Contribution to a public good with altruistic preferences

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Abstract

This paper presents a model of private provision of a public good where individuals in a group have altruistic preferences and care about the private and public good consumption of the other members of the group. I show that increasing the level of altruism increases the Nash level of the public good, demonstrating that caring about others improves public good provision. I then compare the Nash level of the public good to the benchmark level of provision by a social planner who aggregates the preferences of the group. I find that if there are non-contributors to the public good in a Nash equilibrium, income inequality can cause over-provision of the public good as compared to the planner's benchmark. Over-provision can occur because the poorest in the society do not contribute and the richer individuals contribute to the public good as a way to improve the welfare of the poor. These results indicate that public goods cannot substitute the role of income transfers to the poor even when individuals are altruistic if the distribution of incomes is highly unequal.

Keywords: Public goods, Altruism, Private transfers.

JEL classification codes: C72, H41 ,D64

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1 Introduction

Individuals who contribute to a public good are often part of a community and care about the other members that benefit from it. This may be true if the group members are friends, family, or even larger communities - as is the case for local relief aid in the wake of pandemics, or the efforts of environmental or human rights groups. Moreover, the role of the public good may be of particular interest in settings where private transfers of income between individuals are not possible due to transaction costs or social norms, and it is not possible to directly give money to others that one cares about. The current paper addresses this topic by developing a model of private provision of a public good where individuals have altruistic preferences and care about all other members of their group. In particular, it seeks to understand the implications of altruism for the overall level of provision and members' welfare.

The paper shows that while caring about others is beneficial for public good provision, contrary to the common expectation of under-provision under a Nash equilibrium, being altruistic may lead to over-provision of the public good as compared to the welfare benchmark of a social planner. The intuition behind over-provision is connected to the re-distributive role played by a public good in a society where there is income inequality. Over-provision only occurs when there are individuals who benefit from but do not contribute to the public good. When there are non-contributors in equilibrium, the richest members of the group are the contributors and it is the poorest who do not contribute. Since the richer members of the group care about the welfare of the other members (including the poorer members) but have no means of transferring money privately to them, they compensate by contributing more to the public good. However, since the welfare of poorer members depends on their private consumption as well as their consumption of the public good, I show that allowing for private transfers of income may prevent over-provision when the distribution of income is highly unequal.

The paper makes three contributions to the literature. It presents, to my knowledge, one of the first analyses of a model of private provision of a public good with altruistic preferences. Secondly, this research also has policy implications. In terms of the model, over-provision may occur if the poorest in the society do not contribute and the rich give as a way to improve the welfare of the poor. However, since the richer members cannot directly increase the private consumption of others, the outcome may be a level of provision that is both 'too much' and inefficient as compared to a welfare benchmark. This indicates that public goods may not perfectly substitute the role of government income transfers to the poor when there is income inequality. Finally, the paper shows when the re-distributive role of a public good is limited due to income inequality, private transfers of income may play a complementary role to public good provision in the interest of achieving redistribution in a society. This research thus also serves as a preliminary step of a broader agenda that seeks to understand the connections between public good provision and private transfers.

The analysis of the paper unfolds in four steps. I first set up a model of private provision of a public good à la Bergstrom, Blume, and Varian (1986) where members of a group have altruistic preferences. Following Becker (1974), the social utility of a group member is modeled as a linear combination of his own private utility as well as the private utility of other group members. In the second step I show that such a game always has a unique Nash equilibrium and every contributor to the public good in this equilibrium enjoys a common level of consumption of a private good, and they consecrate the rest of their income to the public good. The third step establishes a comparative statics result showing that increasing the level of altruism increases the overall level of provision. Caring about others thereby improves public good provision. Finally, I show that the analysis of altruistic preferences has consequences for the welfare of individuals. I compare the Nash level of the public good to the level provided by a social planner who aggregates the preferences of the group. I show that at a strictly interior equilibrium, the level of the public good provided is always lower in the Nash equilibrium than the social planner's public good allocation. A counter example subsequently provides conditions on the income distribution such that if there are non-contributors, there can be over-provision of the public good as compared to the social planner's level. Under the same conditions on the income distribution, the amount of the public good provided with altruistic preferences is identical to the social planner's level when we allow for private transfers between individuals.

The paper relates to the literature on private provision of public goods, initiated largely by Bergstrom, Blume, and Varian (1986). The paper presented 'altruistic' donations to charity as an example of a public good. Donations to charity however do not directly benefit the donors unless they get some benefit from the 'joy of giving' (Ribar and Wilhelm (2002)). Recognizing this, Andreoni (1988) and Andreoni (1989), among others, proposed models of impure altruism as an alternative motivation for donations [1]. While the model à la Bergstrom, Blume, and Varian (1986) continued to be considered as the benchmark model for altruistic behavior in the subsequent large body of research on alternative motivations for charity [2], the implications of caring about the overall level of well-being of individuals rather than only about the amount of the public good provided were largely neglected. Existing papers consider specific forms of these altruistic preferences, such as Echazu and Nocetti (2015) who construct a model of caring donors who give to a public good that is consumed only by beneficiaries who do not contribute at all, in contrast with the model in the current paper where everyone can contribute to the public good. The analysis in Ley (1997) is the closest to the current paper, where agents' utilities are a weighted sum of two components. The first component is the private utility of the individual from his consumption of a private good and the public good, and the second component is the private utility of other individuals in the group. Ley (1997) however does not establish either existence or uniqueness of the Nash equilibria, nor does he examine the properties of such a Nash equilibrium. The paper shows that Pareto-efficient equilibria of an economy where individuals have such altruistically interlinked preferences remain Pareto-efficient in the selfish economy with no altruism. Ley also shows that for the sub-class of additively separable preferences, the optimal level of the public good does not change with the degree of altruism. On the other hand, the objective of the current paper is to examine the comparative statics of the Nash equilibrium and examine how altruism can impact the level of provision of the public good and welfare.

The paper also relates to the study of altruism in economics, initiated largely by

[1] see also Andreoni (1990), Steinberg (1987), and Cornes and Sandler (1994) for models where individuals gain some private utility from their own contribution to the public good as well as deriving utility from the total amount of contributions Such preferences may exist because individuals get a 'warm glow' from charitable contributions: they may take pride in contributing to charity, or they may want to show off their social status by donations, or simply feel joyful about the intrinsic act of giving (Andreoni (1990)).

[2] such as social pressure (DellaVigna, List, and Malmendier (2012), Name-Correa and Yildirim (2016)), conformism (Rotemberg (2014)), or signaling (Glazer and Konrad (1996)).

the seminal papers by Becker (1974) and Barro (1974)[3]. Recent papers have looked at a number of broad applications of this field, ranging from intergenerational altruism (see Gonzalez, Lazkano, and Smulders (2018) and Galperti and Strulovici (2017)) and games where the payoffs of players are interdependent Ray and Vohra (2020), to private transfers over a fixed social network of individuals with altruistic preferences (Bourlès, Bramoullé, and Perez-Richet (2017)). However, these papers do not include any analysis of public goods. Finally, the paper contributes to the literature on inter-connections between public good provision and private transfers. Buchholz and Konrad (1995) study the case of two countries with different marginal costs of production of the public good, where the country with the higher cost has the incentive to make private transfers to the country with the lower cost, in a two stage game where private transfers are decided in the first stage, and contributions to the public good are decided in the second stage. Their results rely on differing marginal costs, distinguishing it from both the classic Bergstrom, Blume, and Varian (1986) as well as the current paper. The current paper does not rely on differing productivities, and indeed, is the first to establish that transfers can occur even with identical marginal productivities if individuals are altruistic. Very few papers look at connections between public good provision and private transfers in the context of altruism. The implications of altruistic preferences on private transfers were first modeled by Arrow (1981), but his paper has no analysis of public goods. Arora and Sanditov (2016) consider a model where individuals with altruistic preferences, embedded in a fixed network, contribute to a public good for the specific case of constant relative risk aversion utility functions. They do not examine the implications of a general model, and they do not include an analysis of private transfers, as considered by the current paper. Even more recently, Bommier, Goerger, Goussebaile, and Nicolai (2019) consider a model involving altruism, public good provision, and private transfers with one developed country and many developing countries. The developed nation has altruistic preferences towards other developing nations and may transfer money to developing nations. All

[3]Becker's famous 'Rotten Kid' result showed that in the presence of a benevolent family member who transfers income to other members, even selfish 'rotten' members would maximize family income. Some early papers focus solely on whether the presence of a public good may cause the rotten kid theorem to fail (Bergstrom (1989)), or hold (Cornes and Silva (1999)) without considering the larger implications for public good provision.

countries derive utility from abatement of pollution, the public good. Unlike the setting of the current paper where both private transfers and public good provision are determined simultaneously, Bommier, Goerger, Goussebaile, and Nicolai (2019) model their game as a two stage process, where the amount of public good provided is determined by a simultaneous game in the first stage, and private transfers are decided only by the developed country in the second stage.

The rest of the paper is organized as follows: section 2 presents the model and the comparative statics results of increasing the level of altruism. Section 3 presents the analysis on the welfare implications of the model and section 4 concludes.

2 The model

This section discusses a model of contribution to a public good with altruism. There is a group of $N = \{1, 2, ..., n\}$ individuals with $n \geq 2$. A member $i \in N$ has an income $y_i \in [0, \bar{y}]$, where $\bar{y}$ is some strictly positive number. He must choose how to divide his income between his consumption of a private good, x_i, and his contribution to a public good g_i. A given profile of contributions $(g_1, g_2, ..., g_n)$ produces an amount of public good $G = \sum_i g_i$. An individual i's preferences have two components. The first is a private component: he derives utility from his consumption of the private good x_i, as well as the amount of the public good G. This private component is represented by the same utility function $U : [0, \bar{y}] \times [0, n\bar{y}] \to \mathbb{R}$ for all individuals in the group. In addition, individuals are altruistic and their preferences have a social component: they care *equally* about the levels of private utilities of *all* the other members in their group. We assume that private transfers of money between individuals are not possible in this model, either due to transaction costs, or social norms that prevent such transfers. Individual i's preferences are represented by the following function V:

$$V(x_1, ..., x_n, G, \alpha) = U(x_i, G) + \alpha \sum_{j \in N, j \neq i} U(x_j, G) \tag{1}$$

where $\alpha \in (0, 1)$ is a parameter measuring the degree of altruism. I assume that α is positive, meaning that individuals benefit when the private utility of group members increases. The assumption $\alpha < 1$ implies that the individual values his private utility strictly more than his social utility. U is also taken to be increasing and strictly concave [4]. I impose in addition the following assumption on U:

Assumption 1 *For any combination* $(x, G) \in [0, \bar{y}] \times [0, n\bar{y}]$

$$U_{xG}(x, G) \geq 0$$

Assumption 1 means that private consumption and the public good are complementary. This is satisfied for many commonly used utility functions: for instance, for Cobb-Douglas utility where $U(x, G)$ takes the form $U(x, G) = x^a G^{1-a}$ with $0 < a < 1$. Assumption 1, combined with a second assumption outlined below, is a sufficient condition that guarantees the results of this paper.

Assumption 2 *For any* $G \in [0, n\bar{y}]$ *and* $\alpha \in (0, 1)$

$$U_x(0,0)+(1-\alpha)U_G(0,0)+\alpha\sum_{i\in N}U_G(0,0) > 0 > U_x(\bar{y},G)+(1-\alpha)U_G(\bar{y},G)+\alpha\sum_{i\in N}U_G(\bar{y},G)$$

An implication of this assumption is that there exists at least one contributor in any equilibrium. We impose this condition to rule out equilibria where nobody contributes. This assumption will help us in establishing the uniqueness of the Nash equilibria of the model. Denote by $\mathcal{U}$ the class of utility functions that satisfy the assumptions above.

For a given distribution of incomes denoted by $\mathbf{y} = (y_1, y_2, ..., y_n)$ and a coefficient of caring α, denote the normal form game described above by $\mathcal{G}(\mathbf{y}, \alpha)$. The objective of this paper is to examine the properties of the Nash equilibrium of this game. Since the social utility function V is strictly concave, finding the Nash equilibrium corresponds to solving the following maximization problem for every individual i, given the contributions of members other than i:

$$\max_{x_i, g_i} \; U(x_i, g_i + \sum_{j\in N, j\neq i} g_j) + \alpha \sum_{j\in N, j\neq i} U(x_j, g_i + \sum_{j\in N, j\neq i} g_j)$$

subject to

$$x_i + g_i \leq y_i \tag{2}$$

[4]Derivatives are denoted by subscript i.e. U_x denotes the first order (partial) derivative of U with respect to x

$$g_i \geq 0 \text{ and } x_i \geq 0$$

Since the constraint (2) must always hold with equality, it is possible to transform the maximization problem above to

$$\max_{g_i} \; U(y_i - g_i, g_i + \sum_{j \neq i}^{n} g_j) + \alpha \sum_{j \in N, j \neq i} U(x_j, g_i + \sum_{j \neq i}^{n} g_j)$$

subject to:

$$0 \leq g_i \leq y_i$$

The first-order condition (necessary and sufficient for a solution to the above program) for any $i \in N$ is:

$$-U_x(x_i, G) + U_G(x_i, G) + \alpha \sum_{j \in N, j \neq i} U_G(x_j, G) \leq 0 \, \forall \, i \in N \tag{3}$$

and

$$-U_x(x_i, G) + U_G(x_i, G) + \alpha \sum_{j \in N, j \neq i} U_G(x_j, G) = 0 \;\; if \; g_i > 0 \tag{4}$$

Using this setting, we start by noting some properties of the Nash equilibrium (if it exists) will also help us to prove the existence of a unique Nash equilibrium, in the lemma below:

Lemma 1 *At any Nash equilibrium of the game $\mathcal{G}(\mathbf{y}, \alpha)$, all members who contribute to the public good consume the same level of the private good, denoted by $\tilde{x}$. The amount $\tilde{x}$ is also the threshold level of income required for a member to start contributing to the public good. Contributions to the public good are ordered in increasing order of income.*

Proof. We first show all agents who contribute to the public good have the same level of private good consumption, which will be denoted by $\tilde{x}$. Denote by $\mathbf{x}^* = (x_1^*, ..., x_n^*)$ the levels of private good consumption at an equilibrium corresponding to the levels of contribution to the public good $\mathbf{g}^* = (g_1^*, ..., g_n^*)$ for the n group members at the equilibrium. For contradiction, assume that in an equilibrium there exist arbitrary agents 1 and 2 who contribute to the public good and without loss of generality, that $x_1^* < x_2^*$. Since condition 4 holds at the equilibrium for both agents:

$$-U_x(x_1^*, G^*) + U_G(x_1^*, G^*) + \alpha \sum_{j \in N, j \neq 1} U_G(x_j^*, G^*)$$
$$= 0 = -U_x(x_2^*, G^*) + U_G(x_2^*, G^*) + \alpha \sum_{j \in N, j \neq 2} U_G(x_j^*, G^*)$$

Cancelling the common terms out:

$$\begin{aligned} &- U_x(x_1^*, G^*) + U_G(x_1^*, G^*) + \alpha U_G(x_2^*, G^*) \\ &= -U_x(x_2^*, G^*) + U_G(x_2^*, G^*) + \alpha U_G(x_1^*, G^*) \end{aligned}$$

Rearranging

$$U_x(x_2^*, G^*) - U_x(x_1^*, G^*) + (1-\alpha)[U_G(x_1^*, G^*) - U_G(x_2^*, G^*)] = 0$$

Since we assumed that $x_1^* < x_2^*$,

$$U_x(x_2^*, G^*) - U_x(x_1^*, G^*) < 0$$

by strict concavity of U.

By assumption 1

$$U_G(x_1^*, G^*) - U_G(x_2^*, G^*) \leq 0$$

as well, leading to a contradiction. A similar argument holds if $x_1^* > x_2^*$.

We next show that $\tilde{x}$ is also the critical level of income above which an agent starts contributing. We show this in two parts: first, we show that an individual with an income $y_i \leq \tilde{x}$ cannot be a contributor. To show this, suppose for contradiction that $y_i \leq \tilde{x}$ for some i and i is still a contributor. Then the private good consumption for agent i, $x_i^* < y_i$, and hence $x_i^* < \tilde{x}$. Since all contributors at any equilibrium have the same private good consumption, this is violating what was just proved above, the first part of Lemma 1.

We next show that anyone with an income higher than $\tilde{x}$ must be a contributor. To show this, assume for contradiction that there exists some i such that $y_i > \tilde{x}$ and i does not contribute. Then $x_i^* = y_i > \tilde{x}$, again violating what was just proved above.

The final part of Lemma 1 follows easily from the arguments stated just above. If $y_1 > y_2 > \tilde{x}$, for two agents 1 and 2, it follows that $y_1 - \tilde{x} = g_1^* > g_2^* = y_2 - \tilde{x}$. Hence, the contributions of agents are ordered in increasing order of their income. ■

Lemma 1 is driven by the assumption that everyone has the same preferences. This assumption implies that the threshold level of income above which members start contributing is the same for everyone and contributing members consecrate all of their income that crosses this threshold to the public good. Lemma 1 thus reveals two more insights about the model. First, contributing to a public good serves the purpose of redistributing income in an altruistic society. It is easy to see that when all members contribute to the public good, the redistribution achieved is complete - all the members consume same level of the private good, and the public good. Secondly, the threshold $\tilde{x}$ depends upon the distribution of income, and it is possible that there are non-contributors in equilibrium when there is income inequality. In this case, Lemma 1 shows that the wealthier individuals in the society contribute to the public good, and the poorer individuals free-ride. Free-riding behavior in an altruistic society is hence purely based on income inequality.

Lemma 1 establishes that in *any* equilibrium, all contributors must have the same level of private consumption. However, this does not prove existence or uniqueness of the equilibrium since there could be two equilibria with two sets of contributors. In the following proposition, we prove that the equilibrium is indeed unique.

Proposition 1 *The game $\mathcal{G}(\mathbf{y}, \alpha)$ admits a unique Nash equilibrium.*

Proof. We first show that a Nash equilibrium of the game $\mathcal{G}(\mathbf{y}, \alpha)$ always exists. For a given α, $\mathbf{y}$, the function

$$V(g_1, ..., g_n, \mathbf{y}, \alpha) = U(y_i - g_i, g_i + G_{-i}) + \alpha \sum_{j \in N, j \neq i} U(x_j, g_i + G_{-i})$$

is continuous, and given that $g_i \in [0, \bar{\mathbf{y}}]$ is a compact set for any agent i, then by Weierstrass theorem, the maximization problem with V admits a solution. Define for every i, $G_{-i} = \sum_{j \neq i} g_j$ and $\mathbf{x_{-i}} = (x_1, ...x_{i-1}, x_{i+1}, ..., x_n)$ (the profiles of total public good contributions and individual private good consumptions of members other than i). By Berge maximum theorem, the best response functions $g_i(G_{-i}, \mathbf{x_{-i}}, \mathbf{y}, \alpha)$ are continuous. Hence, by Brouwer's fixed point theorem, an equilibrium exists.

Next, we show uniqueness. We show in Lemma 1 that all contributors must have the same level of private consumption - the threshold level of income $\tilde{x}$ which must hold in any equilibrium. For contradiction, suppose that in fact there are two equilibria for a game $\mathcal{G}(\mathbf{y}, \alpha)$. Then in fact, there must exist two distinct thresholds, let us denote them by $\tilde{x}_1 \neq \tilde{x}_2$, with two sets of contributors, denoted by C_1 and C_2. Let us note the number of contributors by c_1 and c_2, and the amount of the public good G as G_1^* and G_2^* respectively as well. Without loss of generality, assume that $\tilde{x}_1 < \tilde{x}_2$.

We first argue that we must have $C_2 \subseteq C_1$. To see this, we use the fact that contributions are ordered in increasing order of income. Suppose we order individuals in increasing order of income i.e. $y_1 \leq y_2 \leq ... \leq y_n$. Then if agent n, the richest individual, is a contributor under the threshold $\tilde{x}_2$, he must be a contributor under $\tilde{x}_1$. Since the threshold $\tilde{x}_1$ is lower than $\tilde{x}_2$, the number of contributors under the threshold $\tilde{x}_1$ must be at least as large as under $\tilde{x}_2$.

We next argue that $G_1^* > G_2^*$. We use that $\tilde{x}_1 < \tilde{x}_2$, hence $y_i - \tilde{x}_1 > y_i - \tilde{x}_2$ for any $y_i \geq 0$. Summing these over the sets C_1 and C_2 and using that $C_2 \subseteq C_1$, we will have that $G_1^* = \sum_{i \in C_1}(y_i - \tilde{x}_1) > \sum_{i \in C_2}(y_i - \tilde{x}_2) = G_2^*$.

We next use that equation 4 must hold for any contributor in an equilibrium. We know from Assumption 2 that there must be at least one contributor in any equilibrium. Therefore, equation 4 will hold with equality for any contributor under the thresholds $\tilde{x}_1$ and $\tilde{x}_2$, giving us:

$$
\begin{aligned}
-U_x(\tilde{x}_1, \sum_{i \in C_1}(y_i - \tilde{x}_1)) + U_G(\tilde{x}_1, \sum_{i \in C_1}(y_i - \tilde{x}_1)) + \\
\alpha \sum_{j \neq i} U_G(x_j, \sum_{i \in C_1}(y_i - \tilde{x}_1)) = 0 = -U_x(\tilde{x}_2, \sum_{i \in C_2}(y_i - \tilde{x}_2)) \\
+U_G(\tilde{x}_2, \sum_{i \in C_2}(y_i - \tilde{x}_2)) + \alpha \sum_{j \neq i} U_G(x_j, \sum_{i \in C_2}(y_i - \tilde{x}_2))
\end{aligned}
$$

We add and subtract a term $\alpha U_G(\tilde{x}_1, \sum_{j \in C_1}(y_i - \tilde{x}_1))$ on the left hand side, and we do the same on the right hand side with the term $\alpha U_G(\tilde{x}_2, \sum_{i \in C_2}(y_i - \tilde{x}_2))$, giving us:

$$
\begin{aligned}
-U_x(\tilde{x}_1, \sum_{i \in C_1}(y_i - \tilde{x}_1)) + (1-\alpha) U_G(\tilde{x}_1, \sum_{i \in C_1}(y_i - \tilde{x}_1)) + \sum_{j \in N} U_G(x_j, \sum_{i \in C_1}(y_i - \tilde{x}_1)) \\
= -U_x(\tilde{x}_2, \sum_{i \in C_2}(y_i - \tilde{x}_2)) + (1-\alpha) U_G(\tilde{x}_2, \sum_{i \in C_2}(y_i - \tilde{x}_2)) + \sum_{j \in N} U_G(x_j, \sum_{i \in C_1}(y_i - \tilde{x}_2))
\end{aligned}
$$

Rearranging, and collecting common terms:

$$\underbrace{U_x(\tilde{x}_2, \sum_{i\in C_2}(y_i - \tilde{x}_2)) - U_x(\tilde{x}_1, \sum_{i\in C_1}(y_i - \tilde{x}_1))}_{<0}$$
$$+(1-\alpha)\{\underbrace{U_G(\tilde{x}_1, \sum_{i\in C_1}(y_i - \tilde{x}_1)) - U_G(\tilde{x}_2, \sum_{i\in C_2}(y_i - \tilde{x}_2))}_{<0}\}$$
$$+\alpha\sum_{j\in N}\underbrace{U_G(x_j, \sum_{i\in C_1}(y_i - \tilde{x}_1)) - U_G(x_j, \sum_{i\in C_2}(y_i - \tilde{x}_2))}_{?} = 0$$

Given our assumptions on the increasingness and concavity of U and the cross-derivative $U_{xG} = U_{Gx} \geq 0$ (the assumption 1), the first two pair of terms of the above equation are clearly negative. For the first pair of terms, we use that

$$\tilde{x}_2 > \tilde{x}_1 \text{ and } G_1^* > G_2^*$$

since U_x is decreasing in x and increasing in G, we have $U_x(\tilde{x}_2, \sum_{i\in C_2}(y_i - \tilde{x}_2)) < U_x(\tilde{x}_1, \sum_{i\in C_1}(y_i - \tilde{x}_1))$. The same type of argument establishes the inequality for the second pair of terms. The only term whose sign is difficult to determine is the third term of the above equation. We already know that $C_2 \subseteq C_1$, or $c_1 \geq c_2$. Any $j \in N$ is in one of three categories:

- j is a contributor under both $\tilde{x}_1$ and $\tilde{x}_2$; in this case the sign of the third pair of terms is negative exactly by the same arguments by which the second pair of terms is negative for that agent j;

- j is a non-contributor under both equilibria; in which case, the third pair of terms is signed negative because $x_j = y_j$ under both thresholds and $G_1^* > G_2*$;

- j is a non-contributor under x_2 and a contributor under x_1. In this case, we use that

$$U_G(x_j, \sum_{i\in C_1}(y_i - \tilde{x}_1)) - U_G(x_j, \sum_{i\in C_2}(y_i - \tilde{x}_2)) = U_G(\tilde{x}_1, \sum_{i\in C_1}(y_i - \tilde{x}_1)) - U_G(y_j, \sum_{i\in C_2}(y_i - \tilde{x}_2)) < 0$$

  using $\tilde{x}_1 < y_j$ and similar arguments as before.

We will then have that the left hand side is strictly negative and the right hand side is 0, giving us a contradiction. Hence, the threshold must be unique for any set of contributors C. ■

Lemma 1 combined with Proposition 1 implies that the amount of the public good provided at the equilibrium does not change with any redistribution of income which keep the set of contributors unchanged. Since every contributor i donates $y_i - \tilde{x}$, if we redistribute income in a way that does not change the set of contributors the threshold $\tilde{x}$ does not change, and hence neither does the amount of the public good provided. This is formally stated without proof in the following corollary:

Corollary 1 *Any redistribution in incomes that does not change the set of contributors at the equilibrium leaves the Nash level of the public good unchanged.*

The neutrality to redistribution of income as predicted in Corollary 1 is not specific to the model of public good provision with altruistic preferences, it holds true as well in particular for the model in Bergstrom, Blume, and Varian (1986). While the model in Bergstrom, Blume, and Varian (1986) had strong neutrality results such as predicting that the total amount of charity does not change when income is redistributed among the set of donors and complete crowding out of private donations by government provision, the joy-of-giving models of altruistic giving were thought to avoid these neutrality results (Ribar and Wilhelm (2002)). Corollary 1 reveals that the specification of altruism can indeed matter for determining the neutrality of income redistribution among the set of contributors. It should also be noted that neutrality may not hold if the current model is modified to the situation where individuals do not have the same level of altruism, making the case where government contributions are neutral an extremely specific case rather than the general one as proposed by Bergstrom, Blume, and Varian (1986).

Thanks to Proposition 1, we can now denote by $\mathbf{g}^*(\mathbf{y}, \alpha) = (g_1^*, ..., g_n^*)$ the unique Nash equilibrium of the game $\mathcal{G}(\mathbf{y}, \alpha)$ and by $G^*(\mathbf{y}, \alpha)$ the aggregate level of the public good at such an equilibrium.

We next examine a comparative statics result that determines how a change in the degree of altruism α affects contributions. The following proposition shows that a weak increase in altruism weakly increases the contributions of all contributors to the public good at the Nash equilibrium.

Proposition 2 *For any two levels of the altruism coefficient $\alpha_1, \alpha_2 \in (0,1)$, whenever $\alpha_1 \leq \alpha_2$, we have that $G^*(\mathbf{y}, \alpha_1) \leq G^*(\mathbf{y}, \alpha_2)$. Moreover, whenever $\alpha_1 < \alpha_2$, we have that $G^*(\mathbf{y}, \alpha_1) < G^*(\mathbf{y}, \alpha_2)$.*

Proof. Lemma 1 tells us that there exists a threshold level of private good consumption that determines the set of contributors at an equilibrium. We name these thresholds with a slight abuse of notation $\tilde{x}_1$ and $\tilde{x}_2$ for the levels of altruism α_1, α_2, and the aggregate levels of the public good by G_1 and G_2 respectively. To show the result it suffices to show that $\tilde{x}_1 \geq \tilde{x}_2$.

Assume for contradiction that $\tilde{x}_1 < \tilde{x}_2$. Since there must be at least one contributor at any Nash equilibrium, and $\tilde{x}_1 < \tilde{x}_2$, the individual contributing in the game $\mathcal{G}(\mathbf{y}, \alpha_2)$ must also be contributing in the game $\mathcal{G}(\mathbf{y}, \alpha_1)$. Condition 4 must hold for the same contributor, giving us:

$$\begin{aligned} &-U_x(\tilde{x_1}, G_1) + U_G(\tilde{x_1}, G_1) + \alpha \sum_{j \in N, j \neq 1} U_G(x^*_{j_1}, G_1) \\ = 0 = &-U_x(\tilde{x_2}, G_2) + U_G(\tilde{x_2}, G_2) + \alpha \sum_{j \in N, j \neq 2} U_G(x^*_{j_2}, G_2) \end{aligned}$$

where $x^*_{j_i}$, $i = 1, 2$ is the equilibrium private contribution of individual j under the altruism level α_i. Or, alternatively,

$$\begin{aligned} U_x(\tilde{x_2}, G_2) - U_x(\tilde{x_1}, G_1) + (1 - \alpha)\{U_G(\tilde{x_1}, G_1) - U_G(\tilde{x_2}, G_2)\} \\ + \alpha_1 \sum_{j \in N} U_G(x^*_{j_1}, G_1) - \alpha_2 \sum_{j \in N} U_G(x^*_{j_2}, G_2) = 0 \quad (5) \end{aligned}$$

By our assumption for the proof $\tilde{x}_1 < \tilde{x}_2$, and this implies straightforwardly that that $G_1 > G_2$. It is clear also that for individuals j who are contributors under both the levels of altruism α_1 and α_2, $x^*_{j_1} = \tilde{x}_1 < \tilde{x}_2 = x^*_{j_2}$. For non-contributors under the altruism regime α_1 who also evidently stay non-contributors under α_2, $x^*_{j_1} = y_j = x^*_{j_2}$. This leaves only the sub-group of non-contributors under the altruism regime α_2 who may become contributors under the regime α_1. For any such person, we have that $x^*_{j_1} = \tilde{x}_1 < y_j = x^*_{j_2}$. Hence we can conclude that $x^*_{j_1} \leq x^*_{j_2}$ for all individuals $j \in N$.

Using the strict concavity of U in x and assumption 1,

$$U_x(\tilde{x_2}, G_2) - U_x(\tilde{x_1}, G_1) < 0$$

and using the strict concavity of U in G and assumption 1,

$$U_G(\tilde{x_1}, G_1) - U_G(\tilde{x_2}, G_2) < 0$$

and finally, using the same reasoning ,for all j

$$U_G(x^*_{j_1}, G_1) - U_G(x^*_{j_2}, G_2) < 0$$

and using that $\alpha_1 \leq \alpha_2$, for all j

$$\alpha_1 U_G(x^*_{j_1}, G_1) - \alpha_2 U_G(x^*_{j_2}, G_2) < 0$$

giving us

$$\alpha_1 \sum_{j \in N} U_G(x^*_{j_1}, G_1) - \alpha_2 \sum_{j \in N} U_G(x^*_{j_2}, G_2) < 0$$

The left hand side of equation 5 is thus strictly less than zero, giving us a contradiction. Similar arguments hold when $\alpha_1 < \alpha_2$.

■

The structure of the proof of Proposition 2 shows that when the level of altruism strictly increases, the threshold level of private consumption strictly decreases, meaning that the individuals who contributed before the increase in altruism strictly increase their contributions, and in addition, there may be new contributors. A strict increase in altruism therefore not only strictly increases the total amount of the public good, it strictly increases the contributions of all existing contributors. Being altruistic in the current model helps improve public good provision.

3 Welfare

We can infer from Proposition 2 that the amount of the public good is higher when the group is more altruistic. Does it imply that when the group is more altruistic, level of welfare for everyone in the group also increases? Comparisons of well-being between Nash equilibria of public good models can be complex (see for instance Banerjee and Gravel (2020)). In case of the current paper, while the level of the public good is higher when the group's level of altruism increases, the levels of some of the individuals' private consumption decrease and hence the net effect on well-being can be ambiguous. A different metric of welfare to consider is the Pareto-efficiency of the Nash equilibrium. While it is widely understood that the Nash equilibrium level of a public good is typically Pareto-inefficient (Cornes and Sandler (1996)), this is true if there exists a Pareto-optimal allocation with a higher level of the public good than in the Nash equilibrium. This may not happen in many cases, such as when the Nash equilibrium is not strictly interior and there are non-contributors in the equilibrium (Buchholz and Peters (2001)).

Despite these complexities, it is nevertheless conceivable to compare the Nash outcome against the metric of a benevolent social planner's preferred allocation of resources. The first step in this endeavor is to define the social planner's objective function. There are two possible ways of formulating the social planner's function. In the first formulation, the benevolent social planner aggregates only the private preferences of the individuals, making the social planner's objective function:

$$\sum_{i \in N} U(x_i, G) \tag{6}$$

Alternatively, one might think that the social planner takes both the private and the social component of the individual utilities into account when aggregating the preferences of all the individuals in the group. The social planner's function can be then written as:

$$\sum_{i \in N} (U(x_i, G) + \alpha \sum_{j \in N, j \neq i} U(x_j, G)) \tag{7}$$

Removing the inner summation sign, the function 7 can rewritten as:

$$(1 + (n-1)\alpha) \sum_{i \in N} U(x_i, G) \tag{8}$$

Since the function in expression (8) is just a monotonic transformation of the function in expression (6), it follows that irrespective of whether we choose the formulation in expression (6) or expression (7) (or any convex combination of the two), the social planner's preferred solution to the public good allocation problem remains unchanged.

The strict concavity of the social planner's function ensures that the social planner's program as presented in expression 6 has a unique interior solution. Hence the social planner would allocate the same, strictly positive level of the private good to all the members of the group, and we denote this level of private good consumption by $\bar{x}$. This also makes intuitive sense because the utility function is identical for all members of the group. We denote the level of public good allocated by the social planner by $\bar{G}$. It is also worth remarking that this solution is by definition Pareto-efficient. The following proposition looks at how the Nash level of provision of the public good compares to the level $\bar{G}$ chosen by the social planner.

Proposition 3 *If the game $\mathcal{G}(\mathbf{y}, \alpha)$ has a strictly interior equilibrium, then $G^* < \bar{G}$. However, if there are non-contributors to the public good in the Nash equilibrium then there may be over-provision of the public good in the Nash equilibrium as compared to the social planner's solution.*

Proof. Proposition 3 has two parts. Here I provide the proof of the first part of Proposition 3 establishing under-provision of the public good under the Nash equilibrium. The strict concavity of the utility functions U ensures that there is a unique maximizer to the expression 6. Using the Lagrangian to solve for the maximum demonstrates quite easily that the social planner solution would allocate the same, strictly positive level of the private good to all the members of the group, and we denote this level of private good consumption by $\bar{x}$. We also denote the level of public good allocated by the social planner by $\bar{G}$.

According to Lemma 1, at a strictly interior Nash solution, every contributor will have the same level of consumption of the private good, and let us denote this by $x_i^* = x^*$ for all i, and the level of the public good is denoted by G^* as before.

Suppose for contradiction that $G^* \geq \bar{G}$. when the Nash is strictly interior. It is then straightforward to show that $x^* \leq \bar{x}$. Since the three different formulations of the social planner's objective function yield the same solution, I use the social planner's objective function as per expression 6:

$$\sum_{i \in N} U(x_i, G)$$

Formulating the Lagrangian to find the maximum of this function gives us that the social planner's solution is characterized by the familiar Samuelson condition:

$$\sum_{i\in N} \frac{U_G(\bar{x},\bar{G})}{U_x(\bar{x},\bar{G})} = 1$$

which can be rewritten as

$$\frac{U_G(\bar{x},\bar{G})}{U_x(\bar{x},\bar{G})} = \frac{1}{n}$$

Using the Lagrangian to solve for the Nash solution gives us a similar condition:

$$\frac{U_G(x^*,G^*)}{U_x(x^*,G^*)} = \frac{1}{1+\alpha(n-1)}$$

$\alpha < 1$ implies that $\frac{1}{1+\alpha(n-1)} > \frac{1}{n}$ and we note

$$\frac{U_G(\bar{x},\bar{G})}{U_x(\bar{x},\bar{G})} - \frac{U_G(x^*,G^*)}{U_x(x^*,G^*)} < 0$$

Given our assumptions, the marginal rate of substitution $\frac{U_G(x,G)}{U_x(x,G)}$ is decreasing in G and increasing in x, meaning that given $G^* \geq \bar{G}$ the expression on the left hand side must be non-negative, giving us a contradiction.

■

The second part of the proposition related to over-provision of the public good can be easily demonstrated with an example of two agents and additively separable preferences, as shown below:

Example 1 *The private utility function of any player i takes the form:*

$$U(x_i, G) = a\log x_i + b\log G \text{ with } 0 < a < 1,\ 0 < b < 1,\ a+b = 1 \tag{9}$$

Two agents 1 and 2 have incomes y_1 and y_2. Assume without loss of generality that $y_1 \leq y_2$. The Nash equilibrium has only one contributor when $y_1 < \frac{ay_2}{a+b(1+\alpha)}$. We know from Lemma 1 that in the case when there is only one contributor, only the richer individual, member 2 will contribute. A comparison of the levels of private and public good provided under the social planner's regime and in the Nash equilibrium is presented in the first two columns of Table 1. The level of provision of the public good by the social planner is strictly lower than the level of provision with altruistic preferences if $y_1 < \frac{\alpha a y_2}{a+b(1+\alpha)}$.

	Social Planner's solution	Nash solution	Model with transfers
		Both members contribute when	
		$y_1 > \frac{ay_2}{a+b(1+\alpha)}$	$y_1 > \frac{ay_2}{a+b(1+\alpha)}$
x_1^*	$\frac{1}{2}\frac{a(y_1+y_2)}{a+b}$	$\frac{a(y_1+y_2)}{2a+b(1+\alpha)}$	$\frac{a(y_1+y_2)}{2a+b(1+\alpha)}$
x_2^*	$\frac{1}{2}\frac{a(y_1+y_2)}{a+b}$	$\frac{a(y_1+y_2)}{2a+b(1+\alpha)}$	$\frac{a(y_1+y_2)}{2a+b(1+\alpha)}$
G^*	$\frac{b(y_1+y_2)}{a+b}$	$\frac{b(1+\alpha)(y_1+y_2)}{2a+b(1+\alpha)}$	$\frac{b(1+\alpha)(y_1+y_2)}{2a+b(1+\alpha)}$
t_{21}			-
		Only member 2 contributes when	
		$y_1 \leq \frac{ay_2}{a+b(1+\alpha)}$	$y_1 \leq \frac{ay_2}{a+b(1+\alpha)}$
			Transfers exist when
			$y_1 \leq \frac{\alpha ay_2}{a+b(1+\alpha)}$
x_1^*	$\frac{1}{2}\frac{a(y_1+y_2)}{a+b}$	y_1	$\frac{1}{1+\alpha}\frac{a(y_1+y_2)}{a+b}$
x_2^*	$\frac{1}{2}\frac{a(y_1+y_2)}{a+b}$	$\frac{ay_2}{a+b(1+\alpha)}$	$\frac{\alpha}{1+\alpha}\frac{a(y_1+y_2)}{a+b}$
G^*	$\frac{b(y_1+y_2)}{a+b}$	$\frac{b(1+\alpha)y_2}{a+b(1+\alpha)}$	$\frac{b(y_1+y_2)}{a+b}$
t_{21}			$\frac{a\alpha y_2-[a+b(1+\alpha)]y_1}{(1+\alpha)(a+b)}$

Table 1: Levels of provision of the public and the private good with Example 1

Proposition 3 offers three key insights into the welfare implications of contributing to a public good with altruistic preferences. Firstly, the intuition behind the under-provision of the public good is closely related to Lemma 1. Since all contributors must have the same level of consumption of the private good, contributing to a public good acts as a means of redistribution in a society. However, given that the objective of redistribution is achieved, as shown in Table 1, the social planner's solution will allocate a larger fraction of the total resources $y_1 + y_2$ to the public good because of the free-riding problem which typically makes the provision of a public good inefficient. Secondly, Proposition 3 also shows that the presence of non-contributors can impede redistribution and lead to inefficiency by over-provision even when individuals are altruistic. While the idea that the amount of public good provided under the Nash equilibrium can exceed the social planner's allocation is by no means new (Buchanan and Kafoglis (1963)), there is very little analysis in the existing literature which explains when over-provision occurs, or why. Buchholz and Peters (2001) suggest that the existence of over-provision might depend on non-normality of preferences, but this explanation does not address agents' preferences for altruism despite the common interpretation of charitable donations as a public good. As further shown by the example with log preferences, the presence of non-contributors is endogenous and may depend on the the distribution of income in the society.

Finally and most importantly, Proposition 3 reveals that a deeper understanding of over-provision relies on a more complex public good model with altruistic preferences involving private transfers. Altruistic individuals may care about both the total amount of the public good as well as the private consumption of other individuals. As demonstrated by Example 1, the extent of redistribution possible is limited when poorer individuals do not contribute to the public good. The most direct way to redistribute income would be through a direct income transfer: however, such transfers may not be possible due to transaction costs. In the absence of such private transfers, if the existing distribution of income is sufficiently skewed, the altruistic rich contribute more to the public good than the social planner in an attempt to improve the welfare of the poor - however, since the welfare of the poor also depends on their consumption of the private good, the resulting level of the public good may be inefficiently high.

To understand why over-provision happens in the absence of private transfers, consider a modification of Example 1. In addition to the existing model of altruistic preferences in public good provision, we now introduce the possibility of the two individuals to make private transfers of income to each other. An agent i can choose how to divide his income y_i between his private consumption x_i, the amount contributed to the public good g_i, and the amount of money he privately transfers to the second agent j, t_{ij}. Let us assume that transfers are non-negative ($t_{ij} \geq 0$), and also that an agent cannot transfer any money to himself ($t_{ii} = 0$). The preferences of the two individuals i and j stay the same, and the private utility functions take the same form of log preferences as seen in Example 1. Adding private transfers changes the model by affecting net income, so the budget constraint of the individuals changes. The budget constraint of individual i is now given by $x_i + g_i = y_i - t_{ij} + t_{ji}$. Example 2 summarizes the new model for an agent i:

Example 2 *The maximization problem for any agent i is given by:*

$$\max_{x_i, t_{ij}, g_i} a \log x_i + b \log G + \alpha a \log x_j + \alpha b \log G \tag{10}$$

subject to:

$$x_i + g_i = y_i - t_{ij} + t_{ji}$$

$$x_i \geq 0,\ g_i \geq 0,\ t_{ij} \geq 0,\ t_{ji} \geq 0,\ t_{ii} = 0$$

The third column of Table 1 shows the levels of private and public good provided under this modified model which allows for private transfers, as well the conditions necessary and sufficient for the existence of transfers. Three results in particular are worth noting:

- The threshold level of income required to become a contributor $\tilde{x} = \frac{ay_2}{a+b(1+\alpha)}$ in our model with altruistic preferences is relatively defined in terms of the income of the richer member (member 2). Moreover, this threshold does not change once we allow for private transfers. Hence when $y_1 > \frac{ay_2}{a+b(1+\alpha)}$, both the model with and without transfers have two contributors, and no private transfers occur between individuals.

- Transfers do not always exist. Transfers exist when $y_1 \leq \frac{a\alpha y_2}{a+b(1+\alpha)}$, and this threshold is strictly lower than the threshold $\tilde{x}$. In other words, it is possible for the distribution of income to be unequal, but enough to not merit a transfer, and in this case, the level of public good provided does not exceed the social planner's level of provision.

- When transfers exist, the level of public good provided under the model with transfers is exactly equal to the social planner's level of provision. In other words, there is no longer over-provision of the public good. Hence, allowing for private transfers removes the need for the level of public good provision to be inefficiently high. It is worth noting that the levels of private good at the equilibrium of the model with transfers is not the same as the social planner's level even when the levels of public good provision under the two models are the same.

The intuition provided above has policy implications for government cash transfers to the poor. If the distribution of income in society is highly unequal and private transfers of income are not possible, an over-provision of private provision may occur when richer individuals contribute to the public good as a way to improve the welfare of the poor. Since the poor care about the consumption of both the private and the public good, the public good may not directly improve the private consumption of the poor. The outcome may be a level of provision that is both 'too much' and inefficient. Proposition 3 thus indicates that public goods may not perfectly substitute the role of government income transfers to the poor.

The question of the efficiency of government transfers of income versus private transfers of income when there is a public good involved is an important one, but beyond the scope of the current analysis. The model outlined in Example 2 with transfers is a very specific case of a more general model of altruistic preferences with transfers, and the connections between the current paper and the model with transfers is a matter of future research.

4 Conclusion

This paper presents a model of altruistic preferences in private provision of a public good where individuals care about both the private consumption as well as the total amount of the public good consumed by the other individuals in their group. Several real life-situations fit this scenario - contributing to local relief efforts in the wake of crises, preventative measures from natural disasters or efforts by human rights groups are all examples of such situations. On one hand, the paper showcases the benefits from being altruistic as it increases contributions to the public good. If people care more, they give more, and this helps in the redistribution of income in society. On the other hand the paper cautions against relying entirely on altruistic tendencies as it cannot completely substitute the role of government income transfers to the poor when there is income inequality in the society even when individuals are caring. The paper shows that altruism in this can lead to an inefficiencies where the rich contribute more as a way to help the poor, but a social planner who can directly transfer money to the poor can redistribute income more efficiently. A natural question to ask in this context would be on the role a public good plays in a society where individuals can make private transfers to other individuals. The paper presents an example where allowing individuals to transfer money privately makes the level of the public good provided identical to the level provided by a social planner, but the interactions between private transfers and the public good are a topic of future research since a complete understanding of the welfare implications would require an analysis of a framework of a public good model with private transfers.

The current paper differs somewhat from reality in that it does not address the case where different people care differently for others. There are several possible variations of this. People may care similarly for their family members (an in-group) and care less about a bigger community (an outgroup). Or alternatively, the model presented in the paper can be modified for every individual to care about other individuals differently, but to regard the other individuals in the same way. Finally, it is possible to think of a fixed network setting where some individuals have a network of people that they care about. Addressing these settings would require additional assumptions on the model and these questions deserve to be considered in future research.

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