Do firms gain from managerial overconfidence? The role of severance pay.

Clara Graziano* and Annalisa Luporini†

This version: August 16, 2021
Preliminary draft

Abstract

We analyze the effects of optimism and overconfidence when the manager's compensation package includes severance pay and the CEO has some bargaining power. We find that overconfidence reduces incentive pay as found by previous literature but (when high enough) increases severance pay, while managerial optimism increases severance pay with no countervailing effect. For high values of overconfidence the incentive compatibility constraint cannot be satisfied and an inefficient level of investment is chosen. Our simple model helps explaining several features of the large severance payments documented by empirical literature. It shows that the optimal contractual severance pay is lower than the actual payment in case of turnover; and that the overall severance payment to the departing CEO is larger than the incentive pay in case of success. Consequently, the attempt to exploit the lower cost of incentives due to overconfidence, may backfire if the CEO is replaced.

JEL classification: J33, M52, L21.

Keywords: overconfidence, managerial compensation, severance pay, entrenchment, firing policy.

1 Introduction

A vast and well-documented empirical evidence suggests that overconfidence is a common phenomenon at the root of many observed behaviors among CEOs

*University of Udine and CESifo; address: University of Udine, Department of Economics and Statistics, via Tomadini 30, 33100 Udine , Italy; email: clara.graziano@uniud.it

†Corresponding author. University of Florence and CESifo. address: Department of Economics and Management, Via delle Pandette 9,50127 Firenze, Italy. e-mail: annalisa.luporini@unifi.it

(Malmendier and Tate, 2015). Theoretical and empirical literature has shown that managerial biases influence contract design and CEO behavior in many different aspects, such as CEO choice of projects (see the survey in Melmandier and Tate 2005), CEO hiring (Goel and Thakor 2008) and CEO compensation (de La Rosa 2011, Gervais et al. 2011 and Otto 2014).

In particular, some authors have suggested that principals can benefit from hiring an overconfident manager. In a standard agency model with moral hazard, the optimal contract trades off risk insurance and incentive provision. Managerial overconfidence (and the resulting divergence of beliefs between principal and agent) affects the trade-off between risk and incentives and makes it easier to satisfy the incentive compatibility constraint. This is so because the manager overestimates the probability of success and this, in turn, makes it cheaper to induces her/him to exert effort. In other words, the amount of the compensation necessary to induce the agent to exert a given level of effort decreases (see de La Rosa 2011, Gervais et al. 2011, Otto 2014, and Humphery-Jenner et al. 2016). Consequently, firms can exploit the misvaluation of their executives by offering a compensation structure with a particularly heavy incentive pay (the so-called exploitation hypothesis empirically investigated in Humphery-Jenner et al. 2016).

Another strand of literature has analyzed severance pay and its role from an optimal contracting point of view. Severance agreements are an important component of managerial contracts given the high turnover among executives, even more so when a manager has to make long-lasting investments and the prospect of being replaced may interfere with her/his decision. If the board cannot commit to retain the manager once the investment is in place, there is room for opportunistic behavior. Severance pay may alleviate the moral hazard problem created by the unobservability and the firm-specific nature of the investment (see for example Almazan and Suarez (2003) and Inderst and Mueller (2010)). On the other hand, the critics of severance pay have pointed out that, by insulating the manager from the consequences of poor performance, severance pay is simply a "reward for failure" that violates the pay-for-performance principle of agency theory (Bebchuk and Fried 2004).

Overall, empirical research on severance agreements offers support to the efficiency explanation of severance agreements (Cadman et al. 2016; Rau and Xu 2013) and shows that the discretionary component in excess of contractual severance agreements is consistent with good governance in forced turnover (Goldman and Huang 2015).[1]Then, a natural question is if and how we can reconcile optimal contracting with the large severance payments offered to departing CEOs that often media interpret as shareholders expropriation. Can a large discretionary pay be optimal when the CEO is forced out because a better CEO has been identified? Though overconfidence may affect severance pay and the replacement decision in many ways, there are no studies of its effects on severance agreements and board's replacement decision. The present paper is a first attempt to fill this gap by analyzing the effect of managerial optimism and overconfidence when the CEO is fired. We follow the previous literature (see de

[1]There are however contrasting view as for example Muscarella and Zhao (2015).

la Rosa 2011, among others) and we distinguish between optimism according to which the manager has a higher subjective belief on the probability of success than the "true" probability, and overconfidence that increases the manager's assessment of the increase in the probability of success due to her/his effort. In our context, effort takes the form of a firm-specific investment. The manager can decide whether to undertake a positive level of investment choosing from two different values (an efficient and an inefficient one) or whether to abstain from investing. We investigate how optimism and overconfidence affect both investment choice and the amount of separation pay necessary to induce the manager to leave and how this in turn affects profits. We build upon the analysis of Almazan and Suarez (2003) who suggest that a sizeable severance pay may be desirable to induce an unbiased manager to take non-contractible actions beneficial for the firm. They show that a mix of severance pay and some degree of entrenchment, rather than just incentive pay, can be a cheaper way to induce the manager to take the desired action. In their model, severance pay is renegotiated at the time when separation occurs, implying that its exact amount is established ex-post when the board knows whether the investment has been made. This costs less than motivating the manager through incentive pay which has to satisfy the incentive compatibility constraint ex-ante, and therefore is only indirectly related to the desired actions. If renegotiation may occur, the manager is motivated to undertake the investment because he anticipates that this will increase his bargaining position at the bargaining stage.

Almazan and Suarez consider an unbiased manager. Overconfidence and optimism create a wedge between board's and manager's belief of expected profit that affects the amount of severance pay asked by the manager. Indeed, if the manager has bargaining power, he leaves the firm only when an "appropriate" amount of severance pay is negotiated and this "appropriate" amount depends on optimism and overconfidence, both directly and through the choice of the investment level. Replacing the manager becomes more costly and results in higher entrenchment.

A crucial assumption in what follows is that the incumbent manager has some bargaining power because she/he can credibly threaten to resist being replaced. The idea underlying this assumption is that the incumbent manager can oppose his replacement by making it a costly and contentious process so that valuable opportunities are missed and firm value decreases. Furthermore, the manager may also take value-destroying decisions. To allow a smooth replacement process, the board is willing to renegotiate the separation agreements and agree to a payment high enough to avoid a costly opposition.

Empirical evidence indicates that payments in excess of contractual severance pay are common, and particularly so, in forced turnover events (Goldman and Huang, 2015). In other words, when the replacement generates a positive surplus, both the board and the CEO may find it advantageous to renegotiate the contractual agreement and increase the severance payment in order to guarantee a smooth turnover. For instance, in 2019 McDonald's CEO Steve Easterbrook, got a very generous exit package of more 37 million $ because his relationship with an employee violated company policy. In this case, the board determined his firing to be not for cause — a discretionary threshold. A commentator explained this decision saying that "litigation in a protracted dispute can be tricky and expensive".[2] Later, when new accusations came to light, the company tried to claw back millions of severance pay. However, the story is indicative of the large severance payment commonly agreed upon and of its discretionary terms.

In what follows, we assume that the manager can oppose replacement if contractual severance pay is smaller than what the manager believes would receive by staying with the firm, an amount that can be considered his "outside options" in the bargaining process. Thus we consider a model where severance pay is renegotiated whenever this is mutually advantageous. We believe this is a reasonably conservative assumption.[3]

The main findings of the paper can be summarized as follows. First, overconfidence and optimism have different impact on managerial compensation and consequently on firm expected profit. While optimism always raises severance payment increasing entrenchment and is thus detrimental for the firm, overconfidence may be either advantageous or detrimental depending on the degree of the bias. Moderate overconfidence makes both incentive and severance pay decrease without affecting investment choice. This results in higher expected profit for the firm. Extreme overconfidence, on the contrary, distorts investment choice and results in and increase in severance pay (and entrenchment). The final effect is a reduction of firm expected profit. Thus, the attempt to exploit executive overconfidence through a heavy use of incentive-pay, documented for example by Humphery-Jenner et al. (2016), can backfire when investment choice and opportunity of replacing the manager are considered. Managerial biases, indicated as beneficial when incentive pay only is considered, may be detrimental to the firm when turnover and severance pay are taken into account. This is also true in case of moderate overconfidence if the latter is coupled with a sufficiently high level of optimism. In such a case, the overall effect depends on the mix of optimism and overconfidence.

Moreover, we show that the discretionary severance pay renegotiated when the CEO leaves, increases with managerial overconfidence and this helps explaining the high level of discretionary severance pay reported by empirical studies and anecdotal evidence. It has indeed been shown that in forced turnover the amount of separation pay largely exceeds the level of contractual severance pay. Our model, rationalizes this practice as a result of the renegotiation at the turnover stage, in the presence of managerial overconfidence.

The rest of the paper is organized as follows: Section 2 presents the model. Section 3 and 4 analyze the contracting problem by backward induction. Sec-

[2] NPR, November 5, 2019 Morning Edition by Alina Selyukh

[3] Many alternatives, however, can be considered. For instance, the manager could be able to appropriate the whole surplus from the replacement (Almazan and Suarez for example consider this case). Of course, assuming a stronger bargaining power resulting in a larger fraction of surplus for the incumbent manager, would make hiring an overconfident manager even more costly for the firm.

tion 5 offers the main results by comparing the value of the firm under diverse assumptions on managerial bias. Finally, Section 6 concludes.

2 The model

Consider a board that perfectly represents the shareholders so that its objective is to maximize firm's value. The board hires a CEO to implement a project. The cash flow generated by the project can take only two values: $r = 0$ and $r = R > 0$. The probability of success of the project, denoted by p_i, depends on a firm-specific investment I_i that the manager can make. With no investment $(I = 0)$ the probability of success is $p_L > 0$, if instead the manager makes the investment, the probability of success increases to $p_M > p_L$ when the investment is I_M, and to $p_H > p_M$ if the larger investment $I_H > I_M$ is chosen. The cost of this investment is borne by the CEO. We assume that $c(I_i) = I_i$ with $i = M, H$ and that $c(0) = 0$. The investment is unverifiable, though it is observable by the board that, consequently, knows the manager's probability of success.

After the manager has decided the level of the investment, a new manager, possibly with a higher probability of success, materializes. We denote the probability of success of the new manager by $q \in [\underline{q}, 1]$. Both the board and the incumbent manager observe the realization of q. When the new manager has a significantly better chance of success, the board may prefer to fire the incumbent manager and hire the replacement.

In order to establish a benchmark, we consider a manager whose subjective beliefs are equal to the "true" probability p_i and we refer to this type of manager as "rational". However, the problem we analyze, arises because incumbent manager and board hold heterogeneous belief regarding the probability of success and are aware of such divergence. Following the previous literature we decompose the managerial bias into two components: optimism, θ, that uniformly biases upward the manager's belief on the probability of success, and overconfidence Δ_i that captures the manager's distorted belief on the productivity of his investment.[4] We then say that the manager is optimistic when he overstates the probability of good outcomes and that the manager is overconfident when he overstates the productivity of his investment. Hence, optimism, is independent of the manager's action and is always at work, while overconfidence is positively related to the level of investment. The effect of such biases on beliefs is made explicit by the following assumption:

<u>*Assumption 1*</u>: i) When no investment is made, the manager's beliefs about the probability of success are $p_L + \theta$.

ii) If the manager makes investment I_M, his beliefs about success are: $p_M + \theta + \Delta_M < 1$,

ii) If the manager makes investment I_H, his beliefs about success are $p_H + \theta + \Delta_H$, where $\Delta_H = \Delta_M(1 + z)$ denotes the higher overconfidence resulting from the

[4] We follow the terminology introduced in de La Rosa, (2011).

higher investment, with $z > \frac{p_H - p_M}{p_M - p_L} > 0$ and $1 - p_H - \theta \geq \Delta_H > 0$.

Assumption 1 states that the effect of overconfidence is higher in case of I_H than in case of I_M,[5] and that, given $z > 0$, the slope of the manager's beliefs of success (considered as a function of overconfidence) is everywhere steeper in case of I_H. In what follows, we consider the parameter z constant and we refer to an increase in Δ_M as an increase in overconfidence. Then, an increase in overconfidence implies a higher increase in the manager's beliefs if the latter chooses I_H than if he chooses I_M.

The board knows that the manager is optimistic and overconfident, and takes this into account when offering the contract. At the same time, the manager believes that the board undervalues his probability of success; he knows that the board only attributes probability p_i to return R, following investment I_i with $i = M, H$, and probability p_L if the investment is not undertaken. This heterogeneity of beliefs affects both the original contract and subsequent renegotiation, if any.

We assume that the CEO can oppose being fired so that replacement can occur only with mutual agreement between board and CEO. Consequently, in order to induce the manager to leave, the board must renegotiate the severance payment whenever the manager deems that contractual terms are unsatisfactory. The following additional assumptions complete the framework.

Assumption 2: If the incumbent CEO does not invest, the probability of success of the replacement is always higher than the incumbent's beliefs of his own probability of success. Conversely, by investing, the incumbent CEO may have higher probability of success than the replacement:

$$p_L + \theta < \underline{q} < p_M < p_H < \overline{q} = 1.$$

Assumption 2 captures the idea that the firm-specific investment is necessary (but not sufficient) to avoid being less productive than the replacement.

Finally, the following two assumptions, determine the preferences of the board as far as the investment is concerned.

Assumption 3: The return R is high enough to guarantee that in the benchmark case with a rational manager, the board always prefers investment I_M to no investment: $R \geq \frac{p_M}{(p_M - p_L)^2} I_M$. This implies that the investment I_M is efficient: $(p_M - p_L)R > I_M$.

Assumption 4: The high investment I_H is inefficient: $(p_H - p_M)R \leq I_H - I_M$.

The above assumption states that the additional cost when choosing I_H rather than I_M is larger than the additional expected return from the investment

[5] As far as the increase in the beliefs of success is concerned, note the level of investment I_M induces an increment in managerial belief equal to $p_M - p_L + \Delta_M$ with respect to the absence of investment, while the additional investment $I_H - I_M$ induces an increment in managerial belief equal to $p_H - p_M + z\Delta_M$.

so that, if the manager were rational, the high investment I_H will never be incentivized by the board. However, as we will see below, this may not prevent an overconfident manager from choosing I_H due to the different assessment of the probability of success. Thus, assumption 4 accounts for the possibility, documented by a large literature (see, among others, Malmendier and Tate 2005), that an optimistic and overconfident manager chooses an investment level higher than the optimal one. Though the cost of the investment is borne by the manager, it affects the outcome of the renegotiation in case of dismissal and it may thus become a cost also for the firm.

Assumptions 3 and 4 together imply: $\frac{I_H - I_M}{p_H - p_M} \geq R \geq \frac{I_M}{p_M - p_L}$ which can be satisfied only if:

$$\frac{I_M}{I_H} \leq \frac{p_M - p_L}{p_H - p_L}.$$

The contract offered by the board maximizes the expected terminal cash flow of the project net of the CEO compensation. Then, the contract aims at providing the manager with the incentive to make the profit-maximizing level of investment even if replacement may occur. To this end, we consider a simple incentive contract with base salary, incentive pay w contingent on the high return R, and severance pay s. We normalize the reservation level of utility to 0, so that the base salary takes value 0. This also represents the compensation of the new manager in case of replacement. Note that severance pay, providing a payment in case the incumbent is replaced, is an essential element of the contract.

The manager is protected by limited liability. If the incumbent CEO remains in office, he enjoys benefit of control $B > 0$. We assume that this value is small in the sense that $B \leq (\underline{q} - p_L)R - \theta w$. Such non-monetary benefit increases the utility of staying with the firm and may thus create a conflict between shareholders and incumbent, but at the same time it makes it easier to induce the manager to invest.

The timing of the model can be summarized as follows:

$t = 0$: The board observes whether the manager is rational or biased (optimistic/overconfident) and offers compensation contract (w, s) tailored to the manager's type. The manager decides whether to accept the offer.

$t = 1$: If the contract is accepted, the manager decides whether and how much to invest.

$t = 2$: The board observes the investment decision (i.e., the probability of success)

$t = 3$: A rival CEO appears. Board and incumbent manager observe the rival's ability. The board evaluates whether it is profitable to replace the incumbent. If this is the case and contractual s is too low for the incumbent to accept replacement, renegotiation occurs and a new level of severance pay, s', is agreed upon.

$t = 4$: Cash flow realizes. The manager is paid the compensation agreed upon.

The model is solved by working backwardly. We first determine the conditions for replacement and we find the outcome of the renegotiation under an arbitrary initial contract. We then discuss the board's replacement decision. Given the replacement decision, we determine the investment level chosen by the manager. Then, we find the incentive compatible contract (w, s) that maximizes the firm final cash flows. Finally, we analyze the effect of the managerial bias on the firm expected profit

3 Renegotiation and replacement decision

Let us first establish the condition under which the board is willing to fire the incumbent manager. Given that our focus is on the incentive to invest, we only consider contracts designed in such a way as to induce the manager to undertake a positive level of investment.

Suppose that the parties have struck a contract designed to induce I_i. The objective of the board is to maximize the firm final cash flow net of managerial compensation. Consequently, the board wants to replace the incumbent manager whenever the expected profit is higher under the new manager:

$$qR - S_i \geq p_i(R - w_{io}), \qquad i = M, H \tag{1}$$

where S_i indicates the severance pay, either contractual (s_i) or renegotiated (s_i'), w_{io} is the incentive pay offered to induce an overconfident manager o to undertake investment $i = M, H$,and the LHS incorporates the fact that no incentive pay is paid to the new CEO when the incumbent is replaced.

The board behaves "rationally" and uses the "right" probability of success. With no constraints on the manager's side, the board would fire him when the probability of success of the replacement is

$$q \geq p_i - \left(\frac{w_{io} - S_i}{R} \right).$$

Note that such decision is based on the difference between the probabilities of success "adjusted" by the difference of the payment to the incumbent manager in case of retention or dismissal. When $S_i > w_{io}$, meaning that the firing cost exceeds the sum that is saved by replacing the manager, the board will make the replacement decision for higher values of q than in the opposite case where $S_i < w_{io}$.

Given the assumption that the manager can oppose being replaced, severance pay S_i must compensate him for the loss suffered in case of replacement. Consider the case in which the manager has undertaken investment I_i. The incumbent believes that his expected compensation is $(p_i + \theta + \Delta_i)w_{io}$, consequently he consents to replacement only if:

$$S_i \geq (p_i + \theta + \Delta_i)w_{io} + B \tag{2}$$

where the LHS is the payment the manager receives in case of replacement and the RHS represents the benefit the CEO expects to receive if he opposes replacement.

For replacement to occur, conditions 1 and 2 must be simultaneously satisfied:

$$(q - p_i)R + p_i w_{io} \geq S_i \geq (p_i + \theta + \Delta_i)w_{io} + B,$$

implying that the increase in the expected return from replacement must be at least as large as what is lost by the incumbent manager when he leaves the firm:

$$(q - p_i)R \geq \theta_i w_i + B.$$

This condition (taken as an equality) determines the cutoff value of q, above which the incumbent is replaced. Let $\widehat{q}_{io}$ denote such a value when the overconfident manager o has made investment I_i:

$$\widehat{q}_{io} = p_i + \frac{B}{R} + \frac{(\theta + \Delta_i)}{R} w_i.$$

When $q > \widehat{q}_{io}$ the incumbent should be replaced even if he has made the firm-specific investment, while he should be retained when $q \leq \widehat{q}_{io}$. It is useful to establish as a benchmark the cutoff value of q for a rational manager (i.e., a manager with $\theta = \Delta_M = 0$)

$$\widehat{q}_{ir} = p_i + \frac{B}{R}$$

Observe that $\widehat{q}_{ir} < \widehat{q}_{io}$ implying that optimism and overconfidence distort the replacement decision. An **entrenchment effect** occurs as the manager is replaced for higher values of q. This is due to the fact that optimism and overconfidence make it more costly to induce the incumbent CEO to accept being replaced. Note also that the cutoff for the overconfident manager $\widehat{q}_{io}$ depends on the incentive pay w_i, while $\widehat{q}_{ir}$ the cutoff for the rational one does not.

To fully characterize the contract offered to the CEO, we have to determine w_{io} and S_{io} (which in turn determine $\widehat{q}_{io}$). We have argued that the incumbent manager will consent to replacement only if the severance pay satisfies (2). But should contractual s be so high as to satisfy such condition? Not necessarily, because a high contractual s may discourage the manager from investing and severance pay can be renegotiated if both parties find it profitable to do so. We will show below that it is optimal to offer the manager a low contractual s and to possibly provide at the renegotiation stage the sum necessary to induce him to quit.

To determine the level of the renegotiated severance pay s' when contractual s is too low to induce the manager to accept replacement, consider that the board wants to replace the incumbent whenever a more productive manager shows up. This occurs when the incumbent made the investment and a replacement with $q > \widehat{q}_{io}$ appears, but this is also what would happen if the incumbent did not invest (so that the probability of success would be $p_L < \underline{q}$). The latter case never occurs in equilibrium but, precisely to determine the appropriate incentives to discourage such behaviour, we need to take into account what payment the incumbent could obtain by not investing and opposing replacement. Thus, the next proposition determines the minimum payments necessary to induce the incumbent to leave both when he has made investment I_i and when he has not invested despite the incentive pay specified in the contract at $t = 0$, was w_i in order to induce I_i

Proposition 1: *Given that contractual s is too low for the biased incumbent to accept being replaced at $t = 3$, then*

i) If the incumbent has made investment I_i, the lowest renegotiated payment necessary to induce him to leave when $q > \widehat{q}_{io}$, is $s'_{io} = (p_i + \theta + \Delta_i)w_i + B$, $i = M, H$.

ii) If the incumbent has not made any investment ($I = 0$), the lowest renegotiated payment necessary to overcome his opposition to replacement is $\underline{s}'_{io} = (p_L + \theta)w_i + B$.

Proof: See Appendix 1.

Given that the incumbent can oppose replacement, he will accept to leave only if the payment is at least as large as what he can gain by staying with the firm. Note that, contrary to contractual severance pay, the renegotiated payments are conditional on the investment because, at the renegotiation stage, the board knows which level of investment (I_M, I_H, or 0) has been chosen by the manager. The following Corollary establishes the minimum renegotiated payments for a rational manager, anticipating the result of Corollary 3 that a rational manager always chooses I_M.

Corollary 1. *If the manager is rational ($\theta = \Delta_M = 0$), the lowest renegotiated payment necessary to induce him to leave when he has chosen investment I_M and $q > \widehat{q}_r$ is $s'_r = p_M w_r + B$. In case the manager had not invested ($I = 0$), the lowest payment necessary to induce him to leave is $\underline{s}'_r = p_L w_r + B$.*

Even in the case of a rational manager, the renegotiated severance pay corresponds to the amount that the manager expects to obtain by staying with the firm. In summary, any payment lower than s'_{io} and $\underline{s}'_{io}$ (or s'_{ir} and $\underline{s}'_{ir}$) would be rejected by the manager. On the other hand, once the investment is made, there is no reason to increase the renegotiated payments above the minimum level necessary to overcome the manager's opposition. This establishes the optimality of the renegotiated severance payments determined in Proposition 1 and Corollary 1. Note that such payments are correctly anticipated by both the manager and the board at the time when the contract is struck and thus contribute to the expected returns calculated by both parties.

4 Manager's investment and optimal compensation

Having established the minimum payment that can be renegotiated when the board wants to replace the manager at $t = 3$, we can determine the incentive pay necessary to induce the latter to make the investment at $t = 1$. Let $E_q[W_{io} + B|I_i]$ denote the expectation with respect to q of the utility expected by the overconfident manager when he invests I_i, $i = M, H$. Recalling that S_{io} is the payment (either contractual or renegotiated) that the manager obtains following his replacement, we can write the participation constraints (PC_i) as:

$$E_q[W_{io}+B|I_i]-I_i = \int_{\underline{q}}^{\widehat{q}_{io}} [(p_i+\theta_i)w_{io}+B]f(q)dq + \int_{\widehat{q}_{io}}^{1} S_{io}f(q)dq - I_i \geq 0 \;\; i = H, M$$

where the reservation utility is normalized to zero as assumed above.

Consider the incentive compatibility constraints. Let $E_q[W_{io}|0]$ represent the expected compensation if the manager does make any investment ($I = 0$) under a contract offering w_{io} and S_{io}. In order to induce the manager to choose investment I_i, $i = M, H$, the following two incentive constraints must be satisfied:

$$E_q[W_{io} + B|I_i] - I_i \geq E_q[W_{io}|0] \qquad \text{(ICC 1)}$$

and

$$E_q[W_{io} + B|I_i] - I_i \geq E_q[W_{io} + B|I_j] - I_j, \quad i, j = M, H, i \neq j. \qquad \text{(ICC2)}$$

(ICC 1) guarantees that the manager prefers I_i to not investing, and (ICC 2) that he prefers I_i to I_j. Note that $E_q[W_{io}|0] \geq 0$ by the limited liability assumption. Consequently, if the incentive compatibility constraint (ICC 1) is satisfied, the participation constraint is satisfied as well.[6] We then focus our attention on the ICCs.

In order to be more specific about the ICCs, we need to know the value of the payment that is made in case of replacement. In other words, we need to know whether contractual s_{io} is lower than s'_{io} thus implying $S_{io} = s'_{io}$ or whether it is higher thus implying $S_{io} = s_{io}$. This is found in the following proposition.

Proposition 2: *The optimal value of the contractual severance pay for an optimistic and overconfident manager ($\theta > 0, \Delta_M > 0$) is $s_{io} = \underline{s}'_{io} = (p_L + \theta)\, w_{io} + B$.*

Proof. See Appendix 2

[6] A positive level of the reservation utility would not affect this result as long as such level is smaller than the expected compensation in case of no investment denoted by $E_q[W_o|0]$.

Corollary 2: *If the manager is rational ($\theta = \Delta_M = 0$), the optimal contractual severance pay is* $s_r = \underline{s}'_r = p_L w_r + B$.

A value of contractual severance pay (s_{io}/s_r) higher than the minimum level of severance pay that would be accepted by a manager who had not invested ($\underline{s}'_{io}/\underline{s}'_r$) is not profitable for the firm because it would make the ICCs more binding. This would raise total expected compensation. On the other hand, there is no point in setting $s_{io} < \underline{s}'_{io}$ (or $s_r < \underline{s}'_r$ in case of a rational manager) because this would not relax the ICCs. Consequently the optimal value for the contractual s_{io} and s_r coincide with $\underline{s}'_{io}$ and $\underline{s}'_r$ implying that, in case the manager is replaced, the severance payment is renegotiated at $t = 3$ and is set either at s'_{io} or at s'_r.

Consider now the incentive-compatibility constraints for the investment level I_M. By substituting $s'_{Mo} = (p_L + \theta_M)w_{Mo} + B$ for S_{io} in $E_q[W_{Mo} + B|I_M]$, we obtain

$$E_q[W_{Mo}+B|I_M] = \int_{\underline{q}}^{\widehat{q}_{Mo}} [(p_M+\theta_M)w_{Mo}+B]f(q)dq + \int_{\widehat{q}_{io}}^{1} [(p_M+\theta_M)w_{Mo}+B]f(q)dq$$

Moreover, we know from Proposition 2 that in case of no investment the manager (who is always replaced) will receive $\underline{s}'_{Mo} = (p_L + \theta)w_{Mo}$, so that it is $E_q[W_{Mo}|0] = (p_L + \theta)\, w_{Mo} + B$. Then, ICC 1 can be written as

$$(p_M + \theta + \Delta_M)w_{Mo} + B - (p_L + \theta)w_{Mo} - B \geq I_M,$$

and the level of the wage that satisfies ICC 1 is:

$$w_{Mo} \geq \frac{I_M}{(p_M + \Delta_M - p_L)}. \tag{3}$$

Analogous substitutions enable to write ICC 2 as

$$(p_H + \Delta_H - p_M - \Delta_M)w_{Mo} \leq I_H - I_M.$$

Therefore, to guarantee that both constraints are satisfied it must be the case that

$$\frac{I_M}{p_M + \Delta_M - p_L} \leq w_{Mo} \leq \frac{I_H - I_M}{p_H + \Delta_H - p_M - \Delta_M}.$$

Recalling that $\Delta_H = \Delta_M(1 + z)$ such inequality can be satisfied only if

$$\frac{I_M}{I_H} \leq \frac{p_M + \Delta_M - p_L}{p_H + \Delta_M(1 + z) - p_L}. \tag{4}$$

Given that $z > \frac{p_H - p_M}{p_M - p_L}$ from Assumption 1, the RHS of 4 is decreasing in Δ_M

making it more difficult to satisfy the constraint as overconfidence rises[7]. Hence, for ICC 2 to be satisfied, the manager's beliefs of success when I_H (instead of I_M) is chosen, must not be too large. This implies that for a high enough level of overconfidence, ICC2 does not hold and the manager, if offered w_{Mo}, will choose I_H. Let $\Delta_M^* = \frac{I_M(p_H-p_L)-I_H(p_M-p_L)}{I_H-I_M(1+z)}$ denote the level of Δ_M such that 4 holds as an equality. This leads to the following definition.

Definition 1: *The manager is moderately overconfident when* $\Delta \leq \Delta_M^*$ *so that (4) is satisfied.*

When the manager is moderately overconfident, a board willing to incentivize I_M offers the lowest possible level of w_{Mo} satisfying 3, that is

$$w_{Mo} = \frac{I_M}{(p_M + \Delta_M - p_L)}.$$

Consider then the incentive compatibility constraints for the high level of investment, I_H. Constraints ICC1 and ICC2 respectively imply:

$$w_{Ho} \geq \frac{I_H}{(p_H + \Delta_H - p_L)} \tag{5}$$

and

$$w_{Ho} \geq \frac{I_H - I_M}{(p_H + \Delta_H - p_M - \Delta_M)} = \frac{I_H - I_M}{p_H - p_M + z\Delta_M}. \tag{6}$$

The incentive pay offered by the board will depend on which constraint is binding. Consider first the case where 5 is the binding constraint and observe that this happens iff $\frac{I_H}{(p_H+\Delta_H-p_L)} > \frac{I_H-I_M}{(p_H+\Delta_H-p_M-\Delta_M)}$ which implies

$$\frac{I_M}{I_H} > \frac{p_M + \Delta_M - p_L}{p_H + \Delta_M(1+z) - p_L}, \tag{7}$$

The above inequality is satisfied only for a high level of overconfidence Δ_M, i.e. for $\Delta_M > \Delta_M^*$. This leads us to the following definition.

Definition 2: *The manager is extremely overconfident when* $\Delta > \Delta_M^*$ *and (7) is satisfied.*

Thus, when the manager is extremely overconfident, I_M is not incentive compatible and the only feasible level of investment is I_H. In this case, the optimal incentive pay is given by

$$w_{Ho} = \frac{I_H}{p_H + \Delta_H - p_L}.$$

[7] Note that $\frac{\partial}{\partial \Delta_M}\left(\frac{p_M+\Delta_M-p_L}{p_H+\Delta_M(1+z)-p_L}\right) = \frac{(p_H-p_L)-(1+z)(p_M-p_L)}{(p_H+\Delta_M(1+z)-p_L)^2} < 0$. If instead $z < \frac{p_H-p_M}{p_M-p_L}$ Assumption 4 is more stringent than 4 and we have that $\frac{p_M-p_L}{p_H-p_L} < \frac{p_M+\Delta_M-p_L}{p_H+\Delta_M(1+z)-p_L}$.In this case Assumption 4 would guarantee that ICC2 is always satisfied.

A high level of overconfidence increases the subjective probability of success to such an extent that, by choosing I_H, the manager expects an increase in his compensation that more than compensates the additional cost of the investment. Therefore he is willing to invest I_H.

Consider now the case where the binding constraint is 6 and note that, for 6 to hold, it must be the case that $\frac{I_H}{(p_H+\Delta_H-p_L)} < \frac{I_H-I_M}{(p_H+\Delta_H-p_M-\Delta_M)}$ or:

$$\frac{I_M}{I_H} < \frac{p_M+\Delta_M-p_L}{p_H+\Delta_H-p_L}.$$

This is the case of moderate overconfidence where also I_M is incentive compatible. To determine which will be the contract offered by the board, observe that under moderate overconfidence, $w_{Ho} = \frac{I_H-I_M}{(p_H+\Delta_H-p_M-\Delta_M)}$ is greater than $w_{Mo} = \frac{I_M}{(p_M+\Delta_M-p_L)}$. Furthermore, the higher bonus also raises $\widehat{q}_{Ho}$. These two effects make I_H generally unprofitable for the firm which will consequently offer the manager w_{Mo} to induce I_M. This is proved in the following proposition which allows us to restrict our attention to two mutually exclusive cases: extreme overconfidence with investment level I_H, and moderate overconfidence with investment level I_M.

Proposition 3: *When the manager is moderately overconfident, the board generally offers* $w_{Mo} = \frac{I_M}{(p_M+\Delta_M-p_L)}$ *and the manager chooses* I_M*. When instead the manager is extremely overconfident, the board offers* $w_{Ho} = \frac{I_H}{(p_H+\Delta_H-p_L)}$ *and the manager chooses* I_H*.*

Proof: See Appendix 3.

Corollary 3. *When the manager is rational (*$\theta = \Delta_M = 0$*), only* I_M *is incentive compatible. The optimal bonus is*

$$w_r = \frac{I_M}{(p_M-p_L)}.$$

Proof. In order to check that I_H is not incentive compatible consider that in this case ICC 2 implies

$$w_{Ho} \geq \frac{I_H-I_M}{(p_H-p_M)},$$

but by Assumption 4 we know that $\frac{I_H-I_M}{(p_H-p_M)} > R$ so that the board will never offer such incentive pay. The rest of the corollary follows from the discussion above, setting $\theta = \Delta_M = 0$.

When the manager is rational and holds correct beliefs about the probability of success, it is not possible to implement I_H because the manager is aware that the increase in the cost with respect to the alternative investment I_M is higher than the increase in expected return (see Assumption 4). Let us then compare the outcome of a rational manager to that of a biased manager choosing I_M. By comparing w_{Mo} to w_r it is immediately evident that overconfidence reduces the incentive pay necessary to induce the manager to choose investment I_M, making it easier to satisfy the ICCs. This is the incentive effect of overconfidence which persists when overconfidence increases to the extreme values that result in the choice of investment I_H. We can then prove the following

Corollary 4. *w_{io} is continuously decreasing in Δ_M, for $i = M, H$.*

Proof. The corollary immediately follows from $w_{Mo} = \frac{I_M}{(p_M+\Delta_M-p_L)}$ and $w_{Ho} = \frac{I_H}{(p_H+\Delta_M(1+z)-p_L)}$, considering that at Δ^*_M it is $\frac{I_M}{(p_M+\Delta_M-p_L)} = \frac{I_H}{(p_H+\Delta_M(1+z)-p_L)}$.□

The result that incentive pay is smaller when the manager is overconfident is in line with previous theoretical literature (De La Rosa, 2011) and with empirical evidence (Otto 2014, Humphery-Jenner et al. 2016). Finally note that, contrary to what happens with overconfidence, optimism (θ) has no impact on the bonus.

Having established the optimal value of the bonus, we are in the position to analyze the relation of the cutoff value of the productivity of the new manager above which replacement occurs, $\widehat{q}_{io}$, with respect to optimism and overconfidence. We already know that $\widehat{q}_r < \widehat{q}_{io}$ meaning that a rational manager will be replaced for lower values of q than a biased manager. The following proposition shows that the entrenchment effect identified in the previous subsection is increasing in managerial biases.

Proposition 4. *The cutoff value, $\widehat{q}_{io}$, is increasing both in optimism and in overconfidence.*

Proof. see Appendix 4.

Optimism and overconfidence introduce a distortion in the replacement decision by increasing the cutoff value above which the board wants to replace the manager, thus inducing an entrenchment effect. Given that $\widehat{q}_{io} = p_i + \frac{B}{R} + \frac{(\theta+\Delta_i)w_{io}}{R}$ this is immediate as far as optimism is concerned. However, it also occurs in the case of overconfidence no matter the fact that the incentive bonus is decreasing in Δ_M (see Corollary 4). In fact the decrease in w_{io} is more than compensated by the increase in the managerial belief of success (reflected in the multiplicative term $(\theta + \Delta_i)$). Note that the increase in the biases occurs not only for a given value of investment but also when the increase in overconfidence induces the shift from I_M to I_H.

4.1 Severance pay

In the previous sections we have determined both the contractual and the renegotiated severance pay. We now discuss their properties, recalling that, in case of replacement, the payment will always be renegotiated. First of all, the relationship between renegotiated severance payment and the two managerial biases is established both for a given level of investment I_i and at Δ_M^* where the shift from I_M to I_H occurs.

Proposition 5. *For a given investment I_i, the severance payment renegotiated in case of replacement, $s'_{io} = (p_i + \theta + \Delta_i)w_i$, is increasing in optimism and decreasing in overconfidence. At Δ_M^* where the shift from I_M to I_H occurs, it is increasing in both optimism and overconfidence.*

Proof. See Appendix 5

For a given level of investment I_i, an increase in optimism always raises the renegotiated severance pay $s'_{io} = (p_i + \theta + \Delta_i)w_{io}$ because w_{io} is not affected by optimism. On the contrary, an increase in the level of overconfidence reduces s'_{io} because the reduction in the bonus w_{io} (see Corollary 4) counterbalances the increase in the managerial belief $(p_i + \theta + \Delta_i)$. Only at Δ_M^*, where the shift from I_M to I_H induces a particularly strong increase in the belief of success, there is a increase in s'_{io}. For $\Delta_M > \Delta_M^*$ however s'_{Ho} is again decreasing in Δ_M.

The difference between renegotiated and contractual severance pay, $s'_{io} - s_{io}$, is defined as **discretionary pay**. Using Propositions 1 and 3 and the expressions for w_{io} $i = M, H$, as well as Corollary 1 and 3 for a rational manager, it is immediate to verify that the discretionary amount paid in addition to the contractual one is given by the cost of the investment.

Result 1: *The discretionary severance pay is equal to the investment: $s'_{io} - s_{io} = I_i$, $i = M, H$, for a biased manager, and $s'_R - s_R = I_M$ for a rational manager.*

The manager anticipates that, in case of replacement, he will be able to recover the investment made, by renegotiating the contractual severance pay. This clearly provides him with the incentive to invest, despite the possibility of being replaced. In other words, renegotiation allows the board to provide the necessary ex-ante incentive by reimbursing the manager for the investment only if this has been undertaken. Note that, in case both of a rational manager and of a moderately overconfident one, the discretionary pay is equal to I_M while the high level I_H obtains in case of extreme overconfidence. Consequently we have the following result.

Result 2: *The expected value of renegotiated severance pay (the sum of discretionary and contractual payments) received in case of turnover is larger than the expected value of incentive pay received in case of success and retention. The difference is increasing in the managerial biases (optimism and overconfidence).*

A natural question is whether the severance pay obtained by a biased manager is larger or smaller than the one received by a rational one. Proposition 5 shows that there are opposing effects at work when the manager is both optimistic and overconfident. Hence the overall effect on the renegotiated severance pay (contractual plus discretionary pay) depends on how the mix of the two biases affects managerial beliefs. A rational manager receives $s'_r = p_M \frac{I_M}{(p_M - p_L)} + B$ and a biased one $s'_{io} = (p_i + \theta + \Delta_i) \frac{I_i}{p_i + \Delta_i - p_L} + B$ where s'_{io} is increasing in optimism and decreasing in overconfidence.Then, by comparing the two payments we obtain the following result.

Result 3: *The renegotiated severance pay of a biased manager is higher than that of a rational one only for a sufficiently high level of the ratio of optimism to overconfidence:*

i) when the manager is moderately overconfident a necessary and sufficient condition is $\frac{\theta}{\Delta_M} \geq \frac{p_L}{p_M - p_L}$.

ii) when the manager is extremely overconfidence, a necessary condition is: $\frac{\theta}{\Delta_M} + \frac{\Delta_M(1+z)+(p_H - p_M)}{\Delta_M} > \frac{p_M}{p_M - p_L}$.

The condition in part *i)* simply says that a biased manager receives a larger renegotiated severance pay than a rational manager when the cost of the investment in terms of increased bias is larger than the benefit in terms of increase in the probability of success. To see this, consider that the LHS numerator is the bias independent of the investment and the numerator on the RHS is probability of success without any investment, while the denominators represent, respectively, the increase in the overall bias due to the investment, Δ_M, and the increase in the probability of success due to the investment, $(p_M - p_L)$. Part *ii)* instead says that when the manager is extremely overconfident, the severance pay can be higher than that of a rational manager even if $\theta = 0$ because the extremely high managerial belief of success tends to counterbalance the decrease in the renegotiated severance pay following the decrease in incentive pay.

5 The effect of optimism and overconfidence on expected profits

So far, we have discussed the characteristics of the optimal payments in each possible state (manager retention or dismissal).We now analyze the overall impact of managerial biases on expected profits V_{io}, considering first the effect of an increase in optimism θ, for a given level of overconfidence Δ_M, and then the effect of an increase in overconfidence Δ_M, taking the level of optimism θ as given. Recalling that the manager will receive s'_{io} when replaced, we can write

$$V_{io} = \int_{\underline{q}}^{\widehat{q}_{io}} [p_i(R - w_{io})] f(q)dq + \int_{\widehat{q}_{io}}^{1} (qR - s'_{io}) f(q)dq \tag{8}$$

The first term on the RHS is the expected profit of the firm when the incumbent is confirmed, while the second term measures the expected profit when the manager is replaced. Consider the effect of an increase in optimism recalling that optimism does not affect the investment choice, so that we can take the level of investment as given.

Proposition 6: *For a given value of overconfidence $\Delta_M \geq 0$, the expected profit of the firm is decreasing in optimism θ.*

Proof: The proposition immediately follows from:

$$\frac{\partial V_{io}}{\partial \theta} = -\int_{\widehat{q}_{io}}^{1} w_{io} f(q)dq - \frac{\partial \widehat{q}_{io}}{\partial \theta} \underbrace{[(\widehat{q}_{io} - p_i)R - ((\theta + \Delta_i)w_{io} + B)]}_{=0} f(\widehat{q}_{io}) < 0, \quad i = M, H.$$

where $(\widehat{q}_{io} - p_i)R - ((\theta + \Delta_i)w_{io} + B) = 0$ is obtained by substituting $\widehat{q}_{io} = p_i + \frac{B}{R} + \frac{(\theta+\Delta_i)w_{io}}{R}$ $\square$

Corollary 3: *If the firm hires an optimistic but not overconfident manager ($\theta > 0$, $\Delta_M = 0$) expected profits are lower than under a rational manager.*

Proposition 6 formalizes that optimism always has a negative effect on profits. Indeed, optimism has no impact on the incentive bonus w_{io} while it raises the severance payment s'_{io}.[8] In the absence of overconfidence, this drives expected profits below the level obtained by a rational manager.

Let us now evaluate the effect of an increase in overconfidence for a given level of optimism. We know that, following an increase in overconfidence for a given level of investment I_i, the incentive bonus paid in case of retention decreases (incentive effect), as well as the severance payment paid in case of dismissal (severance effect).[9] Consequently, the expected cost to induce investment I_i, $i = M, H$, is reduced. However, we cannot ensure that expected profits are continuously increasing in Δ_M because we do not know what happens at Δ_M^* where the increase in overconfidence induces the shift from investment I_M to investment I_H and severance pay has a sudden increase (see Proposition 5). This point is clarified in the following proposition.

[8] We also know that optimism contributes to the entrenchment effect making the cutoff value $\widehat{q}_{io}$ rise but this has no impact on profits because the cutoff value is determined by balancing what is gained from replacement and the payment necessary to have the incumbent leave so that the net effect is equal to zero. In fact the terms concerning the entrenchment effect cancel out in the derivative $\frac{\partial V_{io}}{\partial \theta}$ (maximum theorem).

[9] Again the rise in $\widehat{q}_{io}$ has no effect because the payment necessary to have the incumbent leave cancels out with the sum gained from replacement (see previous footnote).

Proposition 7: *For any given level of optimism $\theta \geq 0$, expected profit increases in Δ_M, as long as $0 < \Delta_M < \Delta_M^*$ and I_M is chosen. At Δ_M^*, the manager is indifferent between I_M and I_H but I_M generally yields higher expected profit and is thus chosen. A further increase to $\Delta_M > \Delta_M^*$, implies a shift to I_H, and leads to a discontinuity: the expected profit has an initial drop and then resumes an increasing trend.*

Proof: See Appendix 6.

Corollary 4: *Expected profit is higher with a moderately overconfident but not optimistic manager ($\theta = 0$, $0 < \Delta_M \leq \Delta_M^*$) than with a rational manager.*

Profit is increasing in overconfidence as long as overconfidence is moderate and does not lead to an inefficiency in the investment level. When overconfidence is so high as to induce the shift from I_M to I_H, there is a discontinuity with a drop in profit because of the sudden increase in the severance payment and in $\widehat{q}_{io}$. Once the new level of the investment I_H is chosen, profit is again increasing in overconfidence. Note however, that there is no guarantee that it will reach again the level corresponding to Δ_M^*, because the increase in Δ_M is bounded by the constraint that the belief of success cannot exceed one. Hence, a moderate level of overconfidence is beneficial for the firm but this may not hold true for extreme levels of overconfidence. In particular, we are certain that a moderately overconfident but not optimistic manager yields higher expected profits than a rational one, but we cannot ensure that this happens also in the case of an extremely overconfident one.

This issue is analyzed by the following proposition. To evaluate the drop in the expected profit occurring at Δ_M^*, we need to specify the probability that a better manager shows up. Given that we have no reason to consider situations where the probability of a particular level of q is higher/lower than other levels, we assume that q is uniformly distributed over the interval $[\underline{q}, 1]$.

Proposition 8. *When $\theta = 0$, the drop in profits occurring at Δ_M^* can be high enough to result in $V_{Ho} < V_r$.*

Proof. See Appendix 7.

Proposition 8 highlights the negative impact on profit resulting from the shift in the investment from I_M to I_H and provides an indication of the size of the drop that takes place at Δ_M^*. In particular, it shows that expected profit can be reduced below the level attained by a rational manager. This leads to the conclusion that, in the absence of optimism, the firm can benefit from hiring a moderately overconfident manager while it may be damaged by an extremely overconfident one.

When the manager is both optimistic and overconfident, the question arises whether the negative effect of optimism can counterbalance the positive effect of (moderate) overconfidence. Is an optimistic and moderately overconfident manager always better than a rational one? The following proposition deals with such a question, maintaining the assumption that q is uniformly distributed over the interval $[\underline{q}, 1]$

Proposition 9. *There always exist a value* $\widetilde{\theta}$*:* $1 - p_H > \widetilde{\theta} \geq 0$ *such that for* $\theta > \widetilde{\theta}$*,* $V_{Mo} < V_R$*.*

Proof. See Appendix 8.

Proposition 9 confirms the detrimental effect of optimism and shows that, for sufficiently high values of θ, the negative effect of optimism prevails on the positive effect of overconfidence, similarly to what we found in Result 3. As a consequence, a biased manager with a high level of optimism generates an expected cash flow smaller than the one obtained by a rational manager, even when the manager is only moderately overconfident. For levels of θ below $\widetilde{\theta}$, the outcome is indeterminate in the sense that it depends on the level of overconfidence. If overconfidence is high enough, it may balance the negative impact of optimism. However, when low levels of overconfidence are coupled with optimism, a rational manager leads to higher expected profits.

Summarizing, if the manager is both optimistic and overconfident, the overall effect of managerial bias is likely to be detrimental for the firm, once severance costs are taken into account.

6 Conclusion

The paper examines the effects of managerial optimism and overconfidence on severance pay and firm expected profit in a setting where the firm aims at motivating the CEO to undertake a firm-specific investment even if a better manager may show up after the incumbent has made the investment. Severance pay helps motivating the manager to invest despite the possibility of replacement is anticipated. However, investment choice may be distorted by overconfidence. In addition to the option of abstaining from investing, two positive levels of investment are available for the manager: the efficient level I_M and the inefficient level $I_H > I_M$. While optimism does not affect investment choice, the degree of overconfidence determines the chosen level of I: under moderate overconfidence the efficient level is implemented but a high degree of the bias results in the shift to the inefficient level. If the board decides to replace the incumbent manager, the latter uses its bargaining power to renegotiate contractual severance pay and a higher payment is agreed upon.

In this context, optimism and overconfidence have different effects on the level of both incentive and severance pay and, consequently, on expected profit. Optimism has a definitely negative effect: it does not affect incentive pay but it raises severance pay (contractual and renegotiated) and this leads to lower expected profit than would be obtained by a rational (i.e. unbiased) manager, both directly and as a result of higher entrenchment. The effect of overconfidence, on the contrary, is more complex and depends on the degree of the bias. Overconfidence always makes incentive pay decrease and this has obviously a positive effect on profit. For any given level of investment, overconfidence also reduces severance pay. However, when the increase in overconfidence is such as to induce the shift to the inefficient investment level, there is a sudden increase in renegotiated severance pay that reduces expected profit. This may drive expected profit below the level attained by a rational manager and there is no guarantee that further increases in overconfidence may reverse the situation. In summary, the firm always benefits from moderate overconfidence while (optimism and) extreme overconfidence may be detrimental to firm profit.

The model shows that it is important to consider the possibility of replacement when studying the effect of managerial biases, because of the impact on the separation payment. In particular, we show that optimism increases contractual severance pay, while the discretionary payment bargained at the replacement stage is determined by overconfidence. Thus, we suggest that the high payments observed in several turnover events may be explained by managerial overconfidence coupled with some managerial bargaining power originated by the manager ability to oppose replacement.

7 References

Almazan, A. and J. Suarez (2003) "Entrenchment and Severance Pay in Optimal Governance Structures", *Journal of Finance*, 58, (2) April, 519–547

Bebchuk, L.A. and J. Fried (2004) *Pay without Performance: the unfulfilled Promise of Executive Compensation*, Cambridge, Harvard University Press.

Cadman, B. D., Campbell J.L., and S. Klasa (2016) "Are Ex Ante CEO Severance Pay Contracts Consistent with Efficient Contracting?" *Journal of Financial Quantitative Analysis,* 51(3), 737–769

de La Rosa, L. E. (2011) "Overconfidence and moral hazard" *Games and Economic Behavior, 73, 429-451.*

Gervais,S., Heaton and Odean (2011) "Overconfidence, Compensation Contracts and Capital Budgeting", *Journal of Finance, 66, (5) 1735-* 1777.

Goel, A.M. and A.V. Thakor (2008) "Overconfidence, CEO selection and Corporate Governance" *Journal of Finance* 63 (6)*,* 2737-2784.

Goldman, E. M. and P. P. Huang (2015) "Contractual vs. Actual Separation Pay Following CEO Turnover". *Management*

Science 61(5):1108-1120.

Humphery-Jenner, M., Lisic, L., and V. Nanda (2016) "Executive overconfidence and compensation structure" *Journal of Financial Economics* , 119, 533-558.

Inderst, R. and H. Mueller (2010). "CEO replacement under private information", *Review of Financial Studies,* 23, 2935-2969.

Kent, D. and D. Hirshleifer (2015) "Overconfident Investors, Predictable Returns, and Excessive Trading" *Journal of Economic Perspectives,* 29 (4), 61-88.

Malmendier, U. and G. Tate (2015) "Behavioral CEOs: The Role of Managerial Overconfidence", *Journal of Economic Perspectives,* 29 (4), 37-60.

Malmendier, U. and G. Tate (2005) "CEO overconfidence and corporate investment, *Journal of Finance* 60, 2661-2700

Muscarella, C.J. and J. Zhao (2015) "Promoting the Quiet Life or Risk-Taking? CEO Severance Contracts and Managerial Decision-Making",(July 1, 2015). AFA 2012 Chicago Meetings Paper. Available at SSRN: https://ssrn.com/abstract=1787221.

Otto, Clemens (2014) "CEO Optimism and Incentive Compensation" *Journal of Financial Economics* , 114, 366-404.

Rau R. P. and J. Xu (2013) "How do ex-ante Severance Pay Contracts Fit into Optimal Executive Incentive Scheme?"*Journal of Accounting Research,* 51(3) , 631-671.

8 Appendix 1: Proof of Proposition 1.

Consider part *i)* in which the manager invests I_i and the probability of success is p_i. Managerial expected utility by staying with the firm is equal to $(p_i + \theta + \Delta_i)w_{io} + B \equiv s'_{io}$. Then, in order to leave, the manager accepts any offer $s'' \geq s'_{io}$.

If the incumbent manager receives s'_{io} and is replaced by a manager of ability q, firm expected profit is $qR - (p_i + \theta + \Delta_i)w_{io} - B$, which is larger than $p_i(R - w_{io})$, the expected profit obtained if the manager stays, for any $q > \widehat{q}_{io}$. Any offer s'' strictly larger than s'_{io} is accepted by the manager but is dominated by s'_{io} since it reduces the firm expected profit. Consider an offer $s'' < s'_{io}$. The manager rejects it because he believes to obtain a higher payoff by staying with the firm and firm profit is smaller with the incumbent than with a replacement. Hence, when the manager makes investment I_i, the offer s'_{io} is optimal for the firm whenever there is a replacement with $q > \widehat{q}_{io}$.

Consider now part *ii)*. If the manager does not invest, his expected payoff by staying with the firms is $(p_L + \theta)w_i + B$. Then, he will reject any offer smaller than $\underline{s}'_{io} = (p_L + \theta)w_i + B$. Firm profit under the incumbent manager is $p_L(R - w_{io})$ while firm profit when the incumbent is offered $\underline{s}'_{io}$ and replaced is $qR-(p_i+\theta+\Delta_i)w_{io}-B$. Assumption 2 and $B<\theta(R-w_{io})$ guarantee that for any value of $q\geq \underline{q}$ expected profit is larger when the manager is replaced because $qR-(p_L+\theta)w_{io}-B\geq p_L(R-w_{io})$. As above, any offer $s''>\underline{s}'_{io}$ is dominated by $\underline{s}'_{io}$ because s'' yields lower expected profit. Any offer $s''<\underline{s}'_{io}$ is rejected by the manager which remains with the firm resulting in lower expected profit than with a replacement.

9 Appendix 2: Proof of Proposition 2

Suppose that contractual s is such that $s\geq s'_{io}>\underline{s}'_{io}$ where $i=M,H$ is the level of investment that the board wants to incentivize. Then, $E_q[W_{io}+B|I_i]=\int_{\underline{q}}^{\widehat{q}_{io}}[(p_i+\theta_i)w_{io}+B]f(q)dq+\int_{\widehat{q}_{io}}^{1}sf(q)dq$ and $E_q[W_{io}+B|I=0]=\int_{\underline{q}}^{1}sf(q)dq$ $i=H,M$ because the manager is always replaced when no investment is made. Consequently, ICC1 can be written as

$$\int_{\underline{q}}^{\widehat{q}_{io}}[(p_i+\theta_i)w_{io}+B]f(q)dq+\int_{\widehat{q}_{io}}^{1}sf(q)dq-I_i\geq\int_{\underline{q}}^{1}sf(q)dq \qquad i=H,M$$

or

$$\int_{\underline{q}}^{\widehat{q}_{io}}[(p_i+\theta_i)w_{io}+B]f(q)dq-I_i\geq\int_{\underline{q}}^{\widehat{q}_{io}}sf(q)dq$$

Clearly the board wants to keep both w_{io} and s as low as possible in order to maximize its profit. Note in fact that besides lowering the expected compensation needed to incentivize I_i, a low level of w_{io} also reduces the distortion in $\widehat{q}_{io}=p_i+\frac{B}{R}+\frac{\theta_i}{R}w_i$ with respect to the efficient level $\widehat{q}_r=p_i+\frac{B}{R}$. This objective can be reached, by setting s at its lowest feasible value that is $s=s'_{io}$.

So far we have constrained s to be weakly higher than s'_{io}. Can the board gain by setting either i) $s'_{io}>s\geq\underline{s}'_{io}$ or ii) $\underline{s}'_{io}>s$?

In case i), ICC1 would take the form

$$\int_{\underline{q}}^{\widehat{q}_{io}}[(p_i+\theta_i)w_{io}+B]f(q)dq+\int_{\widehat{q}_{io}}^{1}s'_{io}f(q)dq-I_i\geq\int_{\underline{q}}^{1}sf(q)dq \qquad i=H,M$$

or, substituting $s'_{io} = (p_i + \theta + \Delta_i)w_{io}$,

$$\int_{\underline{q}}^{\widehat{q}_{io}} [(p_i + \theta_i)w_{io} + B]f(q)dq + \int_{\widehat{q}_{io}}^{1} (p_i + \theta + \Delta_i)w_{io}f(q)dq - I_i \quad = \tag{9}$$

$$\int_{\underline{q}}^{1} [(p_i + \theta_i)w_{io} + B]f(q)dq + \int_{\widehat{q}_{io}}^{1} \Delta_i w_{io}f(q)dq - I_i \quad \geq \quad \int_{\underline{q}}^{1} sf(q)dq \qquad i = H, M$$

Again, the board wants to set s as low as possible in order to reduce w_{io} and minimize both the expected compensation needed to incentivize I_i and the distortion in $\widehat{q}_{io}$. Then it will set $s = \underline{s}'_{io}$. By comparing **??** with $s = s'_{io}$ to 9 with $s = \underline{s}'_{io}$ it is immediately clear that ICC1 is less binding in the latter case thus resulting in both lower expected payment and lower distortion.

The question arises whether the board could further increase profits by setting contractual $s < \underline{s}'_{io}$. Note however that in such a case ICC1 would take the form

$$\int_{\underline{q}}^{\widehat{q}_{io}} [(p_i + \theta_i)w_{io} + B]f(q)dq + \int_{\widehat{q}_{io}}^{1} s'_{io}f(q)dq - I_i \geq \int_{\underline{q}}^{1} \underline{s}'_{io}f(q)dq \qquad i = H, M$$

and we will be back to the previous case, implying that there is no point in setting $s < \underline{s}'_{io}$.

10 Appendix 3: Proof of Proposition 3

In order to prove that the board offers $w_{Mo} = \frac{I_M}{(p_M + \Delta_M - p_L)}$ when the manager is moderately overconfident ($\frac{I_M}{I_H} < \frac{p_M + \Delta_M - p_L}{p_H + \Delta_H - p_L}$), we must prove that the expected profits of the firm are higher under a contract based on $w_{Mo} = \frac{I_M}{(p_M + \Delta_M - p_L)}$ (to induce the choice of I_M) than under a contract based on $w_{Ho} = \frac{I_H - I_M}{p_H + \Delta_H - (p_M + \Delta_M)}$ (to induce the choice of I_H). Note that, substituting the value of s'_{io}, V_{io} can be written as

$$\begin{aligned} V_{io} &= \int_{\underline{q}}^{\widehat{q}_{io}} [p_i(R - w_{io})]\, f(q)dq + \int_{\widehat{q}_{io}}^{1} qR - (p_i + \theta + \Delta_i)\, w_{io} - B)f(q)dq \\ &= p_i(R - w_{io}) + \int_{\widehat{q}_{io}}^{1} [(q - p_i)R - (\theta + \Delta_i)\, w_{io} - B]\, f(q)dq \end{aligned}$$

Recall that $w_{Ho} = \frac{I_H - I_M}{p_H + \Delta_H - (p_M + \Delta_M)} > \frac{I_M}{(p_M + \Delta_M - p_L)} = w_{Mo}$ when $\frac{I_M}{I_H} < \frac{p_M + \Delta_M - p_L}{p_H + \Delta_H - p_L}$. This implies $\widehat{q}_{Mo} = p_M + \frac{B}{R} + \frac{\theta_M}{R} w_{Mo} < p_H + \frac{B}{R} + \frac{\theta_H}{R} w_{Ho} = \widehat{q}_{Ho}$.

Consider the difference

$$
\begin{aligned}
V_{Ho} - V_{Mo} &= \int_{\underline{q}}^{\widehat{q}_{Ho}} p_H (R - w_{Ho}) f(q) dq + \int_{\widehat{q}_{Ho}}^{1} [qR - (p_H + \Delta_H + \theta) w_{Ho} - B] f(q) dq + \\
&\quad - \int_{\underline{q}}^{\widehat{q}_{Mo}} p_M (R - w_{Mo}) f(q) dq - \int_{\widehat{q}_{Mo}}^{1} [qR - (p_M + \Delta_M + \theta) w_{Mo} - B] f(q) dq \\
&= \int_{\underline{q}}^{\widehat{q}_{Mo}} (p_{H-} p_M)(R - w_{Mo}) f(q) dq - \int_{\underline{q}}^{\widehat{q}_{Mo}} (p_{H-} p_M)(w_{Ho} - w_{Mo}) f(q) dq \\
&\quad + \int_{\widehat{q}_{Mo}}^{\widehat{q}_{Ho}} p_H (R - w_{Ho}) f(q) dq - \int_{\widehat{q}_{Ho}}^{1} (p_H + \Delta_H - p_M - \Delta_M) w_{Ho} f(q) dq \\
&\quad - \int_{\widehat{q}_{Mo}}^{\widehat{q}_{Ho}} [qR - (p_M + \Delta_M + \theta) w_{Mo} - B] f(q) dq
\end{aligned}
$$

Considering that $[p_H + \Delta_H - (p_M + \Delta_M)] w_{Ho} = I_H - I_M$ the above expression can be written as

$$
\begin{aligned}
&\int_{\underline{q}}^{\widehat{q}_{Mo}} (p_{H-} p_M)(R - w_{Mo}) f(q) dq - \int_{\underline{q}}^{\widehat{q}_{Mo}} (p_{H-} p_M)(w_{Ho} - w_{Mo}) f(q) dq + \\
&\int_{\widehat{q}_{Mo}}^{\widehat{q}_{Ho}} p_H (R - w_{Ho}) f(q) dq - \int_{\widehat{q}_{Ho}}^{1} (I_H - I_M) f(q) dq - \int_{\widehat{q}_{Mo}}^{\widehat{q}_{Ho}} [qR - (p_M + \Delta_M + \theta) w_{Mo} - B] f(q) \\
= &\int_{\underline{q}}^{\widehat{q}_{Mo}} (p_{H-} p_M)(R - w_{Mo}) f(q) dq + \int_{\widehat{q}_{Mo}}^{\widehat{q}_{Ho}} \{p_H (R - w_{Mo}) - [qR - (\Delta_M + \theta) w_{Mo} - B]\} f(q) dq \\
&- \int_{\underline{q}}^{\widehat{q}_{Mo}} (p_{H-} p_M)(w_{Ho} - w_{Mo}) f(q) dq - \int_{\widehat{q}_{Mo}}^{\widehat{q}_{Ho}} p_H (w_{Ho} - w_{Mo}) f(q) dq - \int_{\widehat{q}_{Ho}}^{1} (I_H - I_M) f(q) dq
\end{aligned}
$$

where $\int_{\widehat{q}_{Mo}}^{\widehat{q}_{Ho}} [qR - (p_M + \Delta_M + \theta) w_{Mo} - B] f(q) > \int_{\widehat{q}_{Mo}}^{\widehat{q}_{Ho}} [qR - (p_H + \Delta_H + \theta) w_{Mo} - B] f(q) dq >$ $\int_{\widehat{q}_{Mo}}^{\widehat{q}_{Ho}} p_H (R - w_{Mo}) f(q) dq$. Considering that $I_H - I_M > (p_{H-} p_M) R$, we then have that

$$
\begin{aligned}
V_{Ho} - V_{Mo} \quad \leq \quad & \int\limits_{\underline{q}}^{\widehat{q}_{Mo}} \left[I_H - I_M - (p_{H-} p_M)\, w_{Mo} \right] f(q) dq - \int\limits_{\widehat{q}_{Ho}}^{1} (I_H - I_M) f(q) dq \\
& - \int\limits_{\underline{q}}^{\widehat{q}_{Mo}} (p_{H-} p_M)\,(w_{Ho} - w_{Mo}) f(q) dq - \int\limits_{\widehat{q}_{Mo}}^{\widehat{q}_{Ho}} p_H (w_{Ho} - w_{Mo}) f(q) dq \\
& - \int\limits_{\widehat{q}_{Mo}}^{\widehat{q}_{Ho}} \left\{ \left[q - (\Delta_M + \theta)\, w_{Mo} - B \right] - \left[p_H R - (p_{H-} p_M)\, w_{Mo} \right] \right\} f(q) dq \\
= \quad & \int\limits_{\underline{q}}^{\widehat{q}_{Mo}} \left[I_H - I_M - (p_{H-} p_M)\, w_{Ho} \right] f(q) dq - \int\limits_{\widehat{q}_{Ho}}^{1} (I_H - I_M) f(q) dq \\
& - \int\limits_{\widehat{q}_{Mo}}^{\widehat{q}_{Ho}} p_H (w_{Ho} - w_{Mo}) f(q) dq \\
& - \int\limits_{\widehat{q}_{Mo}}^{\widehat{q}_{Ho}} \left\{ \left[q - (\Delta_M + \theta)\, w_{Mo} - B \right] - \left[p_H R - (p_{H-} p_M)\, w_{Mo} \right] \right\} f(q) dq
\end{aligned}
$$

Substituting $I_H - I_M = [p_H + \Delta_H - (p_M + \Delta_M)]\, w_{Ho}$, the above inequality becomes

$$\begin{aligned} V_{Ho} - V_{Mo} \quad &\leq \int_{\underline{q}}^{\widehat{q}_{Mo}} [p_H + \Delta_H - (p_M + \Delta_M) - (p_{H-}p_M)]\, w_{Ho} f(q)dq \\ &- \int_{\widehat{q}_{Ho}}^{1} (I_H - I_M) f(q)dq - \int_{\widehat{q}_{Mo}}^{\widehat{q}_{Ho}} p_H(w_{Ho} - w_{Mo}) f(q)dq \\ &- \int_{\widehat{q}_{Mo}}^{\widehat{q}_{Ho}} \{[q - (\Delta_M + \theta)\, w_{Mo} - B] - [p_H R - (p_{H-}p_M)\, w_{Mo}]\}\, f(q)dq \\ &= \int_{\underline{q}}^{\widehat{q}_{Mo}} z\Delta_M w_{Ho} f(q)dq - \int_{\widehat{q}_{Ho}}^{1} (I_H - I_M) f(q)dq \\ &- \int_{\widehat{q}_{Mo}}^{\widehat{q}_{Ho}} p_H(w_{Ho} - w_{Mo}) f(q)dq \\ &- \int_{\widehat{q}_{Mo}}^{\widehat{q}_{Ho}} \{[(q - p_H)\, R + [(p_{H-}p_M) - (\Delta_M + \theta)]\, w - B]\}\, f(q)dq \end{aligned}$$

Considering that $z\Delta_M$ is very small and all the other terms are negative, $V_{Ho} - V_{Mo}$ will generally be negative. Recall in fact that $z\Delta_M \leq \frac{I_H - I_M}{w_{Mo}} - (p_H - p_M)$ for $\Delta_M < \Delta_M^*$,where Δ_M^* is the value of Δ_M where there is the shift from moderately to extreme overconfidence.

11 Appendix 4: Proof of Proposition 4

To evaluate the effect of the biases, substitute $w_{io} = \frac{I_i}{(p_i + \Delta_i - p_L)}$ in the expression for the cutoff value $\widehat{q}_{io}$, obtaining $\widehat{q}_{io} = p_i + \frac{B}{R} + \frac{(\theta + \Delta_i) I_i}{R(p_i + \Delta_i - p_L)}$. Clearly, for a given level of investment, $\widehat{q}_{io}$ is increasing in the optimism component θ and it is is also increasing in the overconfidence parameter Δ_M as

$$\frac{\partial \widehat{q}_{io}}{\partial \Delta_M} = \frac{I_i R\,(p_i + \Delta_i - p_L) - I_i R(\theta + \Delta_i)}{[R\,(p_i + \Delta_i - p_L)]^2} = \frac{I_i(p_i - p_L - \theta)}{R\,(p_i + \Delta_i - p_L)^2} > 0, \; i = M, H$$

because $p_i - p_L > \theta$ by Assumption 1.

To evaluate what happens at Δ_M^* where the shift from I_M to I_H occurs, note that at Δ_M^* it is $w_{Mo} = w_{Ho} \equiv w$ implying that $\widehat{q}_{Mo} = p_M + \frac{B}{R} + \frac{\theta + \Delta_M^*}{R} w <$

$p_H + \frac{B}{R} + \frac{\theta+\Delta_M^*(1+z)}{R}w = \widehat{q}_{Ho}$. Then the cutoff value is increasing also in this point (even if there is a discontinuity).

12 Appendix 5: Proof of Proposition 5

That s'_{io} is increasing in optimism and decreasing in overconfidence for a given level of investment I_i follows immediately from

$$\frac{\partial s'_{io}}{\partial \theta} = (p_i + \Delta_i)\, w_{io} > 0$$

and from

$$\begin{aligned} \frac{\partial s'_{Mo}}{\partial \Delta_M} &= -\left(\frac{I_M}{(p_M + \Delta_M - p_L)}\right)\left(\frac{(p_L+\theta)}{(p_M + \Delta_M - p_L)}\right) < 0. \\ \frac{\partial s'_{Ho}}{\partial \Delta_M} &= -\left(\frac{(1+z)\, I_H}{(p_H + \Delta_M(1+z) - p_L)}\right)\left(\frac{(p_L+\theta)}{(p_H + \Delta_M(1+z) - p_L)}\right) < 0.\square \end{aligned}$$

To verify that s'_{io} is increasing in both optimism and overconfidence at Δ_M^* where the shift from I_M to I_H occurs, note that at at Δ_M^* it is $w_{Mo} = w_{Ho} \equiv w$ implying $s'_{Mo} = (p_M + \theta + \Delta_M^*)w < (p_H + \theta + \Delta_M^*(1+z))w = s'_{Ho}$.

13 Appendix 6: Proof of Proposition 7

In order to prove the proposition we must show that a) when I_i is chosen, profits are increasing in Δ_i and b) when 4 holds as an equality, profits are generally higher if I_M is chosen. Note that, substituting the value of s'_{io}, V_{io} can be written as

$$\begin{aligned} V_{io} &= \int_{\underline{q}}^{\widehat{q}_{io}} [p_i(R - w_{io})]\, f(q)dq + \int_{\widehat{q}_{io}}^{1} qR - (p_i + \theta + \Delta_i)\, w_{io} - B)f(q)dq \\ &= p_i(R - w_{io}) + \int_{\widehat{q}_{io}}^{1} [(q - p_i)R - (\theta + \Delta_i)\, w_{io} - B]\, f(q)dq \end{aligned}$$

a) For $i = M, H$ it is

$$\frac{\partial V_{io}}{\partial \Delta_i} = -p_i \frac{\partial w_{io}}{\partial \Delta_i} - \int_{\widehat{q}_{io}}^{1} [w_{io} + (\theta+\Delta_i)\frac{\partial w_{io}}{\partial \Delta_i}] f(q)dq - \frac{\partial \widehat{q}_{io}}{\partial \Delta_M} \underbrace{[(\widehat{q}_{io} - p_i)R - ((\theta + \Delta_i)w_{io} + B)]}_{=0} f(\widehat{q}_{io}).$$

By substituting $\widehat{q}_{io} = p_i + \frac{B}{R} + \frac{(\theta+\Delta_i)w_{io}}{R}$, we can immediately verify that the square bracket in the last term of the RHS is equal to zero. Substituting $\frac{\partial w_{io}}{\partial \Delta_i} = -\frac{I_i}{(p_i+\Delta_\theta - p_L)^2} = -\frac{w_{io}}{(p_i+\Delta_i-p_L)} < 0$, we then obtain:

$$\frac{\partial V_{io}}{\partial \Delta_i} = p_i \frac{w_{io}}{(p_i+\Delta_i-p_L)} - \int_{\widehat{q}_{io}}^{1} [w_{io} - \frac{(\theta+\Delta_i)w_{io}}{(p_i+\Delta_i-p_L)}] f(q)dq$$

$$\frac{w_{io}}{p_i+\Delta_i-p_L}\left[p_i - \int_{\widehat{q}_{io}}^{1} (p_i-\theta-p_L) f(q)dq\right] > 0,\ i = M, H.$$

b) When 4 holds as an equality, the manager is indifferent between I_M and I_H but profits are generally higher in the former case. Note that in this case $w_{Mo} = w_{Ho} \equiv w$ while $\widehat{q}_{Mo} = p_M + \frac{B}{R} + \frac{\theta_M}{R} w < p_H + \frac{B}{R} + \frac{\theta_H}{R} w = \widehat{q}_{Ho}$. Consider the difference

$$\begin{aligned} V_{Ho} - V_{Mo} &= \int_{\underline{q}}^{\widehat{q}_{Ho}} p_H(R-w)f(q)dq + \int_{\widehat{q}_{Ho}}^{1} [qR - (p_H+\Delta_H+\theta)w - B] f(q)dq + \\ &\quad - \int_{\underline{q}}^{\widehat{q}_{Mo}} p_M(R-w)f(q)dq - \int_{\widehat{q}_{Mo}}^{1} [qR - (p_M+\Delta_M+\theta)w - B] f(q)dq \\ &= \int_{\underline{q}}^{\widehat{q}_{Mo}} (p_{H-}p_M)(R-w)f(q)dq + \int_{\widehat{q}_{Mo}}^{\widehat{q}_{Ho}} p_H(R-w)f(q)dq \\ &\quad - \int_{\widehat{q}_{Ho}}^{1} (p_H+\Delta_H-p_M-\Delta_M) w f(q)dq \\ &\quad - \int_{\widehat{q}_{Mo}}^{\widehat{q}_{Ho}} [qR - (p_M+\Delta_M+\theta)w - B] f(q)dq \end{aligned}$$

Considering that in this case it is

$$w_{Mo} = \frac{I_M}{p_M+\Delta_M-p_L} = \frac{I_H}{p_H+\Delta_H-p_L} = \frac{I_H - I_M}{p_H+\Delta_H-p_M-\Delta_M} = w_{Ho} = w$$

the above expression can be written as

$$\int_{\underline{q}}^{\widehat{q}_{Mo}} (p_{H-}p_M)(R-w)f(q)dq + \int_{\widehat{q}_{Mo}}^{\widehat{q}_{Ho}} p_H(R-w)f(q)dq$$

$$-\int_{\widehat{q}_{Ho}}^{1} (I_H - I_M)f(q)dq - \int_{\widehat{q}_{Mo}}^{\widehat{q}_{Ho}} [qR - (p_M + \Delta_M + \theta)w - B]f(q)$$

$$= \int_{\underline{q}}^{\widehat{q}_{Mo}} (p_{H-}p_M)(R-w)f(q)dq$$

$$+\int_{\widehat{q}_{Mo}}^{\widehat{q}_{Ho}} \{p_H R - (p_{H-}p_M)w - [q - (\Delta_M + \theta)w - B]\} f(q)dq - \int_{\widehat{q}_{Ho}}^{1} (I_H - I_M)f(q)dq$$

where $\int_{\widehat{q}_{Mo}}^{\widehat{q}_{Ho}} [q - (p_M + \Delta_M + \theta)w - B]f(q) > \int_{\widehat{q}_{Mo}}^{\widehat{q}_{Ho}} [q - (p_H + \Delta_H + \theta)w - B]f(q)dq >$ $\int_{\widehat{q}_{Mo}}^{\widehat{q}_{Ho}} p_H(R-w)f(q)dq$. From the definition of discretionary pay we have that in this case $I_H - I_M = (p_{H-}p_M)w + z\Delta_M w$, and considering that $I_H - I_M > (p_{H-}p_M)R$, we have that

$$\begin{aligned} V_{Ho} - V_{Mo} &\leq \int_{\underline{q}}^{\widehat{q}_{Mo}} [I_H - I_M - (p_{H-}p_M)w]f(q)dq - \int_{\widehat{q}_{Ho}}^{1} (I_H - I_M)f(q)dq \\ &\quad - \int_{\widehat{q}_{Mo}}^{\widehat{q}_{Ho}} \{[q - (\Delta_M + \theta)w - B] - [p_H R - (p_{H-}p_M)w]\} f(q)dq \\ &= \int_{\underline{q}}^{\widehat{q}_{Mo}} z\Delta_M w f(q)dq - \int_{\widehat{q}_{Ho}}^{1} (I_H - I_M)f(q)dq \\ &\quad - \int_{\widehat{q}_{Mo}}^{\widehat{q}_{Ho}} \{[(q - p_H)R + [(p_{H-}p_M) - (\Delta_M + \theta)]w - B]\} f(q)dq \end{aligned}$$

Considering that $z\Delta_M$ is very small and both the second and the third term are negative, $V_{Ho} - V_{Mo}$ will generally be negative. Recall that $z\Delta_M \leq \frac{I_H - I_M}{w_{Mo}} - (p_H - p_M)$ per $\Delta_M < \Delta_M^*$.

14 Appendix 7: Proof of Proposition 8

In case of a rational manager, the expected profit is

$$V_r = p_M(R - w_R) + \int_{\widehat{q}_R}^{1} \left[(q - p_M)R - B)\right] f(q)dq.$$

Then, considering 8, $V_{Ho} - V_r$ is equal to

$$V_{Ho} - V_r = p_H(R - w_{Ho}) - p_M(R - w_r) +$$
$$+ \int_{\widehat{q}_{Ho}}^{1} [q - (p_H + \Delta_H + \theta)w_{Ho} - B]f(q)dq - \int_{\widehat{q}_r}^{1} [q - p_M w_r - B]f(q)dq.$$

Under the assumption that q is uniformly distributed over the interval $[\underline{q}, 1]$, such expression becomes

$$V_{Ho} - V_r = \frac{(p_H - p_M)}{2(1-\underline{q})}(\widehat{q}_{Ho} + \widehat{q}_r - 2\underline{q})R - \frac{(2 - \widehat{q}_{Ho} - \widehat{q}_r)}{2(1-\underline{q})}\Delta_H w_{Ho} - p_H w_{Ho} + p_M w_r$$

Given that $(p_H - p_M) R < I_H - I_M$ by assumption 4, a sufficient condition for $V_{Ho} - V_r < 0$ then is

$$(I_H - I_M) \frac{(\widehat{q}_{Ho} + \widehat{q}_r - 2\underline{q})}{2(1-\underline{q})} - \frac{(2 - \widehat{q}_{Ho} - \widehat{q}_r)}{2(1-\underline{q})}\Delta_H w_{Ho} - p_H w_{Ho} + p_M w_r < 0. \quad (10)$$

In order to show that the condition 10 can be satisfied at $\Delta_M = \Delta_M^*$ take into account that for $\Delta_M = \Delta_M^*$ it is:

- $w_{Ho} = \frac{I_H}{p_H + \Delta_H^* - p_L} = \frac{I_M}{p_M + \Delta_M^* - p_L} = w_{Mo}$, where $\Delta_H^* = (1+z)\Delta_M^*$, implying

$$I_H - I_M = \frac{p_H + \Delta_H^* - p_L}{p_M + \Delta_M^* - p_L} I_M$$

- $p_M w_r - p_H w_{Ho} = \frac{p_M I_M}{p_M + \Delta_M^* - p_L} - \frac{p_H I_H}{p_H + \Delta_H^* - p_L} = \left(\frac{p_M \Delta_M^*}{(p_M - p_L)\left(p_M + \Delta_M^* - p_L\right)} - \frac{(p_H - p_M)}{\left(p_M + \Delta_M^* - p_L\right)} \right) I_M$

Substituting these expressions in condition 10 we obtain

$$-(p_H - p_M)(2 - \widehat{q}_{Ho} - \widehat{q}_r) - z\Delta_M^* 2(1 - \widehat{q}_{Ho} - \widehat{q}_r + \underline{q}) + \frac{\Delta_M^*}{(p_M - p_L)} \left[2(1-\underline{q}) - (2 - \widehat{q}_{Ho} - \widehat{q}_r)\right] < 0$$

Considering that, from condition 4, the value of Δ_M^* depends on $\frac{I_M}{I_H}$, and that $1 < \frac{I_M}{I_H} < \frac{p_H - p_L}{p_M - p_L}$ there clearly exist sufficiently low values of the ratio $\frac{I_M}{I_H}$ guaranteeing that the condition is satisfied.

15 Appendix 8: Proof of Proposition 9

From Proposition 7 we have that for any given level of θ, V_{Mo} is increasing in Δ_M. Consider, however, that for any given level of θ, the maximum value that the parameter Δ_M can take is $\Delta_M^M(\theta) = \frac{1-p_H-\theta}{1+z}$. For any level of θ we can then calculate the maximum level of V_{io}, that is the level of expected profit corresponding to $\Delta_M^M(\theta)$. Let us call it $V_{io}^{\Delta_M^M(\theta)}$.

From Assumption 1, the maximum level that can θ can take is to $\theta^M(\Delta_M = 0) = 1-p_H$. We know from Corollary 3 that at such level of θ, it is $V_r > V_{Mo}^{\Delta_M^M(\theta)}$ (note that I_M is chosen when $\Delta_M = 0$). Let θ diminish, which makes V_{Mo} increase. Correspondingly $\Delta_M^M(\theta)$ increases, further increasing $V_{Mo}^{\Delta^M(\theta)}$. Three cases are then possible: i) it may be $V_{Mo}^{\Delta_M^M(\theta)} = V_r$ for some $\Delta_M^M(\theta) < \Delta_M^*$ so that for further decreases in θ, it is $V_{Mo}^{\Delta^M(\theta)} > V_r$; ii) for low enough values of Δ_M^*, it may still be $V_r > V_{Mo}^{\Delta^M(\theta)}$ at $\Delta_M^M(\theta) = \Delta_M^*$; iii) it may also happen that $\theta \to 0$ for $\Delta_M^M(\theta) < \Delta_M^*$, implying that I_H is never chosen because $p_M + \Delta_M^* > 1$. In the first two cases, there clearly exists $\widetilde{\theta} > 0$ such that $V_r > V_{Mo}^{\Delta^M(\theta)}$ for $\theta \geq \widetilde{\theta}$. But even in the third case $V_r > V_{Mo}^{\Delta^M(\theta)}$ will be reached for $\theta > 0$ because we know from Corollary 3 that $V_r > V_{Mo}^{\Delta^M(\theta)}$ for $\theta \to 0$.