}$. We get $m_0 (1 + m_{0,L}) = \frac{4(I-L)}{p(2-p)(H-L)-2(1-p)^2 c}$, $R_{1|\cdot} = \frac{2(I-L)(H-L)}{p(2-p)(H-L)-2(1-p)^2 c} + 2L$.
   We next verify that $m_0 (1 + m_{0,L}) \leq 2$. Indeed, since there are many combinations of monitoring a zero success report, the condition that guarantees that any combination is feasible is that $m_0 (1 + m_{0,L}) = \frac{4(I-L)}{p(2-p)(H-L)-2(1-p)^2 c} \leq 2$. For this, we need $0 < 2(I - L) \leq p(2-p)(H-L) - 2(1-p)^2 c$, which certainly holds under Condition (2).
   We next verify that $R_{1|\cdot} \leq H + L$. This requires that $\left(\frac{1}{2} m_0 (1 + m_{0,L}) - 1\right)(H - L) \leq 0$ which is non-positive iff $m_0 (1 + m_{0,L}) \leq 2$. Thus, $R_{1|\cdot} = H + L$ when Condition (2) holds with equality. Hence, this is a feasible solution which also maximises the borrower's objective.

7. Substituting out $R_2 = R_{1|\cdot}$ derived above in the objective function (27) we get the expected profits (14) as reported in the proposition.

■

Proof of Proposition 4. Starting from programme $\mathcal{P}^{\text{seq}}$, consider the reduced form contract problem $\mathcal{P}^{\text{seq}'}$ in point 3 of the Proof of Proposition 2 with binding incentive constraint (26), repayments $R_2, R_{1|\cdot}, R_{1|L}$, and monitoring probabilities $m_0, m_{0,L}, m_1 \in [0,1]$.[12] In that proof, to prove that $m_1 = 0$, we used a variational argument. Starting from any feasible

[12] The remaining variables, whose value is set by maximum punishment and zero rent in the low state, are independent of the precise feasibility condition being used.

position in the variables, we locally varied them in ways which kept each constraint unchanged, seeing which directions of change improved the objective function. This required reducing m_1 as far as possible. However, the implied optimal values of $m_0(1+m_{0,L})$ and $R_{1|\cdot}$ require that Condition (2) is satisfied. When Condition (2) is not satisfied, the optimal values of $m_0(1+m_{0,L})$ and $R_{1|\cdot}$ implied by $m_1 = 0$ are infeasible under limited liability and monitoring probabilities restricted to the unit interval, namely $R_{1|\cdot} > H+L$ and $m_0(1+m_{0,L}) > 2$ (or, in the simultaneous case, $m_0 > 1$). Nevertheless, the reduced form objective function is still decreasing in m_1 and its lowest possible value is given by setting $m_0(1+m_{0,L})$ and $R_{1|\cdot}$ at their corners and $m_1 > 0$. This in turn implies that there is no pooling of the top two states, i.e., $R_2 > R_{1|\cdot} = H+L$. To determine the actual values of m_1 and R_2, we use the corner values of $R_{1|\cdot} = H+L$ and $m_0(1+m_{0,L}) = 2$ (or, in the simultaneous case, $m_0 = 1$) in the reduced form optimisation problem above $\mathcal{P}^{\text{seq}'}$. The contract problem becomes ($\mathcal{P}^{\text{weak}}$):

$$\max p^2(2H - R_2) + 2p(1-p)\,m_1\left(H + L - R_{1|L}\right)$$

$$\text{st } p^2R_2 + 2p(1-p)\left[H + L - m_1\left(H + L - R_{1|L} + c\right)\right] + (1-p)^2\,2(L-c) = 2I$$

$$R_2 \leq 2H \tag{28}$$
$$R_2 \leq m_1 2H + (1-m_1)(H+L) \tag{29}$$

$$m_1\left(R_{1|L} - H - L\right) = 0 \tag{30}$$

Notice that if constraint (29) is satisfied, then certainly constraint (28) is. So we can ignore (28). Moreover, from the binding incentive constraint (30), we see that $R_{1|L} = H+L$ (since we must have $m_1 > 0$). At this stage the problem is

$$\max p^2(2H - R_2)$$
$$\text{st } p^2R_2 + 2p(1-p)[H + L - m_1c] + (1-p)^2\,2(L-c) = 2I \tag{31}$$
$$R_2 \leq m_1(H-L) + H + L \tag{32}$$

Then (32) must bind since otherwise m_1 could be reduced, allowing a reduction in R_2 without violating (31). Solving (31) and (32), we get the expressions for m_1 and R_2 reported in the proposition, with m_1 positive (because Condition 2 is violated) and less than one ($m_1 - 1 = -2\frac{pH+(1-p)L-I-(1-p)c}{p[p(H-L)-2(1-p)c]} < 0$), and R_2 smaller than $2H$ and larger than $H+L$. Indeed, $R_2 - (H+L) = H + L + \left\{\frac{2(I-L)-p(2-p)(H-L)+2(1-p)^2c}{p[p(H-L)-2(1-p)c]}\right\}(H-L) > 0$. Substituting out R_2 in the objective function we get the expected profits (21) as reported in the proposition. ■