

Distributional aspects of rent seeking activities in a DSGE setup

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Abstract

This paper studies the link between the quality of institutions and economic performance both on a macroeconomic level as well as on the issue of income distribution among different types of agents. For this reason, we introduce heterogeneous agents (capitalists and workers) in a real business cycle model with rent seeking competition. In this framework each type of agent allocates a part of their effort time in order to extract a fraction of a contestable prize. For each type of agent the contestable prize consists of the income of the other type of agents. The degree of extraction depends on the quality of institutions. When studying the behaviour of the model in terms of wedges and propagation mechanism, long-run behaviour, second moment properties and impulse response functions, the introduction of rent seeking in a heterogeneous agents model reveals both qualitative and quantitative differences on the distributional level.

Keywords: Institutions, rent seeking, heterogeneous agents, DSGE

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1 Introduction

The economic and social consequences of the recent crisis that started in 2007-08 have been more severe and deep in countries of Southern Europe compared to countries of Northern Europe. Looking at country data on the quarterly Real per capita GDP for Germany, Greece, Spain, Italy and Portugal and the Eurozone average (EURO19) in Figure 2, we see that there are differences between the countries in the graph both in their level of Real per capita GDP as well as in their behavior after the economic crisis. More specifically, observe that Germany is above the Eurozone average in contrast to Italy, Spain, Greece and Portugal which are below the Eurozone average. It is also clear that the economic crisis has affected all countries negatively; however only in Germany the Real per capita GDP has returned to its pre-crisis levels in the early years after the economic crisis. The remaining countries have a different behavior as they are still recovering from the economic crisis with Greece being the country that experienced the most severe and persistent decrease in Real per capita GDP.

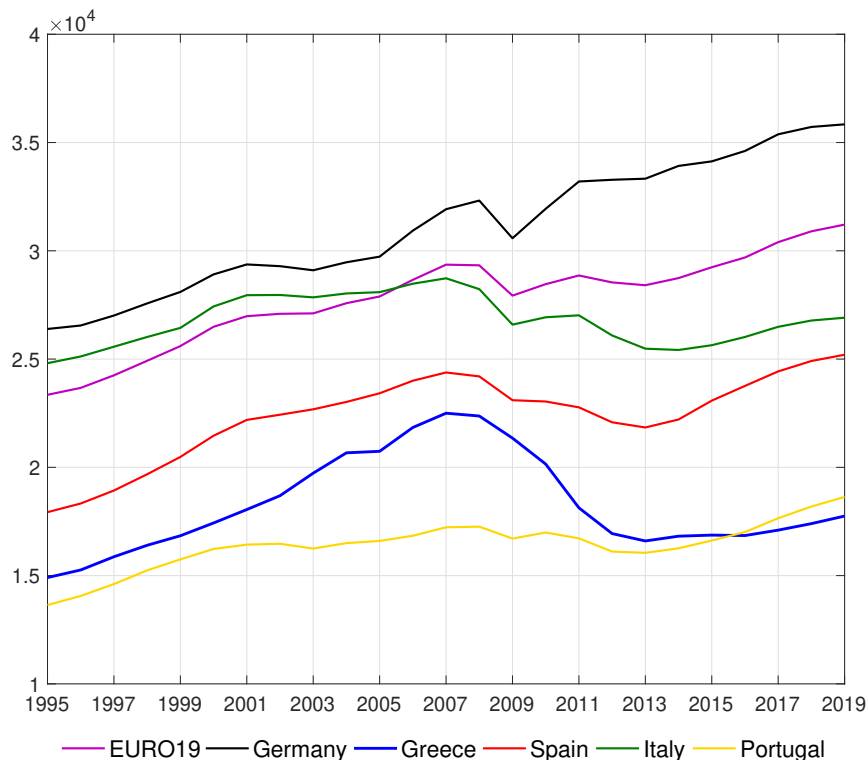


Figure 1: Real per capita GDP

What could lie behind these distinct asymmetries between Eurozone countries? Light on this could be shed by looking at the total factor productivity. In Figure 9 we present the Total factor productivity series for the six countries: Germany, Greece, Spain, Italy and

Portugal.¹ According to Prescott (1986), total factor productivity is a parameter that measures the efficiency in which inputs are used in production. In this figure, we see that Germany is the most efficient country in terms of total factor productivity, in contrast to Greece which is the country with the lowest efficiency in the graph. Looking in more depth, we see that the effects of the 2007-8 economic crisis are manifested in all countries with Greece, Italy and Spain having the sharpest and more persistent decrease. Also, observe that the decrease in total factor productivity in Greece, Italy, Spain and Portugal started much before the economic crisis coinciding with the Eurozone.

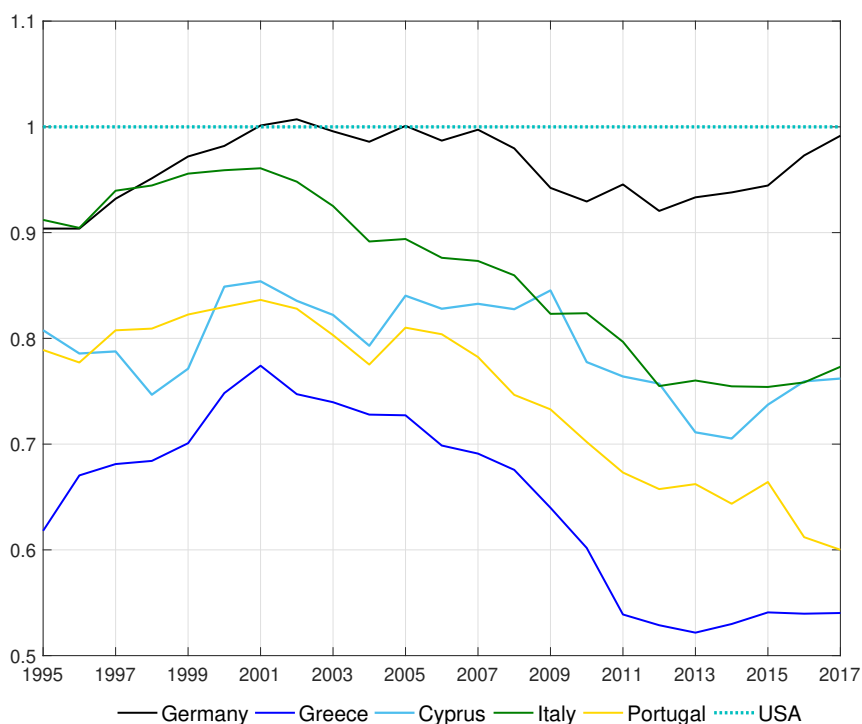


Figure 2: Total factor productivity (USA=1), St. Louis FED

There is plenty of evidence both on theoretical and empirical level on the importance of institutions and macroeconomic performance. Moreover, Christou et al. (2021), find that institutions matter and are fundamental causes of cross-country asymmetries in trends and cycles observed for Eurozone countries. The nexus between macroeconomic performance and institutions is depicted in Figures 3-4. In Figure 3, we present the International Country Risk Guide (ICRG), a widely used index regarding the quality of institutions, for the years 1994-2015, produced by the Political Risk Services (PRS) Group. This comprises 22 variables of risk evaluation grouped in three categories: a) political risk (government stability, socioeconomic

¹We obtain the series for Total Factor Productivity from St. Louis FED; this is an index where USA take the value 1.

conditions, investment profile, internal and external conflict, corruption, military in politics, religious tensions, law and order, ethnic tensions, democratic accountability and bureaucracy quality), b) economic risk (GDP per capita, Real GDP growth, annual inflation rate, budget balance as percentage of GDP and current account as percentage of GDP), c) financial risk (foreign debt as a percentage of GDP, foreign debt service as a percentage of exports of goods and services, current account as a percentage of exports of goods and services, net international liquidity and exchange rate stability). The ICRG index takes a maximum value of 100, where higher values reflect better institutional quality. In our sample of countries, Germany and

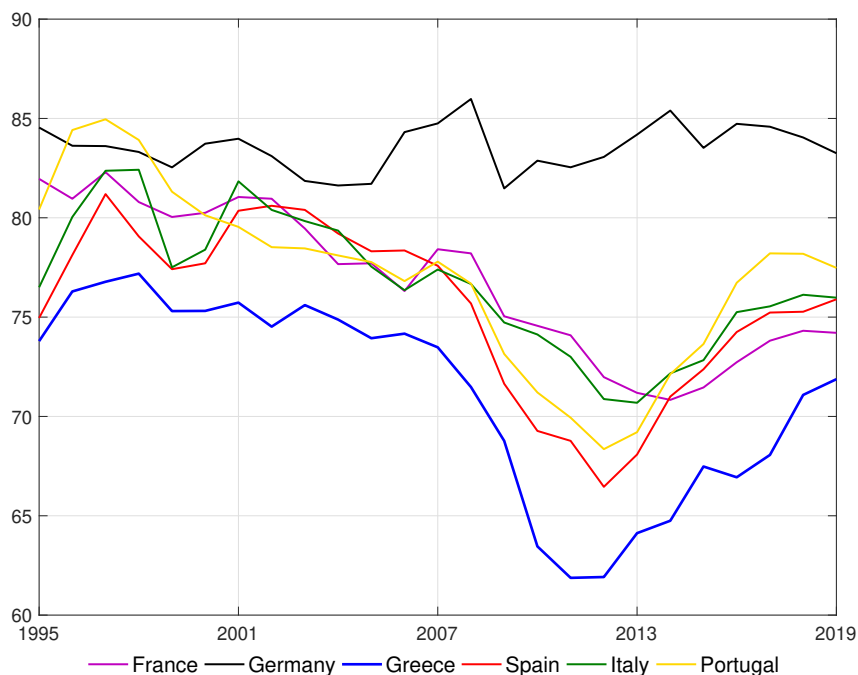


Figure 3: International Country Risk Guide, PRS Group

Greece set the upper and lower bound respectively. It is also clear that Greece has suffered a continuous decrease in the level of institutional quality in the ICRG since 1998, with the lowest level reached in 2010 and 2012 in the aftermath of the crisis eruption. It should not come as a surprise that this index shows an increase in the quality of institutions for Greece after 2012. This is due to the fact that Greece was under an economic adjustment program that reduced the risk of default which is a prominent component of the ICRG index, thus, increasing the index's value for the years after 2012.

Moreover, in Figure 4 we present a scatter plot for the Real per capita GDP and the ICRG index averages over the years 1995-2019. It is evident that there is a positive relationship between real per capita GDP and the ICRG index. Countries ranked highly in the ICRG index, i.e. countries that exhibit higher institutional quality, are characterized by higher real

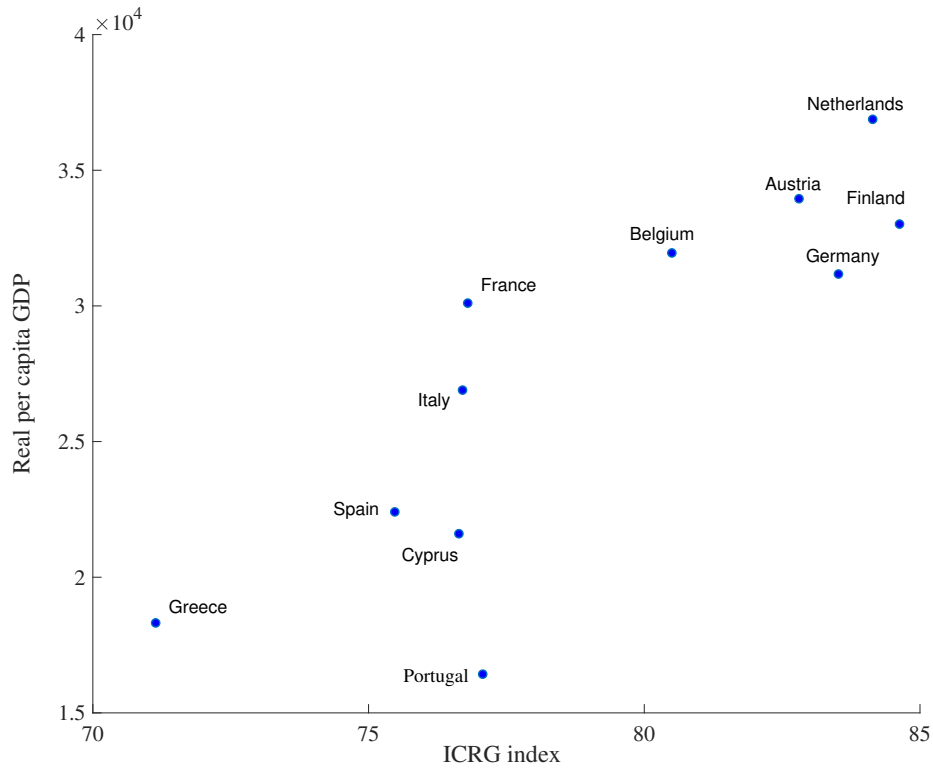


Figure 4: Real per capita GDP vs ICRG index

per capita GDP.

In this paper we extend the model proposed by Christou et al. (2021) in order to assess the nexus between institutions and the distributional aspect of economic performance. Our motivation lies on Figures 5-7. In Figure 5 we present the Gini coefficient, an important social index (covering inequality, poverty, distribution, living conditions etc.) used broadly for cross country comparisons. The Gini coefficient provides a measure of inequality, ranging from 0 (in case of perfect equality) to 1 (in case of no equality), i.e. the higher the index, the greater is the degree of income inequality. As we can see from the figure, there are indeed differences observed among countries. Firstly, observe that Greece, Spain, Cyprus and Italy have on average higher values of the Gini coefficient when compared to France and Germany. Furthermore, after the recent economic crisis in 2008 we observe, with the exemption of Germany, that the Gini index has increased in all countries. A notable example is Cyprus where the Gini coefficient increased from around 31% to almost 35% after 2011.

In Figure 6 we present a scatter plot for the Real per capita GDP and the GINI index averages over the years 2004-2019 for the countries in our sample. This figure clearly shows a negative relationship between Real per capita GDP and the GINI index. We see that in Italy, Spain, Greece and Portugal where the GINI index takes the highest values (i.e. high

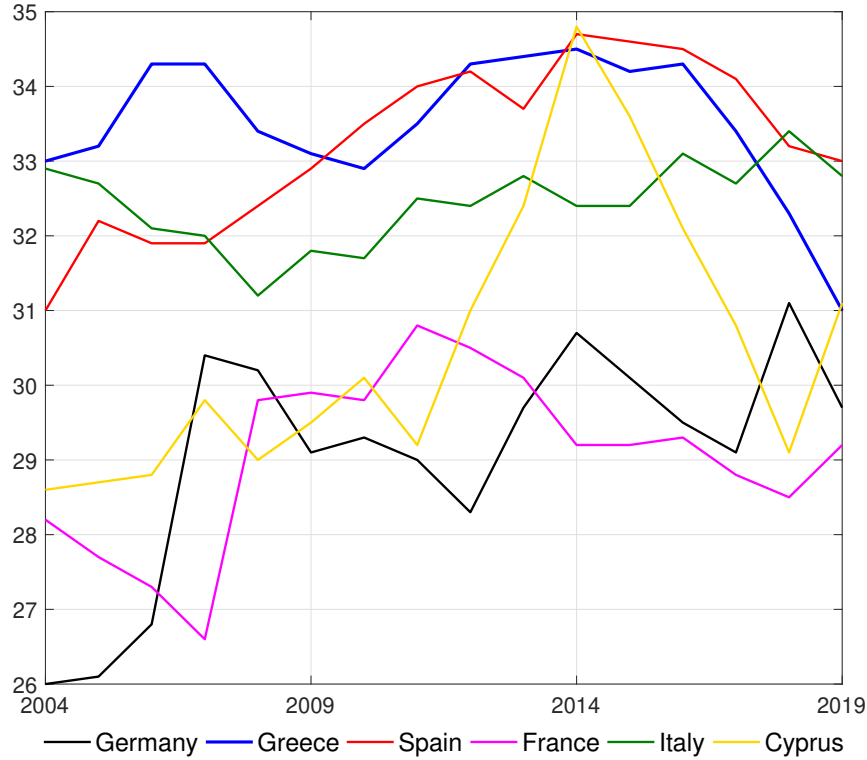


Figure 5: Gini index of equivalised disposable income (%), Eurostat (EU-SILC)

income inequality), Real per capita GDP is the lowest among the countries in our sample. Concerning institutional quality and income inequality in Figure 7 we present the scatter plot for the average over the 2004-2015 period of the ICRG and GINI indices. Clearly, a positive relationship emerges. It is evident from this figure that there are differences among core and periphery countries. Core countries (Finland, Netherlands, Austria, Belgium and Germany) are characterized by low income inequality and high institutional quality in contrast to Italy, Spain, Portugal and Greece where we observe high levels of income inequality and low institutional quality, indicating a negative relationship between institutions and income inequality.

Therefore, in this paper our aim is to study the implications of institutional quality on macroeconomic variables and distribution. To do so, we first introduce heterogeneity in the simple RBC model with two types of agents; capitalists and workers. Then we build on the concept of rent seeking introduced by Tullock (1967) and papers by Park et al. (2005), Angelopoulos, Philippopoulos, Vassilatos (2009), Angelopoulos, Economides, Vassilatos (2011) and Christou et al (2021) and introduce institutions through rent seeking competition in the heterogeneous agents framework. Under this specification, workers and capitalists are engaged in rent seeking activities and compete with agents in each group in order to extract a fraction of a contestable prize, here being the "income of the other agents"; i.e. the contestable prize for workers is the

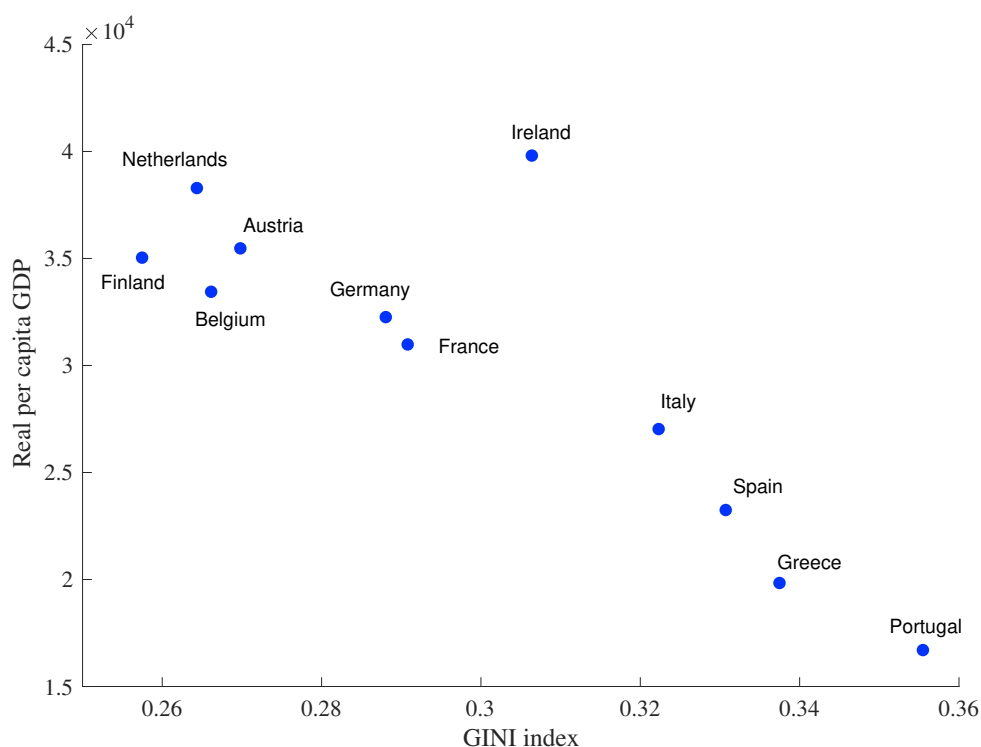


Figure 6: Real per capita GDP vs GINI index

income of capitalists and the contestable prize for capitalists is the income of workers. Our analysis will focus on the qualitative comparison of the two heterogeneous agents models, with and without institutions, in terms of wedges, long-run solution, second moment properties and impulse response functions.

The paper is organized as follows. In section 2, we present the simple RBC model with distortionary taxation and heterogeneous agents (capitalists, workers). In section 3, we extend this model to include institutions in the form of rent seeking activities. Section 4 presents the wedges induced between the two models by the introduction of rent seeking. In section 5, we show the parameterization of the two models. In section 6, we discuss and compare the long run solution of the two models. Second-moment properties are in section 7. Impulse response functions are in section 8. Finally, section 9 closes the paper. An Appendix provides details on data and results.

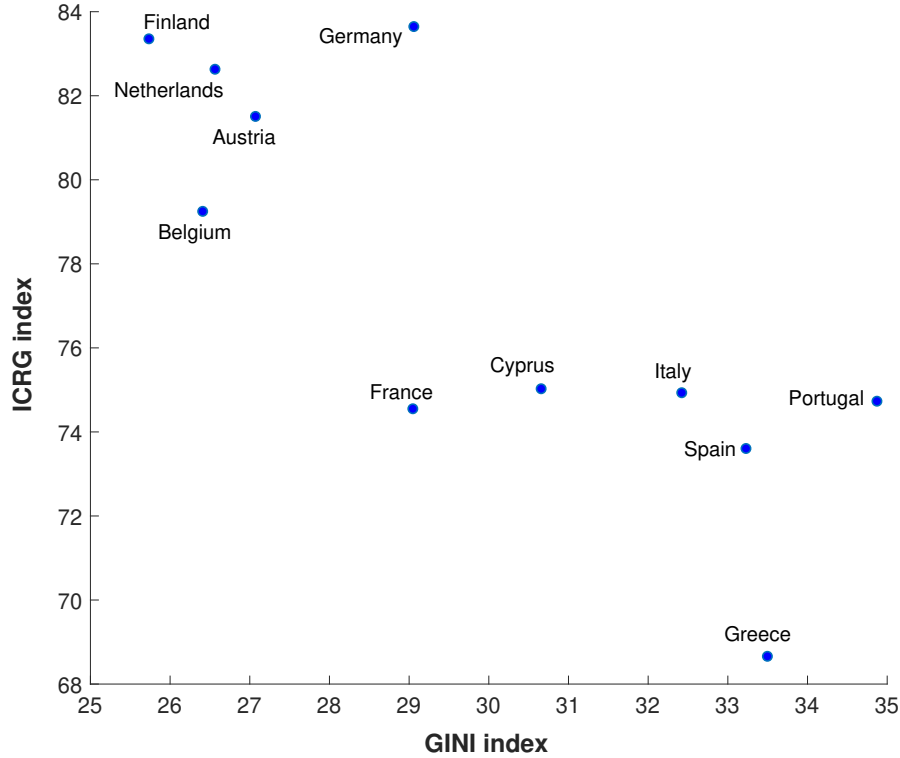


Figure 7: ICRG index vs GINI index

2 A simple RBC model with heterogeneous agents

2.1 Description of the model

We assume that in the economy of population size N_t there are $w = 1, 2, \dots, N_t^w$ households that are workers and $k = 1, 2, \dots, N_t^k$ households that are capitalists.² There are also $f = 1, 2, \dots, N_t^f$ identical firms, where we further assume that $N_t^f = N_t^k$, and the government. We define $v^w \equiv \frac{N_t^w}{N_t}$ and $v^k \equiv \frac{N_t^k}{N_t}$ as the fraction of workers and capitalists in the population respectively. The population size, N_t , is exogenous and evolves according to $N_{t+1} = \gamma_n N_t$; the workers and capitalists household population sizes, N_t^w and N_t^k , evolve according to $N_{t+1}^w = \gamma_n N_t^w$ and $N_{t+1}^k = \gamma_n N_t^k$ respectively where the rate $\gamma_n \geq 1$ is constant and $N_0 > 0$, $N_0^w > 0$ and $N_0^k > 0$ are given. Households of workers supply labour to firms and choose consumption and leisure. Households of capitalists supply both capital and labour to firms and choose consumption, leisure and investment in capital and government bonds. Firms produce a homogeneous product using capital and labor from both types of household. Government uses tax revenue and bonds to finance government consumption and government transfers.

² $N_t = N_t^w + N_t^k$

2.2 Households

2.2.1 Workers

The lifetime utility of each household that is a worker, $w = 1, 2, \dots, N_t^w$, is:

$$E_0 \sum_{t=0}^{\infty} \beta^{*t} U(C_t^w + \psi \bar{G}_t^c, L_t^w) \quad (2.1\text{-CW})$$

where E_0 denotes rational expectations conditional on the information set available at time zero, the time discount factor is $\beta^* \in (0, 1)$, C_t^w is household w 's consumption at time t , \bar{G}_t^c is government consumption of goods and services provided by the government for each household at time t , L_t^w is household w 's leisure time at time t and ψ is a parameter that measures the degree of substitutability between private and government consumption in utility.³

We assume that the instantaneous utility function for each household w takes the following form:

$$U(C_t^w, L_t^w) = \frac{\left((C_t^w + \psi \bar{G}_t^c)^\mu (L_t^w)^{1-\mu} \right)^{1-\sigma}}{1-\sigma} \quad (2.2\text{-CW})$$

where $0 < \mu < 1$ is a weight parameter of consumption in the utility function and $\sigma \geq 0$ is the curvature parameter of the utility function.

In every period t each household w considers the maximization problem of its utility function given its budget constraint. The household has one unit of time in each period divided between leisure L_t^w and effort H_t^w ; thus, $L_t^w + H_t^w = 1$. The household receives income only from labor, $W_t^w H_t^w$, where W_t^w is the wage rate of a worker. Furthermore, each household receives a share of lump-sum government transfers given to all workers, $\bar{G}_t^{t,w}$. Consumption and labour income are taxed at the rates $0 \leq \tau_t^c < 1$ and $0 \leq \tau_t^y < 1$ respectively. Based on the above, the household w 's budget constraint is:

$$(1 + \tau_t^c)C_t^w = (1 - \tau_t^y)W_t^w H_t^w + \bar{G}_t^{t,w} \quad (2.3\text{-CW})$$

Each household w acts competitively by taking prices and government policy as given and chooses $\{C_t^w, H_t^w\}_{t=0}^{\infty}$ to maximize lifetime utility Eq. (2.1-CW) given the definition of instantaneous utility Eq. (2.2-CW), subject to the budget constraint Eq. (2.3-CW) and the time constraint $L_t^w + H_t^w = 1$. The first-order conditions of the maximization problem of the household w include the constraints and the following equation:

$$\frac{\partial U_t^w(\cdot)}{\partial L_t^w} = \frac{1 - \tau_t^y}{1 + \tau_t^c} \frac{\partial U_t^w(\cdot)}{\partial C_t^w} W_t^w \quad (2.4\text{-CW})$$

³If $\psi = 0$ then the household receives no utility from government consumption.

2.2.2 Capitalists

The lifetime utility of each household that is a capitalist, $k = 1, 2, \dots, N_t^k$, is:

$$E_0 \sum_{t=0}^{\infty} \beta^{*t} U(C_t^k + \psi \bar{G}_t^c, L_t^k) \quad (2.5\text{-CW})$$

where E_0 denotes rational expectations conditional on the information set available at time zero, the time discount factor is $\beta^* \in (0, 1)$, C_t^k is household k 's consumption at time t , \bar{G}_t^c is government consumption of goods and services provided by the government for each household at time t , L_t^k is household k 's leisure time at time t and ψ is a parameter that measures the degree of substitutability between private and government consumption in utility.⁴

We assume that the instantaneous utility function for each household k takes the following form:

$$U(C_t^k, L_t^k) = \frac{\left((C_t^k + \psi \bar{G}_t^c)^\mu (L_t^k)^{1-\mu} \right)^{1-\sigma}}{1-\sigma} \quad (2.6\text{-CW})$$

where $0 < \mu < 1$ is a weight parameter of consumption in the utility function and $\sigma \geq 0$ is the curvature parameter of the utility function.

In every period t each household k considers the maximization problem of its utility function given its budget constraint. The household has one unit of time in each period divided between leisure L_t^k and effort H_t^k ; thus, $L_t^k + H_t^k = 1$. The household receives income from labor, $W_t^k H_t^k$, where W_t^k is the wage rate of a capitalist. Each household k decides to invest in capital, I_t^k , and government bonds, D_t^k . This gives each household an interest income $r_t^k K_t^k$ and $r_t^b B_t^k$ from capital and government bonds respectively, where r_t^k and r_t^b are the gross returns to capital and bonds, K_t^k and B_t^k . Additionally, each household receives a share of profits Π_t^k , and a share of lump-sum government transfers given to all capitalists, $\bar{G}_t^{t,k}$. Consumption and both sources of income are taxed at the rates $0 \leq \tau_t^c < 1$ and $0 \leq \tau_t^y < 1$ respectively.

Based on the above, the household k 's budget constraint is:

$$(1 + \tau_t^c)C_t^k + I_t^k + D_t^k = (1 - \tau_t^y)(W_t^k H_t^k + r_t^k K_t^k + \Pi_t^k) + r_t^b B_t^k + \bar{G}_t^{t,k} \quad (2.7\text{-CW})$$

The law of motion of private holding of government bonds evolves according to:

$$B_{t+1}^k = B_t^k + D_t^k \quad (2.8\text{-CW})$$

where the initial B_0^k is given.

⁴If $\psi = 0$ then the household receives no utility from government consumption.

The law of motion of private holding of capital evolves according to:

$$K_{t+1}^k = (1 - \delta^k)K_t^k + I_t^k \quad (2.9\text{-CW})$$

where the parameter $0 < \delta < 1$ is the depreciation rate of capital and the initial K_0^k is given.

Each household k acts competitively by taking prices and government policy as given and chooses $\{C_t^k, H_t^k, K_{t+1}^k, B_{t+1}^k\}_{t=0}^\infty$ to maximize lifetime utility Eq. (2.5-CW) given the definition of instantaneous utility Eq. (2.6-CW), subject to the budget constraint Eq. (2.7-CW), the time constraint $L_t^k + H_t^k = 1$; K_0^k, B_0^k given. The first-order conditions of the maximization problem of the household k include the constraints and the following equations:

$$\frac{\partial U_t^k(\cdot)}{\partial L_t^k} = \frac{1 - \tau_t^y}{1 + \tau_t^c} \frac{\partial U_t^k(\cdot)}{\partial C_t^k} W_t^k \quad (2.10\text{-CW})$$

$$\frac{1}{1 + \tau_t^c} \frac{\partial U_t^k(\cdot)}{\partial C_t^k} = \beta^* E_t \frac{1}{1 + \tau_{t+1}^c} \frac{\partial U_{t+1}^k(\cdot)}{\partial C_{t+1}^k} \left((1 - \tau_{t+1}^y) r_{t+1}^k + 1 - \delta \right) \quad (2.11\text{-CW})$$

$$\frac{1}{1 + \tau_t^c} \frac{\partial U_t^k(\cdot)}{\partial C_t^k} = \beta^* E_t \frac{1}{1 + \tau_{t+1}^c} \frac{\partial U_{t+1}^k(\cdot)}{\partial C_{t+1}^k} (1 + r_{t+1}^b) \quad (2.12\text{-CW})$$

2.3 Firms

Each firm f chooses private capital K_t^f and private labor from workers, $Q_t^{f,w}$, and capitalists, $Q_t^{f,k}$, as to produce a homogeneous product Y_t^f according to the production function:

$$Y_t^f = A_t (K_t^f)^\alpha (Q_t^{f,w} + \phi_t Q_t^{f,k})^{1-\alpha} \quad (2.13\text{-CW})$$

where $0 < \alpha < 1$ is a parameter, $A_t > 0$ is the stochastic total productivity (See section 2.5 for its law of motion), $\phi_t = \frac{Z_t^k}{Z_t^w}$ is the capitalist to workers labor productivity ratio (See section 2.5 for its law of motion), where Z_t^w and Z_t^k are the labor-augmenting technologies of workers and capitalists respectively evolving according to $Z_{t+1}^w = \gamma_z Z_t^w$ and $Z_{t+1}^k = \gamma_z Z_t^k$, where the rate $\gamma_z \geq 1$ is constant, $Z^w > 0$ and $Z^k > 0$ are given. Each firm f acts competitively by taking prices and government policy as given and chooses $K_t^f, Q_t^{f,w}$ and $Q_t^{f,k}$ in order to maximize a series of static profit problems subject to the production function, Eq. (2.13-CW). The profit function is:

$$\Pi_t^f = Y_t^f - r_t^k K_t^f - W_t^w Q_t^{f,w} - W_t^k Q_t^{f,k} \quad (2.14\text{-CW})$$

The first order conditions of the maximization problem of the firm are:

$$W_t^w = \frac{(1 - \alpha) Y_t^f}{Q_t^{f,w} + \phi_t Q_t^{f,k}} \quad (2.15\text{-CW})$$

$$W_t^k = \frac{(1 - \alpha)\phi_t Y_t^f}{Q_t^{f,w} + \phi_t Q_t^{f,k}} \quad (2.16\text{-CW})$$

$$r_t^k = \frac{\alpha Y_t^f}{K_t^f} \quad (2.17\text{-CW})$$

2.4 Government

The government taxes consumption of workers and capitalists at the rate $0 \leq \tau_t^c < 1$ and income of workers and capitalists at the rate $0 \leq \tau_t^y < 1$, and uses these revenue as to finance lump-sum transfers to workers, $G_t^{t,w}$, and capitalists, $G_t^{t,k}$ and government consumption, G_t^c . Thus the government budget constraint is the following:⁵

$$G_t^c + G_t^t + (1 + r_t^b)B_t = B_{t+1} + \tau_t^c(N_t^w C_t^{rw} + N_t^k C_t^{rk}) + \tau_t^y Y_t \quad (2.18\text{-CW})$$

2.5 Exogenous stochastic variables

The exogenous stochastic variables in our model are aggregate productivity, A_t and the capitalist to workers labor productivity ratio, ϕ_t . They both follow a univariate stochastic AR(1) process:

$$\ln A_{t+1} = (1 - \rho_a)\ln A + \rho_a \ln A_t + \epsilon_{t+1}^a \quad (2.19\text{-CW})$$

$$\ln \phi_{t+1} = (1 - \rho_\phi)\ln \phi + \rho_\phi \ln \phi_t + \epsilon_{t+1}^\phi \quad (2.20\text{-CW})$$

where A and ϕ are means of the stochastic process; ρ_a and ρ_ϕ are the first-order autocorrelation coefficients and $\epsilon_{t+1}^a, \epsilon_{t+1}^\phi$ are i.i.d. shocks. The tax rates, τ_t^c and τ_t^y , as well as the shares over GDP of government consumption and government transfers (i.e. $s_t^c = \frac{G_t^c}{Y_t}$ and $s_t^t = \frac{G_t^t}{Y_t}$ respectively) are assumed to be constant over time.

2.6 Decentralized Competitive Equilibrium (DCE)

We solve for the DCE where given market prices $(w_t^w, w_t^k, r_t^k, r_t^b)$, government policy $(S_t^c, s_t^t, \tau_t^c, \tau_t^y)$ and economy-wide variables (A_t, ϕ_t) : (i) each individual household of workers, $w = 1, 2, \dots, N_t^w$, solves its problem defined in section (2.2), (ii) each individual household of capitalists, $k = 1, 2, \dots, N_t^k$, solves its problem defined in section (2.2), (iii) each individual firm, $f = 1, 2, \dots, N_t^f$,

⁵We assume that G_t^t are total transfers distributed to both types of household according to their size in the population, i.e. $G_t^t = G_t^{t,w} + G_t^{t,k} = v_t^w G_t^t + v_t^k G_t^t$, where $G_t^{rw} = \sum_{w=1}^{N_t^w} \bar{G}_t^{rw}$ and $G_t^{rk} = \sum_{k=1}^{N_t^k} \bar{G}_t^{rk}$.

solves its problem defined in section (2.3), (iv) all markets clear: $\sum_{f=1}^{N_t^f} Y_t^f = \sum_{w=1}^{N_t^w} C_t^w + \sum_{k=1}^{N_t^k} C_t^k + \sum_{k=1}^{N_t^k} I_t^k + G_t^c = Y_t$ in the product market, $\sum_{f=1}^{N_t^f} Q_t^{f,w} = \sum_{w=1}^{N_t^w} H_t^w$ and $\sum_{f=1}^{N_t^f} Q_t^{f,k} = \sum_{k=1}^{N_t^k} H_t^k$ in the labour market, $\sum_{f=1}^{N_t^f} K_t^f = \sum_{k=1}^{N_t^k} K_t^k$ in the capital market, profits $\sum_{f=1}^{N_t^f} \Pi_t^f = \sum_{k=1}^{N_t^k} \Pi_t^k = 0$, and (v) all constraints are satisfied. Given that our economy converges to a balanced growth path where consumption, output, capital and investment grow at the rate $\gamma_n \gamma_z$, we express the DCE in terms of variables expressed in per capita and efficient labor units (per capita in the case of labor).⁶ Thus the stationary DCE will be given by Eqs. (2.21-CW)-(2.32-CW):

$$\gamma_n \gamma_z k_{t+1} = (1 - \delta)k_t + i_t \quad (2.21\text{-CW})$$

$$(s_t^c + s_t^t)y_t + (1 + r_t^b)b_t = \gamma_n \gamma_z b_{t+1} + \tau_t^c(v_t^w c_t^w + v_t^k c_t^k) + \tau_t^y y_t \quad (2.22\text{-CW})$$

$$\left(\frac{c_{t+1}^k + \psi s_{t+1}^c y_{t+1}}{c_t^k + \psi s_t^c y_t} \right)^{1-\mu(1-\sigma)} \left(\frac{1 - h_t^k}{1 - h_{t+1}^k} \right)^{(1-\mu)(1-\sigma)} = \beta E_t \left(\frac{1 + \tau_t^c}{1 + \tau_{t+1}^c} \right) (1 + r_{t+1}^b) \quad (2.23\text{-CW})$$

$$(1 + \tau_t^c)v^w c_t^w = (1 - \tau_t^y)w_t^w v^w h_t^w + v^w s_t^t y_t \quad (2.24\text{-CW})$$

$$(1 + \tau_t^c)v^k c_t^k + i_t + \gamma_n \gamma_z b_{t+1} = (1 - \tau_t^y)(w_t^k v^k h_t^k + r_t^k k_t) + (1 + r_t^b)b_t + v^k s_t^t y_t \quad (2.25\text{-CW})$$

$$y_t = A_t k_t^\alpha (v^w h_t^w + \phi_t v^k h_t^k)^{1-\alpha} \quad (2.26\text{-CW})$$

$$\frac{1 - \mu}{\mu} \frac{(c_t^w + \psi s_t^c y_t)}{1 - h_t^w} = \frac{1 - \tau_t^y}{1 + \tau_t^c} w_t^w \quad (2.27\text{-CW})$$

$$\frac{1 - \mu}{\mu} \frac{(c_t^k + \psi s_t^c y_t)}{1 - h_t^k} = \frac{1 - \tau_t^y}{1 + \tau_t^c} w_t^k \quad (2.28\text{-CW})$$

$$\left(\frac{c_{t+1}^k + \psi s_{t+1}^c y_{t+1}}{c_t^k + \psi s_t^c y_t} \right)^{1-\mu(1-\sigma)} \left(\frac{1 - h_t^k}{1 - h_{t+1}^k} \right)^{(1-\mu)(1-\sigma)} = \beta E_t \left(\frac{1 + \tau_t^c}{1 + \tau_{t+1}^c} \right) \left(1 - \delta + (1 - \tau_{t+1}^y)r_{t+1}^k \right) \quad (2.29\text{-CW})$$

⁶Thus, $c_t^w = \frac{N_t^w C_t^w}{N_t^w \gamma_z^w}$ is per worker efficient consumption, $c_t^k = \frac{N_t^k C_t^k}{N_t^k \gamma_z^k}$ is per capitalist efficient consumption, $k_t = \frac{N_t^k K_t^k}{N_t^k \gamma_z^k}$ is per capita efficient capital, $i_t = \frac{N_t^k I_t^k}{N_t^k \gamma_z^k}$ is per capita efficient investment, $y_t = \frac{N_t^k Y_t^k}{N_t^k \gamma_z^k}$ is per capita efficient output.

$$w_t^w = \frac{(1-\alpha)y_t}{v^w h_t^w + \phi_t v^k h_t^k} \quad (2.30\text{-CW})$$

$$w_t^k = \frac{(1-\alpha)\phi_t y_t}{v^w h_t^w + \phi_t v^k h_t^k} \quad (2.31\text{-CW})$$

$$r_t^k = \alpha \frac{y_t}{k_t} \quad (2.32\text{-CW})$$

where $\beta \equiv \beta^* \gamma_z^{\mu(1-\sigma)-1}$. This is an equilibrium of twelve equations in twelve unknown endogenous variables $y_t, c_t^w, c_t^k, h_t^w, h_t^k, i_t, r_t^b, r_t^k, w_t^w, w_t^k, b_{t+1}$ and k_{t+1} , given the paths for A_t, ϕ_t , and the four policy instruments $s_t^c, s_t^t, \tau_t^c, \tau_t^y$. We also define the following variables:

$$c_t = v^w c_t^w + v^k c_t^k \quad (2.33\text{-CW})$$

$$h_t = v^w h_t^w + v^k h_t^k \quad (2.34\text{-CW})$$

$$y_t^w = (1 - \tau_t^y) w_t^w v^w h_t^w + v^w s_t^t y_t - v^w c_t^w \quad (2.35\text{-CW})$$

$$y_t^k = (1 - \tau_t^y) (w_t^k v^k h_t^k + r_t^k k_t) + r_t^b b_t + v^k s_t^t y_t - v^k c_t^k \quad (2.36\text{-CW})$$

where c_t is total consumption, h_t is total non-leisure time and y_t^w and y_t^k are the post taxation and transfers incomes of workers and capitalists respectively.

2.7 Long-run equilibrium

In the long-run, our economy reaches an equilibrium where no shocks exist and variables remain constant but grow at a constant balance growth rate. We remove time subscripts and solve for the equilibrium. Thus, all variables satisfy that $x_{t+1} = x_t = x_{t-1} = x$. The long-run equilibrium is characterized by Eqs. (2.37-CW)-(2.48-CW):

$$(1 + \tau^c) v^w c^w = (1 - \tau^y) w^w v^w h^w + v^w s^t y \quad (2.37\text{-CW})$$

$$(1 + \tau^c) v^k c^k + i + (\gamma_n \gamma_z - (1 + r^b)) b = (1 - \tau^y) (w^k v^k h^k + r^k k) + v^k s^t y \quad (2.38\text{-CW})$$

$$y = A k^\alpha (v^w h^w + \phi v^k h^k)^{1-\alpha} \quad (2.39\text{-CW})$$

$$\frac{1 - \mu (c^w + \psi s^c y)}{\mu} = \frac{1 - \tau^y}{1 + \tau^c} w^w \quad (2.40\text{-CW})$$

$$\frac{1 - \mu (c^k + \psi s^c y)}{\mu} = \frac{1 - \tau^y}{1 + \tau^c} w^k \quad (2.41\text{-CW})$$

$$1 = \beta \left(1 - \delta + (1 - \tau^y) r^k \right) \quad (2.42\text{-CW})$$

$$1 = \beta (1 + r^b) \quad (2.43\text{-CW})$$

$$(\gamma_n \gamma_z - 1 + \delta) \frac{k}{y} = \frac{i}{y} \quad (2.44\text{-CW})$$

$$(s^c + s^t = (\gamma_n \gamma_z - (1 + r_t^b)) \frac{b}{y} + \tau_t^c (v^w \frac{c^w}{y} + v^k \frac{c^k}{y}) + \tau^y) \quad (2.45\text{-CW})$$

$$w^w = \frac{(1 - \alpha) y}{v^w h^w + \phi v^k h^k} \quad (2.46\text{-CW})$$

$$w^k = \frac{(1 - \alpha) \phi y}{v^w h^w + \phi v^k h^k} \quad (2.47\text{-CW})$$

$$r_t^k = \alpha \frac{y}{k} \quad (2.48\text{-CW})$$

where $\beta = \beta^* \gamma_z^{\mu(1-\sigma)-1}$. The above system of equations is an equilibrium system of twelve equations in twelve unknown endogenous variables $y, c^w, c^k, h^w, h^k, i, r^b, r^k, w^w, w^k, b$ and k . We set $b = 0.9y$ (i.e. the government debt-to-GDP ratio is 90% on an annual basis); therefore we choose the long-run government consumption-to-GDP ratio s^c to follow residually and satisfy the government budget constraint Eq. (2.45-CW). We also define the following variables in the long-run equilibrium:

$$c = v^w c^w + v^k c^k \quad (2.49\text{-CW})$$

$$h = v^w h^w + v^k h^k \quad (2.50\text{-CW})$$

$$y^w = (1 - \tau^y) w^w v^w h^w + v^w s^t y - v^w c^w \quad (2.51\text{-CW})$$

$$y^k = (1 - \tau^y) (w^k v^k h^k + r^k k) + r^b b + v^k s^t y - v^k c^k \quad (2.52\text{-CW})$$

3 Weak property rights protection on the "income of the others"

3.1 Description of the model

We assume that in the economy of population size N_t there are $w = 1, 2, \dots, N_t^w$ households that are workers and $k = 1, 2, \dots, N_t^k$ households that are capitalists.⁷ There are also $f = 1, 2, \dots, N_t^f$ identical firms, where we further assume that $N_t^f = N_t^k$, and the government. We define $v^w \equiv \frac{N_t^w}{N_t}$ and $v^k \equiv \frac{N_t^k}{N_t}$ as the fraction of workers and capitalists in the population respectively. The population size, N_t , is exogenous and evolves according to $N_{t+1} = \gamma_n N_t$; the workers and capitalists household population sizes, N_t^w and N_t^k , evolve according to $N_{t+1}^w = \gamma_n N_t^w$ and $N_{t+1}^k = \gamma_n N_t^k$ respectively where the rate $\gamma_n \geq 1$ is constant and $N_0 > 0$, $N_0^w > 0$ and $N_0^k > 0$ are given. Households of workers supply labour to firms and choose in addition to consumption and leisure, how to allocate their non-leisure time between productive work and rent seeking activities. Households of capitalists supply both capital and labour to firms and choose in addition to consumption, leisure, investment in capital and government bonds, how to allocate their non-leisure time between productive work and rent seeking activities. In this model we consider the contestable prize to be the "income of the others"; i.e. the contestable prize of workers is the income of capitalists and the contestable prize of capitalists is the income of workers. Firms produce a homogeneous product using capital and labor from both types of household. Government uses tax revenue and bonds to finance government consumption and government transfers. In the following sections, we present the three blocks of our model: households, firms and the government followed by the decentralized competitive equilibrium, the long-run equilibrium, the parameterization and the long-run solution of the model.

3.2 Households

3.2.1 Workers

The lifetime utility of each household that is a worker, $w = 1, 2, \dots, N_t^w$, is:

$$E_0 \sum_{t=0}^{\infty} \beta^{*t} U(C_t^w + \psi \bar{G}_t^c, L_t^w) \quad (3.1\text{-CWRS})$$

where E_0 denotes rational expectations conditional on the information set available at time zero, the time discount factor is $\beta^* \in (0, 1)$, C_t^w is household w 's consumption at time t , \bar{G}_t^c is government consumption of goods and services provided by the government for each household

⁷ $N_t = N_t^w + N_t^k$

at time t , L_t^w is household w 's leisure time at time t and ψ is a parameter that measures the degree of substitutability between private and government consumption in utility.⁸

We assume that the instantaneous utility function for each household w takes the following form:

$$U(C_t^w, L_t^w) = \frac{\left((C_t^w + \psi \bar{G}_t^c)^\mu (L_t^w)^{1-\mu} \right)^{1-\sigma}}{1-\sigma} \quad (3.2\text{-CWRS})$$

where $0 < \mu < 1$ is a weight parameter of consumption in the utility function and $\sigma \geq 0$ is the curvature parameter of the utility function.

In every period t each household w considers the maximization problem of its utility function given its budget constraint. The household has one unit of time in each period divided between leisure L_t^w and effort H_t^w ; thus, $L_t^w + H_t^w = 1$. We incorporate rent seeking activities in the behaviour of the household, following Angelopoulos, Philippopoulos, Vassilatos (2009) and Angelopoulos, Economides, Vassilatos (2011) and assume that the household w further divides its non-leisure time, H_t^w , between productive work, $\eta_t^w H_t^w$, and rent-extracting or seeking activities, $(1 - \eta_t^w)H_t^w$, where $0 < \eta_t^w \leq 1$ and $0 \leq (1 - \eta_t^w) < 1$ denote the fractions of non-leisure time that the household allocates to productive work and rent extraction or rent seeking activities; thus, $H_t^w = \eta_t^w H_t^w + (1 - \eta_t^w)H_t^w$ in each period. The household receives income from labor, $W_t \eta_t^w H_t^w$, where W_t^w is the wage rate of a worker. Furthermore, each household receives a share of lump-sum government transfers given to all workers irrespective of their rent seeking activities, $\bar{G}_t^{t,w}$. Consumption and labour income are taxed at the rates $0 \leq \tau_t^c < 1$ and $0 \leq \tau_t^y < 1$ respectively.

Based on the above, the household w 's budget constraint is:

$$(1 + \tau_t^c)C_t^w = (1 - \theta_t - \tau_t^y)W_t^w \eta_t^w H_t^w + \bar{G}_t^{t,w} + \frac{(1 - \eta_t^w)H_t^w}{\sum_{w=1}^{N_t^w} (1 - \eta_t^w)H_t^w} \theta_t \sum_{k=1}^{N_t^k} (W_t^k \eta_t^k H_t^k + r_t^k K_t^k + \Pi_t^k) \quad (3.3\text{-CWRS})$$

where $0 \leq \theta_t < 1$ is the economy-wide degree of rent extraction (defined in section 3.5). The last term of the budget constraint indicates that given a contestable prize denoted here as $\theta_t \sum_{k=1}^{N_t^k} (W_t^k \eta_t^k H_t^k + r_t^k K_t^k + \Pi_t^k)$, a self-interested worker attempts to obtain a share of the prize. Due to weak property rights protection, workers receive only a fraction $(1 - \theta_t)$ of their income, whereas the remaining fraction, θ_t , is being extracted by capitalists.

Each household w acts competitively by taking prices, government policy and economy-wide variables as given and chooses $\{C_t^w, H_t^w, \eta_t^w\}_{t=0}^\infty$ to maximize lifetime utility Eq. (3.1-CWRS) given the definition of instantaneous utility Eq. (3.2-CWRS), subject to the budget

⁸If $\psi = 0$ then the household receives no utility from government consumption.

constraint Eq. (3.3-CWRS) and the time constraint $L_t^w + H_t^w = 1$, $H_t^w = \eta_t^w H_t^w + (1 - \eta_t^w)H_t^w$.⁹ The first-order conditions of the maximization problem of the household w include the constraints and the following equations:

$$\frac{\partial U_t(\cdot)}{\partial L_t^w} = \frac{1}{1 + \tau_t^c} \frac{\partial U_t(\cdot)}{\partial C_t^w} \left[(1 - \theta_t - \tau_t^y) W_t^w \eta_t^w + \frac{(1 - \eta_t^w)}{\sum_{w=1}^{N_t^w} (1 - \eta_t^w) H_t^w} \theta_t \sum_{k=1}^{N_t^k} (W_t^k \eta_t^k H_t^k + r_t^k K_t^k + \Pi_t^k) \right] \quad (3.4\text{-CWRS})$$

$$(1 - \theta_t - \tau_t^y) W_t^w H_t^w = \frac{H_t^w}{\sum_{w=1}^{N_t^w} (1 - \eta_t^w) H_t^w} \theta_t \sum_{k=1}^{N_t^k} (W_t^k \eta_t^k H_t^k + r_t^k K_t^k + \Pi_t^k) \quad (3.5\text{-CWRS})$$

3.2.2 Capitalists

The lifetime utility of each household that is a capitalist, $k = 1, 2, \dots, N_t^k$, is:

$$E_0 \sum_{t=0}^{\infty} \beta^{*t} U(C_t^k + \psi \bar{G}_t^c, L_t^k) \quad (3.6\text{-CWRS})$$

where E_0 denotes rational expectations conditional on the information set available at time zero, the time discount factor is $\beta^* \in (0, 1)$, C_t^k is household k 's consumption at time t , \bar{G}_t^c is government consumption of goods and services provided by the government for each household at time t , L_t^k is household k 's leisure time at time t and ψ is a parameter that measures the degree of substitutability between private and government consumption in utility.¹⁰

We assume that the instantaneous utility function for each household k takes the following form:

$$U(C_t^k, L_t^k) = \frac{\left((C_t^k + \psi \bar{G}_t^c)^\mu (L_t^k)^{1-\mu} \right)^{1-\sigma}}{1 - \sigma} \quad (3.7\text{-CWRS})$$

where $0 < \mu < 1$ is a weight parameter of consumption in the utility function and $\sigma \geq 0$ is the curvature parameter of the utility function.

In every period t each household k considers the maximization problem of its utility function given its budget constraint. The household has one unit of time in each period divided between leisure L_t^k and effort H_t^k ; thus, $L_t^k + H_t^k = 1$. We incorporate rent seeking activities in the behaviour of the household, following Angelopoulos, Philippopoulos, Vassilatos (2009) and Angelopoulos, Economides, Vassilatos (2011) and assume that the household k further divides

⁹We assume that each individual household w takes as given the economy-wide variables $\sum_{k=1}^{N_t^k} (W_t^k \eta_t^k H_t^k + r_t^k K_t^k + \Pi_t^k)$, θ_t and $\sum_{w=1}^{N_t^w} (1 - \eta_t^w) H_t^w$.

¹⁰If $\psi = 0$ then the household receives no utility from government consumption.

its non-leisure time, H_t^k , between productive work, $\eta_t^k H_t^k$, and rent-extracting or seeking activities, $(1 - \eta_t^k) H_t^k$, where $0 < \eta_t^k \leq 1$ and $0 \leq (1 - \eta_t^k) < 1$ denote the fractions of non-leisure time that the household allocates to productive work and rent extraction or rent seeking activities; thus, $H_t^k = \eta_t^k H_t^k + (1 - \eta_t^k) H_t^k$ in each period. The household receives income from labor, $W_t^k \eta_t^k H_t^k$, where W_t^k is the wage rate of a capitalist. Each household k decides to invest in capital, I_t^k , and government bonds, D_t^k . This gives each household an interest income $r_t^k K_t^k$ and $r_t^b B_t^k$ from capital and in government bonds respectively, where r_t^k and r_t^b are the gross returns to capital and bonds, K_t^k and B_t^k . Furthermore, each household receives a share of profits Π_t^k , and a share of lump-sum government transfers given to all capitalists irrespective of their rent seeking activities, $\bar{G}_t^{t,k}$. Consumption and both sources of income are taxed at the rates $0 \leq \tau_t^c < 1$ and $0 \leq \tau_t^y < 1$ respectively.

Based on the above, the household k 's budget constraint is:

$$(1 + \tau_t^c)C_t^k + I_t^k + D_t^k = (1 - \theta_t - \tau_t^y)(W_t^k \eta_t^k H_t^k + r_t^k K_t^k + \Pi_t^k) + r_t^b B_t^k + \bar{G}_t^{t,k} + \frac{(1 - \eta_t^k) H_t^k}{\sum_{k=1}^{N_t^k} (1 - \eta_t^k) H_t^k} \theta_t \sum_{w=1}^{N_t^w} W_t^w \eta_t^w H_t^w \quad (3.8\text{-CWRS})$$

where $0 \leq \theta_t < 1$ is the economy-wide degree of rent extraction (defined in section 3.5). The last term of the budget constraint indicates that given a contestable prize denoted here as $\theta_t \sum_{w=1}^{N_t^w} W_t^w \eta_t^w H_t^w$, a self-interested capitalist attempts to obtain a share of the prize. Due to weak property rights protection, capitalists receive only a fraction $(1 - \theta_t)$ of their total income, whereas the remaining fraction, θ_t , is being extracted by workers.

The law of motion of private holding of government bonds evolves according to:

$$B_{t+1}^k = B_t^k + D_t^k \quad (3.9\text{-CWRS})$$

where the initial B_0^k is given.

The law of motion of private holding of capital evolves according to:

$$K_{t+1}^k = (1 - \delta^k) K_t^k + I_t^k \quad (3.10\text{-CWRS})$$

where the parameter $0 < \delta < 1$ is the depreciation rate of capital and the initial K_0^k is given.

Each household k acts competitively by taking prices, government policy and economy-wide variables as given and chooses $\{C_t^k, H_t^k, \eta_t^k, K_{t+1}^k, B_{t+1}^k\}_{t=0}^{\infty}$ to maximize lifetime utility Eq. (3.6-CWRS) given the definition of instantaneous utility Eq. (3.7-CWRS), subject to the budget constraint Eq. (3.8-CWRS) and the time constraint $L_t^k + H_t^k = 1$, $H_t^k = \eta_t^k H_t^k + (1 - \eta_t^k) H_t^k$;

K_0^k, B_0^k given.¹¹ The first-order conditions of the maximization problem of the household k include the constraints and the following equations:

$$\frac{\partial U_t(\cdot)}{\partial L_t^k} = \frac{1}{1 + \tau_t^c} \frac{\partial U_t(\cdot)}{\partial C_t^k} \left[(1 - \theta_t - \tau_t^y) W_t^k \eta_t^k + \frac{(1 - \eta_t^k)}{\sum_{k=1}^{N_t^k} (1 - \eta_t^k) H_t^k} \theta_t \sum_{w=1}^{N_t^w} W_t^w \eta_t^w H_t^w \right] \quad (3.11\text{-CWRS})$$

$$(1 - \theta_t - \tau_t^y) W_t^k H_t^k = \frac{H_t^k}{\sum_{k=1}^{N_t^k} (1 - \eta_t^k) H_t^k} \theta_t \sum_{w=1}^{N_t^w} W_t^w \eta_t^w H_t^w \quad (3.12\text{-CWRS})$$

$$\frac{1}{1 + \tau_t^c} \frac{\partial U_t(\cdot)}{\partial C_t^k} = \beta^* E_t \frac{1}{1 + \tau_{t+1}^c} \frac{\partial U_{t+1}(\cdot)}{\partial C_{t+1}^k} \left((1 - \theta_{t+1} - \tau_{t+1}^y) r_{t+1}^k + 1 - \delta \right) \quad (3.13\text{-CWRS})$$

$$\frac{1}{1 + \tau_t^c} \frac{\partial U_t(\cdot)}{\partial C_t^k} = \beta^* E_t \frac{1}{1 + \tau_{t+1}^c} \frac{\partial U_{t+1}(\cdot)}{\partial C_{t+1}^k} (1 + r_{t+1}^b) \quad (3.14\text{-CWRS})$$

3.3 Firms

Each firm f chooses private capital K_t^f and private labor from workers, $Q_t^{f,w}$, and capitalists, $Q_t^{f,k}$, as to produce a homogeneous product Y_t^f according to the production function:

$$Y_t^f = A_t (K_t^f)^\alpha (Q_t^{f,w} + \phi_t Q_t^{f,k})^{1-\alpha} \quad (3.15\text{-CWRS})$$

where $0 < \alpha < 1$ is a parameter, $A_t > 0$ is the stochastic total productivity (See section 3.6 for its law of motion), $\phi_t = \frac{Z_t^k}{Z_t^w}$ is the capitalist to workers productivity ratio (See section 3.6 for its law of motion), where Z_t^w and Z_t^k are the labor-augmenting technologies of workers and capitalists respectively evolving according to $Z_{t+1}^w = \gamma_z Z_t^w$ and $Z_{t+1}^k = \gamma_z Z_t^k$, where the rate $\gamma_z \geq 1$ is constant and $Z^w > 0$ and $Z^k > 0$ are given. Each firm f acts competitively by taking prices and government policy as given and chooses K_t^f , $Q_t^{f,w}$ and $Q_t^{f,k}$ in order to maximize a series of static profit problems subject to the production function, Eq. (3.15-CWRS). Thus, the profit function is:

$$\Pi_t^f = Y_t^f - r_t^k K_t^f - W_t^w Q_t^{f,w} - W_t^k Q_t^{f,k} \quad (3.16\text{-CWRS})$$

The first order conditions of the maximization problem of the firm are:

$$W_t^w = \frac{(1 - \alpha) Y_t^f}{Q_t^{f,w} + \phi_t Q_t^{f,k}} \quad (3.17\text{-CWRS})$$

¹¹We assume that each individual household h takes as given the economy-wide variables $\sum_{w=1}^{N_t^w} W_t^w \eta_t^w H_t^w$, θ_t and $\sum_{k=1}^{N_t^k} (1 - \eta_t^k) H_t^k$.

$$W_t^k = \frac{(1 - \alpha)Y_t^f \phi_t}{Q_t^{f,w} + \phi_t Q_t^{f,k}} \quad (3.18\text{-CWRS})$$

$$r_t^k = \frac{\alpha Y_t^f}{K_t^f} \quad (3.19\text{-CWRS})$$

3.4 Government

The government taxes consumption of workers and capitalists at the rate $0 \leq \tau_t^c < 1$ and income of workers and capitalists at the rate $0 \leq \tau_t^y < 1$, and uses these revenue as to finance lump-sum transfers to workers, $G_t^{t,w}$, and capitalists, $G_t^{t,k}$. Thus the government budget constraint is the following:¹²

$$G_t^c + G_t^t + (1 + r_t^b)B_t = B_{t+1} + \tau_t^c(N_t^w C_t^w + N_t^k C_t^k) + \tau_t^y Y_t \quad (3.20\text{-CWRS})$$

3.5 Economy-wide rent extraction

As mentioned previously, θ_t is a variable denoting economy-wide rent extraction: higher values of θ_t indicate that the rent-seeking technology becomes more efficient and therefore a larger fraction of the contestable prize can be extracted. We consider θ_t to be a proxy for the quality of institutions in the economy where lower values indicate better institutions. In our model we assume θ_t to be exogenous.¹³

3.6 Exogenous stochastic variables

The exogenous stochastic variable in our model are aggregate productivity, A_t , the capitalist to workers labor productivity ratio, ϕ , as well as the economy-wide degree of rent extraction, θ_t . They all follow a univariate stochastic AR(1) process:

$$\ln A_{t+1} = (1 - \rho_a) \ln A + \rho_a \ln A_t + \epsilon_{t+1}^a \quad (3.21\text{-CWRS})$$

¹²We assume that G_t^t are total transfers distributed to both types of household according to the their size in the population, i.e. $G_t^t = G_t^w + G_t^k = v_t^w G_t^t + v_t^k G_t^t$, where $G_t^w = \sum_{w=1}^{N_t^w} \bar{G}_t^w$ and $G_t^k = \sum_{k=1}^{N_t^k} \bar{G}_t^k$.

¹³Alternatively, one could assume that θ_t is endogenous and increases with per capita rent-seeking activities $\theta_t = \gamma \frac{\sum_{h=1}^{N_t} (1 - \eta_t^h) H_t^h}{N_t}$, $h = w, k$. Furthermore, it could also depend on the fraction of output that the government allocates in securing property rights, s_t^p , (i.e. expenditures on policing, law enforcement etc.), e.g. $\theta_t = \gamma (S_t^p)^{-\xi_2} \left(\frac{\sum_{h=1}^{N_t} (1 - \eta_t^h) H_t^h}{N_t} \right)^{\xi_1}$; γ , ξ_1 and ξ_2 are parameters related to the quality of institutions.

$$\ln\phi_{t+1} = (1 - \rho_\phi)\ln\phi + \rho_\phi\ln\phi_t + \epsilon_{t+1}^\phi \quad (3.22\text{-CWRS})$$

$$\ln\theta_{t+1} = (1 - \rho_\theta)\ln\theta + \rho_\theta\ln\theta_t + \epsilon_{t+1}^\theta \quad (3.23\text{-CWRS})$$

where A, ϕ and θ are means of the stochastic process; ρ_a, ρ_ϕ and ρ_θ are the first-order autocorrelation coefficients and $\epsilon_{t+1}^\alpha, \epsilon_{t+1}^\phi, \epsilon_{t+1}^\theta$ are i.i.d. shocks. The tax rates, τ_t^c and τ_t^y , as well as the shares over GDP of government consumption and government transfers (i.e. $s_t^c = \frac{G_t^c}{Y_t}$ and $s_t^t = \frac{G_t^t}{Y_t}$ respectively) are assumed to be constant over time.

3.7 Decentralized Competitive Equilibrium (DCE)

We solve for the DCE where given market prices $(w_t^w, w_t^k, r_t^k, r_t^b)$, government policy $(S_t^c, s_t^t, \tau_t^c, \tau_t^y)$ and economy-wide variables (A_t, ϕ_t, θ_t) : (i) each individual household of workers, $w = 1, 2, \dots, N_t^w$, solves its problem defined in section (3.2), (ii) each individual household of capitalists, $k = 1, 2, \dots, N_t^k$, solves its problem defined in section (3.2), (iii) each individual firm, $f = 1, 2, \dots, N_t^f$, solves its problem defined in section (3.3), (iv) all markets clear: $\sum_{f=1}^{N_t^f} Y_t^f = \sum_{w=1}^{N_t^w} C_t^w + \sum_{k=1}^{N_t^k} C_t^k + \sum_{k=1}^{N_t^k} I_t^k + G_t^c = Y_t$ in the product market, $\sum_{f=1}^{N_t^f} Q_t^{f,w} = \sum_{w=1}^{N_t^w} \eta_t^w H_t^w$ and $\sum_{f=1}^{N_t^f} Q_t^{f,k} = \sum_{k=1}^{N_t^k} \eta_t^k H_t^k$ in the labour market, $\sum_{f=1}^{N_t^f} K_t^f = \sum_{k=1}^{N_t^k} K_t^k$ in the capital market, profits $\sum_{f=1}^{N_t^f} \Pi_t^f = \sum_{k=1}^{N_t^k} \Pi_t^k = 0$, and (v) all constraints are satisfied. Given that our economy converges to a balanced growth path where consumption, output, capital and investment grow at the rate $\gamma_n \gamma_z$, we express the DCE in terms of variables expressed in per capita and efficient labor units (per capita in the case of labor).¹⁴ Thus the stationary DCE will be given by Eqs. (3.24-CWRS)-(3.37-CWRS):

$$(1 + \tau_t^c)v^w c_t^w = (1 - \theta_t - \tau_t^y)w_t^w \eta_t^w v^w h_t^w + v^w s_t^t y_t + \theta_t (w_t^k \eta_t^k v^k h_t^k + r_t^k k_t) \quad (3.24\text{-CWRS})$$

$$(1 + \tau_t^c)v^k c_t^k + i_t + \gamma_n \gamma_z b_{t+1} = (1 - \theta_t - \tau_t^y)(w_t^k \eta_t^k v^k h_t^k + r_t^k k_t) + (1 + r_t^b)b_t + v^k s_t^t y_t + \theta_t w_t^w \eta_t^w v^w h_t^w \quad (3.25\text{-CWRS})$$

$$y_t = A_t k_t^\alpha (Z^w \eta_t^w v^w h_t^w + Z^k \eta_t^k v^k h_t^k)^{1-\alpha} \quad (3.26\text{-CWRS})$$

¹⁴Thus, $c_t^w = \frac{N_t^w C_t^w}{N_t^w \gamma_z^t}$ is per worker efficient consumption, $c_t^k = \frac{N_t^k C_t^k}{N_t^k \gamma_z^t}$ is per capitalist efficient consumption, $k_t = \frac{N_t^k K_t^k}{N_t^k \gamma_z^t}$ is per capita efficient capital, $i_t = \frac{N_t^k I_t^k}{N_t^k \gamma_z^t}$ is per capita efficient investment, $y_t = \frac{N_t^k Y_t^k}{N_t^k \gamma_z^t}$ is per capita efficient output.

$$\frac{1 - \mu}{\mu} \frac{(v^w c_t^w + \psi v^w s_t^c y_t) h_t^w}{1 - h_t^w} = \frac{1}{1 + \tau_t^c} [(1 - \theta_t - \tau_t^y) w_t^w \eta_t^w v^w h_t^w + \theta_t (w_t^k \eta_t^k v^k h_t^k + r_t^k k_t)] \quad (3.27\text{-CWRS})$$

$$\frac{1 - \mu}{\mu} \frac{(v^k c_t^k + \psi v^k s_t^c y_t) h_t^k}{1 - h_t^k} = \frac{1}{1 + \tau_t^c} [(1 - \theta_t - \tau_t^y) w_t^k \eta_t^k v^k h_t^k + \theta_t w_t^w \eta_t^w v^w h_t^w] \quad (3.28\text{-CWRS})$$

$$\left(\frac{c_{t+1}^k + \psi s_{t+1}^c y_{t+1}}{c_t^k + \psi s_t^c y_t} \right)^{1-\mu(1-\sigma)} \left(\frac{1 - h_t^k}{1 - h_{t+1}} \right)^{(1-\mu)(1-\sigma)} = \beta E_t \left(\frac{1 + \tau_t^c}{1 + \tau_{t+1}^c} \right) (1 - \delta + (1 - \theta_{t+1} - \tau_{t+1}^y) r_{t+1}^k) \quad (3.29\text{-CWRS})$$

$$\gamma_n \gamma_z k_{t+1} = (1 - \delta) k_t + i_t \quad (3.30\text{-CWRS})$$

$$\left(\frac{c_{t+1}^k + \psi s_{t+1}^c y_{t+1}}{c_t^k + \psi s_t^c y_t} \right)^{1-\mu(1-\sigma)} \left(\frac{1 - h_t^k}{1 - h_{t+1}} \right)^{(1-\mu)(1-\sigma)} = \beta E_t \left(\frac{1 + \tau_t^c}{1 + \tau_{t+1}^c} \right) (1 + r_{t+1}^b) \quad (3.31\text{-CWRS})$$

$$w_t^w = \frac{(1 - \alpha) y_t}{\eta_t^w v^w h_t^w + \phi_t \eta_t^k v^k h_t^k} \quad (3.32\text{-CWRS})$$

$$w_t^k = \frac{(1 - \alpha) \phi_t y_t}{\eta_t^w v^w h_t^w + \phi_t \eta_t^k v^k h_t^k} \quad (3.33\text{-CWRS})$$

$$r_t^k = \alpha \frac{y_t}{k_t} \quad (3.34\text{-CWRS})$$

$$(S_t^c + s_t^t) y_t + (1 + r_t^b) b_t = \gamma_n \gamma_z b_{t+1} + \tau_t^c (v_t^w c_t^w + v_t^k c_t^k) + \tau_t^y y_t \quad (3.35\text{-CWRS})$$

$$\theta_t (w_t^k \eta_t^k v^k h_t^k + r_t^k k_t) = (1 - \theta_t - \tau_t^y) (1 - \eta_t^w) w_t^w v^w h_t^w \quad (3.36\text{-CWRS})$$

$$\theta_t w_t^w \eta_t^w v^w h_t^w = (1 - \theta_t - \tau_t^y) w_t^k (1 - \eta_t^k) v^k h_t^k \quad (3.37\text{-CWRS})$$

where $\beta \equiv \beta^* \gamma_z^{\mu(1-\sigma)-1}$. This is an equilibrium of fourteen equations in fourteen unknown endogenous variables $y_t, c_t^w, c_t^k, h_t^w, h_t^k, \eta_t^w, \eta_t^k, i_t, r_t^b, r_t^k, w_t^w, w_t^k, b_{t+1}$ and k_{t+1} , given the paths for A_t, ϕ_t, θ_t and the four policy instruments $s_t^c, s_t^t, \tau_t^c, \tau_t^y$. We also define the following variables:

$$c_t = v^w c_t^w + v^k c_t^k \quad (3.38\text{-CWRS})$$

$$h_t = v^w h_t^w + v^k h_t^k \quad (3.39\text{-CWRS})$$

$$\eta_t = v^w \eta_t^w + v^k \eta_t^k \quad (3.40\text{-CWRS})$$

$$y_t^w = (1 - \theta_t - \tau_t^y) w_t^w \eta_t^w v^w h_t^w + v^w s_t^y y_t - v^w c_t^w + \theta_t (w_t^k \eta_t^k v^k h_t^k + r_t^k k_t) \quad (3.41\text{-CW})$$

$$y_t^k = (1 - \theta_t - \tau_t^y) (w_t^k \eta_t^k v^k h_t^k + r_t^k k_t) + r_t^b b_t + v^k s_t^y y_t - v^k c_t^k + \theta_t (w_t^w \eta_t^w v^w h_t^w) \quad (3.42\text{-CWRS})$$

where c_t is total consumption, h_t is total non-leisure time and y_t^w and y_t^k are the post taxation and transfers incomes of workers and capitalists respectively.

3.8 Long-run equilibrium

In the long-run, our economy reaches an equilibrium where no shocks exist and variables remain constant but grow at a constant balance growth rate. We remove time subscripts and solve for the equilibrium. Thus, all variables satisfy that $x_{t+1} = x_t = x_{t-1} = x$. The long-run equilibrium is characterized by Eqs. (3.43-CWRS)-(3.56-CWRS):

$$v^w c^w = (1 - \theta - \tau^y) w^w \eta^w v^w h^w + v^w s^y y + \theta (w^k \eta^k v^k h^k + r^k k) \quad (3.43\text{-CWRS})$$

$$v^k c^k + i = (1 - \theta - \tau^y) (w^k \eta^k v^k h^k + r^k k) + v^k s^y y + \theta w^w \eta^w v^w h^w \quad (3.44\text{-CWRS})$$

$$y = A k^\alpha (\eta^w v^w h^w + \phi \eta^k v^k h^k)^{1-\alpha} \quad (3.45\text{-CWRS})$$

$$(\gamma_n \gamma_z - 1 + \delta) k = i \quad (3.46\text{-CWRS})$$

$$\frac{1 - \mu (v^w c^w + \psi v^w s^y y) h^w}{\mu (1 - h^w)} = \frac{1}{1 + \tau^c} [(1 - \theta - \tau^y) w^w \eta^w v^w h^w + \theta (w^k \eta^k v^k h^k + r^k k)] \quad (3.47\text{-CWRS})$$

$$(1 - \theta - \tau^y) w^k (1 - \eta^k) v^k h^k = \theta w^w \eta^w v^w h^w \quad (3.48\text{-CWRS})$$

$$\frac{1 - \mu (v^k c^k + \psi v^k s^y y) h^k}{\mu (1 - h^k)} = \frac{1}{1 + \tau^c} [(1 - \theta - \tau^y) w^k \eta^k v^k h^k + \theta w^w \eta^w v^w h^w] \quad (3.49\text{-CWRS})$$

$$\theta(w^k \eta^k v^k h^k + r^k k) = (1 - \theta - \tau^y)(1 - \eta^w)w^w v^w h^w \quad (3.50\text{-CWRS})$$

$$1 = \beta(1 - \delta + (1 - \theta - \tau^y)r^k) \quad (3.51\text{-CWRS})$$

$$w^w = \frac{(1 - \alpha)y}{\eta^w v^w h^w + \phi \eta^k v^k h^k} \quad (3.52\text{-CWRS})$$

$$w^k = \frac{(1 - \alpha)\phi y}{\eta^w v^w h^w + \phi \eta^k v^k h^k} \quad (3.53\text{-CWRS})$$

$$r^k = \alpha \frac{y}{k} \quad (3.54\text{-CWRS})$$

$$1 = \beta(1 + r_{t+1}^b) \quad (3.55\text{-CWRS})$$

$$s^t y = \tau^y (w^w \eta^w v^w h^w + w^k \eta^k v^k h^k + r^k k) \quad (3.56\text{-CWRS})$$

The above system of equations is an equilibrium system of fourteen equations in fourteen unknown endogenous variables $y, c^w, c^k, h^w, h^k, \eta^w, \eta^k, i, r^b, r^k, w^w, w^k, b$ and k . We set $b = 0.9y$ (i.e. the government debt-to-GDP ratio is 90% on an annual basis); therefore we choose the long-run government consumption-to-GDP ratio s^c to follow residually and satisfy the government budget constraint Eq. (3.56-CWRS). We also define the following variables in the long-run:

$$c = v^w c^w + v^k c^k \quad (3.56\text{-CWRS})$$

$$h = v^w h^w + v^k h^k \quad (3.57\text{-CWRS})$$

$$\eta = v^w \eta^w + v^k \eta^k \quad (3.58\text{-CWRS})$$

$$y^w = (1 - \theta - \tau^y)w^w \eta^w v^w h^w + v^w s^t y - v^w c^w + \theta(w^k \eta^k v^k h^k + r^k k) \quad (3.59\text{-CWRS})$$

$$y^k = (1 - \theta - \tau^y)(w^k \eta^k v^k h^k + r^k k) + r^b b + v^k s^t y - v^k c^k + \theta(w^w \eta^w v^w h^w) \quad (3.60\text{-CWRS})$$

4 Wedges

In what follows, we focus on the DCE equations that change after the introduction of rent seeking activities in the simple RBC model with heterogeneous agents, namely the first order conditions with respect to effort of workers and capitalists, η_t^w and η_t^k respectively, non-leisure time of workers and capitalists, h_t^w and h_t^k respectively, and capital of capitalists, k_{t+1}^k , as well as the respective household budget constraints of workers and capitalists and the production function. Note that equations from the simple RBC model with heterogeneous agents are labeled as CW whereas the equations from the simple RBC model with heterogeneous agents and rent seeking activities are labelled as CWRS.

Thus, the first order condition for the effort level of workers, η_t^w , and the effort level of capitalists, η_t^k , are Eqs. (3.36-CWRS) and (3.37-CWRS) respectively.¹⁵

$$\theta_t(w_t^k \eta_t^k v^k h_t^k + r_t^k k_t) = (1 - \theta_t - \tau_t^y)(1 - \eta_t^w)w_t^w v^w h_t^w \quad (3.36-CWRS)$$

$$\theta_t w_t^w \eta_t^w v^w h_t^w = (1 - \theta_t - \tau_t^y)w_t^k (1 - \eta_t^k) v^k h_t^k \quad (3.37-CWRS)$$

the household budget constraint for workers is:

$$(1 + \tau_t^c)v^w c_t^w = (1 - \tau_t^y)w_t^w v^w h_t^w + v^w s_t^t y_t \quad (2.24-CW)$$

$$(1 + \tau_t^c)v^w c_t^w = (1 - \tau_t^y)w_t^w \eta_t^w v^w h_t^w + v^w s_t^t y_t + \theta_t(w_t^k \eta_t^k v^k h_t^k + r_t^k k_t - w_t^w \eta_t^w v^w h_t^w) \quad (3.24-CWRS)$$

the household budget constraint for capitalists is:

$$(1 + \tau_t^c)v^k c_t^k + i_t + \gamma_n \gamma_z b_{t+1} = (1 - \tau_t^y)(w_t^k v^k h_t^k + r_t^k k_t) + (1 + r_t^b)b_t + v^k s_t^t y_t \quad (2.25-CW)$$

$$(1 + \tau_t^c)v^k c_t^k + i_t + \gamma_n \gamma_z b_{t+1} = (1 - \tau_t^y)(w_t^k \eta_t^k v^k h_t^k + r_t^k k_t) + (1 + r_t^b)b_t + v^k s_t^t y_t + \theta_t(w_t^w \eta_t^w v^w h_t^w - w_t^k \eta_t^k v^k h_t^k - r_t^k k_t) \quad (3.25-CWRS)$$

the production function is:

$$y_t = A_t k_t^\alpha (v^w h_t^w + v^k \phi_t h_t^k)^{1-\alpha} \quad (2.26-CW)$$

$$y_t = A_t k_t^\alpha (v^w \eta_t^w h_t^w + v^k \phi_t \eta_t^k h_t^k)^{1-\alpha} \quad (3.26-CWRS)$$

¹⁵In the simple RBC model with heterogeneous agents η_t is not a choice variable ($\theta_t = 0$).

the first order condition for non-leisure time of workers, h_t^w , is:

$$\frac{1 - \mu}{\mu} \frac{(c_t^w + \psi s_t^c y_t)}{1 - h_t^w} = \frac{1 - \tau_t^y}{1 + \tau_t^c} w_t^w \quad (2.27\text{-CW})$$

$$\begin{aligned} \frac{1 - \mu}{\mu} \frac{(c_t^w + \psi s_t^c y_t)}{1 - h_t^w} &= \frac{1 - \tau_t^y}{1 + \tau_t^c} w_t^w \eta_t^w \\ &+ \frac{\theta_t}{(1 + \tau_t^c) v^w h_t^w} (w_t^k \eta_t^k v^k h_t^k + r_t^k k_t - w_t^w \eta_t^w v^w h_t^w) \end{aligned} \quad (3.27\text{-CWRS})$$

the first order condition for non-leisure time of capitalists, h_t^k , is:

$$\frac{1 - \mu}{\mu} \frac{(c_t^k + \psi s_t^c y_t)}{1 - h_t^k} = \frac{1 - \tau_t^y}{1 + \tau_t^c} w_t^k \quad (2.28\text{-CW})$$

$$\frac{1 - \mu}{\mu} \frac{(c_t^k + \psi s_t^c y_t)}{1 - h_t^k} = \frac{1 - \tau_t^y}{1 + \tau_t^c} w_t^k \eta_t^k + \frac{\theta_t}{(1 + \tau_t^c) v^k h_t^k} (w_t^w \eta_t^w v^w h_t^w - w_t^k \eta_t^k v^k) \quad (3.28\text{-CWRS})$$

the first order condition for capital of capitalists, k_{t+1}^k , is:

$$\left(\frac{c_{t+1}^k + \psi s_{t+1}^c y_{t+1}}{c_t^k + \psi s_t^c y_t} \right)^{1-\mu(1-\sigma)} \left(\frac{1 - h_t^k}{1 - h_{t+1}} \right)^{(1-\mu)(1-\sigma)} = \beta E_t \left(\frac{1 + \tau_{t+1}^c}{1 + \tau_{t+1}^c} \right) \left(1 - \delta + (1 - \tau_{t+1}^y) r_{t+1}^k \right) \quad (2.29\text{-CW})$$

$$\begin{aligned} &\left(\frac{c_{t+1}^k + \psi s_{t+1}^c y_{t+1}}{c_t^k + \psi s_t^c y_t} \right)^{1-\mu(1-\sigma)} \left(\frac{1 - h_t^k}{1 - h_{t+1}} \right)^{(1-\mu)(1-\sigma)} \\ &= \beta E_t \left(\frac{1 + \tau_{t+1}^c}{1 + \tau_{t+1}^c} \right) \left(1 - \delta + (1 - \tau_{t+1}^y) r_{t+1}^k \right) - \beta E_t \left(\frac{1 + \tau_{t+1}^c}{1 + \tau_{t+1}^c} \right) \theta_{t+1} r_{t+1}^k \end{aligned} \quad (3.29\text{-CWRS})$$

The introduction of frictions (i.e. distortionary taxation, market power, sticky prices and sticky wages) in the simple RBC model manifests itself in the terminology of Chari et al. (2007) as wedges affecting labor, investment and government consumption outcomes. In our model, we introduce a friction in the form of rent seeking that implies a wedge similar to a labor and an investment wedge in Chari et al. (2007). To see this we compare the respective DCE conditions implied by the simple RBC model with heterogeneous agents and rent seeking activities with the ones from the simple RBC model with heterogeneous agents.

In the absence of rent seeking activities (i.e. $\theta_t = 0$), our model is nothing but a simple RBC model with heterogeneous agents. When $\theta_t > 0$, it is evident from Eqs. (3.36-CWRS) and (3.37-CWRS), that the existence of rent seeking activities affects the first order condition with respect to the effort level of workers and capitalists, η_t^w and η_t^k respectively. Furthermore,

looking at the production function in the two models, Eqs. (2.26-CW) and Eqs. (3.26-CWRS), we can see that in the case $\theta_t > 0$ (i.e. $0 < \eta_t < 1$) production is affected as well; this is what we define as a production wedge.

Moreover, comparing the household budget constraints between the two models with and without rent seeking activities, i.e. Eqs. (2.24-CW) vs Eq. (3.24-CWRS), and then for capitalists, i.e. Eqs. (2.25-CW) vs Eq. (3.25-CWRS), we see that rent seeking via θ_t affects both household budget constraints in two directions: firstly, negatively as a fraction θ_t of the household's income is extracted, secondly, positively as the household increases its income in engaging in rent seeking activities and this way manages to extract a fraction θ_t of the contestable prize, i.e. the income of the other type of household.

Comparing the first order conditions of the models with and without rent seeking activities for non-leisure time of workers and capitalists, i.e. Eqs. (2.27-CW) and (2.28-CW), and Eqs. (3.27-CWRS) and (3.28-CWRS), we observe that θ_t also exerts two opposite effects. In the terminology of Chari et al. (2007), the additional terms $\frac{\theta_t}{(1+\tau_t^c)h_t^w}(w_t^k \eta_t^k v^k h_t^k + r_t^k k_t - w_t^w \eta_t^w v^w h_t^w)$ in Eq. (3.27-CWRS) and $\frac{\theta_t}{(1+\tau_t^c)h_t^k}(w_t^w \eta_t^w v^w h_t^w - w_t^k \eta_t^k v^k)$ in Eq. (3.28-CWRS) induce a wedge that resembles to a labor tax that further distorts the marginal rate of substitution between consumption and leisure of both workers and capitalists.

Finally, the capitalist's decision with respect to capital in the model with rent seeking activities, Eq. (3.29-CWRS), deviates from the respective condition of the simple RBC model with heterogeneous agents, Eq. (2.29-CW), in the term $\beta E_t \left(\frac{1+\tau_t^c}{1+\tau_{t+1}^c} \right) \theta_{t+1} r_{t+1}^k$. This works like an investment tax in Chari et al. (2007) and induces a wedge in the intertemporal marginal rate of substitution.

Thus, the introduction of rent seeking activities in our model in the terminology of Chari et al. (2007) introduces an additional labor, investment and production wedge. In other words, the introduction of rent seeking induces a labor, an investment, a household budget constraint and production wedge which are richer compared to the simple RBC model with heterogeneous agents and depend on the level of institutional quality.

5 Parameterization

We choose not to calibrate the two models as there are no data on sectoral variables appearing in the model. Instead, we set each parameter equal to the average of the respective parameter value for the countries selected in Christou et al. (2020), (i.e. for Austria, Belgium, Germany, France, Finland, Netherlands, Cyprus, Greece, Ireland, Italy, Portugal and Spain). We present these values in Table 1.

Following usual practise in the literature, we set the curvature parameter in the utility

function, σ , equal to 2 and the degree of substitutability between private and government consumption in the utility function, ψ , equal to zero.¹⁶ Next, we set the population growth rate, γ_n , to 1.0074 (the average growth rate of the population of all countries) and the growth rate of the exogenous labor-augmenting technology to 1.024, equal to the average growth rate of the United States. We follow King and Rebelo (1999) and normalize the initial level of technical progress, Z_0 , to 1.

Table 1: Parameters

Parameters	Description	Value
v^w	Worker's share in population	0.80
v^k	Capitalist's share in population	0.20
θ	Long-run value of the economy-wide degree of rent extraction	0.0418
μ	Consumption weight in utility function	0.4449
α	Capital share in production	0.3486
β	Discount factor	0.9656
N_0	Total population initial level	1
γ_z	Growth rate of labor-augmenting technology	1.0240
γ_n	Growth rate of population	1.0074
Z^w	Workers Labor-augmenting technology initial level	1
Z^k	Capitalists labor-augmenting technology initial level	2
A	Long-run aggregate productivity	0.8879
δ	Depreciation rate	0.0562
s^t	Share of government transfers	0.1923
τ_t^c	Consumption tax rate	0.2082
τ_t^y	Total income tax rate	0.3467
σ	Curvature parameter in utility function	2

We further normalize the labor-augmenting technology initial level of workers, Z^w , to 1 and set the respective parameter of capitalists, Z^k , equal to 2. Also, we set the level of long-run aggregate productivity, A , to 0.8879 (the average value of all countries of the period 2001-2014 of the Total Factor Productivity series from the St. Louis FED).¹⁷ We set the annual rate of depreciation rate, δ , to 0.0562, the time preference rate, β , to 0.9656 and the capital share in production, α , to 0.3486. In the heterogeneous agents model with rent seeking activities the

¹⁶We assume that government consumption provides no utility to either workers or capitalists.

¹⁷The series we use for the Total Factor Productivity from St. Louis FED, is an index where USA take the value 1.

long-run value of the economy-wide degree of rent extraction, θ , is set to 0.0418.¹⁸ Finally, in what concerns the volatility and persistence of the stochastic exogenous variables of our model we set the persistence parameters of ρ_θ and ρ_ϕ to 0.99, and the standard deviation of the shocks σ_θ and σ_ϕ to be 0.01. In what concerns, ρ_α and σ_a we set these values so as the cyclical component of the model generated series of the economy-wide output is characterized by the same volatility and persistence in the average GDP over all countries in our sample.

6 Long-run solution and sensitivity analysis

6.1 Long-run solution

In Table 2, we present the long-run solution of the simple RBC model with heterogeneous agents (See section 2) and that of the simple RBC model with heterogeneous agents as well as rent seeking activities (See section 3).

The introduction of rent seeking activities in the heterogeneous agents framework has a negative impact on the macroeconomy, in the sense that the long-run values of output, consumption, investment and capital, both on the economy-wide and sectoral level fall, with the exception of non-leisure time which remains more or less unaffected. Looking at workers and capitalists, the introduction of rent seeking although reduces the income of both workers and capitalists, the fall is relatively larger for capitalists; hence the increase in the y^w/y^k ratio. Another feature that stands out is that given the current parameterization the effort level of capitalists is smaller than the effort level of workers indicating that capitalists engage far more in rent seeking activities compared to workers. Note that as the relative labor productivity of capitalists to workers, ϕ_t , increases, the value of η^k increases and the gap between η^k and η^w becomes smaller and even leads to a value of η^k larger than η^w . Also, as v^k increases this also increases η^k . A possible explanation lies in the fact that as labor productivity ratio ϕ_t increases, this works as an incentive to devote more time to productive work since rent seeking becomes more costly. On the other hand, when v^k increases, competition among capitalists becomes harsher while the number of workers that contribute to the creation of the contestable prize falls. Both these factors contribute positively on the incentives of capitalists to allocate time to productive activities.

In order to investigate the distributional implications of our model we calculate the following "coefficient of variation"-type inequality index: $I^{CW} = \frac{(y^w - y^k)^2}{y^w} + \frac{(y^w - y^k)^2}{y^k} + \frac{(y^w - y^k)^2}{y^w + y^k}$. Observe that the introduction of rent seeking reduces inequality. However, in the presence of rent seeking although the inequality gap among workers and capitalists narrows, both types of

¹⁸In the heterogenous agents model without rent seeking activities, $\theta = 0$.

Table 2: Long-run solution

Variable	Description	CW	CWRS
y	Output	0.5701	0.5018
y^w/y^k	Sectoral income ratio	0.8559	0.8970
y^w	Income of workers	0.2021	0.1819
y^k	Income of capitalists	0.2362	0.2028
ICW	Inequality index	0.0133	0.0057
c	Consumption	0.2980	0.2681
c^w	Consumption of workers	0.2527	0.2273
c^k	Consumption of capitalists	0.4793	0.4312
k	Capital	1.4139	1.1650
i	Investment	0.1241	0.1023
η	Fraction of non-leisure time allocated to productive work	1	0.9096
η^w	Fraction of non-leisure time of workers allocated to productive work	1	0.9154
η^k	Fraction of non-leisure time of capitalists allocated to productive work	1	0.8862
s^c	Share of government consumption	0.2596	0.2620
h	Non-leisure time	0.3461	0.3489
h^w	Non-leisure time of workers	0.3393	0.3421
h^k	Non-leisure time of capitalists	0.3733	0.3760
w^w	Wage rate of workers	0.8824	0.8518
w^k	Wage rate of capitalists	1.7649	1.7035
r^k	Return on capital	0.1406	0.1502
r^b	Return on bonds	0.0356	0.0356

agents enjoy lower income, consumption and leisure time. The economy therefore is characterized by a situation where all agents are worse off but the distribution is more equal. Clearly, in terms of long-run welfare the absence of rent seeking would be Pareto improving for both workers and capitalists despite the increase in inequality.

6.2 Sensitivity analysis

We now focus on the long-run of our model with rent seeking activities, and examine the distributional repercussions of changes in the quality of institutions (θ), the share of capitalists in the population (v^k) and the relative labor productivity ratio of capitalists to workers (ϕ). Our results are presented in Table 3. According to our model, a deterioration of the quality of institutions (i.e. an increase in θ) decreases inequality. However, this comes at the cost of lower

Table 3: Long-run solution (CWRS): Sensitivity analysis with respect to θ , v^k and ϕ

	$\theta = 0.05$	$\theta = 0.1$	$v^k = 0.2$	$v^k = 0.4$	$\phi = 2$	$\phi = 3$
IC,W	0.0046	0.0007	0.0057	0.3595	0.0057	0.0650
y	0.4891	0.4156	0.5018	0.5855	0.5018	0.5855
y^w	0.1780	0.1551	0.1819	0.1400	0.1819	0.1866
y^k	0.1966	0.1620	0.2028	0.3088	0.2028	0.2621
c	0.2624	0.2287	0.2681	0.3128	0.2681	0.3128
h	0.3494	0.3524	0.3489	0.3448	0.3489	0.3382
η	0.8926	0.7946	0.9096	0.8743	0.9096	0.8902
c^w	0.2225	0.1939	0.2273	0.2333	0.2273	0.2333
c^k	0.4220	0.3677	0.4312	0.4321	0.4312	0.6309
h^w	0.3426	0.3456	0.3421	0.3249	0.3421	0.3249
h^k	0.3766	0.3796	0.3760	0.3747	0.3760	0.3913
η^w	0.8997	0.8099	0.9154	0.8147	0.9154	0.8793
η^k	0.8643	0.7334	0.8862	0.9638	0.8862	0.9335

output, consumption and leisure for both types of agents which now find themselves in a Pareto inferior long-run equilibrium. The fact that the deterioration in institutional quality comes with lower inequality seems to be at odds with what we observe in Figure 13 where the scatter plot among the GINI coefficient and the ICRG index implies a negative relationship among quality of institutions and inequality. Note however that the relationship among institutions and inequality is much more complex than the unconditional correlation implied by a simple scatter plot. For example, the fall in inequality comes as a result of the counterproductive for the macroeconomy decrease in labor effort. This has to do with factors such as, among others, the size of the contestable prize, the share of capitalists or workers in the population, as well as the ease with which rent seeking takes place. The latter is what the change in θ grabs. So it is equally important to see what happens when v^k or ϕ changes. An increase in the share of capitalists in the population v^k , increases inequality. Economy-wide output and consumption are higher and so is the income and consumption of capitalists. On the other hand, the position of workers becomes marginalized with both workers' output and consumption falling. This deterioration in workers position takes place despite their increased rent seeking effort (i.e. decrease in their labor effort η^w) in order to take advantage of the substantially bigger contestable prize created by capitalists. However, the extra income obtained through rent seeking is not enough to avoid the overall decrease in workers income.

A somewhat different picture emerges when ϕ , the relative labor productivity of capitalists

to workers, increases. An increase in ϕ increases inequality as in the case of an increase in v^k above. However, in this case both economy-wide and sectoral output and consumption increase following the increase in ϕ . As one would naturally expect, the relative increase for capitalists is substantially bigger. Workers, once more, increase rent seeking effort (i.e. decrease in their labor effort η^w) in order to take advantage of the substantially bigger contestable prize created by capitalists. Although, the extra income obtained through rent seeking allows workers income to increase, this increase is not enough to avoid an increase in inequality.

7 Second moment properties

In this section we evaluate the performance of the heterogeneous model with rent seeking activities. For this reason we try to answer the following questions: firstly, how does the introduction of rent seeking in a model with heterogeneous agents affect its qualitative behaviour. To this end we compute the second moment properties characteristics of a heterogeneous model with rent seeking activities (labelled CWRs) relative to those without rent seeking activities (labelled CW). The second question we try to answer is how does heterogeneous model with rent seeking activities (CWRs) perform relative to the data. To this end we compute the second moment properties characteristics of the data, focusing uniquely on economy-wide variables, and compare them with the respective characteristics of the heterogeneous model with rent seeking activities (CWRs).

We linearize the DCE of each model around its respective long-run solution. The linearized DCE can be written in the form $E_t[A_1\hat{x}_{t+1} + A_0\hat{x}_t + B_1\hat{z}_{t+1} + B_0\hat{z}_t] = 0$, where we define \hat{x}_t to be the endogenous variables, \hat{z}_t to be the exogenous variables, and A_1, A_0, B_1, B_0 are constant matrices of dimension 16x16, 16x16, 16x2 and 16x2 for the simple RBC model with heterogeneous agents; for the simple RBC model with heterogeneous agents and rent seeking activities A_1, A_0, B_1, B_0 are constant matrices of dimension 18x18, 18x18, 18x3 and 18x3 respectively. The elements of \hat{z}_t follow the AR(1) processes in Eqs. (2.19-CW)-(2.20-CW) and Eqs. (3.21-CWRs)-(3.23-CWRs) respectively for the simple RBC model with heterogeneous agents and the simple RBC model with heterogeneous agents and rent seeking activities, and tax rates the shares of government policy to GDP are assumed to be constant. Thus, for the simple RBC model with heterogeneous agents we end up with a linear stochastic difference equation system in sixteen variables; two are predetermined (\hat{k}_t, \hat{b}_t) and the remaining fourteen are forward-looking ($\hat{y}_t, \hat{y}_t^w, \hat{y}_t^k, \hat{c}_t, \hat{c}_t^w, \hat{c}_t^k, \hat{i}_t, \hat{h}_t, \hat{h}_t^w, \hat{h}_t^k, \hat{w}_t^w, \hat{w}_t^k, \hat{r}_t^k, \hat{r}_t^b$). For the simple RBC model with heterogeneous agents and rent seeking activities, we end up with a linear stochastic difference equation system in eighteen variables; two are predetermined (\hat{k}_t, \hat{b}_t) and the remaining sixteen are forward-looking ($\hat{y}_t, \hat{y}_t^w, \hat{y}_t^k, \hat{c}_t, \hat{c}_t^w, \hat{c}_t^k, \hat{i}_t, \hat{\eta}_t^w, \hat{\eta}_t^k, \hat{h}_t, \hat{h}_t^w, \hat{h}_t^k, \hat{w}_t^w, \hat{w}_t^k, \hat{r}_t^k, \hat{r}_t^b$). Given

the calibrated parameter values each model is characterized by saddle-path stability.

Coming to the first question, we try to answer how does the introduction of rent seeking in a model with heterogeneous agents affect its behaviour. To this end we compute and compare the second moment properties of the simple RBC model with heterogeneous agents (labelled CW) with the second moment properties of the simple RBC model with heterogeneous agents and rent seeking activities (labelled CWRS). To do so, we solve and simulate each model as to generate series for each of the endogenous variables. In what concerns the volatility and persistence of the stochastic exogenous variables of our model we set the persistence parameters of ρ_θ and ρ_ϕ to 0.99, and the standard deviation of the shocks σ_θ and σ_ϕ to be 0.01. Regarding ρ_α and σ_a we set these values so as the cyclical component of the model generated series of the economy-wide output is characterized by the same volatility and persistence as the average GDP over all countries in our sample. We calculate the trend using the HP filter with a smoothing parameter of 100 and then obtain the cyclical component. The second moment properties for the key economy-wide variables (y, c, i, h, k and η) as well as sectoral variables ($y^w, y^k, c^w, c^k, i, h^w, h^k, k, \eta^w$ and η^k) in the two models are presented in Tables 4-6.

We first compare relative volatility between the two models (See Table 4). Looking at the economy-wide variables we see that the two models share similar qualitative characteristics. Consumption, non-leisure time, capital and the effort level are less volatile than output, whereas investment is more volatile than output. However, close inspection of the relative volatility to

Table 4: Relative volatility, $x \equiv s_x/s_y$

x	CW	CWRS	x	CW	CWRS
c	0.8307	0.8376	y^w	0.9046	0.9928
i	1.4107	1.4333	y^k	1.6844	2.0093
h	0.0547	0.1023	c^w	0.9046	0.8635
k	0.2290	0.2314	c^k	0.7129	0.8401
η	Na	0.1306	h^w	0.0553	0.1459
σ^y	0.0252	0.0252	h^k	0.3916	0.2988
			η^w	Na	0.1091
			η^k	Na	0.2382

output in the two models reveals some quantitative differences. Concerning economy-wide variables, the biggest effect is that of the relative volatility of non-leisure time to output which doubles in the model with rent seeking activities, albeit still remaining small. Moreover, when it comes to the sectoral variables, we observe significant quantitative differences once rent seeking is introduced. The introduction of rent seeking increases volatility of income to output of both workers and capitalists with the increase being much more spectacular in the case of

capitalists income where relative volatility to output becomes around 20% higher. In what concerns consumption and non-leisure time in sectoral variables, we see that the effects in these variables work in opposite directions for workers and capitalists. The introduction of rent seeking reduces the volatility of consumption of workers to output and increases that of capitalists with the change for capitalists being substantially larger. Looking at non-leisure time we see that the introduction of rent seeking activities in the model, increases the volatility of non-leisure time to output of workers and decreases that of capitalists, with the changes being equally large. Furthermore, while both the effort level of workers and the effort level of capitalists series are less volatile than output, the relative volatility of the effort of capitalists is larger. A possible explanation could lie on the fact that the share of capitalists in the population ($v^k = 0.20$), is smaller than the respective share of workers ($v^w = 0.80$). The lower the value of v^k , the higher the stakes for capitalists over the contestable prize, i.e. the share of contestable prize extracted by each capitalist is larger. This may lead the high relative volatility of the effort level for capitalists.¹⁹

We present persistence of the series generated by the two models in Table 5 where we see that series generated for the economy-wide variables by both models are characterized by high persistence. Observe that with the exception of y^k , all other sectoral variables behave in a similar manner whether rent seeking is included or not. When it comes to y^k , observe first of all that this variable is characterized by considerably low persistence relatively to y^w and other economy-wide variables (less than half). Moreover, the introduction of rent seeking activities in the model reduces persistence of y^k dramatically to less than 10%.

Table 5: Persistence, $\rho(x_t, x_{t-1})$

x	CW	CWRS	x	CW	CWRS
y	0.4822	0.4769	y^w	0.4854	0.4791
c	0.4890	0.4829	y^k	0.2100	0.0780
i	0.4744	0.4693	c^w	0.4854	0.4799
h	0.4651	0.4643	c^k	0.4974	0.4878
k	0.8484	0.8462	h^w	0.4651	0.4686
η	Na	0.4646	h^k	0.4645	0.4627
			η^w	Na	0.4656
			η^k	Na	0.4636

Finally, we present the contemporaneous co-movement of the endogenous variables with output in Table 6. When it comes to the co-movement of economy-wide variables with output,

¹⁹In fact, increasing the value of v^k from 0.20 to 0.50 ceteris paribus decreases the relative volatility of η^k as well as that of η^w and their differences.

the introduction of rent seeking activities does not seem to have any significant qualitative effect in the model with heterogeneous agents with co-movement of economy-wide variables being quantitatively similar. Consumption, investment and hours at work are contemporaneous procyclical, whereas capital is contemporaneous countercyclical.²⁰ Looking at sectoral variables this also holds for consumption and income. Note that in what concerns capitalists, the introduction of rent seeking activities in the model, reduces y^k co-movement with output substantially. On the other hand, where we observe substantial differences are in sectoral non-leisure time variables. The effect on non-leisure time is not only quantitative but also qualitatively substantial with h^w transformed from contemporaneous countercyclical in the model without rent-seeking to contemporaneous procyclical after the introduction of rent seeking activities.

Table 6: Contemporaneous co-movement with output, $\rho(y_t, x_{t+1})$

x	CW	CWRS	x	CW	CWRS
c	0.9992	0.9986	y^w	0.9927	0.9851
i	0.9983	0.9968	y^k	0.9332	0.7951
h	0.9474	0.8057	c^w	0.9927	0.9901
k	-0.0648	-0.0685	c^k	0.9631	0.9494
η	Na	0.0642	h^w	-0.6828	0.3540
			h^k	0.9646	0.6511
			η^w	Na	0.0043
			η^k	Na	0.1680

We now come to answer how does heterogeneous model with rent seeking activities (CWRS) perform relative to the data. To answer this question we focus on the comparison in terms of second moment properties characteristics of the series generated by the simple RBC model with heterogeneous agents and rent seeking activities (labelled as CWRS) with the respective second moment properties characteristics of the series in the data, focusing uniquely on economy-wide variables. We choose ρ^α and σ^α as to match the volatility and persistence of the output series generated by the model with the volatility and persistence of the GDP series in the data. We present our results for the key macroeconomic variables in Tables 7-9; the first line consists of the average of volatility, persistence and co-movement in the data of 12 Eurozone countries in our sample for each economy-wide variable whereas the second line presents the results of the heterogeneous agents model and rent seeking activities.²¹

²⁰As can be seen in Appendix B, the highest value of the co-variance of capital with output obtains one period ahead, i.e. capital is leading procyclical in both models with and without rent seeking activities.

²¹We take the average values of the second moment properties in the data for 12 Eurozone countries

Table 7: Relative volatility, $x \equiv s_x/s_y$

	x					
	c	i	h	k	η	σ^y
Data	0.8548	3.0691	0.3458	0.3718	Na	0.0252
CWRS	0.8376	1.4333	0.1023	0.2314	0.1306	0.0252

We begin our analysis with the second moment properties in the data. As shown in the first line of Table 7, consumption, hours at work and capital series are less volatile than output, whereas investment is more volatile than consumption in the data. Concerning persistence (See Table 8) of the series in the data, we find that all series are persistent however output is the most persistent (0.8054) whereas capital is the least persistent series (0.3456). Finally, looking at the contemporaneous co-movement of the key macroeconomic variables with output (See Table 9), we find that consumption, investment and hours at work are contemporaneously procyclical and capital is lagging procyclically.²²

Table 8: Persistence, $\rho(x_t, x_{t-1})$

	x					
	y	c	i	h	k	η
Data	0.8054	0.5501	0.4504	0.5496	0.3456	Na
CWRS	0.4769	0.4829	0.4693	0.4643	0.8462	0.4646

Table 9: Contemporaneous co-movement with output, $\rho(y_t, x_{t+1})$

	x	c	i	h	k	η
Data	0.5171	0.8769	0.3578	0.3446	Na	
CWRS	0.9986	0.9968	0.8057	-0.0685	0.0642	

In terms of comparison with the data, the model with rent seeking activities behaves well. More specifically, the model with rent seeking can clearly match the qualitative characteristics we observe in the data when it comes to relative volatility. As in the data, the model with rent seeking activities generates consumption, hours at work and capital series that are less volatile than output, whereas the investment series generated are more volatile than output. In more depth we see that the relative volatility of consumption is closely matched by the model with heterogeneous agents and rent seeking activities. Moreover, comparing the persistence of the series generated by the model with rent seeking activities with the persistence of the series in

following the same logic we adopted in the parameterization of the model in the previous section.

²²See Appendix B for the detailed table of co-movement with output in the data.

the data, we see that this is closely matched. Co-movement with output in the model with rent seeking activities implies throughout that consumption, investment, non-leisure time are contemporaneously procyclical while capital lags procyclically and the effort level is contemporaneous countercyclical.²³ This is also the picture in the data; however, the contemporaneous cross-correlations with output are much higher in the model.

8 Impulse response functions

In order to investigate the dynamic implications of the introduction of rent seeking activities in a heterogeneous agents model we compute impulse responses to the key economy-wide and sectoral variables for the heterogeneous agents model with and without rent seeking activities. Thus, we compute for each model the responses of the key endogenous variables (measured as percentage deviations from their model-consistent long-run value) to a unit shock to A_t and ϕ_t in the simple RBC model with heterogeneous agents and to A_t , ϕ_t and θ_t in the simple RBC model with heterogeneous agents and rent seeking activities. In what follows we present the responses of the endogenous variables to each of the shocks and compare the two models. In what concerns the shocks in A_t and ϕ_t we present the responses on impact while for θ_t we present the responses through time.²⁴

We first look at the effects of a positive shock in A_t in Table 10. An increase in A_t increases labour and capital productivity which leads to an increase in the demand of firms for more inputs. As you can see in Table 10, this leads to an increase in the real wage and the return on capital which in turn increases the household income and then current and future consumption (through consumption smoothing) and investment in capital increase. In what leisure is concerned there are two opposite effects taking place. The substitution effect, where leisure decreases given the increase of the wage rate (i.e. non-leisure time increases) and the income effect where the increase in income causes leisure to increase; in this case the substitution effect dominates and non-leisure time increases.

Looking at the economy-wide variables we find that the two models have a similar qualitative behaviour. However, looking in more depth, we find some quantitative differences between the two models regarding non-leisure time where after a productivity shock, the effect of non-leisure time in the model with rent seeking activities is much larger. In what concerns sectoral variables in the two models, we find that they all follow a similar qualitative behaviour as economy-wide variables apart from non-leisure time of workers and capitalists. Moreover, we find that the behavior of the η^w and η^k after an increase in A_t is different in the model with rent

²³See Appendix B.

²⁴For the detailed impulse response functions tables see Appendix C.

Table 10: Positive shock in A_t : Response on impact

Variables	CW	CWRS	Variables	CW	CWRS
y	1.0759	1.0643	y^w	1.0142	1.2065
c	0.8965	0.9109	y^k	1.3674	1.1995
h	0.0566	0.1294	c^w	0.9780	0.8721
k	0.1282	0.1295	c^k	0.7246	0.9926
i	1.5066	1.5221	h^w	-0.0362	0.1770
η	Na	0.0128	h^k	0.3941	-0.0441
w^w	0.9594	0.9656	η^w	Na	0.0398
w^k	0.9594	0.9656	η^k	Na	-0.0948
r^k	1.0759	1.0643			

seeking activities, with η^w increasing and η^k decreasing. A possible explanation of the behavior of the opposite response to an increase in A_t in the effort of capitalists and workers is that the bigger contestable prize provides an incentive to capitalists to increase rent seeking effort. On the other hand, the responses of non-leisure time to a shock in total factor productivity are qualitatively different in the model with and without rent seeking activities. A positive shock in A_t in the absence of rent seeking reduces non-leisure time of workers and increases the non-leisure time of capitalists while the opposite happens in the presence of rent seeking. Given the increase in productivity, non-leisure time of capitalists increases in the model without rent seeking activities, yet the income effect for workers dominates and non-leisure time of workers decreases. In the model with rent seeking activities, the observed fall in non-leisure time of capitalists and increase in investment and in the return on capital implies that in the presence of rent seeking, capitalists take advantage of the increased productivity by focusing on the source of income from capital. On the other hand, the only way workers can take advantage of increased productivity is via increasing effort and non-leisure time.

Finally, looking at sectoral consumption and income we see that the picture is more mixed. Sectoral consumption and income may respond in a positive shock in A_t in a similar qualitative manner but quantitatively we observe differences among capitalists and workers, with the introduction of rent seeking implying the response of capitalists and workers moving in opposite direction. More specifically, in the model with rent seeking a positive shock in total factor productivity, implies a bigger response of the income for workers and a lower response for capitalists, while the opposite holds for consumption.

We now look at Table 11 at the responses of an increase in ϕ_t (i.e. an increase in the labor productivity of capitalists to workers ratio).²⁵ We see that an increase in ϕ_t shares a

²⁵We have normalized labor productivity of workers to 1, thus an increase in ϕ_t comes from an increase in the labor productivity of capitalists.

similar qualitative picture between the two models in the economy-wide variables, with the only exception of non-leisure time and of the effort level (in the model with rent seeking activities). Output, consumption, investment increase in both models; however the response of non-leisure time and the response of η is qualitatively different. A possible explanation of this effect lies

Table 11: Positive shock in ϕ_t : Response on impact

Variables	CW	CWRS	Variables	CW	CWRS
y	0.2520	0.2496	y^w	-0.0586	0.0666
c	0.1696	0.1721	y^k	0.6034	0.5318
h	-0.0277	0.0289	c^w	-0.0730	-0.1490
k	Na	0.0409	c^k	0.6811	0.8495
i	0.4499	0.4808	h^w	-0.1203	0.0293
w^w	-0.1349	-0.1336	h^k	0.3089	0.0276
w^k	0.8651	0.8664	η^w	Na	-0.1795
r^k	0.2520	0.2496	η^k	Na	0.4281
η	Na	-0.0580			

in the inspection of the behaviour of sectoral variables. The increase in labor productivity of capitalists provides an incentive to capitalists to work more (this is mirrored also in the increased wage of capitalists). This explains the increase in non-leisure time of capitalists in the model without rent seeking. In the presence of rent seeking, non-leisure time increases yet at a much lower level; however, this is counterbalanced by a substantial increase in the effort level. The fact that now workers are less productive relative to capitalists is reflected in the fall in workers wage. This provides an incentive to workers to work less. In the model without rent seeking activities this is manifested in the falling non-leisure time. In the model with rent seeking activities, this disincentive to work is manifested by the fall in the effort level of workers, although non-leisure time of workers slightly increases possibly to take even more advantage of the substantial increase to the income of capitalists which consists of the contestable prize. Given that a positive shock in ϕ_t is beneficial for capitalists but not for workers, the opposite effects on sectoral variables are not surprising. Consumption and income of capitalists both increase in the models with and without rent seeking. In the absence of rent seeking activities, the fact that workers productivity has become relatively smaller leads to the fall in both consumption and income of workers. However, in the presence of rent seeking activities, an increase in ϕ_t increases output albeit at a much smaller level relative to the increase of the income of capitalists. This reflects the fact that workers take advantage of the now bigger contestable prize. However, this is not enough to avoid a fall in the consumption of workers.

Finally, in Table 12 we present an increase in θ_t (i.e. a deterioration in institutional quality) in the model with rent seeking activities. The increase in θ_t clearly has an overall negative impact in the economy-wide macroeconomic performance with output, consumption, investment and capital economy-wide to fall. This is also the case for sectoral variables with the

Table 12: Positive shock in θ_t
Periods

Variables	1	2	3	10	20	30	100
θ	1	0.9900	0.9801	0.9135	0.8262	0.7472	0.3697
y	-0.1681	-0.1776	-0.1861	-0.2258	-0.2437	-0.2391	-0.1284
c	-0.0898	-0.1020	-0.1131	-0.1669	-0.1973	-0.2004	-0.1111
h	0.0587	0.0581	0.0576	0.0537	0.0485	0.0439	0.0217
k	0	-0.0342	-0.0656	-0.2224	-0.3226	-0.3494	-0.2040
i	-0.4018	-0.4031	-0.4040	-0.4016	-0.3821	-0.3546	-0.1803
η	-0.2121	-0.2105	-0.2088	-0.1971	-0.1801	-0.1637	-0.0815
y^w	0.0016	-0.0111	-0.0226	-0.0806	-0.1177	-0.1277	-0.0747
y^k	-0.3737	-0.3745	-0.3750	-0.3710	-0.3516	-0.3257	-0.1653
c^w	-0.1712	-0.1808	-0.1895	-0.2301	-0.2484	-0.2437	-0.1309
c^k	0.0817	0.0642	0.0481	-0.0337	-0.0896	-0.1091	-0.0691
h^w	0.1523	0.1488	0.1455	0.1266	0.1076	0.0941	0.0448
h^k	-0.2819	-0.2721	-0.2628	-0.2119	-0.1666	-0.1389	-0.0623
η^w	-0.1302	-0.1305	-0.1307	-0.1294	-0.1227	-0.1137	-0.0577
η^k	-0.5396	-0.5304	-0.5215	-0.4680	-0.4096	-0.3640	-0.1766
w^w	0.0900	0.0767	0.0645	0.0018	-0.0422	-0.0590	-0.0405
w^k	0.0900	0.0767	0.0645	0.0018	-0.0422	-0.0590	-0.0405
r^k	-0.1681	-0.1434	-0.1205	-0.0034	0.0789	0.1103	0.0756

exception of the consumption of capitalists, while the effect on impact for income of workers is positive but negligible; however, after the second period the response on the income of workers becomes negative and increases in absolute value (See Table 12). When it comes to effort level, given that a deterioration in institutional quality makes rent seeking more effective it comes as no surprise that effort level falls both on the economy-wide and sectoral level with the fall in the effort level of capitalists being by far the largest. When it comes to non-leisure time the picture is more mixed. Economy-wide non-leisure time and non-leisure time of workers increase after a positive shock in θ_t while non-leisure time of capitalists falls with its fall being considerably larger relative to the responses of economy-wide non-leisure time and non-leisure

time of workers in absolute value. Clearly, the deterioration of institutions creates an incentive to rent seek and this is bad for the economy.

9 Conclusions

In this paper, we build on the simple RBC model and introduce heterogeneity with two types of households: workers and capitalists. We also build on the concept of rent seeking introduced by Tullock (1967) and papers by Park et al. (2005), Angelopoulos, Philippopoulos, Vassilatos (2009) and Angelopoulos, Economides, Vassilatos (2011) and use the simple RBC model with heterogeneous agents and distortionary taxation in order to introduce institutions through rent seeking competition. Under this specification, workers and capitalists are engaged in rent seeking activities and compete with agents in each group in order to extract a fraction of a contestable prize, here being the income of other agents; i.e. the contestable prize for workers is the income of capitalists and the contestable prize for capitalists is the income of workers. In the terminology of Chari et al. (2007), we observe that introducing rent seeking activities in the simple RBC model with heterogeneous agents introduces an additional friction to the simple RBC model with heterogeneous agents and distortionary taxation that induces wedges which distort agents decisions and depend on the level of institutional quality.

Moreover, looking at the long-run solution of the two models we find that the introduction of rent seeking activities in the heterogeneous agents framework has a negative impact on the macroeconomy.

We also investigate the qualitative implications, in terms of second moment properties, of introducing rent seeking activities in the simple RBC model with heterogeneous agents. In terms of the economy-wide variables, we find a similar qualitative behavior in the two models however quantitative differences arise mostly in sectoral variables. Moreover, we investigate how does the heterogeneous model with rent seeking activities (CWRS) perform relative to the data. A notable finding is that the heterogeneous agents model with rent seeking activities performs quite well in terms of relative volatility, persistence and co-movement.

Finally, in order to investigate the dynamic implications of the introduction of rent seeking activities in a heterogeneous agents model we compute impulse responses to the key economy-wide and sectoral variables for the heterogeneous agents model with and without rent seeking activities. We find that the two models share similar qualitative characteristics overall, yet looking at sectoral variables reveals a more interesting picture as quantitative and qualitative differences arise.

Overall, comparison of the dynamic characteristics among heterogeneous agents model with and without rent seeking activities reveals a similar qualitative behaviour on the economy-wide level while differences are observed on the distributional level.

References

- Acemoglu, D., 2009, Introduction to Modern Economic Growth. Princeton University Press.
- Acemoglu, D., Johnson, S., 2005, Unbundling institutions. *Journal of Political Economy* 113, 949-995.
- Alesina, A., Özler, S., Roubini N., Swagel P., 1996, Political instability and economic growth. *Journal of Economic Growth*, Volume 1, Issue 2, 189-211.
- Angelopoulos, K., Economides, G., Vassilatos, V., 2011, Do institutions matter for economic fluctuations? Weak property rights in a business cycle model for Mexico. *Review of Economic Dynamics*, Volume 14, Issue 3, 511-531.
- Angelopoulos, K., Philippopoulos, A., Vassilatos, V., 2009, The social cost of rent seeking in Europe. *European Journal of Political Economy*, Volume 25, 280-299.
- Aron, J., 2000, Growth and institutions: A review of the evidence. *The World Bank Research Observer*, Volume 15, Issue 1, 99-135.
- Becker, G.S., 1983, A Theory of Competition Among Pressure Groups for Political Influence. *The Quarterly Journal of Economics*, vol. 98, no. 3, pp. 371-400.
- Bourguignon, F., 2018, World changes in inequality: An overview of facts, causes, consequences, and policies. *CESifo Economic Studies*.
- Brunt, L., 2007. Property rights and economic growth: Evidence from a natural experiment. *CEPR DP 6404*.
- Chari, V.V., Kehoe J.T., McGrattan E.R., 2007, Business Cycle Accounting. *Econometrica*, Volume 75, Issue 3, 781-836.
- Christou, T., Philippopoulos A., Vassilatos V., 2019, Modeling rent seeking activities: quality of institutions, macroeconomic performance and the economic crisis. Athens University of Economics and Business, Department of Economics, Working paper series, 06-19.
- Christou, T., Philippopoulos A., Vassilatos V., 2021, Institutions and Macroeconomic Performance: Core vs Periphery Countries in the Eurozone. *Oxford Economic Papers* (forthcoming).

Conesa J.C., Kehoe J.T., Ruhl J.K. (2007), Modeling Great Depressions: The Depression in Finland in the 1990s. Federal Reserve Bank of Minneapolis, Quarterly Review (November 2007).

Dellas, H., Malliaropoulos, D., Papageorgiou, D., Vourvachaki, E., 2017, Fiscal policy with an informal sector. Bank of Greece, Working paper series, No. 235.

Drazen, A., 2000, Political Economy in Macroeconomics. Princeton University Press.

Dixit, A.K., 2004, Lawlessness and Economics Alternative Modes of Governance, Princeton University Press.

Effendic, A., Pugh, G., Adnett N., 2011, Institutions and economic performance: A meta-regression analysis. European Journal of Political Economy, Volume 27, 586-599.

European Commission, Eurostat database and AMECO database, 2019.

Esteban J. and Ray D., 2011. Linking Conflict to Inequality and Polarization. American Economic Review, American Economic Association, vol. 101(4), pages 1345-1374, June.

Federal Reserve Bank of St. Louis Database, 2018, USA.

Grief, A., 2000, The fundamental problem of exchange: A research agenda in Historical Institutional Analysis. European Review of Economic History, vol. 4, issue 03, 251-284.

Hillman, A.L., (2009), Public Finance and Public Policy: A Political Economy Perspective on the Responsibilities and Limitations of Government. Cambridge University Press.

International Country Risk Guide (ICRG) index, 2015. Political Risk Services (PRS) Group Inc.

King, R.G., Rebelo, S.T., 1999, Resuscitating real business cycles. In: Taylor, J., Woodford, M. (Eds.), North-Holland, Amsterdam, 927-1007.

Kollintzas, T., Vassilatos, V., 2000, A small open economy model with transaction costs in foreign capital. European Economic Review, Volume 44, 1515-1541.

Malley, J., Philippopoulos, A., Woitek, U., 2009, To react or not? Technology shocks, fiscal policy and welfare in the EU-3. European Economic Review, Volume 53, 689-714.

Murphy, K.M., Shleifer, A., Vishny R.V., 1991, The Allocation of Talent: Implications for Growth, *The Quarterly Journal of Economics*, Volume 106, Issue 2, May 1991, Pages 503–530.

North, D.C., 1990. *Institutions, Institutional Change and Economic Performance*. Cambridge University Press, Cambridge.

Park, H., Philippopoulos, A., Vassilatos V., 2005, Choosing the size of the public sector under rent seeking from state coffers. *European Journal of Political Economy*, Volume 21, 830-850.

Tullock, G., 1967, The Welfare Costs of Tariffs, Monopolies and Theft. *Western Economic Journal*, Volume 5:3, 224.

Total Economy Database, 2018. The Conference Board.

Worldwide Governance Indicators (WGI) Project, 2019, The World Bank, Washington DC, USA.

Appendices

A Data

A.1 Country data

We consider the following two sets of countries: a) Core countries, consisting of Austria (AT), Belgium (BG), Germany (DE), France (FR), Finland (FI), Netherlands (NL) and b) Periphery countries, consisting of Cyprus (CY), Greece (GR), Ireland (IR), Italy (IT), Portugal (PT) and Spain (ES). Data are of annual frequency and cover the period 2001-2016. Our main data source for macroeconomic variables is Eurostat. We also use data from the Total Economy Database, the St. Louis FED and AMECO, the International Country Risk Guide from the PRS Group and the World Governance Indicators from the World Bank. To find the share of hours at work in available time, h_t , we use the ratio of the 'annual hours worked per worker' series to the 'total available time per worker' from the Total Economy Database.²⁶ We use the 'Net Capital stock' series from AMECO for real capital in our model.²⁷

Table 13: Taking the model to the data

Code	Description	Variable
$DC.1 = I_t$	Total investment	$I_t = D.4 \frac{D.1}{D.2}$
$DC.2 = C_t$	Total consumption	$C_t = [D.2 + (D.7 - D.8)] \frac{D.1}{D.2}$
$DC.3 = H_t$	Hours at work	$H_t = \frac{D.16}{52 \times 14 \times 7}$

Matching the model with the data

To match the economy-wide variables of our closed economy model for each model with the variables observed in the data we follow usual practise (e.g. see Kehoe and Prescott (2002, 2007) and Conesa et al. (2007)), and define output in our model to be the real gross domestic product in the data. We also allocate real net exports to real consumption in the data, and investment and capital in our model to be total investment and total capital respectively in the data.

²⁶Total available time per worker is calculated as 52 weeks x 14 hours x 7 days.

²⁷We use the GDP deflator to transform nominal variables to real variables.

Table 14: Data

Code	Variable	Unit	Source
D.1	Gross domestic product	Millions of euros	Eurostat
D.2	Gross domestic product	Millions of 2010 euros	Eurostat
D.3	Final consumption expenditure	Millions of euros	Eurostat
D.4	Gross fixed capital formation	Millions of euros	Eurostat, AMECO
D.5	Consumption of fixed capital	Millions of euros	Eurostat
D.6	Net capital stock	Millions of 2010 euros	AMECO
D.7	Exports of goods and services	Millions of euros	Eurostat
D.8	Imports of goods and services	Millions of euros	Eurostat
D.9	Final consumption expenditure of general government	Millions of euros	Eurostat
D.10	Gross fixed capital formation of general government	Millions of euros	Eurostat
D.11	Social benefits other than social transfers in kind and social transfers in kind purchased market production, payable	Millions of euros	Eurostat
D.11	Population	Thousands of people	TED
D.12	Annual hours worked per worker	Hours	TED
D.13	Total annual hours worked	Hours	TED
D.14	EMU convergence criterion bond yields	Rate	Eurostat
D.15	Total factor productivity (USA=1)	Index	St. Louis FED
D.16	Composite Risk Rating	Index	ICRG, PRS Group
D.19	GINI coefficient	Index	Eurostat

B Second moment properties

B.1 Volatility and relative volatility

Table 15: Relative volatility, $x \equiv s_x/s_y$

x	CW	CWRS	x	CW	CWRS
c	0.8307	0.8376	c^w	0.9046	0.8635
i	1.4107	1.4333	c^k	0.7129	0.8401
h	0.0547	0.1023	h^w	0.0553	0.1459
k	0.2290	0.2314	h^k	0.3916	0.2988
η	Na	0.1306	y^w	0.9046	0.9928
σ^y	0.0252	0.0252	y^k	1.6844	2.0093
			η^w	Na	0.1091
			η^k	Na	0.2382

B.2 Persistence

Table 16: Persistence, $\rho(x_t, x_{t-1})$

x	CW	CWRS	x	CW	CWRS
y	0.4822	0.4769	y^w	0.4854	0.4791
c	0.4890	0.4829	y^k	0.2100	0.0780
i	0.4744	0.4693	c^w	0.4854	0.4799
h	0.4651	0.4643	c^k	0.4974	0.4878
k	0.8484	0.8462	h^w	0.4651	0.4686
η	Na	0.4646	h^k	0.4645	0.4627
			η^w	Na	0.4656
			η^k	Na	0.4636

B.3 Co-movement with output

Table 17: Co-movement with output, $\rho(y_t, x_{t+1})$

Co-movement with output			
	t-1	t	t+1
c	0.4651	0.9986	0.4931
i	0.4925	0.9968	0.4496
h	0.4113	0.8057	0.3429
k	-0.3633	-0.0685	0.4662
c^w	0.4657	0.9901	0.4817
c^k	0.4321	0.9494	0.4842
h^w	0.1550	0.3540	0.1891
h^k	0.3782	0.6511	0.2093
y^w	0.4641	0.9851	0.4780
y^k	0.4301	0.7951	0.0760
η	0.0323	0.0642	0.0285
η^w	-0.0050	0.0043	0.0129
η^k	0.0977	0.1680	0.0545

Table 18: Co-movement with output in the data, $\rho(y_t, x_{t+1})$

Data			
	t-1	t	t+1
c	0.2819	0.5171	0.2350
i	0.4496	0.8789	0.5758
h	0.2224	0.3578	0.0069
k	-0.0549	0.3446	0.6327
η	Na	Na	Na

C Impulse response functions

Shock in A_t

Table 19: Positive shock in A_t , CW

A	1	0.9900	0.9801	0.9135	0.8262	0.7472	0.3697
y	1.0759	1.1053	1.1315	1.2409	1.2584	1.2003	0.6251
y^w/y^k	-0.3532	-0.3264	-0.3016	-0.1732	-0.0779	-0.0344	0.0010
y^w	1.0142	1.0466	1.0755	1.1995	1.2288	1.1771	0.6155
y^k	1.3674	1.3730	1.3771	1.3727	1.3067	1.2115	0.6144
c	0.8965	0.9383	0.9759	1.1451	1.2077	1.1708	0.6188
c^w	0.9780	1.0142	1.0466	1.1887	1.2307	1.1842	0.6217
c^k	0.7246	0.7783	0.8269	1.0533	1.1590	1.1426	0.6127
h	0.0566	0.0527	0.0491	0.0302	0.0160	0.0093	0.0020
h^w	-0.0362	-0.0337	-0.0314	-0.0194	-0.0102	-0.0060	-0.0013
h^k	0.3941	0.3669	0.3417	0.2105	0.1115	0.0648	0.0140
k	0	0.1282	0.2455	0.8230	1.1783	1.2640	0.7250
i	1.5066	1.5063	1.5049	1.4710	1.3802	1.2711	0.6404
w^w	0.9594	0.9969	1.0305	1.1787	1.2254	1.1811	0.6210
w^k	0.9594	0.9969	1.0305	1.1787	1.2254	1.1811	0.6210
r^k	1.0759	0.9771	0.8860	0.4180	0.0801	-0.0638	-0.0998

Table 20: Positive shock in A_t , CWRS

	1	2	3	10	20	30	100
A	1	0.9900	0.9801	0.9135	0.8262	0.7472	0.3697
y	1.0643	1.0957	1.1238	1.2459	1.2763	1.2252	0.6446
y^w/y^k	0.0069	0.0374	0.0654	0.2051	0.2941	0.3174	0.1849
y^w	1.2065	1.2422	1.2741	1.4128	1.4474	1.3896	0.7311
y^k	1.1995	1.2047	1.2087	1.2077	1.1534	1.0721	0.5462
c	0.9109	0.9512	0.9875	1.1536	1.2188	1.1856	0.6317
c^w	0.8721	0.9063	0.9371	1.0760	1.1258	1.0905	0.5787
c^k	0.9926	1.0458	1.0938	1.3170	1.4152	1.3862	0.7434
h	0.1294	0.1281	0.1268	0.1182	0.1068	0.0966	0.0478
h^w	0.1770	0.1826	0.1875	0.2092	0.2151	0.2068	0.1090
h^k	-0.0441	-0.0703	-0.0943	-0.2134	-0.2875	-0.3048	-0.1751
η	0.0128	0.0147	0.0164	0.0245	0.0292	0.0297	0.0165
η^w	0.0398	0.0455	0.0507	0.0760	0.0903	0.0920	0.0511
η^k	-0.0948	-0.1084	-0.1208	-0.1812	-0.2155	-0.2194	-0.1218
k	0	0.1295	0.2484	0.8425	1.2222	1.3237	0.7730
i	1.5221	1.5270	1.5304	1.5216	1.4477	1.3433	0.6831
w^w	0.9656	1.0028	1.0362	1.1865	1.2393	1.1997	0.6363
w^k	0.9656	1.0028	1.0362	1.1865	1.2393	1.1997	0.6363
r^k	1.0643	0.9662	0.8754	0.4034	0.0541	-0.0985	-0.1284

Shock in ϕ_t

Table 21: Positive shock in ϕ_t , CW

ϕ	1	0.9900	0.9801	0.9135	0.8262	0.7472	0.3697
y	0.2520	0.2615	0.2700	0.3073	0.3186	0.3067	0.1611
y^w/y^k	-0.6619	-0.6484	-0.6355	-0.5601	-0.4830	-0.4261	-0.2054
y^w	-0.0586	-0.0453	-0.0331	0.0280	0.0683	0.0814	0.0502
y^k	0.6034	0.6031	0.6024	0.5881	0.5513	0.5075	0.2556
c	0.1696	0.1830	0.1952	0.2523	0.2796	0.2763	0.1485
c^w	-0.0730	-0.0586	-0.0453	0.0214	0.0659	0.0808	0.0507
c^k	0.6811	0.6925	0.7024	0.7391	0.7301	0.6884	0.3548
h	-0.0277	-0.0285	-0.0291	-0.0318	-0.0321	-0.0306	-0.0159
h^w	-0.1203	-0.1185	-0.1167	-0.1058	-0.0935	-0.0836	-0.0409
h^k	0.3089	0.2988	0.2894	0.2375	0.1912	0.1622	0.0749
k	0	0.0383	0.0733	0.2458	0.3519	0.3775	0.2165
i	0.4499	0.4498	0.4494	0.4393	0.4122	0.3796	0.1912
w^w	-0.1349	-0.1194	-0.1052	-0.0329	0.0178	0.0379	0.0297
w^k	0.8651	0.8706	0.8749	0.8806	0.8440	0.7851	0.3994
r^k	0.2520	0.2232	0.1967	0.0615	-0.0333	-0.0708	-0.0554

Table 22: Positive shock in ϕ_t , CWRS

	1	2	3	10	20	30	100
ϕ	1	0.9900	0.9801	0.9135	0.8262	0.7472	0.3697
y	0.2496	0.2604	0.2701	0.3145	0.3316	0.3223	0.1716
y^w/y^k	-0.4652	-0.4509	-0.4375	-0.3622	-0.2933	-0.2490	-0.1144
y^w	0.0666	0.0810	0.0942	0.1590	0.1974	0.2039	0.1147
y^k	0.5318	0.5319	0.5317	0.5212	0.4907	0.4529	0.2291
c	0.1721	0.1860	0.1986	0.2588	0.2895	0.2881	0.1568
c^w	-0.1490	-0.1340	-0.1200	-0.0479	0.0049	0.0273	0.0259
c^k	0.8495	0.8610	0.8708	0.9057	0.8899	0.8384	0.4330
h	0.0289	0.0286	0.0283	0.0264	0.0239	0.0216	0.0107
h^w	0.0293	0.0313	0.0331	0.0417	0.0459	0.0454	0.0246
h^k	0.0276	0.0189	0.0109	-0.0294	-0.0565	-0.0653	-0.0399
η	-0.0580	-0.0568	-0.0556	-0.0489	-0.0420	-0.0369	-0.0177
η^w	-0.1795	-0.1757	-0.1722	-0.1514	-0.1301	-0.1144	-0.0549
η^k	0.4281	0.4192	0.4107	0.3612	0.3103	0.2729	0.1309
k	0	0.0409	0.0785	0.2662	0.3861	0.4182	0.2442
i	0.4808	0.4824	0.4835	0.4807	0.4573	0.4243	0.2158
w^w	-0.1336	-0.1175	-0.1026	-0.0259	0.0292	0.0513	0.0388
w^k	0.8664	0.8725	0.8775	0.8877	0.8553	0.7985	0.4086
r^k	0.2496	0.2195	0.1916	0.0483	-0.0545	-0.0958	-0.0726

Shock in θ_t

Table 23: Positive shock in θ_t , CWRs

	1	2	3	10	20	30	100
θ	1	0.9900	0.9801	0.9135	0.8262	0.7472	0.3697
y	-0.1681	-0.1776	-0.1861	-0.2258	-0.2437	-0.2391	-0.1284
y^w/y^k	0.3753	0.3635	0.3524	0.2904	0.2340	0.1980	0.0906
y^w	0.0016	-0.0111	-0.0226	-0.0806	-0.1177	-0.1277	-0.0747
y^k	-0.3737	-0.3745	-0.3750	-0.3710	-0.3516	-0.3257	-0.1653
c	-0.0898	-0.1020	-0.1131	-0.1669	-0.1973	-0.2004	-0.1111
c^w	-0.1712	-0.1808	-0.1895	-0.2301	-0.2484	-0.2437	-0.1309
c^k	0.0817	0.0642	0.0481	-0.0337	-0.0896	-0.1091	-0.0691
h	0.0587	0.0581	0.0576	0.0537	0.0485	0.0439	0.0217
h^w	0.1523	0.1488	0.1455	0.1266	0.1076	0.0941	0.0448
h^k	-0.2819	-0.2721	-0.2628	-0.2119	-0.1666	-0.1389	-0.0623
η	-0.2121	-0.2105	-0.2088	-0.1971	-0.1801	-0.1637	-0.0815
η^w	-0.1302	-0.1305	-0.1307	-0.1294	-0.1227	-0.1137	-0.0577
η^k	-0.5396	-0.5304	-0.5215	-0.4680	-0.4096	-0.3640	-0.1766
k	0	-0.0342	-0.0656	-0.2224	-0.3226	-0.3494	-0.2040
i	-0.4018	-0.4031	-0.4040	-0.4016	-0.3821	-0.3546	-0.1803
w^w	0.0900	0.0767	0.0645	0.0018	-0.0422	-0.0590	-0.0405
w^k	0.0900	0.0767	0.0645	0.0018	-0.0422	-0.0590	-0.0405
r^k	-0.1681	-0.1434	-0.1205	-0.0034	0.0789	0.1103	0.0756