Public Health Investments and the Direction of Technological Progress: A Theory of Deskilling during the British Industrial Revolution

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Abstract

The British Industrial Revolution was characterized by the decline in the average level of skills of workers as technological progress was unskill-biased, and the stagnation of life expectancy despite economic development. In this paper, I rationalize these two features of British industrialization in a two-sector growth model in which the direction of technological progress is endogenous and public health investments are the result of profit-maximization by the capitalist class. I show that improvements in life expectancy can generate a switch from unskill- to skill-biased technological progress and a transition to a regime of sustained economic growth. However, unskill-biased technological progress

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initially reduces capitalists' incentives to undertake investments in public health measures and thereby delays the take-off. The theory is consistent with observations of a declining skill-premium, and simulations of the model provide a convincing account of the dynamics of the Industrial Revolution.

Keywords: Industrial Revolution, Directed Technological Change, Public Health, Unified Growth Theory

1 Introduction

While the role mortality reductions and increases in longevity played in the transition from a regime of Malthusian stagnation to one of sustained economic development has been extensively studied [de la Croix and Licandro (1999), Boucekkine et al. (2003), Cervellati and Sunde (2005)], less focus has been put on the source of such improvements. Most long-run growth models assume that health is a by-product of economic development and gradually improves as output grows [Kalemli-Ozcan (2002), Chakraborty (2004), Soares (2005)]. Through the Ben-Porath mechanism, lower mortality then raises the returns to education and fosters human capital accumulation, eventually allowing the economy to take-off to a modern growth regime [Galor (2011), Cervellati and Sunde (2015), Yasui (2016)]. Looking at the evolution of both health and human capital during the British Industrial Revolution however casts doubt on such mutually supporting forces driving progress. First, the average level of human capital of the labor force declined [Mitch (1993), de Pleijt (2018)]. The relative returns to skills actually fell over the course of industrialization, as a result of *unskill-biased* technological progress [Van Zanden (2009), de Pleijt and Weisdorf (2017)]. Second, life expectancy stagnated at a time of unprecedented economic development [Wrigley and Schofield (1981), Szreter and Mooney (1998)]. While better standards of living may very well have a positive effect on the health of the population, the stagnation of real wages at the time [Allen (2009)] calls for another explanation of improvements in health. A large body of empirical evidence suggests that substantial public investments in health-related infrastructure and amenities at the end of the 19th century were the main driver of longevity increases instead [Easterlin (1999), Cutler et al. (2006), Chapman (2019)].

The objective of this paper is to rationalize both the stagnation of life expectancy and the decline in the average level of human capital of the labor force that characterized the British Industrial Revolution, and provide an explanation for the subsequent take-off that occurred in the late 19th century. To do so, I build a two-sector growth model in which the direction of technological progress is endogenous and public investments in health-related infrastructure are the result of profit-maximization by the capitalist class. In particular, I emphasize how the health of the labor force, by influencing its skill composition through the Ben-Porath effect, is crucial to the direction of technological progress because of the market size effects that drive innovation. In turn, by altering the returns to human capital in aggregate production, the direction of technological progress affects capitalists' incentives to support and fund costly public health measures that would entice workers to invest in education.

Specifically, because of a low life expectancy and thus limited incentives to acquire education, technological progress is initially directed towards the unskilled-labor-using sector. The skill-premium declines, shifting the skill composition of the labor force towards unskilled labor, which fuels unskill-biased technological progress even further in a somewhat vicious circle. This generates the deskilling process that describes the first stage of the Industrial Revolution. Furthermore, by lowering the role of human capital in aggregate production and thereby the incentives of capitalists to undertake investments in sanitary infrastructure, unskill-biased technological progress delays improvements in life expectancy. As in Galor and Moav (2006), such investments occur only when the diminishing returns to physical capital eventually turn lower than those of human capital, as profit-maximizing capitalists eventually find it beneficial to have a more educated workforce and consent to raise taxes to fund public health measures. Life expectancy finally increases, and the resulting shift in the skill composition of the labor force induces an endogenous switch from unskill- to skill-biased technological progress that triggers a take-off to a regime of human capital accumulation and sustained economic growth.

Aidt et al. (2010) and Chapman (2019) describe the two features of England's institutional framework at the time that motivates the focus of this paper on capitalists' behavior *vis-à-vis* public investments in health-related amenities. First, such investments had to be financed locally by taxes on property[1]. The burden of their cost was thus not equally shared between social classes as it fell almost exclusively on upper- and middle-class landlords, entrepreneurs, and capitalists who owned property. Second, while the working class did not participate to their funding, it was also excluded from the political process through which such taxes were agreed upon. Indeed, the franchise allowing men to vote in local elections at the time was confined to those duly paying this property tax. England was in practice a 'taxpayer democracy' in which only those bearing the cost of public health investments could participate in the decision to levy taxes to finance them. Importantly, they were also those who would benefit the less from the better sanitary environment: Szreter (2002) describes how the propertied classes escaped the unsanitary conditions of the urban centers by physically moving away to more salubrious areas. de la Croix and Sommacal (2009) indeed document the stark health inequalities between the bourgeoisie and the proletariat that characterized the Industrial Revolution. Ruling out the hypothesis that the provision of public health measures by (relatively) healthy capitalists was driven by pure altruism, I analyze it through the prism of economic incentives instead.

Although the *political* incentives behind the provision of public goods or any kind of redistribution by the elite have been largely examined [Bourguignon and Verdier (2000), Acemoglu and Robinson (2001), Lizzeri and Persico (2004)], this paper sets in a flourishing literature studying how the *economic* incentives of profit-maximizing elites shape long-run development. The seminal contribution of Galor and Moav (2006) shows how the complementarity between human and physical capital prompts the capitalist class to eventually support the provision of public schooling. Despite being funded through the taxation of their wealth, the education of the working class increases capitalists' revenues when the returns to human capital surpass those of physical capital. Galor et al. (2009) as well as Leite et al. (2020) study the opposition to such reforms by a landed aristocracy, and Goñi (2021) empirically corroborates their results. While such papers focus on investments in public schooling as a mean for human capital accumulation, I instead consider investments in health-related amenities that foster the acquisition of skills through improvements in longevity. I do so because the lack of investments in urban sanitary infrastructure in 19th century England warrants inquiry *per se*, but also because of the tight link between life expectancy and education. Sufficient health levels can indeed be a pre-requisite to the accumulation of human capital even when the financial cost of education is low, especially when the legality of child labor entails additional opportunity costs [Doepke and Zilibotti (2005), Humphries (2010)]. Furthermore, the fact that investments in both public health infrastructure and public schooling in England occurred in the same period – the late stage of the industrialization process – indicates that they may share similarities. I therefore argue that the profit incentives of the capitalist class can also explain (the lack of) public health investments in industrializing England.

Nevertheless, I go further than the capital-skill complementarity at the heart of Galor and Moav's theory by endogenizing the direction of technological progress, allowing it to be biased towards unskilled rather than skilled labor. In addition to explaining the poor performance of life expectancy in England throughout most of the 19th century, considering unskill-biased technological progress also allows to reconcile theories of long-run growth with the peculiar evolution of the skill composition of the labor force during the British Industrial Revolution. While a rich empirical literature has documented the historical rise in the supply of unskilled workers in 19th century England [Nicholas and Nicholas (1992), Humphries (2013), de Pleijt and Weisdorf (2017)], theory typically depicts a monotonic increase in the returns to skills due to technological innovation [Galor (2011)]. The gradual accumulation of human capital that theoretically ensues seems at odds with observations of a declining skill premium [Clark (2005), Van Zanden (2009)]. Although Rahman (2017) cleverly rationalizes this by assuming that human capital is a consumption as well as an investment good, a steady rise in the level of human capital still characterizes his theory of the industrialization process. O'Rourke et al. (2013) manage to solve this puzzle by incorporating an innovation sector into a unified growth model to account for unskill-biased technological progress. While they focus on explaining the demographic transition, they emphasize how the substitutability of skilled and unskilled labor in aggregate production makes technological progress biased towards the unskilled-labor-using sector in the first stage of industrialization because of the market size effects behind innovation. Individuals respond to such economic incentives by raising fertility, which increases the supply of unskilled, uneducated children. In their model, it is only when innovation starts to occur in the skilled-labor-using sector as well that the terms of the children quality-quantity trade-off shift and education takes off.

In this paper, although I do not consider fertility decisions, I propose an alternative explanation of the transition from an unskilled-labor-intensive industrialization process to a regime of human capital accumulation driven by skilled-biased technological progress. Instead of being the result of *ex ante* changes in technology, the take-off of education stems from improvements in longevity that raise the returns of human capital for workers, when capitalists eventually concede to fund investments in public sanitary infrastructure. It is only when the skill composition of the labor force shifts towards skilled labor because of the resulting incentives to invest education that innovation, although not explicitly modeled but captured in a reduced-form assumption, reacts to such market size effects. The rise in the number of skilled workers subsequently fosters technological progress in the skilled-labor-using sector and the vicious circle of deskilling becomes virtuous: the skill premium finally increases, reinforcing the accumulation of human capital, and the economy takes off to a regime of sustained growth. However, I emphasize that, contrary to most unified growth models, such a take-off is not inevitable. If increases in longevity and the resulting educational incentives are too modest, they may not be enough to reverse the course of unskill-biased technological progress and the economy can get stuck in a poverty trap in which the skilled-labor-using technology eventually disappears.

To summarize, this paper provides a theoretical analysis of the close interaction between the health of the labor force, its skill composition, and the direction of technological progress; as well as of the profit incentives of the capitalist class to undertake (or not) costly investments in public health-related amenities. I then show that numerical simulations of the model convincingly capture salient features of the British Industrial Revolution: the stagnation of life expectancy and the deskilling process of the working class during the first stage of the industrialization process, the endogenous emergence of public health measures funded via the consensual taxation of the capitalist class, and the ensuing switch from unskill- to skill-biased technological progress that triggers the take-off to a regime of sustained economic growth.

The paper is organized as follow: Section 2 provides empirical evidence about life expectancy and the direction of technological progress during the British Industrial Revolution that motivate the analysis. Section 3 sets up the model. Section 4 investigates how life expectancy, the skill composition of the labor force, and the direction of technological progress interact. Section 5 endogenizes the provision of public health and describes capitalists' decisions about taxation. Section 6 provides a numerical simulation of the model that replicates features of the industrialization of England. Section 7 briefly concludes.

2 Historical context and empirical motivations

In this section, I provide empirical evidence for the two surprising features of the British Industrial Revolution that motivate the theory: (1) the stagnation of life expectancy and the crucial role of public health investments in improving the condition of the working class, and (2) the process of deskilling driven by unskill-biased technological progress.

2.1 Life expectancy during the Industrial Revolution in England

In most theoretical growth models about the role of health on human capital accumulation, improvements in health are a by-product of economic growth. Life expectancy is often assumed to be an increasing function of output or technological progress. The underlying argument is that the rise in the standards of living associated to economic development naturally improves population health. This is the thesis of Thomas McKeown's influential book, *The Mod-*

ern Rise of Population [McKeown (1976)], in which he argues that greater per capita nutritional intakes permitted by higher real incomes were the main factor behind increases in longevity in both England and Wales in 19th century.

Taking a closer look at the evolution of life expectancy in England during the Industrial Revolution however casts doubt on the causal link between income and health. Wrigley and Schofield (1981) show how the gradual increase in life expectancy during the 18th century stopped at the beginning of the 19th century. It is striking that, at the time when England was experiencing unprecedented economic growth, the life expectancy of the vast majority of the population indeed stagnated. Szreter and Mooney (1998) describe an even grimmer situation in British cities where negative externalities associated with urbanization, they argue, exerted a penalty on the health of the population. Life expectancy in England surpassed the levels at the start of the Industrial Revolution only after 1870, as can be seen in Figure 1.

[FIGURE 1 SHOULD APPEAR HERE]

Steckel (1999) and Steckel and Floud (1997) confirm that population health might have decreased during periods of industrialization and urbanization in the first half of the 19th century (with the notable exception of Sweden). Recent empirical evidence would have come as no surprise to acute contemporary observers who already documented the dire living conditions and the immiseration of the working class in industrializing countries. Friedrich Engels' magnum opus, *The Condition of the Working Class in England* [Engels (1993)], paints a dark picture of the industrial proletariat at the time. The hardship of new labor along with urbanization and the lack of public health infrastructures, Engels argues, took a serious toll on the health of the working class.

Although it may seem natural to think that economic growth translates into better standards of living and higher life expectancy, Szreter (1997) argues that the growth process itself has disruptive effects on society and may pose threats to population health. While the negative causal effect of industrialization and urbanization on health is still debated in the historical literature, it is accepted that substantial improvements in life expectancy in England only occurred in the late 19th century [Hinde and Harris (2019), Davenport (2020)]. Szreter (1988, 2004) nevertheless emphasizes how addressing the potentially negative externalities of rapid development requires substantial investments in health-related infrastructure and amenities.

At the time of the publication of McKeown's arguments, demographer, and sociologist Samuel H. Preston was already stressing the role of such public health investments in the major improvements in longevity at the end of the 19th and the beginning of the 20th century [Preston (1975)]. Looking thoroughly at British history, Easterlin (1999) documents how, rather than economic development and market forces, it is the combination of scientific discoveries and their application through collective action that were the main factors responsible for the take-off of life expectancy in England that he himself situates around 1871. Indeed, big public works in cities such as building sewers and sanitation systems, providing clean water, and paving streets allowed to curb mortality from infectious diseases and had an unprecedented positive impact on life expectancy – an episode called the sanitation revolution. More specifically, Chapman (2019) quantitatively assesses the contribution of sanitation infrastructure to the decline in mortality rates in British cities during the second half of the 19th century and finds that, by largely eliminating mortality from waterborne diseases and reducing airborne diseases transmission, public health investments account for 60% of the mortality decline.

Furthermore, the prominent role played by public health measures in the improvements of population health is not a British peculiarity: looking at the US, Cutler and Miller (2005) estimate that water purification alone can account for half the mortality decline in the early 20th and Costa (2015), also emphasizing the role played by scientific advances, highlights how substantial public health measures were necessary to their application. Cutler et al. (2006) note that the importance of public health investments in the rise of life expectancy is proof that *"health comes from institutional ability and political willingness to implement known technologies, neither of which is an automatic consequence of rising incomes"* (p.116). One objective of this paper is therefore to break down the direct link between aggregate output and population health that is often assumed in theoretical growth models, and instead study the endogenous provision of public health measures over the course of development, focusing on the British Industrial Revolution.

2.2 Deskilling and the direction of technological progress

Although technological progress is commonly seen today as being skill-biased, thus increasing the relative demand for skilled labor, this view probably stems from our recent experience in the modern world. Classical economists however, from Adam Smith to Karl Marx, viewed technological progress as a process inherently biased towards unskilled labor [Brugger and Gehrke (2018)]. Indeed, during the Industrial Revolution, physical capital and unskilled labor substituted for skilled labor with the division of labor, mechanization, and the rise of the factory system. Allen (2017) describes how, during the 18th and 19th centuries in England, skilled artisans and craftsmen who used to supervise the whole production process gradually disappeared in the face of machine competition. O'Rourke et al. (2013) emphasize how Luddites were in fact those skilled workers who opposed not technological progress *per se*, but their replacement by new inventions that could be operated by unskilled workers. The rise in the demand for child labor that Humphries (2010) documents, looking at autobiographies of men who worked through the period, clearly indicates the decline of the role of skills in aggregate production. The deskilling nature of technological change during the Industrial Revolution, in addition to having been noticed by contemporary economists, is also depicted in the literature of the 19th century, be it in Charles Dickens' *Oliver Twist* for England, or Emile Zola's *Germinal* for France. However, the most salient piece of evidence of the decline in the role of skills in production during the Industrial Revolution is probably the gradual fall in the skill-premium documented by Clark (2005) and Van Zanden (2009). The lack of a marked increase in the market returns to human capital is hard to reconcile with theories of industrialization based on incentives to acquire education. Technological innovation raised the demand for unskilled workers instead, and the labor force transitioned from the workshop to the factory [de Pleijt and Weisdorf (2017)]. The complementarity between human and physical capital at the heart of unified growth theory only emerged around the turn of the 20th century [Goldin and Katz (1998)].

A vast literature quantifies the process of deskilling of the labor force by looking at educational attainment or literacy rates during the industrialization period. Mitch (1993) documents a relatively low performance in terms of literacy during the British Industrial Revolution, with rates fluctuating around 60% between 1750 and 1850, without any significant improvement. Similarly, Nicholas and Nicholas (1992) emphasize how literacy really was an investment in human capital at the time. They show that, while England had attained a sufficient level of literacy on the eve of the Industrial Revolution to allow growth to set in, there was a stagnation and even a decline in male literacy rates in the late 18th and early 19th century, as further measures to decrease illiteracy were deemed unnecessary. They stress the unskilled-labor intensive nature of production in industrializing England, and describe how the factory system and the introduction of power-driven machinery did not require much educational investment on the part of workers.

Literacy rates, however, are a very imperfect measure of the level of human capital of the population as they proxy primary schooling only, and therefore do not differentiate a low-skill but literate factory worker from a skilled engineer. In recent work, de Pleijt (2018) goes further by studying the evolution of human capital in England as measured in average years of education instead, disentangled into primary, secondary, and tertiary schooling. She finds that years of schooling increased substantially from the 16th century onwards, which is consistent with Boucekkine et al. (2007), who document a surge in the building of new schools during the period. However, she shows how starting from the early 18th century, years of formal education started to decline, driven by a stagnation in years of primary schooling while secondary and tertiary educational attainment decreased significantly from 1700 to 1880. A direct implication of this evolution is a remarkable increase in the share of unskilled workers in England from the onset of the Industrial Revolution onwards, documented in de de Pleijt and Weisdorf (2017). Using occupational titles to classify English workers according to the skills encompassed by their work, they find that the share of unskilled workers was as low as 20% in the 16th century, but that it rose strikingly after 1700 to reach almost 40% in the 19th century. This process of deskilling of the labor force illustrated in Figure 2 is a clear indication that the demand for educated workers fell sharply during the British Industrial Revolution and the objective of this paper is to replicate this feature of development.

[FIGURE 2 SHOULD APPEAR HERE]

It is important to note however that the deskilling hypothesis is not necessarily antagonistic to another view of the Industrial Revolution that emphasizes the role of knowledge in the process of industrialization, by fostering both the invention and the application of new technologies. In the words of Joel Mokyr, the main proponent of this view, *"the essence of the Industrial Revolution was technological, and technology is knowledge"* [Mokyr (2005), p.19]. According to Mokyr and Voth (2010), technological advances were a close collaboration between formally educated scientists trying to understand the nature of the world in the spirit of the Enlightenment, and skilled craftsmen who searched to apply this knowledge to production, driven by profit. Squicciarini and Voigtländer (2015) show that disentangling the 'upper-tail' knowledge of an elite at the top of the skill distribution from that of the average workers reinstates the role of human capital in the Industrial Revolution. It is not in contradiction however with a decline in the average level of skills in workforce, as new innovations may very well have been biased towards unskilled labor, raising the demand for uneducated workers. Mokyr (1990) himself mentions the replacement of skilled artisans by unskilled workers using new and superior technology.

3 The Model

3.1 Production

The economy is composed of two sectors Y^s and Y^u using respectively skilled L^s and unskilled labor L^u, both coupled with capital K. I assume both sector are perfect substitute in the constitution of aggregate output Y, such that:

$$Y_t = z_t \left\{ (1 - x_t) Y_t^u + x_t Y_t^s \right\} \tag{1}$$

Where x_t is the relative productivity of the skilled sector and z_t is Total Factor Productivity (TFP). The production function of each sector is a Cobb-Douglas with the same capital intensity α to make things as tractable as possible. The sector-specific level of technology is constant and denoted by A^j for $j = u, s$.

$$Y_t^u = A^u (K_t^u)^\alpha (L_t^u)^{1-\alpha} \tag{2}$$

$$Y_t^s = A^s (K_t^s)^\alpha (L_t^s)^{1-\alpha} \tag{3}$$

Factors of production are paid their marginal product, therefore:

$$w_t^u = \frac{\partial Y_t}{\partial L_t^u} = z_t(1 - x_t) A^u (1 - \alpha) \left(\frac{K_t^u}{L_t^u}\right)^\alpha \tag{4}$$

$$w_t^s = \frac{\partial Y_t}{\partial L_t^s} = z_t x_t A^s (1 - \alpha) \left(\frac{K_t^s}{L_t^s}\right)^\alpha \tag{5}$$

$$r_t^u = \frac{\partial Y_t}{\partial K_t^u} = z_t(1 - x_t) A^u \alpha \left(\frac{K_t^u}{L_t^u}\right)^{\alpha-1} \tag{6}$$

$$r_t^s = \frac{\partial Y_t}{\partial K_t^s} = z_t x_t A^s \alpha \left(\frac{K_t^s}{L_t^s}\right)^{\alpha-1} \tag{7}$$

Capital is fully mobile across sectors and both interest rates therefore equalize. This gives:

$$\frac{K_t^s / L_t^s}{K_t^u / L_t^u} = \left(\frac{A^s}{A^u} \frac{x_t}{1 - x_t}\right)^{\frac{1}{1-\alpha}} \tag{8}$$

Denoting the fraction of the aggregate capital stock K_t employed in the low-skill sector by γ_t such that $K_t^u = \gamma_t K_t$ and $K_t^u = (1 - \gamma_t) K_t$, the above equation gives:

$$\gamma_t = \frac{(A^u[1 - x_t])^{\frac{1}{1-\alpha}} L_t^u}{(A^u[1 - x_t])^{\frac{1}{1-\alpha}} L_t^u + (A^s x_t)^{\frac{1}{1-\alpha}} L_t^s} \tag{9}$$

Let us rewrite the production function as follow:

$$Y_t = z_t K_t^\alpha \left\{ (A^u[1 - x_t])^{\frac{1}{1-\alpha}} L_t^u + (A^s x_t)^{\frac{1}{1-\alpha}} L_t^s \right\}^{1-\alpha} \tag{10}$$

The skill premium can be expressed as a function of sectoral productivity levels:

$$\omega_t \equiv \frac{w_t^s}{w_t^u} = \left(\frac{A^s}{A^u} \frac{x_t}{1 - x_t}\right)^{\frac{1}{1-\alpha}} \tag{11}$$

3.2 The direction of technological progress

To model the direction of technological progress, I draw inspiration from several papers. First, Cervellati and Sunde (2015) interpret the increase of the relative weight of the skill-intensive sector in aggregate production x_t as *skill-biased technological change*. A feature of their model however is that the relative productivity of the skilled sector can only grow, which leaves no room to the matter of interest here, *unskill-biased technological progress*. In this paper, I therefore consider that sectoral relative productivities can go in either directions, and technological change is be skill-biased when x increases, and unskill-biased when x decreases. What then determines in which sector innovation occurs? I follow the basic intuition underlying the canonical directed technological change model, in which innovators decide upon the amount of research and development in a given sector according to profit incentives [Acemoglu (2002)]. Two countervailing forces influence the direction of technological progress: a price effect that encourages innovation of more expensive goods produced using scarce factors, and a market size effect that fosters innovation in the technology using the abundant factors. It is the elasticity of substitution between the factors that ultimately determines which effect dominates. O'Rourke et al. (2013) embed such an innovation process in which profit-maximizing innovators decide in which sector to innovate in a unified growth model. They argue that, given that skilled and unskilled labor are grossly substitutable, innovation was directed towards the unskilled-labor-using sector in the first stage of the Industrial Revolution because of the market size effect. In this paper, I do not explicitly model the innovation sector. Instead, I make the reduced form assumption that the relative productivity of each sector is a function of the contemporary skill composition of the labor force. This captures market size effects that prompt innovation in the sector using the abundant factor. Specifically, I assume that the weight in aggregate production of the skilled (unskilled) sector is a decreasing (increasing) and linear function of the share of unskilled workers $\mu_t = \frac{L_t^u}{L_t^u + L_t^s}$. Formally:

$$x_t \equiv x(\mu_t) = 1 - \mu_t \tag{12}$$

Furthermore, I assume that the growth rate of TFP is positively affected by the number of skilled workers in the previous period. This captures the process of accumulation of *upper-tail* knowledge driven by an elite at the top of the skills distribution described by Mokyr (2005). Cervellati and Sunde (2015) make a similar assumption, interpreting the share of skilled workers $(1-\mu_t)$ as the amount of labor involved in research. It is defined as follow, with $\eta > 0$:

$$g_t^z = \frac{z_{t+1} - z_t}{z_t} = \eta(1 - \mu_{t-1}) \tag{13}$$

3.3 Individuals

There are two types of individuals in the economy: workers and capitalists. Both have different objective functions and constraints. I describe each of them in the next subsections.

3.3.1 Workers

Each period, a continuum of workers with unit mass $L = 1$ is born. Each worker lives with certainty through the first period of her life – childhood – and survives with a probability $\phi \in (0, 1)$ to the second period – adulthood. Two things are worth noting. First, since my objective is not to explain the demographic transition, I do not consider fertility decisions and instead assume that the number of children born each period is exogenous and constant. Second, the survival probability ϕ enters workers' utility function by multiplying utility in adulthood, effectively acting as a discount factor. Since an individual is expected to live for $1 + \phi$ periods, it is common in the theoretical literature on studying the links between health and growth to interpret ϕ as longevity [Chakraborty (2004), de la Croix and Sommacal (2009), Raffin and Seegmuller (2014)]. In this paper, I therefore refer to ϕ as life expectancy.

In the first period, workers decide whether to make an indivisible investment in education that is costly in effort. Children are heterogenous in ability, and therefore face a different cost of education accordingly. In the second period of their life, the workers who made the educational investment are employed in the skilled-labor-using sector, while those who did not work in the unskilled-labor-using sector. They provide inelastically one unit of labor at the prevailing wage in the sector they are employed in and consume the whole of their resulting labor income.[2]

Lifetime utility for a worker with ability i is of the following form:

$$U_i = -\Delta^i E_t^i + \phi \log c_{t+1}^w \tag{14}$$

Where Δ^i is the effort worker i needs to put in if he undertakes education, hence the lower Δ^i, the more able she is (Δ^i can therefore be interpreted as the inverse of a worker's capacity). E_t^i represents the indivisible investment and takes a value of one if she decides to go to school, and zero otherwise. The budget constraint of any worker is simply:

$$c_{t+1}^w \leq \begin{cases} w_{t+1}^u & \text{iff} \quad E_t^i = 0 \\ w_{t+1}^s & \text{iff} \quad E_t^i = 1 \end{cases} \tag{15}$$

A worker with 'disability' Δ^i therefore decides to invest in education if and only if it increases her lifetime utility. Formally, if $U_i(E_t^i = 1) \geq U_i(E_t^i = 0)$. When taking such a decision, each worker therefore compares the wage differential between the two sectors to his educational effort cost. However, since workers do not observe the future level of wages as it depends on the skill composition of the labor force the next period, I assume that they base their expectations on the current skill premium ω_t. This defines a threshold level $\Delta^\star$ for ability at which $U_i(E_t^i = 1) = U_i(E_t^i = 0)$:

$$\Delta_t^\star = \phi \log(\omega_t) \tag{16}$$

3.3.2 Capitalists

Each period, a number N^k of identical capitalists are born. For simplicity, let us normalize N^k to one. They receive bequests from their parents b_t in the first period. The whole of those bequests is saved. In the second period, they do not work and use their rental income to consume c_{t+1} and leave bequests b_{t+1} to their children in turn. To capture the substantial health inequalities between the bourgeoisie and the proletariat in England during the Industrial Revolution [de la Croix and Sommacal (2009)], I assume that they face a probability of surviving to the second period ϕ^k greater than that of workers ϕ. Specifically:

$$\phi^k = 1 > \phi$$

This assumption that capitalists already enjoy the maximum level of health will simplify the analysis when decisions about the provision of public health are introduced in Section 4, but more importantly it captures the fact that they benefit from public investments in health-related amenities only through their effects on production, hence on their income.

Capitalists derive utility from both consumption and the bequest they leave to their children. They therefore choose c_{t+1} and b_{t+1} to maximize their utility subject to their budget constraint:

$$\max_{c_{t+1};b_{t+1}} U_k = (1-\beta)\log c_{t+1}^k + \beta \log b_{t+1}^k \tag{17}$$

$$s.t. \quad c_{t+1}^k + b_{t+1}^k \leq b_t^k r_{t+1} \equiv I_{t+1}^k \tag{18}$$

Where I_{t+1}^k is capitalists' total wealth. This simply gives:

$$c_{t+1}^k = (1-\beta) I_{t+1}^k \tag{19}$$

$$b_{t+1}^k = \beta I_{t+1}^k \tag{20}$$

3.4 Physical capital accumulation

Capitalists are the only individuals who save. As mentioned above, their savings are equal to the bequest they receive from their parents. Denoting aggregate bequest by $B_t = b_t^k$ and assuming full depreciation for simplicity, the aggregate stock of capital in $t+1$ is therefore:

$$K_{t+1} = B_t \tag{21}$$

4 The skill composition of the labor force

I assume that workers' ability $\Delta \in (0; \bar{\Delta})$ follows a uniform distribution with a probability density function $g(\Delta)$. Recall that a child undertakes the indivisible effort investment in education if and only if her ability is lower than the threshold $\Delta_t^\star = \phi \log(\omega_t)$. The threshold is therefore a function of workers' life expectancy and the skill premium: $\Delta_t^\star \equiv \Delta^\star(\phi, \omega_t)$.

4.1 The share of unskilled workers

In $t+1$, the share of unskilled workers born in t is:

$$\mu_{t+1} \equiv \int_{\Delta_t^\star}^{\bar{\Delta}} g(\Delta) d\Delta = 1 - G(\Delta_t^\star) \tag{22}$$

Where $G(\Delta_t^\star)$ is the cumulative distribution function:

$$G(\Delta_t^\star) = \begin{cases} 0 \quad \text{for} \quad \Delta_t^\star < 0 \\ \Delta_t^\star/\bar{\Delta} \quad \text{for} \quad 0 < \Delta_t^\star \leq \bar{\Delta} \\ 1 \quad \text{for} \quad \Delta_t^\star \geq \bar{\Delta} \end{cases}$$

It follows that the share of unskilled workers is equal to 0 when $\Delta_t^\star \geq \bar{\Delta}$. It is straightforward to show that this is the case for:

$$\mu_t \leq \frac{A}{e^{(1-\alpha)\bar{\Delta}/\phi} + A} \equiv \underline{\mu}$$

Where $A = A^s/A^u$. Conversely, this share is equal to 1 when $\Delta_t^\star < 0$, hence when:

$$\mu_t > \frac{A}{1+A} \equiv \overline{\mu}$$

Since $\underline{\mu} < \overline{\mu}$ that the total share of unskilled workers in $t+1$ is defined by the following function:

$$\mu_{t+1} \equiv \Lambda(\omega_t, \phi) = \begin{cases} 0 & \text{for} \quad \mu_t \leq \underline{\mu} < \overline{\mu} \\ \int_{\Delta_t^\star}^{\bar{\Delta}} g(\Delta) d\Delta & \text{for} \quad \underline{\mu} < \mu_t \leq \overline{\mu} \\ 1 & \text{for} \quad \underline{\mu} < \overline{\mu} < \mu_t \end{cases} \tag{23}$$

The share of unskilled workers is therefore constant for $\mu_t \leq \underline{\mu}$ and $\mu_t > \overline{\mu}$. Partial derivatives for $\mu_t \in]\underline{\mu}; \overline{\mu}]$ are as follow:

$$\begin{aligned} \frac{\partial \Lambda(.)}{\partial \phi} &= -\frac{1}{\bar{\Delta}} \log(\omega_t) < 0 \\ \frac{\partial \Lambda(.)}{\partial \omega_t} &= -\frac{\phi}{\bar{\Delta}} \frac{1}{\omega_t} < 0 \end{aligned}$$

Improvements in life expectancy therefore reduce the share of unskilled workers by lengthening the horizon over which individuals benefit from the educational investment undertaken when young. This is the standard Ben-Porath mechanism that is common in the theoretical literature on health and economic growth. The skill-premium also raises the returns to education, it is therefore natural that it increases the average level of skills of the population.

4.2 Dynamics: the (de)skilling process

Recall that the relative productivities of each sector directly depend on the skill composition of the workforce. The current skill premium on which individuals base their expectations can therefore be expressed as a function of the share of unskilled workers: $\omega_t = \{A \cdot (1-\mu_t)/\mu_t\}^{\frac{1}{1-\alpha}}$. This makes $\mu_{t+1} = \Lambda(\mu_t, \phi)$ an autonomous, first-order, non-linear difference equation, *conditional on* ϕ. In this section, I study the dynamics of the skill composition of the workforce and investigate how it crucially depends on workers' life expectancy ϕ. To this purpose, I explicitly treat ϕ as an argument of the function Λ, which allows for a better exposition. Let us express the share of unskilled workers in $t+1$ as a function of that in the previous period and life expectancy:

$$\mu_{t+1} \equiv \Lambda(\mu_t, \phi) = \begin{cases} 0 & \text{for} \quad \mu_t \leq \underline{\mu} < \overline{\mu} \\ 1 - \frac{\phi}{(1-\alpha)\bar{\Delta}} \log\left(A \cdot \frac{1-\mu_t}{\mu_t}\right) & \text{for} \quad \underline{\mu} < \mu_t \leq \overline{\mu} \\ 1 & \text{for} \quad \underline{\mu} < \overline{\mu} < \mu_t \end{cases} \tag{24}$$

As a first step, let us study the dynamics of μ_{t+1} conditional on life expectancy, that is, for a given ϕ.

$$\frac{\partial \Lambda(.)}{\partial \mu_t} = \begin{cases} 0 & \text{for} \quad \mu_t \leq \underline{\mu} < \overline{\mu} \\ \frac{\phi}{(1-\alpha)\bar{\Delta}} \frac{1}{(1-\mu_t)\mu_t} > 0 & \text{for} \quad \underline{\mu} < \mu_t \leq \overline{\mu} \\ 0 & \text{for} \quad \underline{\mu} < \overline{\mu} < \mu_t \end{cases}$$

The share of unskilled workers in the current period is an increasing function of that of the previous period. This comes from the interplay between the skill composition of the labor force and the direction of technological progress. The lower (greater) the current share of unskilled workers, the more technology becomes biased towards skilled (unskilled) labor because of market size effects behind innovation. Skill-biased (unskill-biased) technological progress in turn raises (reduces) incentives to invest in education and thereby decreases (increases) the future share of skilled (unskilled) workers, setting up a virtuous (vicious) circle of human capital accumulation (decumulation).

Looking at the second derivative of the function Λ allows us to characterize the dynamics of the share of the skill composition of the labor force.

$$\frac{\partial^2 \Lambda(.)}{\partial \mu_t^2} = \begin{cases} 0 & \text{for} \quad \mu_t \leq \underline{\mu} < \overline{\mu} \\ \frac{\phi}{(1-\alpha)\bar{\Delta}} \frac{2\mu_t - 1}{(1-\mu_t)^2 \mu_t^2} & \text{for} \quad \underline{\mu} < \mu_t \leq \overline{\mu} \\ 0 & \text{for} \quad \underline{\mu} < \overline{\mu} < \mu_t \end{cases}$$

It is possible to show that on the interval $]\underline{\mu}; \overline{\mu}]$, $\partial^2 \Lambda(.)/\partial \mu_t^2 < 0$ for $\mu_t < 1/2$, $\partial^2 \Lambda(.)/\partial \mu_t^2 = 0$ for $\mu_t = 1/2$ and $\partial^2 \Lambda(.)/\partial \mu_t^2 > 0$ for $\mu_t > 1/2$. $\mu_t = 1/2$ is therefore a unique inflexion point. The study of the function Λ leads to the following proposition:

Proposition 1. *If $A < e^2$,*

(i) There exists two stable steady states $\mu_0^\star = 0$ and $\mu_1^\star = 1$ and a unique unstable steady state $\mu_u^\star \in]\underline{\mu}; \overline{\mu}]$.

(ii) Over time, the share of unskilled workers increases if $\mu_t > \mu_u^\star$ and decreases if $\mu_t < \mu_u^\star$.

(iii) $\mu_u^\star$ increases with workers' life expectancy ϕ.

Proof: see Appendix.

This proposition describes the evolution of the share of unskilled workers over time, conditional on life expectancy, and contains the mechanism behind the deskilling process that characterized the British Industrial Revolution.[3] Due to the interaction of the direction of technological progress and the skill composition of the labor force described above, the economy will converge in the long-run to a steady state where either every worker is skilled ($\mu_0^\star = 0$), or one in which everyone is unskilled ($\mu_1^\star = 1$). Initial conditions play a prominent role in the direction the economy will take at first.

If $\mu_0 < \mu_u^\star$, the relative weight of the skill-intensive sector in aggregate production starts to rise and so does the skill premium. Skill-biased technological progress therefore increases the returns to skills and provides workers with incentives to invest in education. The share of unskilled workers begins to decrease, which in turn reinforces the direction of technological progress towards the skilled-labor-using sector. The skill premium keeps on increasing and the share of unskilled workers converges to $\mu_0^\star = 0$. The unskilled-labor-using sector eventually disappears, every worker is skilled and employed in the skill-intensive sector, and the productivity of the labor input in the production function is high (since $A^s > A^u$).

On the contrary, if $\mu_0 > \mu_u^\star$, the economy takes another route: the unskilled-labor-using sector starts to gain weight in aggregate production relative to the skill-intensive sector, and the skill-premium declines. The unskill-biased nature of technological progress therefore lowers the incentives of workers to undertake the schooling investment required to work in the skilled sector and earn the associated premium, and the share of unskilled workers increases. Through the same interplay between the skill composition of the labor force and the direction of technological progress, the relative importance of the unskilled sector strengthens, while the skilled sector starts to shrink. The evolution of the labor force is therefore characterized by a progressive loss of skills as the number of educated workers falls. This mirrors the episode of deskilling that occurred for most of the British Industrial Revolution, with the disappearance of skilled artisans and the transition from workshop to factories, the reduction in literacy rates, and more generally, the surge in the demand for unskilled labor. Provided nothing happens, the skilled sector ultimately vanishes and the share of unskilled workers converges to $\mu_1^\star = 1$.

As the value of $\mu_u^\star$ depends on ϕ, the health of the labor force is crucial in determining which path the economy initially engages in: for a given μ_0, a low life expectancy as in Figure 3 makes it more likely that $\mu_0 > \mu_u^\star$ and, as a result, technological progress is unskill-biased and the share of unskilled workers increases. Low levels of life expectancy – and hence poor incentives to invest in education – in the first stage of England's industrialization may have triggered the deskilling process observed during the 18th and 19th centuries.

[FIGURE 3 SHOULD APPEAR HERE]

Importantly, and this is one of the main claims of this paper, once the economy goes down the path of deskilling, improvements in the health of the labor force may provide enough educational incentives to shift its skill composition, and thereby change the direction of technological progress. Formally, it is indeed possible that for a given given μ_t, a sufficient rise in ϕ increases $\mu_u^\star$ such that $\mu_u^\star > \mu_t$, as illustrated in Figure 4. More intuitively, when life expectancy is initially low and the share of unskilled is increasing because of unskill-biased technological progress, a sufficient increase in the time-horizon over which workers can reap the fruits of education can offset the adverse effect of the declining skill premium. When that is the case, due to those new incentives, the share of unskilled workers starts to fall and, because of market size effects, technological innovation reacts accordingly. This triggers a switch from unskill- to skill-biased technological progress and the positive feedback loops set in: as the skilled-labor-using sector progressively gains weight in aggregate production, workers are enticed with more incentives to acquire human capital, which skews the skill composition of the workforce further towards skilled labor and reinforces the direction of technological change until the economy converges to the steady state in which every worker is skilled.

[FIGURE 4 SHOULD APPEAR HERE]

5 The provision of public health

Now that we get a sense of how life expectancy affects the dynamics of the economy, I investigate how it is endogenously determined by public health investments. Let us consider that the life expectancy of a worker born in t is a function of public health investments undertaken when she was young m_t, such that $\phi_{t+1} = \phi(m_t) \equiv \frac{\bar{\phi}+m_t}{1+m_t}$. This functional form, often used in OLG models with endogenous longevity such as Raffin and Seegmuller (2014), ensures that ϕ is an increasing and concave function of m_t, and $\phi(0) = \bar{\phi}$, $\phi(\infty) < 1$ and $\infty > \phi'(0) = 1 - \bar{\phi} > 0$. As in Galor and Moav (2006), the government taxes capitalists' wealth at rate τ_t to fund public health investments.[4] This makes capitalists the only ones bearing the cost of workers' improvements in health. Their budget constraints is modified in the following way:

$$c_{t+1}^k + b_{t+1}^k \leq (1 - \tau_t) b_t^k r_{t+1} \equiv I_{t+1}^k \tag{25}$$

And the law of motion of the capital stock is now:

$$K_{t+1} = (1 - \tau_t)B_t \tag{26}$$

Finally, the resource constraint of the government is:

$$m_t = \tau_t B_t \tag{27}$$

As mentioned in the introduction, England's institutional framework during the Industrial Revolution is often referred to as a 'taxpayer democracy,' as only those duly paying taxes were granted with voting rights [Szreter (1997), Aidt et al. (2010)]. To capture this in the model, I assume that capitalists detain the political power to set taxes on their own wealth. τ_t, hence public health investments, is therefore the result of capitalists' utility maximization. Plugging the first order conditions of their optimization problem back into their utility function yields:

$$\tau_t = \text{argmax}\{\log I^k_{t+1} + C\} \tag{28}$$

Where $C = (1-\beta)\log(1-\beta) + \beta \log \beta < 0$ is a constant. Let us assume that $\log I^k_{t+1} + C > 0$ to ensure positive utility, and note that capitalists choose τ_t to maximize their total wealth – or capital income $I^k_{t+1} = (1 - \tau_t)b_t r_{t+1}$. By definition $I^k_t = \alpha Y_t$, so this amounts to maximizing output in $t + 1$. However, while capitalists expect workers to react to changes in life expectancy and decide on the level of public health investments accordingly, I assume that they do not internalize the effect shifts in the skill composition of the labor force have on the direction of technological progress. Just as workers, they base their expectations of future technology $x_{t+1} = x(\mu_{t+1})$ on current technology $x_t = x(\mu_t)$ instead, in a myopic fashion. Two things are worth noting. First, the tax rate chosen by capitalists will therefore only maximize expected output rather than actual future output. Second, it will also not maximize welfare as capitalists do not incorporate workers' utility gain from improvements in life expectancy in their decision. Recalling that $B_t = \alpha\beta Y_t$, let us rewrite capitalists' *expected* output Y^k as follows:

$$\begin{aligned} Y^k_{t+1} &= z_{t+1} \cdot (\alpha\beta(1-\tau_t)Y_t)^{\alpha} \\ &\times \left\{ \phi(\tau_t Y_t) \cdot \left[(A^u \mu_t)^{\frac{1}{1-\alpha}} \Lambda(\phi(\tau_t Y_t), \mu_t) + (A^s(1-\mu_t))^{\frac{1}{1-\alpha}} (1 - \Lambda(\phi(\tau_t Y_t), \mu_t)) \right] \right\}^{1-\alpha} \end{aligned} \tag{29}$$

Capitalists therefore choose τ_t to maximize expected output Y^k_{t+1}, taking μ_t and Y_t as given. A greater tax burden has a negative effect on output by reducing the capital stock, but this may be offset by two positive effects on the labor factor: public health investments directly increase the labor supplied by each worker[5] and indirectly, by raising the returns to education, reallocate workers from the unskilled-labor-using sector to the more productive skill-intensive sector.

Solving capitalists' maximization program, let us recall the Kuhn-Tucker conditions:

$$\frac{\partial Y^k_{t+1}}{\partial \tau_t} \leq 0 \quad ; \quad \tau_t \geq 0 \quad ; \quad \tau_t \cdot \frac{\partial Y^k_{t+1}}{\partial \tau_t} = 0$$

Considering the cases where $\tau_t = 0$ and $\tau_t > 0$, we obtain:

for $\tau_t = 0$,

$$Y_t \leq \frac{\bar{\phi}}{\beta(1-\alpha)(1-\bar{\phi})} \frac{\omega(\mu_t) - [\omega(\mu_t) - 1]\,\Lambda(\bar{\phi}, \mu_t)}{\omega(\mu_t) - [\omega(\mu_t) - 1]\left(\Lambda(\bar{\phi}, \mu_t) + \bar{\phi}\Lambda_{\phi}(\mu_t)\right)}$$

for $\tau_t > 0$,

$$\frac{(1-\tau_t)Y_t}{(1+\tau_t\alpha\beta Y_t)} = \frac{\bar{\phi} + \tau_t\alpha\beta Y_t}{\beta(1-\alpha)(1-\bar{\phi})} \frac{\omega(\mu_t) - [\omega(\mu_t) - 1]\,\Lambda(\phi(\tau_t, Y_t), \mu_t)}{\omega(\mu_t) - [\omega(\mu_t) - 1]\left[\Lambda(\phi(\tau_t, Y_t), \mu_t) + \phi(\tau_t, Y_t)\Lambda_{\phi}(\mu_t)\right]}$$

Where $\omega(\mu_t) = \left(\frac{A^s}{A^u}\frac{1-\mu_t}{\mu_t}\right)^{\frac{1}{1-\alpha}}$ is the current skill-premium on which capitalists base their expectations.

As in Galor and Moav (2006), this defines a threshold level of aggregate bequests below which $\tau_t = 0$ because capitalists perceive that the returns to skills are lower than the returns to capital. Rearranging, I express the result in the next proposition:

Proposition 2. *The tax rate in period t, τ_t, is is given by*

$$\tau_t = \tau(\mu_t, Y_t)\begin{cases} = 0 & \text{for} \quad Y_t \leq \tilde{Y}_t \\ > 0 & \text{for} \quad Y_t > \tilde{Y}_t \end{cases}$$

$$\textit{where } \tilde{Y}_t = \frac{\bar{\phi}}{\beta(1-\alpha)(1-\bar{\phi})}\frac{\omega(\mu_t)-[\omega(\mu_t)-1]\Lambda(\bar{\phi},\mu_t)}{\omega(\mu_t)-[\omega(\mu_t)-1]\left(\Lambda(\bar{\phi},\mu_t)+\bar{\phi}\Lambda_{\phi}(\mu_t)\right)} \equiv \tilde{Y}(\mu_t)$$

The first stage of development is therefore characterized by a lack of investments in public health measures as capitalists do not find it beneficial to entice workers to invest in education. The novelty relative to Galor and Moav (2006) is that, because the direction of technological progress alters the role of skills in aggregate production – and hence the returns to human capital, the threshold level of output above which such investments eventually occur, $\tilde{Y}(\mu_t)$, is not constant. To see how changes in relative productivities of the skilled- and unskilled-labor-using sectors therefore influence the provision of public health measures, let us investigate how this threshold moves with the current skill premium ω_t according to which capitalists set the tax rate.

Corollary. *Unskill-biased technological change reduces capitalists' incentives to provide public health.*

Proof: see Appendix.

It is indeed possible to show that $\partial\tilde{Y}_t/\partial\omega_t < 0$ for $\mu_t \in]\underline{\mu};\overline{\mu}]$, and $\partial\tilde{Y}_t/\partial\omega_t = 0$ otherwise: the output threshold below which public health investments are profitable to capitalists increases (decreases) with unskill-biased (skill-biased) technological progress.[6] Specifically, unskill-biased technological progress narrows the productivity differential between the two sectors and therefore lowers the benefits, for capitalists, of workers reallocating from the unskilled-labor-using sector to the more productive skilled-labor-using sector. Unskill-biased technological progress therefore deters profit-maximizing capitalists to support tax increases to fund the provision of public health investments that would entice workers to invest in education. Because unskill-biased technological progress increases this threshold above which the returns to human capital surpass the diminishing returns to physical capital, such investments occur in the later stage of the industrialization process. This result is the basis of my claim that the direction of technological progress towards unskilled-labor-using technologies during the Industrial Revolution delayed improvements in the life expectancy of the working class in England.

Interestingly, even though output eventually passes the threshold $\tilde{Y}(\mu_t)$ because physical capital accumulation is subject to diminishing returns, the timing at which it does is crucial to the dynamics of the economy in the long-run. If public health investments happen when the ensuing improvements in longevity provide enough educational incentives to shift the skill composition of the labor force towards skilled labor, they can trigger the switch to skill-biased technological progress and put a halt to the deskilling process. In this case, the virtuous process described in the previous section makes the economy converge to the 'good' steady state, in which every worker is skilled and TFP grows at a constant rate. However, if taxes are raised too late in development, the incentives to invest in education are not enough to change the direction of technological change. Increases in life expectancy only delay the convergence to a steady state with only unskilled workers and no TFP growth, in which the economy gets stuck. As a consequence, and contrary to most unified growth models, the take-off to a regime of sustained economic growth is not inevitable.

6 Deskilling during the British Industrial Revolution

After having closed the model with the endogenous provision of public health investments, let us see how well it accounts for the features of the British Industrial Revolution that motivated this paper: the stagnation of the life expectancy of the working class and the deskilling process that characterized most of the 18th and 19th centuries. The dynamics of the model is fully described by the following system of equations:

$$\begin{aligned}
Y_{t+1} &= Y(Y_t, \mu_t, \tau_t, z_{t+1}) \\
\mu_{t+1} &= \Lambda(Y_t, \mu_t, \tau_t) \\
\tau_t &= \tau(Y_t, \mu_t) \\
z_{t+1} &= (1 + \eta(1 - \mu_t)) \cdot z_t
\end{aligned}$$

Given the nonlinear nature of the system, it is solved numerically. In addition to the dynamics of industrialization in England, the model should also generate an endogenous switch from unskill- to skill-biased technological progress at the turn of the 20th century. As described in the previous sections, the transition is triggered by public investments in health-related infrastructure and the ensuing improvements in life expectancy. For capitalists to consent to fund such investments by raising taxes, the returns to skills in aggregate production must eventually exceed those of physical capital. If this occurs too late in the development process however, it may not be enough to change the direction of technological progress and put a halt on the deskilling process, and the economy can get stuck in a poverty trap.

There is a range of parameters values that could yield the desired qualitative results. Capitalists' propensity to save β is an important driver of physical capital accumulation. A reasonable saving rate ensures indeed the steady growth of the capital-to-labor ratio. Given the diminishing returns to physical capital, the greater the savings rates, the earlier workers' skills become profitable to capitalists. In this baseline calibration, I set $\beta = 0.2$. To force the trajectory of the share of unskilled workers not to be too steep, the ratio of sector-specific productivities A^s/A^u needs to be high enough. I set $A^s = 4$ and $A^u = 1$. One caveat is that given the functional forms chosen on the production side to allow the analytical results of the previous sections, the skill-premium will take values that will not match existing data.[7] While such values are of unrealistic magnitude in the simulation, the trajectory of the skill-premium matches historical patterns.

The other parameters are chosen for various reasons. $\bar{\Delta}$ is the most arbitrary and influences the trajectory of the share of unskilled workers. I set $\bar{\Delta} = 2.82$ to ensure it does not either surge or plunge too fast. The capital intensity of the production function is set to $\alpha = 0.5$. The parameter governing the growth rate of TFP, $\eta = 0.7$, is set such that an economy that has converged to a balanced growth path with only skilled workers grows at a rate around 2.7%. I normalize z_0 to one. I set $\mu_0 = 0.2$ to match observations by de Pleijt and Weisdorf (2017) of a share of unskilled workers around 20% in at the start of the 18th century. To ensure $\mu_0 > \mu_u^\star$, I choose the lower bound for life expectancy to be $\bar{\phi} = 0.4$. Finally, $Y_0 = 0.5$ such that $Y_0 < \tilde{Y}_0 \approx 1$ and there are no public health investments at the beginning. Finally, I consider that each period corresponds to roughly 25 years and solve the model for ten periods to cover a time span of 250 years, starting from the first quarter of the 18th century, when the deskilling episode began. The resulting dynamics of the economy is given in the following figures.

Figures 5 and 6 show the crux of the theory: the interplay between the skill composition of the labor force and the direction of technological change. Initial conditions are such that the share of unskilled workers starts to increase, slowly but steadily, at the beginning of the 18th century. Recall that in order to capture the market size effects that drive technological innovation, it is assumed that the relative weight of each sector in aggregate production depends on the number of workers employed there. An increase in the number of unskilled workers therefore raises the relative productivity of the unskilled-labor-using sector relative to the skill-intensive sector. Technological progress is *unskill-biased* and the skill-premium declines. As the returns to skill fall, workers' incentives to invest in education follow and the share of unskilled workers further increases, fueling the unskill-biased march of technological progress.

[FIGURE 5 SHOULD APPEAR HERE]

[FIGURE 6 SHOULD APPEAR HERE]

As both the share of unskilled workers and the prominence of the unskilled-labor-using sector rise, output grows from physical capital accumulation, rising productivity of unskilled labor, and TFP growth. The current direction of technological progress reduces the returns to skills in aggregate production, and hence capitalists' incentives to finance any public investment that would prompt workers to acquire education, including investments in health-related amenities. As a result of this lack of investment, the first stage of the industrialization process is characterized by the stagnation of the life expectancy of the labor force in addition to the deskilling process described above (Figure 7).

[FIGURE 7 SHOULD APPEAR HERE]

It is only when capitalists consent to undertake public health investments that workers' life expectancy finally takes off. Because physical capital accumulation is nevertheless subject to diminishing returns, capitalists eventually find it profitable to entice workers to reallocate from the unskilled- to the skilled-labor-using sector. Therefore, when output crosses the threshold above which the returns to human capital surpass those of physical capital in the middle of the 19th century, capitalists set a positive tax rate on their wealth to fund public health measures (Figures 8 and 9). Modest at first, improvements in longevity raise the returns to education, which partially offsets the negative effect of unskill-biased technological progress on workers' educational incentives. It does not immediately reverse the trend of deskilling, but slows down the rise of the share of unskilled workers and thereby curb the direction of technological progress.

[FIGURE 8 SHOULD APPEAR HERE]

[FIGURE 9 SHOULD APPEAR HERE]

Taxes increase substantially in the last quarter of the 19th century, which causes expenditure on public health infrastructure to surge. The marked increase in the life expectancy of the working class fosters educational attainment, and after a long period of deskilling, the share of unskilled workers starts to decline. The turn of the 20th century is therefore the tipping point in the transition to a regime of sustained economic growth. The shift in the skill composition of the labor force spurs innovation in the skilled-labor-using sector, and technological progress switches from being unskill- to skill-biased.

This episode can be interpreted as the emergence of the capital-to-skill complementarity documented by Goldin and Katz (1998) that characterizes the modern growth regime. As the skill-intensive sector gains prominence in aggregate production, the skill premium rises, which supplements the positive effects of better life expectancy on workers incentives to invest in education. The positive feedback loops between the direction of technological progress and the skill composition of the labor force set in, reinforcing skill-biased technological progress. Ultimately, the unskilled-labor-using sector vanishes and the economy evolves along a balanced growth path with skilled workers only and a high growth rate of TFP (Figure 10).

[FIGURE 10 SHOULD APPEAR HERE]

As there are no longer benefits from reallocating workers across sectors once the unskilled sector has disappeared, the tax rate progressively declines. Public health expenditure nevertheless grow with output and life expectancy eventually converges to its maximum level.

7 Conclusion

In this paper, I propose a theory of the British Industrial Revolution that accounts for both the stagnation of life expectancy and the decline in the average level of skills of the labor force observed during the 18th and 19th centuries. In particular, I consider the endogenous direction of technological progress in a growth model in which public health investments are the result of profit-maximization by the capitalist class.

The direction of technological progress allows to reconcile unified growth theory with the downward trajectory of the skill-premium at the time. Although technological progress was indeed the crux of the Industrial Revolution, it was directed towards unskilled rather than skilled labor. The introduction of new machinery displaced the production of manufactured goods from the workshop to the factory, and artisans and craftsmen were progressively replaced by less skilled workers, including children, who could operate those new inventions with only a limited education. Such shifts in the skill composition of the labor force prompted further innovation to improve the productivity of unskilled labor, which gradually became an abundant factor of production, through market size effects. As a result, industrialization in England was characterized by unskill-biased technological progress and the gradual deskilling of the labor force.

In turn, the lesser role of skills in aggregate production deterred the capitalist class to consent to costly public investments in health-related infrastructure that would have incentivized workers to invest in education. Unskill-biased technological progress therefore delayed improvements in the life expectancy of the British working class. When capitalists finally undertook sanitary investments in British cities at the end of the 19th century because the returns to an educated workforce nevertheless surpassed the diminishing returns to physical capital, the resulting incentives to acquire human capital put a halt on the deskilling process. The number of skilled workers started to increase, and innovation reacted accordingly: technological progress eventually turned from being unskill- to skill-biased. The vicious dynamics of the skill composition of the labor force and technology became virtuous, which allowed the economy to take-off to the modern regime of sustained growth at the turn of the 20th century.

More generally, the theory relaxes the direct link between economic development and the health of the population that is often assumed in growth theory.

Life expectancy is instead the result of political decisions on public investments in health-related amenities and may stagnate despite economic growth, provided the costs of its provision outweigh the benefits for the ruling class. In this paper I assume that public health investments are undertaken if and only if they maximize the income of the capitalist class, but many other factors played a role in the decision to engage in such big public works. Economic historians such as Peter Lindert or Simon Szreter have notably emphasized the enfranchisement of the working class that pushed towards its own interests, while acute politicians seized on the electoral opportunity. Both explanations are not mutually excludable as it is likely that an interplay between the bargaining power of the working class coupled with capitalists' own interest led to the provision of public goods such as health infrastructure and education.

Notes

[1]Municipalities also funded spending in urban amenities using outstanding loans, secured on future tax revenues. For the sake of simplicity, I assume that public health investments are financed by raising taxes only.

[2]The consumption of a child is implicitly contained in that of her parent.

[3]Note that if $A \geq e^2$, the proposition is the same for $\phi < \phi_1$ and $\phi \geq \phi_2$. When $\phi \in [\phi_1; \phi_2[$ however, there exists a unique stable steady state $\mu_s^\star$ and two unstable steady states $\mu_{u,a}^\star < \mu_{u,b}^\star$ such that μ_t converges to $\mu_s^\star$ when $\mu_t \in]\mu_{u,a}^\star; \mu_{u,b}^\star[$, to $\mu_0^\star$ when $\mu_t < \mu_{u,a}^\star$ and to $\mu_1^\star$ when $\mu_t > \mu_{u,b}^\star$. The share of unskilled workers may therefore converge to a stable steady state $0 < \mu_s^\star < 1$ for an intermediate level of health. I do not focus on this case as it serves no purpose to the story told in the next sections and will not appear in later simulations as the productivity differential between the two sector will indeed satisfy $A < e^2$.

[4]As discussed above, public health investments in 19th century England were not funded by the taxation of inheritance, but rather by property taxes. However, despite formally being a tax on bequest, τ can also be interpreted as a tax on property, capital, or wealth. The latter interpretation is retained here.

[5]As stated earlier, ϕ_{t+1} can be interpreted as either the number of surviving workers in $t+1$ or as their life expectancy. In any case, an increase in ϕ_{t+1} raises total labor supply: either one considers more people are working, or that each individual works a longer period of time. The latter interpretation is favored here.

[6]When μ_t is outside the interval $]\underline{\mu}; \overline{\mu}]$ however, one of the two sectors has vanished and the only profit capitalists derive from investing in public health is through the direct effect such investments have on workers' productivity.

[7]Evidence for the evolution of the skill-premium during the Industrial Revolution are scarce and obviously imperfect, but the most convincing are probably Clark (2005) and Van Zanden (2009).

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Appendix

Proof of proposition 1

(i) To look for potential steady states conditional on the value of ϕ, let us set $\mu_{t+1} = \mu_t = \mu^\star$ and look at $\mu^\star = \Lambda(\mu^\star, \phi)$. Two trivial steady states are $\mu^\star = 0$ and $\mu^\star = 1$. To find the others, let us consider the function $F(\mu^\star, \phi) \equiv \Lambda(\mu^\star, \phi)/\mu^\star$ and find the solutions to the equation $F(\mu^\star, \phi) = 1$ in the interval $]\underline{\mu}; \overline{\mu}]$:

$$\frac{1}{\mu^\star}\left[1 - \frac{\phi}{(1-\alpha)\bar{\Delta}} \log\left\{A \cdot \frac{1-\mu^\star}{\mu^\star}\right\}\right] = 1$$

First, note that F is continuous, $F(\underline{\mu}, \phi) = 0$ and $F(\overline{\mu}, \phi) = 1 + 1/A > 1$. There is therefore at least one possible solution. Furthermore, $F_\phi < 0$. Now, let us study the sign of the derivative of F with respect to $\mu^\star$:

$$F_{\mu^\star} = \frac{1}{\mu^{\star 2}}\left\{\frac{\phi}{(1-\alpha)\bar{\Delta}}\left[\log\left\{A \cdot \frac{1-\mu^\star}{\mu^\star}\right\} + \frac{1}{1-\mu^\star}\right] - 1\right\}$$

It follows that $F_{\mu^\star} = 0$ when $\log\left\{A \cdot \frac{1-\mu^\star}{\mu^\star}\right\} + \frac{1}{1-\mu^\star} = (1-\alpha)\bar{\Delta}/\phi$. For clarity, denote the left-hand side by $H(\mu^\star)$ and the right-hand side by $\kappa(\phi)$ and look for the solutions of $H(\mu^\star) = \kappa(\phi)$. By definition, $\underline{\mu}$ is such that $\log\left\{A \cdot \frac{1-\mu^\star}{\mu^\star}\right\} > (1-\alpha)\bar{\Delta}/\phi$, hence $H(\underline{\mu}) > \kappa(\phi)$, and $H(\overline{\mu}) = 1 + A$. $H'(\mu^\star) = (2\mu^\star - 1)/[\mu^\star(1-\mu^\star)^2]$, therefore $H'(\mu^\star) < 0$ when $\mu^\star < 1/2$, $H'(\mu^\star) = 0$ when $\mu^\star = 1/2$, and $H'(\mu^\star) > 0$ when $\mu^\star > 1/2$. There are several cases to consider, depending on the value of $\kappa(\phi)$, with $\kappa'(\phi) < 0$.

When $\phi < \phi_1 \equiv (1-\alpha)\bar{\Delta}/(1+A)$, $H(\overline{\mu}) < \kappa(\phi)$ and the equation $H(\mu^\star) = \kappa(\phi)$ has a unique solution $\mu_a < 1/2$. $F(\mu^\star, \phi)$ is therefore increasing in the interval $]\underline{\mu}; \mu_a[$ and decreasing in $]\mu_a; \overline{\mu}]$. This implies that the equation $F(\mu^\star, \phi) = 1$ has a unique solution $\mu^\star < 1/2$.

When $\phi \geq \phi_2 \equiv (1-\alpha)\bar{\Delta}/(2+\log A)$, $H(\mu^\star) \geq \kappa(\phi)$ since $H(1/2) \geq \kappa(\phi)$, and $F_{\mu^\star} \geq 0$. The equation $F(\mu^\star, \phi) = 1$ therefore has a unique solution.

When $\phi_1 \leq \phi < \phi_2$, the equation $H(\mu^\star) = \kappa(\phi)$ has two solutions, μ_1 and μ_2, with $\mu_1 < 1/2 < \mu_2$. $F_{\mu^\star}$ is positive for $\mu^\star < \mu_1$, negative for $\mu_1 < \mu^\star < \mu_2$, and positive again for $\mu^\star > \mu_2$. The equation $F(\mu^\star, \phi) = 1$ may therefore have a unique solution if $F(\mu_1, \phi) > F(\mu_2, \phi) > 1$ or $1 > F(\mu_1, \phi) > F(\mu_2, \phi)$, or three solutions if and only if $F(\mu_1, \phi) > 1 > F(\mu_2, \phi)$. A necessary and sufficient condition for having three steady states is $A > e^2$.
Proof: suppose $F(1/2, \phi_2) < 1$, which is the case when $A > e^2$. Since the function F is continuous and decreasing in ϕ for all $\mu^\star$, there exists a $\epsilon > 0$ such that for $\phi = \phi_2 - \epsilon$, $F(\mu_1, \phi_2 - \epsilon) > 1 > F(1/2) > F(\mu_2, \phi_2 - \epsilon)$. $A > e^2$ is therefore sufficient to ensure three solutions. To show it is also necessary, suppose $F(1/2, \phi_2) = 1$. In this case, $F_{\mu^\star} > 0$ for $\mu^\star > 1/2$ and therefore $F(\mu^\star, \phi_2) > 1$ for $\mu^\star > 1/2$. Now consider another ϵ such that for $\phi = \phi_2 - \epsilon$, $F(\mu_1, \phi_2 - \epsilon) > F(1/2, \phi_2) > F(\mu_2, \phi_2 - \epsilon)$. Since $\mu_2 > 1/2$ and F is decreasing with ϕ for all $\mu^\star$, it follows that $F(\mu_2, \phi_2 - \epsilon) > F(\mu^\star, \phi_2) > 1$. There is a unique steady state: the condition $A > e^2$ is therefore necessary. QED.

(ii) For simplicity, let us make the assumption that $A < e^2$, so that there are three solutions to the equation $\Lambda(\mu^\star) = \mu^\star$. There are two stable steady states $\mu_0^\star = 0$ and $\mu_1^\star = 1$ and one unstable steady state $\mu_u^\star$ in $]\underline{\mu}; \overline{\mu}]$. The stability of both $\mu_0^\star = 0$ and $\mu_1^\star = 1$ follows from $\Lambda'(\mu_0^\star) = \Lambda'(\mu_1^\star) = 0$. $\mu_u^\star$ is unstable because $\Lambda'(\mu_u^\star) > 1$.
Proof: $\Lambda'(\mu_u^\star) > 1 \Leftrightarrow \phi/[(1-\alpha)\bar{\Delta}] > (1-\mu_u^\star)\mu_u^\star$. Now, rearranging $\Lambda(\mu_u^\star) = \mu_u^\star$ yields $(1-\mu_u^\star)\mu_u^\star = \phi/[(1-\alpha)\bar{\Delta}] \log\{A(1-\mu_u^\star)/\mu_u^\star\}$. Plugging this into $\Lambda'(\mu_u^\star) > 1$, we see that it amounts to $1/\mu_u^\star - \log\{(1-\mu_u^\star)/\mu_u^\star\} > \log A$. Let us denote the left hand side by $h(\mu_u^\star)$. A sufficient condition is therefore $h(\mu_u^\star) \geq 2$ since $\log A < 2$ by assumption. $h'(\mu_u^\star) < 0$ for $\mu_u^\star < 1/2$, $h'(\mu_u^\star) = 0$

for $\mu_u^\star = 1/2$, and $h'(\mu_u^\star) > 0$ for $\mu_u^\star > 1/2$. $h''(1/2) > 0$ so $h(1/2)$ is a global minimum and $h(1/2) = 2 \geq 2$, therefore $h(\mu_u^\star) \geq 2$, which concludes the proof. QED.

(iii) To investigate the effect of workers' life expectancy on the dynamics of the skill composition of the labor force, let us look at how $\mu_u^\star$ varies with ϕ. To do so, consider $\mu_u^\star - \Lambda(\mu_u^\star) = 0$ and use the implicit function theorem to get $\partial\mu_u^\star/\partial\phi = \frac{\partial\Lambda/\partial\phi}{1-\Lambda'(\mu_u^\star)}$. Since $\Lambda/\partial\phi < 0$ and $\Lambda'(\mu_u^\star) > 1$, it follows that $\partial\mu_u^\star/\partial\phi > 0$. QED.

Proof of proposition 2 corollary

Let us investigate how the direction of technological progress, proxied by the skill premium $\omega(\mu_t) \equiv \omega_t$, influences the output threshold above which the returns to human capital surpass those of physical capital.

When μ_t is outside the interval $]\underline{\mu};\overline{\mu}]$, $\tilde{Y}_t = \frac{\bar{\phi}}{\beta(1-\alpha)(1-\bar{\phi})}$, hence $\partial\tilde{Y}_t/\partial\omega_t = 0$.

When $\mu_t \in]\underline{\mu};\overline{\mu}]$,

$$\tilde{Y}_t = \frac{\bar{\phi}}{\beta(1-\alpha)(1-\bar{\phi})} \frac{1 + \frac{\bar{\phi}}{\Delta}(\omega_t - 1)\log\omega_t}{1 + 2\frac{\bar{\phi}}{\Delta}(\omega_t - 1)\log\omega_t}$$

Hence:

$$\partial\tilde{Y}_t/\partial\omega_t = \frac{\bar{\phi}}{\beta(1-\alpha)(1-\bar{\phi})} \frac{\frac{\bar{\phi}}{\Delta}\left(\frac{\omega_t-1}{\omega_t} + \log\omega_t\right)}{\left[1 + 2\frac{\bar{\phi}}{\Delta}(\omega_t - 1)\log\omega_t\right]^2} > 0$$

As long as $\omega_t > 1$.

Figures

Figure 1. Life expectancy at birth in England in the 19th century

Sources: Szreter and Mooney (1998) for British cities, Wrigley and Schofield (1981) for national average in England and Wales

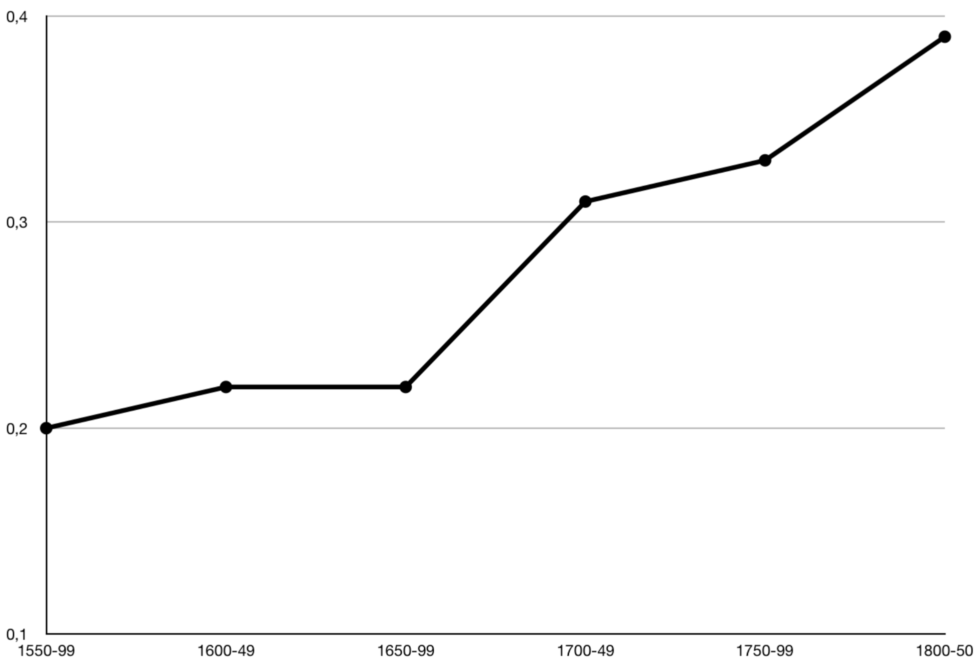

Figure 2. Share of unksilled workers in England: 1550–1850

Source: de Pleijt and Weisdorf (2017)

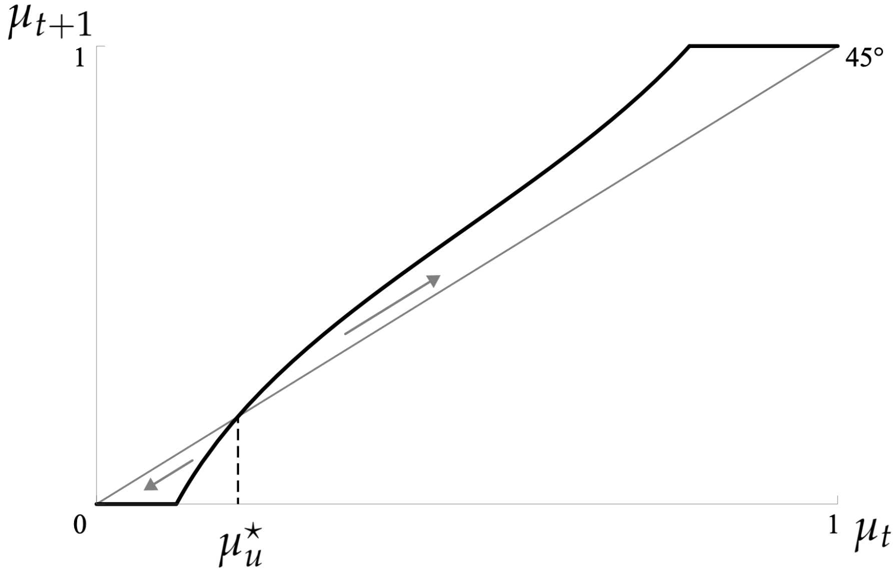

Figure 3. Dynamics for a low life expectancy

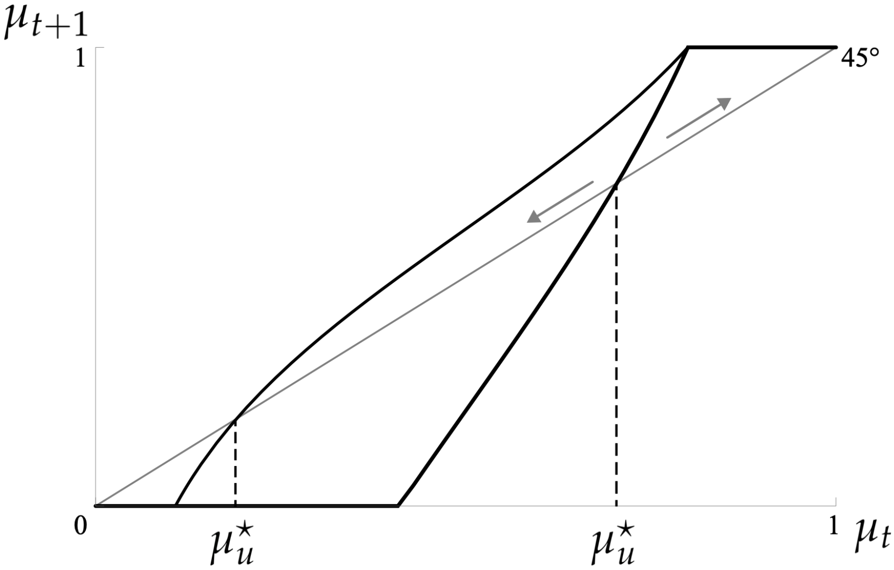

Figure 4. Effect of an increase in life expectancy

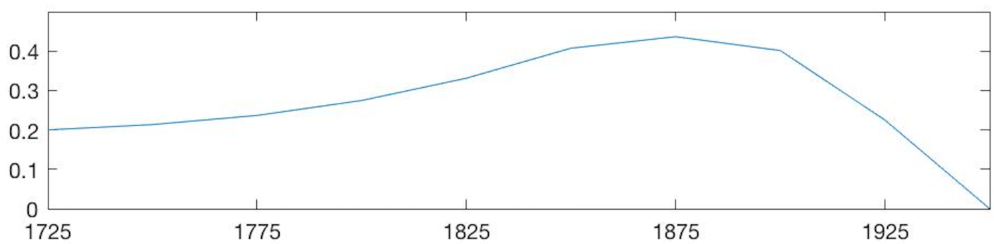

Figure 5. The share of unskilled workers

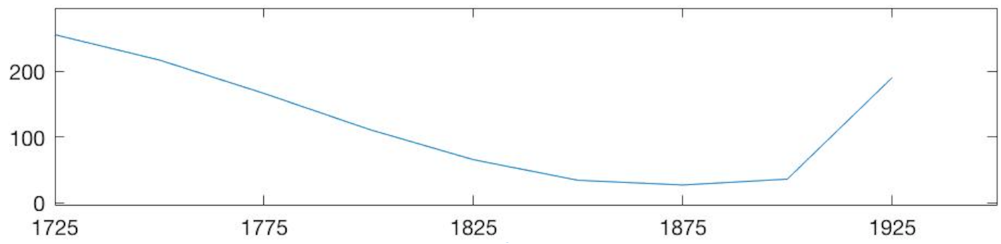

Figure 6. Skill-premium

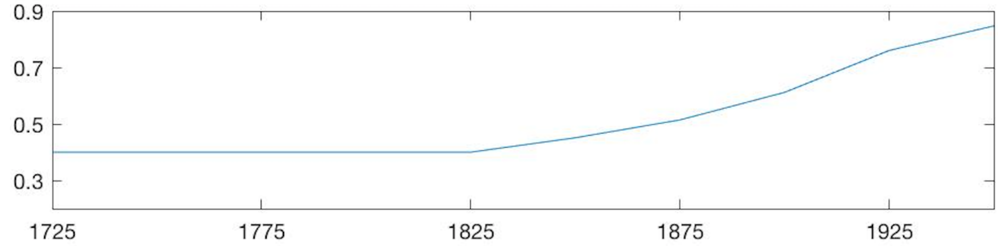

Figure 7. Workers' life expectancy

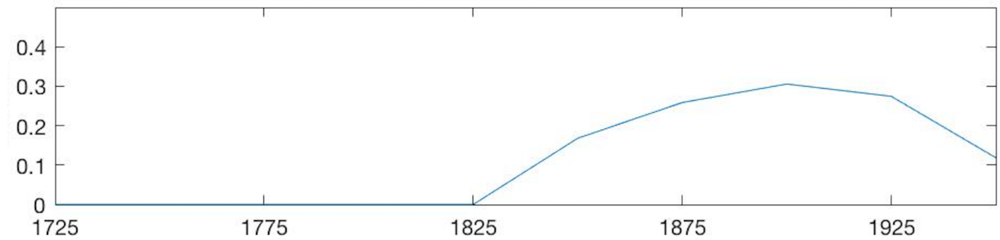

Figure 8. Tax rate on capital

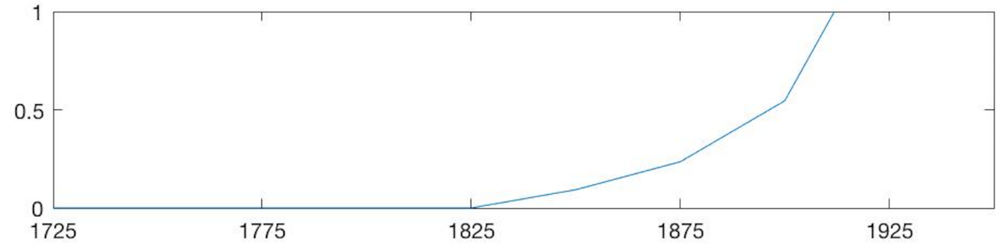

Figure 9. Public health investments

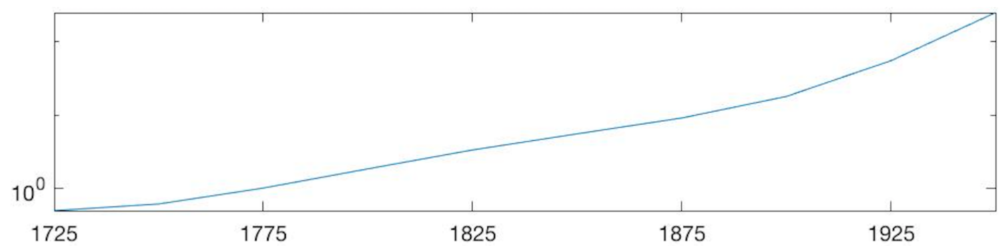

Figure 10. Output