Single sourcing from a supplier with unknown efficiency and capacity

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Abstract

We characterize a retailer's optimal order when its supplier's unknown marginal cost of production either has a low intercept but is increasing, or is constant, or when a lower intercept is associated to a steeper marginal cost and reciprocally. Asymmetric information results in two kinds of distortions. When demand is large enough, the retailer under-purchases to all types except to the least capacity constrained one, and the retail price increases. When demand is low enough, the retailer over-purchases to all types except to the most efficient one, and when the product is perishable the retail price decreases. For an intermediate demand, the retailer over- or under-purchases except from the two extreme types as well as an intermediate one. Bunching occurs for an interval of types next to the latter. The optimal order is not always continuous with respect to the seller's type.

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1 Introduction

As pointed out by many empirical analysis, firms display an enormous amount of heterogeneity even within the same industry, be it on their production technology or on the cost of their inputs. This heterogeneity also prevails geographically, no matter whether U.S. firms or firms from developing countries are considered[1]. Productivity or cost functions estimations[2] also show that firms operate neither at the same scale nor with the same level of efficiency, and often face increasing marginal costs of production, that is decreasing returns-to-scale[3].

Although it is not their focus, all these estimations also allow to emphasize an important issue. Once a firm has chosen its short-run capacity of production and once its operations are planned, generally based on demand forecasts and given longer run investments whose costs are sunk[4], a firm's short run cost structure is fixed. Moreover when the production and sales phases start, given the orders already confirmed, each new order a firm receives exhausts its planned capacity, and, once it is fully used, additional orders force this firm to acquire extra inputs which can be more costly at the margin than what had been planned ex-ante. How low and how steep a firm's marginal cost curve is, is given, and differences in efficiencies as well as in residual capacities coexist. As the most efficient operators attract customers more easily than the least efficient ones, the residual capacity of the least efficient firms can be larger than the residual capacity of the most efficient ones at any point in time. That is efficiencies and residual capacities can evolve in opposite directions, the most efficient operators being more capacity constrained than the least efficient ones.

Whether a supplier's residual capacity is large or not, and whether a firm's marginal cost is low or not, is generally not observable to a buyer[5]. When deciding

[1]See for example Baily et al [4], Bartelsman and Dhrymes [5], and Roberts and Tybout [14].

[2]See amongst others Beard et al [6], Röller [15], Van Biesebroeck [17] or Kim and Knittel [11].

[3]For example Van Biesebroeck [17] points out that these differences, and in particular decreasing returns, may come from the use of a lean manufacturing system instead of a mass production system which generates more economies of scale.

[4]In capital, such as in a plant or in the machines needed. Planning production is of concern to management scientists but also to economists, at least as early as in Holt, Modigliani, Muth and Simon [9]. The rationale for sharing production plans with suppliers in the automobile industry is studied in Doyle and Snyder [8].

[5]For example see again Van Biesebroeck [17].

how much to order, a buyer may face an efficient but capacity constrained seller, or on the contrary an inefficient but capacity unconstrained seller, or any combination between these two extremes. Due to potential diseconomies of scale which differ, differences in the total cost of these different types of suppliers may give each of them the opportunity to raise some profits: a supplier efficient at the margin but experiencing large decreasing returns-to-scale could pretend it is less efficient but less capacity constrained when it is offered to produce a small quantity, while oppositely when it is offered to produce a large quantity, a supplier less efficient at the margin but less capacity constrained could pretend it is the opposite. Depending on the demand it faces, a buyer (be it a retailer or a downstream manufacturer) therefore faces a tension between procuring the quantity of product it needs as efficiently as possible, and ordering an optimal quantity which allows to serve the downstream market optimally. This tension is particularly important when the product traded is perishable, e.g. when the product is a service, and when purchases must be renewed regularly, when the service purchased is not durable.

In this paper, we explore how a buyer (or retailer) should optimally purchase the product it resales, when it does not observe the true characteristics of its single supplier, and when this supplier can either be efficient but faces steep decreasing returns-to-scale, or on the contrary is less efficient but faces constant returns, or any combination of the two such that the steeper the marginal cost of production the smaller its intercept it. In such a setting where the unobserved characteristic of the supplier affects the determinants of its variable cost of production, three classical assumptions satisfied by the basic textbook principal-agent model do not hold any more. First, a supplier's total variable cost of production may increase or decrease as its types changes, and hence countervailing incentives are present[6]. Second, a supplier's marginal cost of production may also increase or decrease as its type changes, and hence the Spence-Mirrlees condition fails to be satisfied[7]. Last, when it determines the menu of contracts it offers to its supplier, the retailer's expected profit, which depends on how types affect the total variable cost of production, may fail to be concave with respect to the purchase order it sends to the supplier.

[6]See Lewis and Sappington [12], Biglaiser and Mezzeti [7], Maggi and Rodriguez-Clare [13], and Jullien [10].

[7]See Araujo and Moreira [1], [2] and Schottmüller [16].

In such a model, we characterize the distortions the retailer acting as a principal chooses on the quantity it purchases, compared to what a fully informed monopoly would purchase and resell. We demonstrate that these distortions depend on the determinants of a supplier's cost of production, as well as the market demand the retailer faces. When demand is large enough, asymmetric information leads the retailer to under-purchase compared to what a fully informed monopolist would do. Asymmetric information therefore results in additional social losses compared to a monopoly, even if double marginalization is absent. On the contrary when demand is low enough, asymmetric information leads the retailer to over-purchase compared to what a fully informed monopolist would do, which reduces social losses compared to the situation where a monopoly operates. Last in the interim situation, the retailer over-purchases from efficient but capacity constrained sellers, while it under-purchases from inefficient but capacity unconstrained sellers. We analyze the situation where bunching occurs, and we demonstrate that the quantity profile offered in the contract is not always continuous with respect to the seller's type.

Our paper crosses two streams of research in the principal-agent literature which, to our knowledge, have been examined separately so far. In Lewis and Sappington [12], affine total costs of production change with the agent's type and countervailing incentives follow from the tension between misreporting one's fixed and one's unit cost of production. This tension occurs more generally when the agent's outside option depends on its type, which has been analyzed comprehensively in Jullien [10]. We show that countervailing incentives may also follow from the tension between the determinants of the variable cost of production of a firm, namely the intercept and the slope of the marginal cost, in a model where fixed costs are sunk and hence outside options can be normalized to zero for all types. Over-production, under-production, and the pooling of different suppliers on the output value such that all variable costs are equal occur in equilibrium, which mirrors in our setting the seminal result obtained by Lewis and Sappington [12]. When countervailing incentives emerge from tensions between the determinants of a variable cost of production, the Spence-Mirrlees condition can also fail to be satisfied. Our paper therefore relates to Araujo and Moreira [1] and [2], and Schottmüller [16]). Under the assumption that marginal costs functions rotate around each other as the supplier's type changes, we demonstrate that the non local incentive constraints never bind when the purchase order is monotonic with respect to the supplier's type. Hence, local conditions are sufficient to determine the second best purchase orders, and the usual distortions coming from countervailing incentives are still present. This is in contrast with Schottmüller [16], who analyzes the case where quadratic cost functions depend on the agent's type through the variable total cost of production but also through type-dependant fixed costs, in such a manner that countervailing incentives are absent. In such a setting, he characterizes the effect of non local incentive constraints when the purchase order is otherwise monotonic in the agent's type. In particular he shows that distortions may occur including for the type realizing the first best when only local conditions are considered. Last, the negative correlation between the determinants of an agent's variable cost of production introduce non-concavities in the retailer's virtual surplus, so that the optimal quantity schedule is not continuous anymore, and the set of types for which pooling occurs increases.

2 The Model

A downstream retailer D sells to its customers a (non-negative) quantity of product q, which it procures from a single upstream supplier U. The consumers' inverse demand is denoted $P(q)$, which is linear and strictly decreasing in q,

$$P(q) = \max\{a - bq, 0\} \quad \text{with } a > 0,\ b > 0. \tag{1}$$

We let $P'(q)$ denote the first order derivative of the inverse demand[8]. The upstream supplier U cannot directly access the market, and produces the quantity q with a technology of production whose investment costs are sunk and normalized to 0, and whose variable cost of production is continuous and convex in the quantity q. This variable cost also depends on a parameter θ, and is denoted $C(q;\theta)$ given by[9]

$$C(q;\theta) = \theta q + \frac{1}{2} d(\theta) q^2 \quad \text{for all } q \geq 0, \text{ with } \theta \geq 0,\ d(\theta) \geq 0. \tag{2}$$

[8]When a function depends on one variable only, we use a $'$ to indicate its total derivative with respect to this variable.

[9]Assuming that fixed costs are sunk allows us to focus on countervailing incentives which come from increasing marginal costs of production, and not from type dependent participation constraints as in Jullien [10]. This cost function also differs from the running example in Schottmüller [16], in which non sunk type-dependent fixed costs are introduced to ensure that informational rents are monotonic in the agent's type and countervailing incentives are absent.

Supplier U's variable cost of production is private information to this firm: the parameter θ is the realization of a random variable Θ which is drawn on a bounded support $[0, \bar{c}]$ and is revealed only to U; the function $d(\theta)$ is continuous and strictly decreasing in θ, $d'(\theta) < 0$, and is common knowledge to U and R. To fix the ideas we assume $d(\theta)$ is linear and given by

$$d(\theta) = \bar{d}\left(1 - \frac{\theta}{\bar{c}}\right). \tag{3}$$

The cumulative distribution function and the density function of Θ are continuous on $[0, \bar{c}]$ and given respectively by

$$F(\theta) \in [0,1] \text{ and } f(\theta) \geq 0 \quad \text{for } \theta \in [0, \bar{c}], \tag{4}$$

where moreover we assume that $F(\theta)$ verifies[10]

$$\frac{\partial}{\partial\theta}\left(\frac{F(\theta)}{f(\theta)}\right) \geq 0 \geq \frac{\partial}{\partial\theta}\left(\frac{1 - F(\theta)}{f(\theta)}\right) \quad \text{for } \theta \in [0, \bar{c}]. \tag{5}$$

Under these assumptions firm U's type is given by $\theta \in [0, \bar{c}]$, which determines immediately the slope of its marginal cost $d(\theta) \in [0, \bar{d}]$. Figure 1 illustrates this assumption. When θ tends to 0, the slope of the marginal cost tends to $\bar{d}$ while, when θ tends to $\bar{c}$ the slope of the marginal cost tends to 0. Moreover marginal costs rotate around a unique value $q^0 = \frac{\bar{c}}{\bar{d}}$, while the total costs intersect twice, at $q = 0$ where there are all nil, and at an output level equal to $2q^0 = 2\frac{\bar{c}}{\bar{d}}$. Therefore a highly specialized supplier ($c = 0$) is strongly capacity constrained ($d = \bar{d}$) but can operate very efficiently when demand is low, while a supplier with a large capacity does not suffer decreasing returns-to-scale ($d = 0$), faces a constant cost per unit ($c = \bar{c}$) which allows him or her to operate more efficiently than other types of suppliers when demand is large[11]. We denote $C(2q^0)$ the total cost of producing $2q^0$ which is identical across all types.

[10] As explained below, the consequence of the cost structure we consider is that the objective function of the principal (here retailer D) is not necessary concave in the purchase order q. Under this assumption on $F(\theta)$, there exists a threshold type θ such that the objective function is convex only for types higher than this threshold, and concave else. This assumption is satisfied if $F(\theta)$ is log-concave (see Bagnoli and Bergstrom [3]).

[11] When $d(\theta)$ is not linear in θ, marginal costs do not intersect at the same quantity q^0 and total costs do not intersect at the same quantity $2q^0$ anymore. We study the consequences of this alternative assumption in section 5.

We denote $C_q(q;\theta)$ the first order partial derivative with respect to q, i.e. the marginal cost of supplier U to produce q when it is of type θ

$$C_q(q;\theta) = \theta + \bar{d}\left(1 - \frac{\theta}{\bar{c}}\right) q. \tag{6}$$

The first order and the cross-partial derivatives with respect to θ (and respectively θ and q) are equal to

$$C_\theta(q;\theta) = q - \frac{\bar{d}}{2\bar{c}}q^2 \text{ and } C_{q\theta}(q;\theta) = 1 - \frac{\bar{d}}{\bar{c}}q. \tag{7}$$

These derivatives do not have a constant sign as q varies: $C_\theta(q;\theta) \geq 0$ if $q \leq \frac{2\bar{c}}{\bar{d}}$ and negative else, and similarly $C_{q\theta}(q;\theta) \geq 0$ if $q \leq \frac{\bar{c}}{\bar{d}}$ and negative else. Therefore when the technology of production is heterogenous across the population of suppliers, and the fundamentals of each supplier's variable cost of production are negatively correlated with each other, countervailing incentives are present while the Spence-Mirrlees condition is not satisfied. The second and third order derivatives for $\theta \in [0,\bar{c}]$ are $C_{qq}(q;\theta) = \bar{d}\left(1 - \frac{\theta}{\bar{c}}\right) \geq 0$ and nil for $\theta = \bar{c}$, $C_{q\theta\theta}(q;\theta) = 0$, and $C_{qq\theta}(q;\theta) = -\frac{\bar{d}}{\bar{c}} < 0$ for all q and θ.

To procure the quantity q it markets, retailer D is able to offer a menu of binding contracts to its upstream supplier U, from which U can choose[12]. Each contract consists in a payment and a quantity the supplier has to produce, if it accepts the contract, which depends on the type supplier U announces to the retailer D. Let $\tilde{\theta}$ be the message supplier U of type θ reports to the principal, and let $\left(T(\tilde{\theta}), q(\tilde{\theta})\right)$ be the contract offered to supplier U when it reports $\tilde{\theta}$. Then retailer D earns an expected profit equal to

$$\pi_D^e(\tilde{\theta}) = \mathrm{E}\left(P(q(\tilde{\theta}))q(\tilde{\theta}) - T(\tilde{\theta})\right) \tag{8}$$

where the expectation is computed on the distribution of types $F(\theta)$. The ex-post profit of supplier U of type θ reporting $\tilde{\theta}$ is given by:

$$\pi_U(q(\tilde{\theta});\theta) = T(\tilde{\theta}) - C(q(\tilde{\theta});\theta) \text{ for } \theta \in [0,\bar{c}]. \tag{9}$$

[12]We assume that the retailer R cannot sell less than the quantity it has procured. Leftover inventories can be infinitely costly to dispose (be it for economic or reputation reasons, as the recent scandals on Amazon leftovers inventories showed), and impossible to resell (e.g. because the product is perisable). We leave the study of the optimal procurement of a storable good for another paper.

If it does not accept the contract offered by the downstream retailer D, the upstream supplier U earns no profit. The timing of the game is standard:

1. Nature draws the type θ of supplier U and informs this firm;
2. The downstream retailer D offers a menu of binding contracts to supplier U which specifies a payment and a production which depends on the type $\tilde{\theta}$ the upstream supplier U reports, $\left(T(\tilde{\theta}); q(\tilde{\theta})\right)$;
3. Supplier U reports its type or refuses the contract offered;
4. Supplier U produces according to the contract it has accepted, and the total quantity q is sold on the market by the downstream retailer R, so that payoffs are realized.

We can restrict our attention to direct mechanisms in which supplier U sends a message $\tilde{\theta}$ to the retailer D which consists in a type, $\tilde{\theta} \in [0, \bar{c}]$, and in which the contract offered by the retailer to the supplier is contingent to the type it reports. If the game above possesses an equilibrium in which supplier U announces its type truthfully, then the revelation principle insures that payment and production can be directly conditioned to the types the supplier reports, and truth-telling occurs when the contract offered by the retailer satisfies the following set of incentive-compatibility constraints:

$$\pi_U(q(\theta); \theta) \geq \pi_U(q(\tilde{\theta}); \theta) \text{ for } \tilde{\theta} \neq \theta \tag{10}$$

As discussed above, the assumptions made on supplier U's cost function imply that the Spence-Mirrlees condition is not satisfied and U's profit is not monotonic in its type[13].

To conclude this section, let us recall the optimal purchasing strategy of the downstream retailer when its supplier's cost is known. Let the industry profit when the supplier U's cost is common knowledge be given by

$$\Pi(q; \theta) = P(q)q - C(q; \theta) \tag{11}$$

[13]Indeed $\frac{\partial^2 \pi_U(q;\theta)}{\partial q \partial \theta} = -C_{q\theta}(q; \theta) > 0$ if $q > \frac{\bar{c}}{d} = q^0$ and negative else; similarly $\frac{\partial \pi_U(q;\theta)}{\partial \theta} = -C_\theta(q; \theta) > 0$ if $q > \frac{2\bar{c}}{d} = 2q^0$.

and let $\Pi_q(q;\theta) = P(q) + qP'(q) - C_q(q;\theta)$ be its derivative with respect to q. Under our assumptions, $\Pi(q;\theta)$ is continuous, differentiable and concave in q. If D cannot discriminate consumers, we have:

Definition 1 (Monopolistic purchases) *The non discriminating integrated monopoly production $q^M(\theta)$ is the unique solution of $\Pi_q(q^M(\theta);\theta) = 0$, equal to*

$$q^M(\theta) = \frac{a-\theta}{2b + \frac{\bar{d}}{\bar{c}}(\bar{c}-\theta)}, \quad \textit{with derivative} \quad q^{M'}(\theta) = \frac{-2b - \bar{d} + \frac{\bar{d}}{\bar{c}}a}{\left(2b + \frac{\bar{d}}{\bar{c}}(\bar{c}-\theta)\right)^2}.$$

We refer to this threshold as being the *monopoly* one hereafter. Note that when $q^M(0) \leq q^0$, then $q^M(\theta)$ is decreasing in θ, while when $q^M(0) \geq q^0$, then $q^M(\theta)$ is increasing[14]. The optimal solution to the problem above is therefore strictly monotonic in θ.

The first best level of production for the entire economy obtains when the market price is equal to the marginal cost θ. It also obtains when D is able to perfectly discriminate consumers demand (case in which D earns the total welfare production and sales generate in the economy). We have,

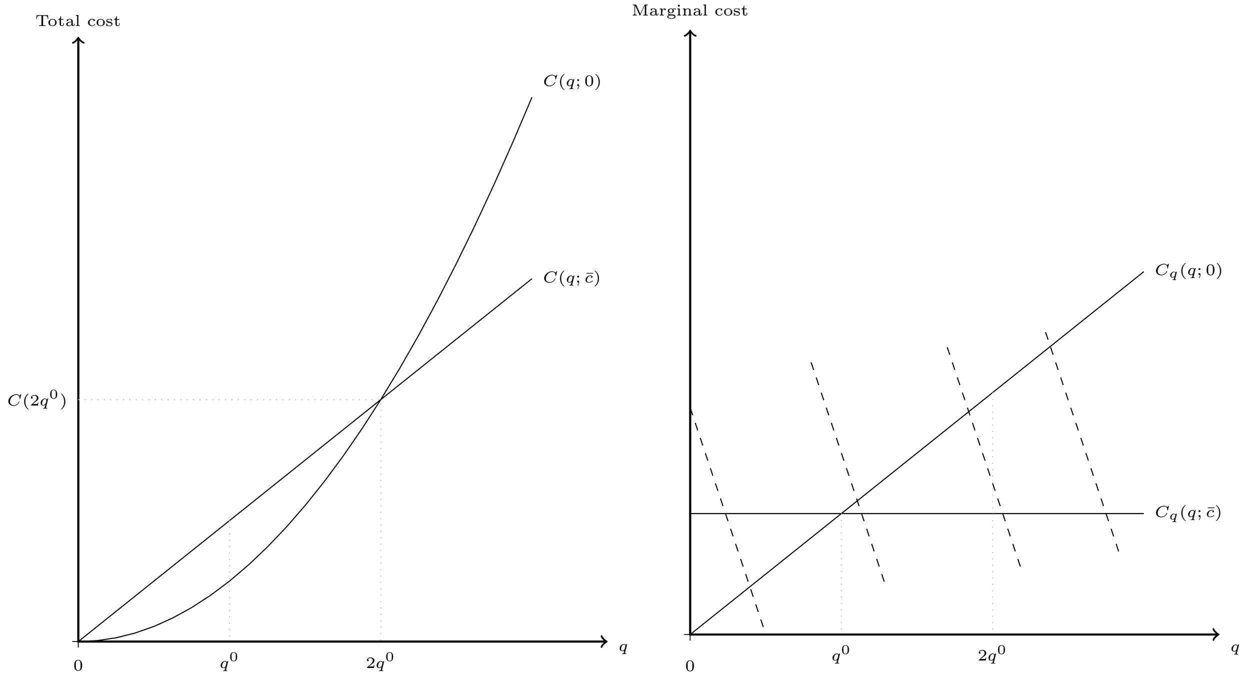
Definition 2 (Competitive purchases) *The first best production $q^{FB}(\theta)$ is the unique solution of $P(q^*(\theta)) - C_q(q^*(\theta);\theta) = 0$, equal to*

$$q^{FB}(\theta) = \frac{a-\theta}{b + \frac{\bar{d}}{\bar{c}}(\bar{c}-\theta)}, \quad \textit{with derivative} \quad q^{FB'}(\theta) = \frac{-b - \bar{d} + \frac{\bar{d}}{\bar{c}}a}{\left(b + \frac{\bar{d}}{\bar{c}}(\bar{c}-\theta)\right)^2}.$$

We refer to this threshold as being the *competitive* one hereafter. Again this solution is monotone in the type θ, increasing or decreasing depending on the demand parameters[15]. Figure 1 below concludes this section with a graphical illustration of the levels an integrated and non discriminating monopoly would produce depending on the size of market demand.

[14] Indeed $q^{M'}(\theta)$ is negative if $a \leq \frac{\bar{c}}{\bar{d}}(2b + \bar{d})$, while $q^M(0) = \frac{a}{2b+\bar{d}} \leq q^0 = \frac{\bar{c}}{\bar{d}}$ if $a \leq \frac{\bar{c}}{\bar{d}}(2b + \bar{d})$, which are the same conditions. The opposite inequalities hold when $q^M(0) \geq q^0$.

[15] $q^{FB}(\theta)$ does not increase or decrease in the same range of parameters than $q^M(\theta)$.



(a) Total costs for $\theta = \{0, \bar{c}\}$

(b) Marginal costs for $\theta = \{0, \bar{c}\}$ and marginal revenues ((X:Y)=(1:2))

Figure 1: Supplier's costs and monopoly production as types and demand change

3 Local and global incentive constraints

The retailer D optimization problem consists in choosing the quantity order $q(\theta)$ and a profit level for its supplier $\pi_U(\theta;\theta)$ which maximizes its expected profit

$$\pi_D^e(\theta) = E\left(P(q(\theta))q(\theta) - \pi_U(q(\theta);\theta) - C(q(\theta);\theta)\right) \tag{12}$$

subject to the individual rationality (IR) and the incentive compatibility (IC) constraints:

$$(IR)\ \pi_U(q(\theta);\theta) \geq 0 \text{ for all } \theta \in [0, \bar{c}] \tag{13}$$

$$(IC)\ \pi_U(q(\theta);\theta) \geq \pi_U(q(\tilde{\theta});\theta) \ \text{ for all } (\tilde{\theta}, \theta) \in [0, \bar{c}] \times [0, \bar{c}] \tag{14}$$

The payment offered by the retailer and the quantity ordered to each supplier's type θ must be such that reporting $\tilde{\theta} = \theta$ is optimal for each type of supplier U. This feature has the following consequence.

Lemma 1 *In any local optimum of the retailer which satisfies the (IR) and (IC) constraints, the purchase order $q(\theta)$ to supplier U must be non increasing with θ ($q'(\theta) \leq 0$) when $q(\theta) \leq q^0$, and non decreasing with θ ($q'(\theta) \geq 0$) when $q(\theta) \geq q^0$.*

Proof. See Appendix A.1.‖

Whether the purchase order $q(\theta)$ is increasing or decreasing with the supplier's type θ therefore changes in the (quantity, type) plane. A second result characterizes the evolution of the net profit of supplier U when θ varies, that is of its informational rent. We have

Lemma 2 *Supplier U's profit increases with its type θ when $q(\theta) \geq 2q^0$, and decreases when $q(\theta) \leq 2q^0$. Moreover its derivative to θ is equal to $\pi'_U(q(\theta);\theta) = -C_\theta(q(\theta);\theta)$.*

Proof. See Appendix A.2.‖

The immediate consequence is that the profit of supplier U is minimal for the type θ^{00} such that $q(\theta^{00}) = 2q^0$ when $q(\theta)$ is such that some types produce less than $2q^0$ and some others produce more.

We can express the rent of supplier U of type θ, $\pi_U(q(\theta);\theta)$, as a function of the cross-partial derivative of its cost function, $C_{q\theta}(q;\theta)$, and establish the condition under which there are no global deviation in the type space supplier U would prefer to announce. Using the first order condition of the revelation game,

$$\pi'_U(q(\theta);\theta) = T'(\theta) - q'(\theta)C_q(q(\theta);\theta) - C_\theta(q(\theta);\theta) = -C_\theta(q(\theta);\theta), \tag{15}$$

so that the rent can be rewritten as

$$\begin{aligned}\pi_U(q(\theta);\theta) =& T(\theta) - C(q(\theta);\theta) = \pi_U(q(\bar{c});\bar{c}) - \int_\theta^{\bar{c}} \pi'_U(q(\tilde{\theta});\tilde{\theta})d\tilde{\theta} \\ =& \pi_U(q(\bar{c});\bar{c}) + \int_\theta^{\bar{c}} C_\theta(q(\tilde{\theta});\tilde{\theta})d\tilde{\theta}.\end{aligned} \tag{16}$$

The *non local* (or *global*) *incentive constraint* can then be obtained[16]: announcing θ must be more profitable than any other type $\hat{\theta}$ in $[0,\bar{c}]$. In our setting,

$$\pi_U(q(\theta);\theta) - T(\hat{\theta}) + C(q(\hat{\theta});\theta) \geq 0. \tag{17}$$

Then this expression can be rewritten using the different expressions of the rent in

[16]See Araujo and Moreira [1] and Schottmüller [16].

(16) above. To fix the ideas, suppose that $\hat{\theta} \leq \theta$. Then:

$$
\begin{aligned}
&\pi_U(q(\theta);\theta) - T(\hat{\theta}) + C(q(\hat{\theta});\theta) = \pi_U(q(\theta);\theta) - \pi_U(q(\hat{\theta});\hat{\theta}) - C(q(\hat{\theta});\hat{\theta}) + C(q(\hat{\theta});\theta) \\
&= \int_\theta^{\bar{c}} C_\theta(q(\tilde{\theta});\tilde{\theta})d\tilde{\theta} - \int_{\hat{\theta}}^{\bar{c}} C_\theta(q(\tilde{\theta});\tilde{\theta})d\tilde{\theta} - C(q(\hat{\theta});\hat{\theta}) + C(q(\hat{\theta});\theta) \\
&= -\int_{\hat{\theta}}^{\theta} C_\theta(q(\tilde{\theta});\tilde{\theta})d\tilde{\theta} + \int_{\hat{\theta}}^{\theta} C_\theta(q(\hat{\theta});\tilde{\theta})d\tilde{\theta} = \int_{\hat{\theta}}^{\theta} \Big(C_\theta(q(\hat{\theta});\tilde{\theta}) - C_\theta(q(\tilde{\theta});\tilde{\theta})\Big)\, d\tilde{\theta} \\
&= -\int_{\hat{\theta}}^{\theta}\int_{q(\hat{\theta})}^{q(\tilde{\theta})} C_{q\theta}(\tilde{q};\tilde{\theta})d\tilde{q}d\tilde{\theta}. \qquad (18)
\end{aligned}
$$

The same result holds true if $\hat{\theta} \geq \theta$[17]. The non linear incentive constraint therefore requires

$$
-\int_{\hat{\theta}}^{\theta}\int_{q(\hat{\theta})}^{q(\tilde{\theta})} C_{q\theta}(\tilde{q};\tilde{\theta})d\tilde{q}d\tilde{\theta} = \int_{\hat{\theta}}^{\theta}\int_{q(\hat{\theta})}^{q(\tilde{\theta})} \left(-1 + \frac{\bar{d}}{\bar{c}}\tilde{q}\right) d\tilde{q}d\tilde{\theta} \geq 0 \qquad (19)
$$

for any θ and $\hat{\theta}$ in $[0, \bar{c}]$. This constraint states that the positive area below the curve $C_{q\theta}(\tilde{q};\tilde{\theta})$, computed for values of (q, θ) in the half plane $q > q^0$, exceeds the area above this curve computed for values of (q, θ) in the half plane $q < q^0$. To summarize, the retailer's problem consists in maximizing

$$
\pi_D^e = \int_0^{\bar{c}} \Pi(q(\theta);\theta) - \pi_U(q(\theta);\theta)dF(\theta) \qquad (20)
$$

with respect to $(q(\theta), \pi_U(q(\theta);\theta))$ for all $\theta \in [0, \bar{c}]$, subject to

$$
\begin{aligned}
&\pi_U(q(\theta);\theta) \geq 0 \quad \forall\theta \in [0, \bar{c}] && \text{(IR)} \\
&\pi_U'(q(\theta);\theta) = -C_\theta(q(\theta);\theta) \quad \forall\theta \in [0, \bar{c}] && \text{(LIC)} \\
&-\int_{\hat{\theta}}^{\theta}\int_{q(\hat{\theta})}^{q(\tilde{\theta})} C_{q\theta}(\tilde{q};\tilde{\theta})d\tilde{q}d\tilde{\theta} \geq 0 \quad \forall(\theta, \hat{\theta}) \in [0, \bar{c}]^2 && \text{(NLIC)}
\end{aligned}
$$

The solution of the retailer's maximisation problem above is denoted $(q^*(\theta), \pi_U^*(\theta))$ for $\theta \in [0, \bar{c}]$.

4 Optimal sourcing when only local incentive constraints matter

In this section we examine the case of a large market, and we assume that variable costs are linear in the supplier's type. That is, the demand parameters (a, b) are

[17]Indeed, $\pi_U(q(\theta);\theta) - T(\hat{\theta}) + C(q(\hat{\theta});\theta) = \int_\theta^{\hat{\theta}} C_\theta(q(\tilde{\theta});\tilde{\theta})d\tilde{\theta} - \int_\theta^{\hat{\theta}} C_\theta(q(\hat{\theta});\tilde{\theta})d\tilde{\theta} = -\int_{\hat{\theta}}^{\theta} C_\theta(q(\tilde{\theta});\tilde{\theta})d\tilde{\theta} + \int_{\hat{\theta}}^{\theta} C_\theta(q(\hat{\theta});\tilde{\theta})d\tilde{\theta} = -\int_{\hat{\theta}}^{\theta}\int_{q(\hat{\theta})}^{q(\tilde{\theta})} C_{q\theta}(\tilde{q};\tilde{\theta})d\tilde{q}d\tilde{\theta}$.

such that given the support of the distribution of types $[0, \bar{c}]$, the first best level of purchases $q^M(\theta)$ for $\theta \in [0, \bar{c}]$ are "much" larger than q^0 for all types (but can be larger or smaller than $2q^0$), and the slope of the marginal cost of production is linear and decreasing in θ,

$$q^M(0) \gg q^0 \quad \text{and} \quad d(\theta) = \bar{d}\left(1 - \frac{\theta}{\bar{c}}\right). \tag{21}$$

Our assumptions imply that all cost and marginal cost functions intersect at the same type-independent values $2q^0$ and q^0 respectively[18], i.e. are such that marginal costs rotate around each other. These assumptions therefore mirror to the case where asymmetric information lies on variable costs of production, the assumptions made by Araujo and Moreira [2], who analyze the case of a rotation of demand functions.

Let us start our analysis by showing that the non local incentive constraint is never binding under the assumptions we imposed above. When the variable cost function of each type of supplier is linear in θ, with $d(\theta)$ being decreasing, q^0 is independent from θ, i.e. defines an horizontal line in the plane (θ, q). Therefore for any $(\theta, \hat{\theta})$, if the solution to the retailer's problem $q^*(\theta)$ is monotonic in the type θ, then all possible pairs (θ', q') which belong to the set $\{(\theta', q') \in [\theta, \hat{\theta}] \times [q(\theta), q(\hat{\theta})] \mid q' \leq q^*(\theta')\}$, do also belong to the half-plane where $C_{q\theta}$ is of constant sign. Since the set of pairs (θ', q') is the one over which the cross-partial derivative $C_{q\theta}$ is integrated and is equal to the left-hand-side term of the non local incentive constraint, then this left-hand-side term is of constant sign. Hence the non-local incentive compatibility constraint is satisfied by any monotonic solution to the principal's problem, and the multiplier of this constraint is therefore nil. To summarize,

Lemma 3 *When $d(\theta)$ is linear and decreasing in θ, if the solution $q^*(\theta)$ to retailer D's problem is monotone in θ, then the non-local incentive compatibility constraint is not binding.*

The second step in our analysis consists in remarking that $2q^0$, the quantity around which the derivative of the agent profit changes sign, can be part of a contract which is particularly attractive to the retailer. Indeed, the contract $(q(\theta), \pi_U(q(\theta); \theta)) =$

[18] In the case where there are only two types, the linearity of $d(\theta)$ does not matter any more as the two cost functions intersect only once.

$(2q^0, 0)$ is implementable and satisfies the individual rationality constraint of all types $\theta \in [0, \bar{c}]$, i.e. is feasible[19]. This contract is such that the IR constraint of any type offered this contract is binding. Moreover as the industry profit $\Pi(2q^0; \theta) \geq 0$ for all θ, the participation of all types is granted in our model: purchasing the product from any type of supplier is profitable to the retailer.

From Lemma 3 above, the non local incentive constraint can be neglected. Then the expected virtual surplus can be determined and simplified as in Jullien [10], starting from the integral of the Hamiltonian determined when maximizing π_D^e in (20) with respect to the (IR) and (LIC) constraints. Let $\mu(\theta)$ be the non negative multiplier of the (IR) constraint of a type θ, which we assume to be an integrable function of θ[20], and let $1 - F(\theta)$ be a primitive of $-f(\theta)$ and $1 - M(\theta)$ be a primitive of $-\mu(\theta)$. Then by integrating by part the integrals which depend on the supplier's profit $\pi_U(q(\theta); \theta)$, we have

$$\begin{aligned} V_D^e &= \int_0^{\bar{c}} \Pi(q(\theta); \theta) - \pi_U(q(\theta); \theta) dF(\theta) + \int_0^{\bar{c}} \mu(\theta) \pi_U(q(\theta); \theta) d\theta \\ &= \int_0^{\bar{c}} \Pi(q(\theta); \theta) dF(\theta) + [(1 - F(\theta)) \pi_U(q(\theta); \theta)]_0^{\bar{c}} - \int_0^{\bar{c}} \frac{1 - F(\theta)}{f(\theta)} \pi'_U(q(\theta); \theta) dF(\theta) \\ &\quad - [(1 - M(\theta)) \pi_U(q(\theta); \theta)]_0^{\bar{c}} + \int_0^{\bar{c}} \frac{1 - M(\theta)}{f(\theta)} \pi'_U(q(\theta); \theta) dF(\theta). \end{aligned} \tag{22}$$

To simplify this last expression, it is necessary to interpret $M(\theta) = \int_0^\theta \mu(t) dt$. First, $\mu(\theta)$ interprets as the opportunity gain for the retailer to reduce $\pi_U(q(\theta); \theta)$ from an infinitesimal (positive) amount to 0, holding the quantity $q(\theta)$ unchanged. As $\mu(\theta)$ is positive or nil, $M(\theta)$ cannot decrease, and interprets as the opportunity gain D obtains by reducing uniformly the profits left to all types between 0 and θ, from an infinitesimal (positive) amount to 0, holding all quantities unchanged. Then, keeping quantities unchanged, a uniform reduction of profits across all types continuously distributed over $[0, \bar{c}]$ has a cumulated opportunity gain given by $M(\bar{c}) = 1$, and consequently $M(\theta)$ has the property of a cumulated distribution function[21].

[19] Here $q(\theta) = 2q^0$ is constant and the supplier's reservation utility is nil for all θ. Moreover the rate of growth of this reservation utility is nil, and the derivative of the supplier's profit with respect to θ, equal to $-C_\theta(q; \theta)$, evaluated at $q = 2q^0$, is nil too. Hence Jullien [10]'s *homogeneity* property is (weakly) verified in our model.

[20] Strictly speaking we should consider the possibility that $\mu(\theta)$ is a non integrable function of θ. This occurs for example when one particular type only sees its (IR) constraint binding, while the (IR) constraints of the other types do not. We neglect this issue for the moment.

[21] There are some specificities related to the cumulation of $\mu(\theta)$: $M(\theta)$ remains constant on every

Still maintaining the assumption that $\mu(\theta)$ is integrable (and in particular that it is not strictly positive for only one θ), and using the local incentive constraint (LIC), $M(0) = 0$ and the expected virtual surplus the retailer maximizes with respect to $q(\theta)$ simplifies into:

$$V_D^e = \int_0^{\bar{c}} \Pi(q(\theta);\theta) - \frac{F(\theta) - M(\theta)}{f(\theta)} C_\theta(q(\theta);\theta) dF(\theta) \tag{23}$$

The point-wise optimization of the virtual surplus with respect to $q(\theta)$ for each θ gives the first and second order conditions

$$\Pi_q(q(\theta);\theta) - \frac{F(\theta) - M(\theta)}{f(\theta)} C_{\theta q}(q(\theta);\theta) = 0 \tag{24}$$

and

$$\Pi_{qq}(q(\theta);\theta) - \frac{F(\theta) - M(\theta)}{f(\theta)} C_{\theta qq}(q(\theta);\theta) \leq 0. \tag{25}$$

The virtual surplus is not always concave in the purchases $q(\theta)$[22], even if $\Pi_{qq}(q(\theta);\theta) = 2P'(q(\theta) - C_{qq}(q(\theta);\theta) = -2b - d(\theta)$ is negative under our assumptions. Indeed, the second term of the second order condition above is not always negative: under the assumption that the marginal costs curves cross once in the graph (q,cost) as the type changes, the second order cross partial derivative $C_{\theta qq}(q(\theta);\theta) = d'(\theta)$ is strictly negative. Since the support of $M(\theta)$ belongs to the support of $F(\theta)$ then when $M(\theta) = 1$, $F(\theta) - 1 \leq 0$ and the second term of (25) is negative, but when $M(\theta) = 0$, the second term of (25) is positive. Consequently the second order condition (25) is negative only for types close enough to $\bar{c}$, but when θ is closer to 0, $F(\theta) - M(\theta)$ may be positive and the second order condition can fail to be satisfied. Let

$$(q^*(\theta), M^*(\theta))_{\theta \in [0,\bar{c}]} \equiv \arg \max_{(q(\theta),M(\theta))} V_D^e \tag{26}$$

be the optimal quantity scheme and cumulated opportunity gain of saturating IR constraints which maximize the expected virtual surplus. We now characterize the

interval of types for which $\mu(\theta) = 0$, and hence when $\mu(\theta)$ can be integrated, the support of types for which $M(\theta) \in (0,1)$ is included in $[0,\bar{c}]$; moreover when only the IR constraint of $\theta = 0$ binds, then $M(0) = 1$ and 0 else, while when the IR constraint of $\theta = \bar{c}$ binds, then $M(\bar{c}) = 1$ and 0 else.

[22] And therefore our model does not satisfy assumption 2 in Jullien [10], so that the property called *potential separation* cannot be verified here. However similarly to Jullien we do obtain an interval of types offered a contract which leaves them no rent, and, as $2q^0$ is independent of θ, where all firms produce the same quantity.

assumptions under which $q^*(\theta), M^*(\theta))_{\theta\in[0,\bar{c}]}$ exists and is unique for all demand satisfying $q^M(0) \gg q^0$, and we characterize this solution.

Let us start with the determination of the quantities $\tilde{q}(\theta,1)$ and $\tilde{q}(\theta,0)$ which maximize V_D^e for $M(\theta) = 1$ and when $M(\theta) = 0$ for all $\theta \in [0, \bar{c}]$, irrespectively of the fact that $M(\theta)$ should have the same property than a cumulative distribution function. The solution in (26) above can correspond to one of these two solutions. To do so, we need to impose the following assumption.

Assumption 1 *In our model,*

(i) There exists a maximum quantity $\bar{q}$ that the retailer's order $q(\theta)$ cannot exceed for any $\theta \in [0, \bar{c}]$;

(ii) The demand is such that $P(0) - G(\bar{c}) \geq 0$, where $G(\theta) = \theta + \frac{F(\theta)}{f(\theta)}$ is strictly increasing in θ.

The first part of this assumption (i) states that it exists a maximal quantity which can be ordered by the retailer, unrelated to the cost of production of the supplier and possibly large[23], but bounded. The second part of this assumption (ii) is satisfied if (sufficient condition) $f(\theta) \neq 0$ for every θ and is particular at the bounds $\theta = 0$ and $\theta = \bar{c}$. As the following result demonstrates, the first part (i) of the assumption above ensures that whenever the retailer's profit is convex and increasing in q, the quantity ordered can be determined (it is equal to $\bar{q}$). The second part (ii) ensures that the retailer's marginal profit is strictly positive whenever it is convex, i.e. it ensures that the retailer is better off asking all types to produce a strictly positive quantity. Assumption 1 therefore ensures that the quantity ordered by the retailer exists and is continuous in θ even if the retailer's profit is convex in q. We have

Lemma 4 *The unique values $\tilde{q}(\theta,1)$ and $\tilde{q}(\theta,0)$ which maximize V_D^e respectively for $M = 1$ and $M = 0$ are such that:*

(i) $\tilde{q}(\theta,1)$ is continuous and increasing in θ,

(ii) $\tilde{q}(\theta,0)$ is continuous and increasing in θ for $\theta \in [0, \bar{\Gamma}]$, and jumps upward to $\bar{q}$ for $\theta \in [\bar{\Gamma}, \bar{c}]$, where $\bar{\Gamma}$ solves $\bar{\Gamma} + \frac{F(\bar{\Gamma})}{f(\bar{\Gamma})} = \frac{(2b+\bar{d})\bar{c}}{\bar{d}}$,

[23]e.g. this can be a logistic constraint, such as the maximal quantity which can be transported.

(iii) $\tilde{q}(\theta,0) \geq q^M(\theta) \geq \tilde{q}(\theta,1)$ *for all* $\theta \in [0,\bar{c}]$*, with* $\tilde{q}(0,0) = q^M(0)$ *and* $\tilde{q}(\bar{c},1) = q^M(\bar{c})$.

Proof. See Appendix A.3‖.

We can illustrate this result in the graph below.

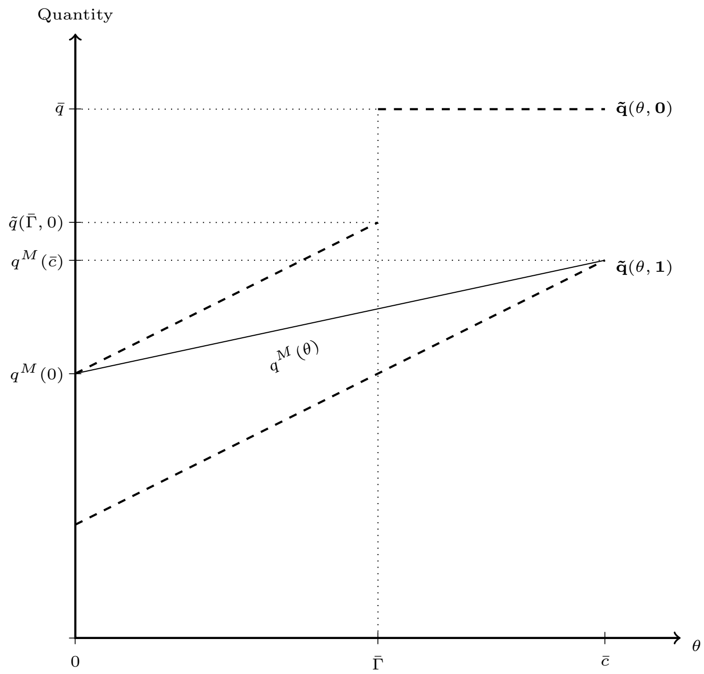


Figure 2: Monopolistic purchases (solid line) and virtual surplus optima (dashed lines) when $M = 1$ or $M = 0$ for all $\theta \in [0, \bar{c}]$

From Lemma 2 and definition 1, it exists a unique value θ^0 such that $2q^0P'(2q^0) + P(2q^0) = C_q(2q^0;\theta^0)$. Depending on how large the market demand is, this type belongs to $[0,\bar{c}]$ or not: $\theta^0 \in [0,\bar{c}]$ if $2q^0P'(2q^0) + P(2q^0) - C_q(2q^0;0) \leq 0$ and $2q^0P'(2q^0) + P(2q^0) - C_q(2q^0;\bar{c}) \geq 0$, that is if the marginal revenue at $2q^0$ is in between the marginal cost of the lowest type $\theta = 0$ computed at $2q^0$, and the marginal cost of the highest type $\theta = \bar{c}$. Then the other two cases we need to address are $2q^0P'(2q^0) + P(2q^0) - C_q(2q^0;0) > 0$, case in which the marginal revenue at $2q^0$ is strictly above all marginal costs, or $2q^0P'(2q^0) + P(2q^0) - C_q(2q^0;\bar{c}) < 0$, case in which the marginal revenue at $2q^0$ is strictly below all marginal costs. In these two last cases, it is possible that the IR constraint of only one type, either $\theta = 0$ or $\theta = \bar{c}$, binds at equilibrium. We have:

Lemma 5 *When $2q^0$ is strictly smaller than $\tilde{q}(0,1)$ (respectively strictly larger than $\max\{\tilde{q}(\bar{\Gamma},0);\tilde{q}(\bar{c},1)\}$), then if it exists, the solution to (26) must be such that $M^*(0) = 1$ and 0 else (resp. $M^*(\bar{c}) = 1$ and 0 else).*

Proof. See Appendix A.4.‖

Then suppose that the demand is such that $\tilde{q}(0,1) \leq 2q^0 \leq \max\{\tilde{q}(\bar{\Gamma},0);\tilde{q}(\bar{c},1)\}$: we characterize below the contract offered by the retailer at equilibrium, in each of the different configurations of the marginal revenue we detailed previously. These configurations are such that the IR constraint binds for a convex subset of types.

Proposition 1 *When $C_q(2q^0;\bar{c}) \leq P(2q^0) + 2q^0 P'(2q^0) \leq C_q(2q^0;0)$, the equilibrium $(q^*(\theta), M^*(\theta))$ is such that the IR of all types $\theta \in [\min\{\theta_1,\bar{\Gamma}\},\theta_2]$ bind, where θ_1 (respectively θ_2) is the unique type which solves $\tilde{q}(\theta_1,0) = 2q^0$ (resp. $\tilde{q}(\theta_2,1) = 2q^0$). Moreover $(q^*(\theta), M^*(\theta))$ is given by:*

$$(q^*(\theta), M^*(\theta)) = \begin{cases} (\tilde{q}(\theta,0),0) & \text{if } \theta < \min\{\theta_1,\bar{\Gamma}\} \\ (2q^0, F(\theta) + f(\theta)\frac{\Pi_q(2q^0;\theta)}{C_{\theta q}(2q^0;\theta)}) & \text{if } \theta \in [\min\{\theta_1,\bar{\Gamma}\},\theta_2] \\ (\tilde{q}(\theta,1),1) & \text{if } \theta > \theta_2 \end{cases}$$

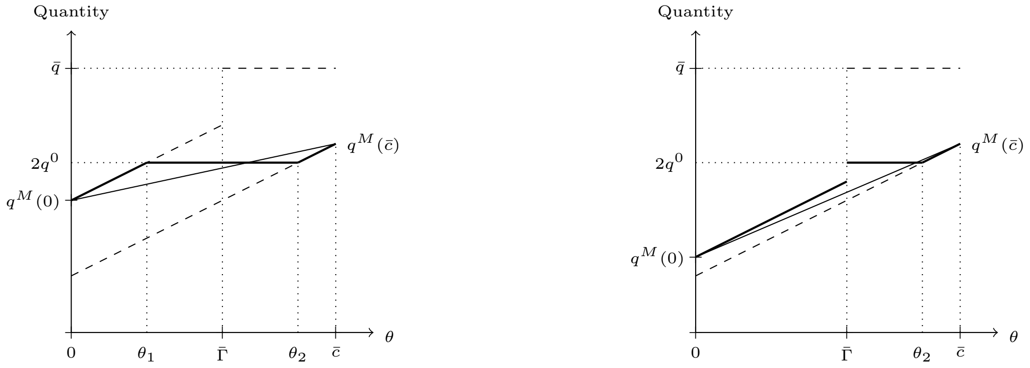
Proof. See Appendix A.5.‖

From the previous analysis, the following corollary comes immediately.

Corollary 1 (to Proposition 1) *The equilibrium order $q^*(\theta)$ is larger (respectively lower) than $q^M(\theta)$ for θ lower (resp. larger) than θ^0, where θ^0 solves $2q^0P'(2q^0) + P(2q^0) = C_q(2q^0;\theta^0)$. Moreover if $\theta_1 > \bar{\Gamma}$, $q^*(\theta)$ jumps upward at $\theta = \bar{\Gamma}$; else it is continuous.*

Figures 3a and 3b illustrate the equilibrium described in proposition 1 above, by representing the scheme $q(\theta)$ with a thick black line and the monopoly purchases $q^M(\theta)$ by a thin black line going from $q^M(0)$ to $q^M(\bar{c})$.

The next two propositions characterize the equilibrium in the cases where countervailing incentives do not affect the retailer's order scheme as much as in proposition 1 above, so that a "no distortion at the top" result emerges. First, consider the



(a) Continuous quantity for $\theta_1 \leq \bar{\Gamma}$ (b) Discontinuous quantity for $\theta_1 > \bar{\Gamma}$

Figure 3: Equilibrium when $C_q(2q^0;\bar{c}) \leq P(2q^0) + 2q^0P'(2q^0) \leq C_q(2q^0;0)$

case in which demand is large enough, so that the retailer is better off employing optimally a supplier with a large capacity of production. We have,

Proposition 2 *When $P(2q^0) + 2q^0P'(2q^0) > C_q(2q^0;0)$ and $2q^0 \geq \tilde{q}(0,1)$, the equilibrium $(q^*(\theta), M^*(\theta))$ is such that the IR constraint of all types $\theta \in [0, \max\{\theta_2, 0\}]$ bind, where θ_2 solves $\tilde{q}(\theta_2, 1) = 2q^0$. Moreover $(q^*(\theta), M^*(\theta))$ is given by:*

$$(q^*(\theta), M^*(\theta)) = \begin{cases} (2q^0, F(\theta) + f(\theta)\frac{\Pi_q(2q^0;\theta)}{C_{\theta q}(2q^0;\theta)}) & \text{if } \theta \in [0, \max\{\theta_2, 0\}) \\ (\tilde{q}(\theta,1), 1) & \text{if } \theta \in [\theta_2, \bar{c}]. \end{cases}$$

In the extreme case of a large enough demand, the most attractive supplier for retailer D is the one with the largest capacity, $\theta = \bar{c}$, and the IR constraint of a type $\theta = 0$ is the only one binding.

Corollary 2 (to Proposition 2) *The equilibrium order $q^*(\theta)$ is strictly lower than (respectively equal to) $q^M(\theta)$ when θ is strictly lower than (respectively equal to) $\bar{c}$. Moreover when $2q^0 < \tilde{q}(0,1)$, then $q^*(\theta) = \tilde{q}(\theta,1)$ and $M^*(0) = 1$ and 0 for all $\theta \neq 0$.*

Proof. See Appendix A.6.‖

Oppositely, consider the case demand is small enough so that the retailer is better off employing optimally the supplier whose marginal cost of production has the lowest intercept. We have,

Proposition 3 *When $P(2q^0)+2q^0P'(2q^0) < C_q(2q^0;\bar{c})$, the equilibrium $(q^*(\theta), M^*(\theta))$ is such that the IR constraint of all types $\theta \in [\min\{\bar{\Gamma}, \bar{c}\}, \bar{c}]$ bind. Moreover $(q^*(\theta), M^*(\theta))$ is given by:*

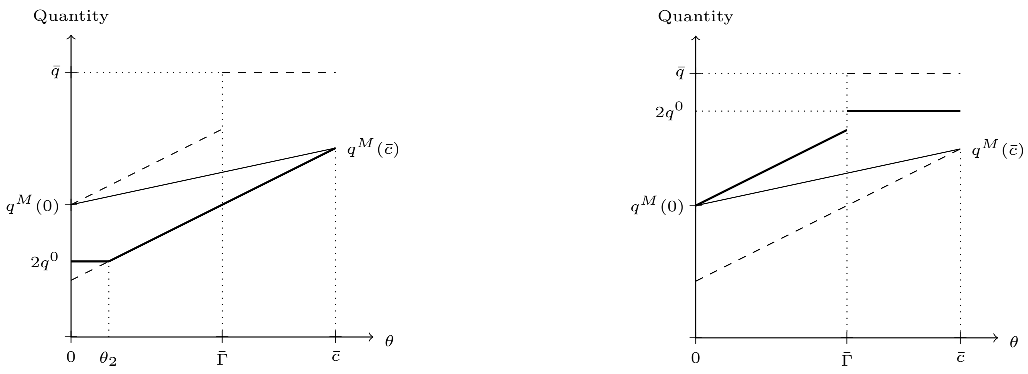
$$(q^*(\theta), M^*(\theta)) = \begin{cases} (\tilde{q}(\theta, 0), 0) & \text{if } \theta \in [0, \min\{\bar{\Gamma}, \bar{c}\}) \\ \\ (2q^0, F(\theta) + f(\theta)\frac{\Pi_q(2q^0;\theta)}{C_{\theta q}(2q^0;\theta)}) & \text{if } \theta \in [\min\{\bar{\Gamma}, \bar{c}\}, \bar{c}]. \end{cases}$$

In the extreme case of a small enough demand, the most attractive supplier for retailer D is the most efficient one, $\theta = 0$, and the IR constraint of a type $\theta = \bar{c}$ is the only one binding.

Corollary 3 (to Proposition 3) *The equilibrium order $q^*(\theta)$ is strictly larger than (respectively equal to) $q^M(\theta)$ when θ is strictly larger than (respectively equal to) 0. Moreover when $2q^0 > \max\{\tilde{q}(\bar{\Gamma}, 0), \tilde{q}(\bar{c}, 1)\}$, then $q^*(\theta) = \tilde{q}(\theta, 0)$ and $M^*(\bar{c}) = 1$ and 0 for all $\theta \neq \bar{c}$.*

Proof. See Appendix A.7.‖

The figure below illustrate these cases.



(a) $P(2q^0) + 2q^0P'(2q^0) > C_q(2q^0; 0)$ (b) $P(2q^0) + 2q^0P'(2q^0) < C_q(2q^0; \bar{c})$

Figure 4: Equilibria when the market size points to one extreme type

5 Optimal sourcing and global incentives

As we demonstrated in the previous section, non local incentive constraints can be neglected when marginal costs rotate around each other as the agent' type increases.

Let us consider instead the following marginal cost function

$$C_q(\theta) = d_1(\theta) + d_2(\theta)q, \tag{27}$$

where the intercept increases with θ, $d_1'(\theta) > 0$, the slope decreases, $d_2'(\theta) < 0$, with

$$d_1(0) = 0,\ d_2(0) = \bar{d},\ d_1(\bar{c}) = \bar{c},\ d_2(\bar{c}) = 0 \tag{28}$$

and where the intersection between the marginal costs of two different types $\tilde{\theta} < \hat{\theta}$ is given by

$$q^0(\tilde{\theta}, \hat{\theta}) = \frac{d_1(\hat{\theta}) - d_1(\tilde{\theta})}{d_2(\tilde{\theta}) - d_2(\hat{\theta})}, \tag{29}$$

with

$$\lim_{(\hat{\theta},\tilde{\theta})\to(0,0)} q^0(\tilde{\theta}, \hat{\theta}) = \underline{q}^0 < \lim_{(\hat{\theta},\tilde{\theta})\to(\bar{c},\bar{c})} q^0(\tilde{\theta}, \hat{\theta}) = \overline{q}^0. \tag{30}$$

Under these assumptions, the derivatives of the cost function become

$$C_\theta = d_1'(\theta)q + d_2'(\theta)q^2 \quad \text{and} \quad C_{q\theta} = d_1'(\theta) + 2d_2'(\theta)q. \tag{31}$$

In this case, we demonstrate now that the non local incentive constraint change the distortions which result from the presence of countervailing incentives.

6 Conclusion

To conclude, note that when the demand parameters (a, b) are such that the first best quantities are "much" smaller than q^0 for all types, $q^M(\bar{c}) < q^M(0) \ll q^0$, then the local conditions are sufficient to determine the optimal schedule offered by the retailer to its supplier. As the Spence-Mirrlees condition is verified, and since no countervailing incentives are present in that situation, the standard analysis applies.

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A Proofs

A.1 Proof of Lemma 1

When maximizing its profit with respect to the type $\tilde{\theta}$ it reports, supplier U faces the following necessary and sufficient conditions (for possible values of $C_q \in [0, \bar{c}]$)

$$T'(\tilde{\theta}) - q'(\tilde{\theta})C_q(q(\tilde{\theta});\theta)|_{\tilde{\theta}=\theta} = 0 \tag{32}$$

and

$$T''(\tilde{\theta}) - q''(\tilde{\theta})C_q(q(\tilde{\theta});\theta) - (q'(\tilde{\theta}))^2 C_{qq}(q(\tilde{\theta});\theta) \leq 0 \tag{33}$$

at $\tilde{\theta} = \theta$, where $C_{qq}(q(\tilde{\theta});\theta)$ is the second order partial derivative of the total cost $C(q;\theta)$ with respect to q. Rewriting the necessary condition at $\tilde{\theta} = \theta$ and differentiating it with respect to θ gives

$$T'(\theta) - q'(\theta)C_q(q(\theta);\theta) = 0 \tag{34}$$

and

$$T''(\theta) - q''(\theta)C_q(q(\theta);\theta) - (q'(\theta))^2 C_{qq}(q(\theta);\theta) - q'(\theta)C_{q\theta}(q(\theta);\theta) = 0 \tag{35}$$

which can be replaced in the sufficient condition to obtain

$$q'(\theta)C_{q\theta}(q(\theta);\theta) \leq 0. \tag{36}$$

The cross partial derivative of total cost with respect to q and θ evaluated at $q(\theta)$ is equal to $C_{q\theta}(q(\theta);\theta) = 1 - \frac{\bar{d}}{\bar{c}}q(\theta)$, whose sign is given by

$$C_{q\theta}(q(\theta);\theta) \geq 0 \Leftrightarrow 1 - \frac{\bar{d}}{\bar{c}}q(\theta) \geq 0 \Leftrightarrow q(\theta) \leq q^0 \tag{37}$$

while $C_{q\theta}(q(\theta);\theta) \leq 0$ when $q(\theta) \geq q^0$. Therefore $q'(\theta) \leq 0$ for θ such that $q(\theta) \leq q^0$ while $q'(\theta) \geq 0$ for θ such that $q(\theta) \geq q^0$.

A.2 Proof of Lemma 2

The derivative of the supplier's profit with respect its type θ when U reports it truthfully is equal to

$$\pi'_U(\theta;\theta) = T'(\theta) - q'(\theta)C_q(q(\theta);\theta) - C_\theta(q(\theta);\theta) = -C_\theta(q(\theta);\theta) \tag{38}$$

once the first order condition of the reporting game has been cancelled out. Using the expression of the total cost, it comes

$$C_\theta(q(\theta);\theta) = q(\theta) + \frac{1}{2}\bar{d}\left(-\frac{1}{\bar{c}}\right)(q(\theta))^2 = q(\theta)\left(1 - \frac{\bar{d}}{2\bar{c}}q(\theta)\right). \tag{39}$$

As $q(\theta) \geq 0$, the sign of this derivative is given by the sign of $1-\frac{\bar{d}}{2\bar{c}}q(\theta)$. Consequently $\pi'_U(c;c) \geq 0$ if $1-\frac{\bar{d}}{2\bar{c}}q(\theta) \leq 0$, i.e. if $q(\theta) \geq \frac{2\bar{c}}{\bar{d}} = 2q^0$, and $\pi'_U(\theta;\theta) \leq 0$ if $1-\frac{\bar{d}}{2\bar{c}}q(\theta) \geq 0$, i.e. if $q(\theta) \leq \frac{2\bar{c}}{\bar{d}} = 2q^0$.

A.3 Proof of Lemma 4

As the analysis above demonstrated, the virtual surplus is concave in q for $M = 1$ and hence the solution $\tilde{q}(\theta, 1)$ solves the first order condition which is necessary and sufficient:

$$\Pi_q(\tilde{q}(\theta,1);\theta) - \frac{F(\theta)-1}{f(\theta)}C_{\theta q}(\tilde{q}(\theta,1);\theta) = 0. \tag{40}$$

Since all functions are continuous, $\tilde{q}(\theta, 1)$ is continuous in θ. Using the Implicit Function Theorem and omitting the functions arguments, we have

$$\frac{d\tilde{q}}{d\theta}(\theta,1) = -\frac{\Pi_{q\theta} + \frac{d\left(\frac{1-F(\theta)}{f(\theta)}\right)}{d\theta}C_{\theta q}}{\Pi_{qq} + \frac{1-F(\theta)}{f(\theta)}C_{\theta qq}} \tag{41}$$

Under our assumptions, $\Pi_{qq} < 0$, $C_{\theta qq} < 0$, and $\Pi_{q\theta} = -C_{q\theta}$, where $C_{q\theta} < 0$ for $q > q^0$ which is the hypothesis under which we are working. The numerator of the above simplifies as

$$\Pi_{q\theta} + \frac{d\left(\frac{1-F(\theta)}{f(\theta)}\right)}{d\theta} C_{\theta q} = C_{\theta q}\left(-1 + \frac{d\left(\frac{1-F(\theta)}{f(\theta)}\right)}{d\theta}\right) \tag{42}$$

which is positive under the assumption that $\frac{1-F(\theta)}{f(\theta)}$ is decreasing. As the denominator is negative, we have $\frac{d\tilde{q}}{d\theta}(\theta, 1) \geq 0$. Therefore $\tilde{q}(\theta, 1)$ is non-decreasing in θ.

As discussed before, the virtual surplus is not concave in q for $M = 0$. When $M = 0$, under our assumptions the second order condition rewrites

$$\Pi_{qq}(q(\theta); \theta) - \frac{F(\theta)}{f(\theta)} C_{\theta qq}(q(\theta); \theta) = -2b - \bar{d} + \frac{\bar{d}}{\bar{c}}\left(\theta + \frac{F(\theta)}{f(\theta)}\right). \tag{43}$$

Let $G(\theta) = \theta + \frac{F(\theta)}{f(\theta)}$ which is strictly increasing in θ under our assumptions, and let $\Gamma(x)$ be its reciprocal, the second derivative of V_R^e is negative if and only if

$$-2b - \bar{d} + \frac{\bar{d}}{\bar{c}}\left(\theta + \frac{F(\theta)}{f(\theta)}\right) \leq 0 \Leftrightarrow \theta \leq \Gamma\left((2b + \bar{d})\frac{\bar{c}}{\bar{d}}\right) \equiv \bar{\Gamma}. \tag{44}$$

As the virtual surplus is concave for $\theta = 0$, $\bar{\Gamma} > 0$. However it can be larger or smaller than $\bar{c}$: when $\theta = \bar{c}$, the second order derivative of V_D^e is equal to $-2b + \frac{\bar{d}}{\bar{c}f(\bar{c})}$ which can be positive when cost functions are very convex in q, or negative when $\bar{d}$ is not sufficiently large. The virtual surplus V_D^e is therefore convex for $M = 0$ when

$$\theta \in [\min\{\bar{c}; \bar{\Gamma}\}, \bar{c}]. \tag{45}$$

Consequently when $\theta \in [0, \min\{\bar{c}; \bar{\Gamma}\}]$, the first order condition is sufficient to determine the optimum $\tilde{q}(\theta, 0)$, which solves

$$\Pi_q(\tilde{q}(\theta, 0); \theta) - \frac{F(\theta)}{f(\theta)} C_{\theta q}(\tilde{q}(\theta, 0); \theta) = 0. \tag{46}$$

A straightforward application of the Implicit Function Theorem allows to determine the variation of $\tilde{q}(\theta, 0)$ when $\theta \in [0, \bar{\Gamma}]$. It comes,

$$\frac{d\tilde{q}}{d\theta}(\theta, 0) = -\frac{\Pi_{q\theta} - \frac{d\left(\frac{F(\theta)}{f(\theta)}\right)}{d\theta} C_{\theta q}}{\Pi_{qq} - \frac{F(\theta)}{f(\theta)} C_{\theta qq}} \tag{47}$$

where $\Pi_{qq} < 0$, $C_{\theta qq} < 0$, and $\Pi_{q\theta} = -C_{q\theta}$, where $C_{q\theta} < 0$ for $q > q^0$. As $\frac{F(\theta)}{f(\theta)}$ increases with θ, the numerator is positive, and as the second order condition is negative, the denominator negative, to that $\frac{d\tilde{q}}{d\theta}(\theta, 0) \geq 0$. Therefore $\tilde{q}(\theta, 0)$ increases with $\theta \in [0, \bar{\Gamma}]$.

On the other hand when $\theta \in [\min\{\bar{c}; \bar{\Gamma}\}, \bar{c}]$, the virtual surplus is convex and the marginal virtual surplus increases in q. Note that when $q = 0$, $\Pi_q(0;\theta) - \frac{F(\theta)}{f(\theta)} C_{\theta q}(0;\theta) = P(0) - C_q(0;\theta) - \frac{F(\theta)}{f(\theta)} C_{\theta q}(0;\theta) = P(0) - \theta - \frac{F(\theta)}{f(\theta)}$. For the virtual surplus to be convex for $\theta \in [\bar{\Gamma}, \bar{c}]$, the previous analysis has showed that $\frac{(2b+\bar{d})\bar{c}}{d} \leq G(\theta) \leq G(\bar{c})$. Consequently $P(0) - \theta - \frac{F(\theta)}{f(\theta)} = P(0) - G(\theta) \geq P(0) - G(\bar{c})$. Let us assume that $P(0) - G(\bar{c}) \geq 0$, then the marginal virtual is positive for $q = 0$ and increasing for all θ such as the virtual surplus V_D^e is convex. Therefore for types θ such that V_D^e is convex, the marginal virtual surplus is strictly positive for all $q \geq 0$. Hence the solution consists in increasing the quantity ordered to a maximum value $\bar{q}$, which we assume to be finite[24].

To summarize, when $M = 0$ the quantity which maximizes the virtual surplus is equal to $\tilde{q}(\theta, 0)$ when $\theta \in [0, \bar{\Gamma}]$, and to $\bar{q}$ else.

The comparison of $\tilde{q}(\theta, 0)$ with $\tilde{q}(\theta, 1)$ is straightforward from the comparison of the first order conditions which determine these quantities: when the virtual surplus is concave in q for $M = 1$ and $M = 0$, both marginal virtual surpluses defining these two solutions are strictly decreasing in q. Since, in the case of large market, the quantity produced is such that $C_{\theta q} < 0$ for all θ, the left-hand-side of the first order condition which determines $\tilde{q}(\theta, 1)$ defines a function of θ which is below that of the first order condition which determines $\tilde{q}(\theta, 0)$ for all $\theta \in [0, \bar{\Gamma}]$. Then when $M = 0$ the quantity order jumps upward to $\bar{q}$, and is *a fortiori* above $\tilde{q}(\theta, 1)$. Therefore the solution which maximizes the virtual surplus for $M = 0$ is strictly above the solution the solution for $M = 1$. Last the monopoly solution is strictly increasing with θ, and such that $\Pi_q(q^M(\theta); \theta) = 0$. Inspecting the first order conditions determining $\tilde{q}(\bar{c}, 1)$ and $\tilde{q}(0, 0)$ above, it is immediate to verify that these solutions coincide with $q^M(\bar{c})$ and $q^M(0)$ respectively. Therefore $q^M(\theta)$, which increases with θ, lies in between the solutions for $M = 1$ and $M = 0$.

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[24] Since the market demand is strictly decreasing and linear, there is a quantity above which the total income of the retailer is nil. When q exceeds this quantity, the virtual surplus is therefore equal to $-C(q;\theta) - \frac{F(\theta)}{f(\theta)} C_\theta(q;\theta)$. As $C(q;\theta)$ is linear in θ, at every θ^0 we have $C(q;\theta) = C(q;\theta^0) + C_\theta(q;\theta^0)(\theta - \theta^0)$. Therefore $C(q;\theta)$ exceeds its derivative at every θ^0, and if $\frac{F(\theta)}{f(\theta)}$ is bounded everywhere, then the virtual surplus becomes negative when q is large. Consequently there is an upper bound on the order q of the retailer when the virtual surplus is convex. Alternatively, we can directly assume that there is an upward limit to the order the retailer can buy from its supplier.

A.4 Proof of Lemma 5

Consider first the case $2q^0 < \tilde{q}(0,1)$. In that case, from Lemma 2, all quantity schemes $q(\theta)$ which are feasible (including $q^*(\theta)$ if it exists) are such that the supplier's profit increases with θ. Suppose that the IR constraint binds for some type $\tilde{\theta} \in (0, \bar{c}]$. As to satisfy Lemma 1 the quantity scheme must be non decreasing with θ, then for $\theta < \tilde{\theta}$ the profit of supplier θ must be strictly negative, and the quantity produced (weakly) lower than the quantity requested from $\tilde{\theta}$. This is in contradiction with the IR constraint of this type. This is true for every $\tilde{\theta} \in (0, \bar{c}]$ and therefore the only IR constraint which is binding is the one of a type 0, where all mass in $M^*(\theta)$ is located. Therefore the solution to (26) must be such that $M^*(0) = 1$.

Consider now the case $2q^0 > \max\{\tilde{q}(\bar{\Gamma}, 0); \tilde{q}(\bar{c}, 1)\}$. This is a situation in which from Lemma 2, all quantity schemes $q(\theta)$ which are feasible (including $q^*(\theta)$ if it exists) are such that the supplier's profit decreases with θ. Again if the IR constraint was saturated for any $\tilde{\theta} \in [0, \bar{c})$, the IR constraint of supplier of type $\theta > \tilde{\theta}$ would not be respected. Therefore the solution to (26) must be such that $M^*(\bar{c}) = 1$ and 0 else.

A.5 Proof of Proposition 1 and its corollary

We must consider two situations in turn: first, the case where $2q^0 \leq \tilde{q}(\bar{\Gamma}, 0)$, and then the case $2q^0 > \tilde{q}(\bar{\Gamma}, 0)$.

Start with $2q^0 \leq \tilde{q}(\bar{\Gamma}, 0)$. In that case from Lemma 4, it exists a unique $\theta_1 \leq \bar{\Gamma}$ (respectively $\theta_2 > \bar{\Gamma}$) which solves $\tilde{q}(\theta_1, 0) = 2q^0$ (resp. $\tilde{q}(\theta_1, 0) = 2q^0$). From Lemma 2, the supplier payoff decreases with θ when the quantity is lower than $2q^0$, while it with θ when q exceeds $2q^0$. Suppose the retailer sets the rent Π_U to 0 only for all types $\theta \in [\theta_1, \theta_2]$ which produce $2q^0$. Consider types $\theta \in [0, \theta_1]$: from Lemma 1 and 2, the quantity ordered at equilibrium must be smaller than $2q^0$ and hence must be such that rents are left. Therefore the cumulated value $M^*(\theta)$ must be nil for these types, and the optimization of the virtual surplus leads the retailer to offer $q^*(\theta) = \tilde{q}(\theta, 0) < 2q^0$. Consider types in $[\theta_2, \bar{c}]$: by a symmetric argument rents must be left to all these types which must produce more than $2q^0$. The cumulated value $M^*(\theta)$ must therefore be equal to 1 when $\theta = \theta_2$. Then the profit of the retailer increases by equating $q^*(\theta) = \tilde{q}(\theta, 1) > 2q^0$. Then $M^*(\theta)$ is equal to

$$M^*(\theta) = F(\theta) + f(\theta)\frac{\Pi_q(2q^0;\theta)}{C_{\theta q}(2q^0;\theta)} \tag{48}$$

which equates the first order condition of the optimization of the virtual surplus to 0. It remains to check whether $2q^0$ maximizes the expected profit of the retailer given the value of the multiplier. For $\theta \in [\theta_1, \theta_2]$, we have

$$\frac{F(\theta) - M^*(\theta)}{f(\theta)} = -\frac{\Pi_q(2q^0;\theta)}{C_{\theta q}(2q^0;\theta)} \tag{49}$$

where as $2q^0 > q^0$, $C_{\theta q}(2q^0;\theta) < 0$, and where by construction $\Pi_q(2q^0;\theta) = 0$ for $\theta^0 \in (\theta_1, \theta_2)$, with $\Pi_q(2q^0;\theta) > 0$ for $\theta > \theta^0$ and $\Pi_q(2q^0;\theta) < 0$ for $\theta < \theta^0$ again by construction. The second order condition simplifies as

$$\Pi_{qq}(2q^0;\theta) - \frac{F(\theta) - M^*(\theta)}{f(\theta)} C_{qq\theta}(2q^0;\theta) = \Pi_{qq}(2q^0;\theta) + \frac{\Pi_q(2q^0;\theta)}{C_{\theta q}(2q^0;\theta)} C_{qq\theta}(2q^0;\theta). \tag{50}$$

Consider the case $\theta \in [\theta_1, \theta^0]$. As $\Pi_{qq}(2q^0;\theta) < 0$, $C_{qq\theta}(2q^0;\theta) < 0$, and $\Pi_q(2q^0;\theta) < 0$, the second order condition is clearly negative for $\theta < \theta^0$, and therefore $q(\theta) = 2q^0$ maximizes the profit of the retailer. Consider the case $\theta \in [\theta^0, \theta_2]$. In that case the second order condition is potentially not satisfied, and the profit of the retailer convex. However consider the difference in the marginal profit $\Pi_q(q;\theta) - \Pi_q(2q^0;\theta)$ which is equal to the value of the marginal profit for any other quantity than $2q^0$ when the multiplier is $M^*(\theta)$. By construction when $q < 2q^0$ and $\theta > \theta^0$, the marginal revenue is larger than its value at $2q^0$, and the marginal cost of type θ is smaller than its value at $2q^0$. Consequently this difference in marginal profits is strictly positive for any $q < 2q^0$. Therefore the profit of the retailer increases when q tends to $2q^0$. Therefore $q = 2q^0$ maximizes the retailer's profit when $\theta > \theta^0$ and the retailer's profit is convex.

Suppose that a subset of types in $[\theta_1, \theta_2]$ earn rents: this is impossible as it require either the quantity to increase above $2q^0$ and then to decrease again, which breaks the local incentive constraints, or it would break the monotonicity of the supplier's utility for quantities strictly lower (or strictly larger) than $2q^0$. Suppose that the individual rationality constraint is binding for types $\theta < \theta_1$. Then to respect the local incentive constraints the quantity $q(\theta) < 2q^0$, but in that case the profit left to this type would be strictly negative and the individual rationality constraint not respected anymore, which is impossible. Similarly suppose that the individual rationality constraint is binding for types $\theta > \theta_2$: respecting the local incentive constraints asks to increase $q(\theta)$ above $2q^0$, but in that case by lowering the type towards θ_2 the profit of the supplier would become negative, which is again impossible.

The analysis is similar when $2q^0 > \tilde{q}(\bar{\Gamma}, 0)$, except for the fact that the virtual surplus is not concave in q anymore when $\theta > \bar{\Gamma}$. Suppose the retailer sets the rent Π_U to 0 for all types $\theta \in [\bar{\Gamma}, \theta_2]$ which produce $2q^0$. This value is lower than $\bar{q}$ and since the virtual surplus is convex while the marginal virtual surplus is strictly positive, this quantity is indeed the order which maximizes the virtual surplus and respect the individual rationality of these types of suppliers. Then types in $[0, \bar{\Gamma}]$ are offered to produce $\tilde{q}(\theta, 0)$, which is lower than $2q^0$ and hence satisfy the local incentive constraints, and earn some rents so that $M^*(\theta) = 0$. A symmetric argument applied to types in $[\theta_2, \bar{c}]$ reveals that rents must be left to all these types which must produce more than $2q^0$. The cumulated value $M^*(\theta)$ must therefore be equal to 1 when $\theta = \theta_2$, and the profit of the retailer is maximal by equating $q^*(\theta) = \tilde{q}(\theta, 1) > 2q^0$. Last the cumulated multiplier must be $M^*(\theta) = F(\theta) + f(\theta) \frac{\Pi_q(2q^0;\theta)}{C_{\theta q}(2q^0;\theta)}$ for $\theta \in [\bar{\Gamma}, \theta_2]$.

A.6 Proof of Proposition 2 and its corollary

Start with Proposition 2. When $2q^0 P'(2q^0) + P(2q^0) - C_q(2q^0; 0) > 0$, $q^M(\theta) > 2q^0$ for all θ. From Lemma 2, the supplier's profit is strictly increasing in q, and hence the retailer is better off minimizing informational rents by setting the order of a type 0 supplier as close as it can to $2q^0$. The limit to the reduction of the quantity is set by $2q^0$ offered for $\Pi_U = 0$, below which the supplier would earn a negative profit. Therefore the IR constraint of at least a type $\theta = 0$ bind, and when $\theta > \theta_2$ the retailer has saturated the IR constraint of all the types it could, $M(\theta) = 1$. Then by concavity of the virtual surplus, it offers $q^*(\theta) = \tilde{q}(\theta, 1)$ to all types $\theta > \theta_2$.

A.7 Proof of Proposition 3 and its corollary

Consider now Proposition 3. When $2q^0 P'(2q^0) + P(2q^0) - C_q(2q^0; \bar{c}) < 0$, $q^M(\theta) < 2q^0$ for all θ. From Lemma 2, the supplier's profit is strictly decreasing in q, and hence the retailer is better off minimizing informational rents by setting the order of a supplier of type $\bar{c}$ as close as it can to $2q^0$. Therefore the IR constraint of at least a type $\theta = \bar{c}$

bind, and as the virtual surplus is convex and increasing in q for $\theta \geq \bar{\Gamma}$ this choice makes the retailer better off. When $\theta < \bar{\Gamma}$, the retailer must lower the quantity to respect the IR constraints as well as the local incentive compatibility constraints. As the retailer is better off setting $\Pi_U(2q^0; \theta) = 0$ for types in $[\min\{\bar{\Gamma}, \bar{c}\}, \bar{c}]$, the profit of the supplier must be positive for $q(\theta) < 2q^0$. Consequently $M(\theta) = 0$ for all $\theta < \bar{\Gamma}$. As the virtual surplus is concave for $\theta < \bar{\Gamma}$, and maximal at $\tilde{q}(\theta, 0)$, the retailer offers $q(\theta) = \tilde{q}(\theta, 0)$ to all $\theta < \bar{\Gamma}$.