Merge to monopoly in a vertically differentiated market with positional effects

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October 5, 2021

Abstract

We provide an alternative behavioral approach to the well-known argument that a merge to monopoly is desirable only when it is accommodated by a (supply-side) technology transfer and/or cost-reduction. When consumers are subject to positional effects, like status or envy, there are instances where a merge to monopoly may be welfare-enhancing due to a (demand-side) utility-increasing argument. Our results hold under different modes of product market competition or cost functions.

JEL Classification: L11; D11; D42.

Keywords: vertical differentiation; positional good; positional effect; status effect; envy effect; luxury goods market; merge to monopoly.

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*Skartados acknowledges financial support from AUEB's "Action II" Postdoctoral research grant EP-2991/01-01.

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1. Introduction

Harsanyi (1980) argues that: "*apart from economic payoffs, social status seems to be the most important incentive and motivating force of social behavior.*" The pursuit of social status may lead to the need of possessing goods, both as a medium of self-esteem and as a signal to society.[1] But this works the opposite as well. Possessions could trigger enviousness from those who have not, or have the worst, to those who have the best.

Research on possession of a good as a measure of social status goes back, at least to Veblen (1899) who introduced the term *conspicuous consumption*, i.e., the consumption of goods to publicly display economic power. This type of good is named "*positional good*".[2] Alongside Hirsch (1977), they argued that the consumption of such goods satisfies both utilitarian and social needs (i.e., status display).

In our analysis, we care not only for conspicuous consumption (what we call the *status effect*) but also for invidious consumption (what we call the *envy effect*). The latter refers to the consumption of goods to provoke envy of other people. The combination of these two effects form the *positional effect*: except for the status effect enjoyed by the owner of the positional good, there is also an envy effect on behalf of all those who don't own it, but they would like to have it (Parrott and Smith, 1993). As we will show later, this behavioral aspect of the consumers' utility could be as important in decision-making as the economic payoffs.

Even though there is an almost unanimous consent among prominent economists that these positional effects are important, they lack solid microeconomic foundations, while their market implications are somewhat still unclear (Grilo et al., 2001).

Our research scope is multi-fold. First, we wish to provide a characterization of a duopoly model of vertically differentiated goods in a market where consumers have positional effects. Through this, we wish to poke researchers to enrich the scarce literature on this subject, both in terms of empirical research and theoretical microfoundation. Second, given our analysis, we wish to provide some (initial) results that seem to contradict the literature. To do so, we use limit convergence and numerical methods.

In particular, and in contrast to the literature, which recommends technology transfer and cost-reduction as the main reasons for an antitrust authority to allow a merge to monopoly (see Farrell and

[1]As Levy (1959) quoted: "*People buy things not only for what they can do, but also for what they mean.*"
[2]Positional (or status) goods are a subset of economic goods whose consumption (and subsequent utility) depends negatively on the consumption of others (Vatiero, 2011).

Shapiro, 1990; Spector, 2003; Motta, 2004), we show that there are instances (non-empty sets of parameters) where the consumer surplus and total welfare is higher in case of monopoly compared to a duopoly. Therefore, a merge to monopoly in luxury goods markets should not be a priori blocked based solely on concentration indices and cost-reducing arguments. It should rather be scrutinized by antitrust authorities since it may be welfare-enhancing.

Moreover, and in contrast to existing literature (see Gabszewicz et al., 1986; Gabszewicz et al., 2016; Gabszewicz et al., 2017; Marini, 2018) our model exhibit instances where a monopoly may choose to produce both qualities (i.e., “high” and “low”). Interestingly, these two results apply in cases where the envy effect is very high compared to the status effect, which may be even zero. In other words, in both cases, it is the enviousness of the low-quality buyers and no-buyers that drives our results.

To model this, we consider a vertically differentiated goods market with two qualities, say “low” and “high”, where the consumers’ utility is the sum of two parts: an *intrinsic* and a *positional* part. The former accounts for the functionality, usability, and usefulness of the product. The latter derives from the *positional effect* of the product, which is a sum of two parts: a (positive) status effect enjoyed by the consumers having the high-quality product, and a (negative) envy effect felt by consumers having the low-quality product or even not buying at all.[3]

The rest of the paper is structured as follows. Section 2 offers a comprehensive literature review. Section 3 offers a description of the model. Section 4 deals with the analysis of the equilibrium results. Section 5 checks the robustness of our results by offering a short discussion over two possible extensions. Section 6 concludes. All proofs are relegated into the Appendix.

2. Literature review

Our analysis aims at luxury goods markets. Industry analysis indicates that its size exceeds \$1.2 trillion globally in 2018 (Bain & Company, 2018). Interestingly, the market has experienced more than 126 multi-billion-dollar mergers in Europe alone since 2000 many of which attracted a lot of attention from antitrust authorities (de Maurengnault, 2020).[4] This market has two characteristics. First, consumers buy

[3]Empirical evidence suggests that especially in urban youth, the material possessions of a high-status individual may strongly influence the preferences of the low-status surroundings (Shi and Xie, 2013).
[4]Recent examples: in October 2020 LVMH acquired Tiffany & Co for \$15.8 billion; in November 2020 VF Group acquired Supreme for \$2.1 billion; the same month, Moncler acquired Stone Island for \$2.1 billion.

goods with costly features that are hard to justify by their intrinsic value (Bagwell and Bernheim, 1996). And second, the retail price high mark-ups for such goods are hard to justify based only on the firms' market power (Feenstra and Levinshon, 1995).

Our paper also contributes to the growing literature on monopolistic competition between firms whose products are vertically differentiated by a single attribute. This literature is based on Mussa and Rosen (1978), Gabszewicz and Thisse (1979), and Shaked and Sutton (1982). Tirole (1988) offers a textbook treatment and many insights into such models.

We are in line with many papers considering the economic impact of consumption under behavioral considerations. Leibenstein (1950) shows that the demand for a positional good is decreasing in the number of consumers purchasing it. Regarding the idea that status may generate envy, Winkelmann (2012) shows that living in a neighborhood with a higher number of Ferrari's and Porsche's harms own income satisfaction. Bellet (2019) shows that new constructions of houses at the top of the house size distribution in a suburb, lower the satisfaction that neighbors derive from their house size.

In terms of modeling, we are closely related to two papers: Deltas and Zacharias (2018) and Karakosta and Zacharias (2021). The former analyzes the pricing strategy and the conditions under which positional considerations influence a monopoly to introduce a second variant of its product. The latter considers the impact of positional considerations on optimal tax and welfare when a monopoly produces one or two variants. Even though the two papers are close in terms of modeling utility, there are many differences in research scope and market structure. They both consider monopoly, and they care for product's variants. Our scope is to show that a merge to monopoly in the luxury goods market may be welfare-increasing therefore should be scrutinized and not a priori rejected by antitrust authorities.

Besides these two papers, several theoretical and empirical papers consider various positional considerations albeit without our modeling or research questions. Lambertini and Orsini (2005) model a vertically differentiated duopoly, where only the high-quality good has a (status) positional externality, while the low-quality good is non-positional. They compare the welfare performance of the duopoly against a social planner operating the same bundle of goods. Duopoly earns more from the high-quality good, while the social planner imposes a price premium (a quality tax) and acts independently of positional concerns. We deprive of their model by assuming both goods have status and envy effects.

Friedrichsen (2018), inspired by Bernheim (1994)'s model, explores quality provision, prices, and optimal product lines when consumers care for the social image attached to products. Under a monopoly setting, products have identical quality and differ only in price and social image. Under competition, average quality and market coverage are higher than monopoly. Our main difference is that her social image somewhat resembles our status effect, without having an envy effect to counterbalance. She also encounters instances (non-empty sets of parameters) where monopoly may produce more welfare compared to the competition. Grilo et al. (2001) deal with consumers' conformity or vanity in a spatial model of horizontal product differentiation and they highlight the importance of social attributes associated with a good's consumption.

In the empirical literature, Bursztyn et al. (2018) worked together with a large Indonesian bank distributing credit cards and showed that the demand for the fancy-looking platinum card exceeds the demand for the nondescript card with the same benefits. Transaction data revealed that platinum cards are more likely used in social contexts, implying social image motivations.

Lin et al. (2018) gathered data from Facebook to show that social media posting about experiential purchases (e.g., a traveling experience) is more frequent and it could trigger more envy compared to material purchases (e.g., a gadget with a price comparable to the travel). In a lab experiment, Banuri and Nguyen (2020) show that consumption increases when it is observable and signals status.

3. The model

The supply side of the market consists of two firms, denoted by the subscripts L and H, each constrained to offer only one of the two available qualities of a single product (which are exogenously determined). Firm L sells a product of low-quality $\sigma_L \geq 0$ at a price $p_L > 0$. On the other hand, firm H sells a product of high-quality $\sigma_H \geq \sigma_L \geq 0$ at a price $p_H > 0$.[5] We assume that the marginal cost for both firms is zero while each firm endures a fixed (*sunk*) quadratic cost of quality improvement $\frac{\sigma_i^2}{2}$ (which is paid in a pre-stage), i.e., we consider a "*pure*" vertically differentiated industry.[6] So, firms' *net* profits are:

[5]As in Shaked and Sutton (1982), we may impose an arbitrary ceiling on quality, i.e., $\sigma_i \in [0, \bar{\sigma}]$, where $\bar{\sigma}$ can be latter normalized to one.

[6]Shaked and Sutton (1983) define a pure vertically differentiated industry as one in which the cost of quality improvement is increasing and fall primarily into fixed costs. If there exists a unit variable cost of quality improvement, should be modest, or even equal to zero. Further, as Motta (1993) argues, when firms engage in R&D expenses or advertising activities to improve their quality, it is better to model these costs as quality-related fixed costs. As in Moorthy (1988), the quadratic form allows us to capture the idea that quality-related costs increase at a faster rate than any consumers willingness to pay.

$$\pi_i(p_L, p_H) = p_i\, q_i(p_L, p_H), \;\; i = L, H \tag{1}$$

where $q_i > 0$ is the quantity sold per firm. In case of a merge to monopoly, profits are:

$$\pi_M(p_L, p_H) = \sum_i p_i\, q_i(p_L, p_H) \tag{2}$$

The demand side of the market consists of a continuum of consumers, uniformly distributed in $[0, 1]$, each of a quality type ("taste") indexed by $\theta \in [0,1]$. A higher θ corresponds to a lower marginal utility of income and therefore a higher income (Tirole, 1988).[7] Consumers observe qualities and prices before deciding whether to buy or not.

As is standard in these models, we assume the existence of a marginal consumer positioned in θ_L who is indifferent between no-buying or buying the low-quality good. In the same spirit, we assume the existence of a marginal consumer positioned in θ_H who is indifferent between buying the low-quality or the high-quality good. Note that: $0 \leq \theta_L \leq \theta_H \leq 1$.

The *tweak* of our model is in the following lines. The consumers who don't buy anything incur a (pure) *envy effect* due to those who buy (no matter the quality). The utility function of the non-buyers is:

$$\mathcal{U}_0 = -\lambda_e\,(1 - \theta_L) \tag{3}$$

where $\lambda_e \geq 0$ is exogenous and shows the intensity of the envy effect.[8] In other words, the non-buyers experience a positive network effect (an increase in θ_L increases $\mathcal{U}_0$).[9]

The consumers who buy the low-quality product incur a *positional effect*, i.e., an envy effect due to those possessing the high-quality product, *and* a status effect over the no-buyers. The utility of a low-quality buyer of type θ is:

$$\mathcal{U}_L = \theta\,\sigma_L - p_L + \lambda_s\,\theta_L - \lambda_e\,(1 - \theta_H) \tag{4}$$

where $\lambda_s \geq 0$ is exogenous and it shows the intensity of the status effect.[10] The low-quality buyers experience a negative network effect over the no-buyers (an increase in θ_L reduces $\mathcal{U}_L$) and a positive

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[7]"Tastes" may be viewed as markets of social status because they constitute consumers' capacity to differentiate and appreciate these practices and products (Bourdieu, 1984).

[8]The non-buyers could be considered as buyers of a normalized good with zero quality σ_0 and price p_0.

[9]Our results remain qualitatively the same even if we assume $\mathcal{U}_0 \equiv 0$.

[10]Positional effects $\lambda_i, i = e, s$ are exogenously determined since they are influenced by individual's economic position, education, family background and socialization (see Bourdieu, 1984).

network effect over the high-quality buyers (an increase in $\theta_H - \theta_L$ increases $\mathcal{U}_L$). It is not a priori clear which effect dominates.

Finally, the consumers who buy the high-quality product incur a (pure) *status effect* over those possessing the low-quality product or not buying at all. They experience a negative network effect, since the less they are the more status they experience. The utility of a high-quality buyer of type θ is:

$$\mathcal{U}_H = \theta\, \sigma_H - p_H + \lambda_s\, \theta_H \tag{5}$$

Given the specifications of the model, firms' face the following demands: (i) for the low-quality firm $D_L = \theta_H - \theta_L$, and (ii) for the high-quality firm $D_H = 1 - \theta_H$. Consumers with $\theta \in [0, \theta_L)$ don't buy anything and have a demand equal to: $D_0 = \theta_L$ (i.e., the market is uncovered a priori). Obviously: $D_0 + D_L + D_H = 1$.

We assume that each consumer derives utility only from the first unit of the product she buys. So, demand equal quantity sold per firm:

$$D_i = q_i, i = L, H \tag{6}$$

Firms engage in a single-stage game with observable actions, where they simultaneously choose prices to maximize their respective profits. To solve the game, we use the Nash equilibrium solution concept over pure strategies.

4. Equilibrium analysis

The marginal consumer θ_L satisfies $\mathcal{U}_0 = \mathcal{U}_L$, while marginal consumer θ_L satisfies $\mathcal{U}_L = \mathcal{U}_H$. Solving the system of (3)=(4) and (4)=(5), we get marginal consumers as a function of prices:

$$\theta_L^*(p_L, p_H) = \frac{\lambda_e\,(\lambda_e - p_H) + p_L(\sigma_H - \sigma_L + \lambda_s)}{(\sigma_L - \lambda_e)\,(\sigma_H - \sigma_L - \lambda_e) + \lambda_s\,(\sigma_H - \lambda_e) + \lambda_s^2}$$

$$\theta_H^*(p_L, p_H) = \frac{(\sigma_L - \lambda_e)(p_L - p_H + \lambda_e) + \lambda_s(\lambda_e - p_H)}{(\sigma_L - \lambda_e)\,(\sigma_H - \sigma_L - \lambda_e) + \lambda_s\,(\sigma_H - \lambda_e) + \lambda_s^2} \tag{7}$$

An increase in p_L would move θ_L^* upward (since $\sigma_H > \sigma_L$), but it may also move θ_H^* downward if $\sigma_L > \lambda_e$. On the other hand, an increase in p_H would move θ_L^* downward (fewer non-buyers). Again, its effect on θ_H^* depends on the sign of $\sigma_L - \lambda_e$. Moving forward, we will examine the duopoly and monopoly cases separately.[11]

4.1 Duopoly

Each firm chooses its price p_i to maximize its net profits $\pi_i(p_L, p_H), i = L, H$. The first-order conditions give rise to the following reaction functions:

$$R_L^D(p_H) = \frac{(p_H-\lambda_e)(\sigma_L+\lambda_s)}{2\,(\sigma_H+\lambda_s-\lambda_e)}$$

$$R_H^D(p_L) = \frac{(p_L+\sigma_H-\sigma_L)(\sigma_L-\lambda_e)+\sigma_H\,\lambda_s+\lambda_s^2}{2\,(\sigma_L+\lambda_s-\lambda_e)} \tag{8}$$

The low-quality firm's reaction function is always upward sloping. But for $\sigma_L < \lambda_e$ the high-quality firm's reaction function may be downward sloping. That is, when the envy effect is relatively strong, the high-quality firm finds it optimal to reduce its price. Solving the system of these two reaction functions, we get the equilibrium prices:

$$p_L^D = \frac{1}{\phi}\left((\sigma_L-\lambda_e)\,(\sigma_H-\sigma_L-2\,\lambda_e)+(\sigma_H-2\,\lambda_e)+\lambda_s^2\right)$$

$$p_H^D = \frac{1}{\phi}\Big((\sigma_L-\lambda_e)(2\,\sigma_H(\sigma_H-\lambda_e)-\sigma_L(2\,\sigma_H-\lambda_e))+\ \lambda_s(2\,\sigma_H^2-2\sigma_L^2-4\,\sigma_H\lambda_e+\lambda_e^2+ \\ \sigma_L\,(2\,\sigma_H+\lambda_e))+2\,\lambda_s^2\,(2\,\sigma_H-\lambda_e)+2\,\lambda_s^3\Big) \tag{9}$$

where: $\phi = (\sigma_L-\lambda_e)(4\,\sigma_H-\sigma_L-4\,\lambda_e)+(3\,\sigma_L+4\,\sigma_H-7\,\lambda_e)+4\,\lambda_s^2$.

Substituting (9) back to (1) and (6), we get the respective equilibrium results. Equilibrium profits are stated in the Appendix. There are constellations of parameters where the duopoly finds negative demands. Due to the algebraic complexity of the equilibrium values, these loci could not be explicitly stated in a closed-form formula. Instead, we use numerical methods to illustrate them graphically. This allows us to have a qualitative view of the duopoly's permissible areas.

Consider the following **Figure 1**. On the horizontal axis, we measure the envy effect λ_e and in the vertical axis, we measure the status effect λ_s. As in Motta (1993), we set $\mu = \frac{\sigma_L}{\sigma_H}$ in the equilibrium results to measure the (relevant) distance between qualities. Since $0 \leq \sigma_L \leq \sigma_H$, then by construction $\mu \in [0,1]$.

[11]Case III of Motta (1993) could be resembled as a benchmark case of our model with $\lambda_e = \lambda_s = 0$.

Figure 1's shaded areas are loci where both duopolists have positive demands and profits. We represent three such loci, for $\mu = 0.1, 0.5$, and 0.9. On the contrary, the blank areas represent loci where one of the duopolists suffers from negative demand and/or profits.

Interestingly, as the quality difference shrinks (i.e., $\mu \to 1$), the market can hold both firms for a wider variety of positional effects. Therefore, an increase in quality difference (i.e., $\mu \to 0$) may hinder the foreclosure of one duopolist. We can qualitatively state that our numerical approximations show that in most of the cases it is the low-quality firm that faces the market exit first. Consider the following numerical example: let $\mu = 0.9$, $\lambda_e = 1$, and $\lambda_s = 2$. Duopoly's equilibrium profits are: $\pi_L^D = -0.3228$ while $\pi_H^D = 0.6385$. All in all, these qualitative results rely on both the finiteness property (Gabszewicz and Thisse, 1980) (i.e., the number of firms with non-negative profits is bounded) and the behavioral effect caused by both status λ_s and envy λ_e (i.e., a relative increase in envy effect, most often, drives the low-quality firm out of the market).

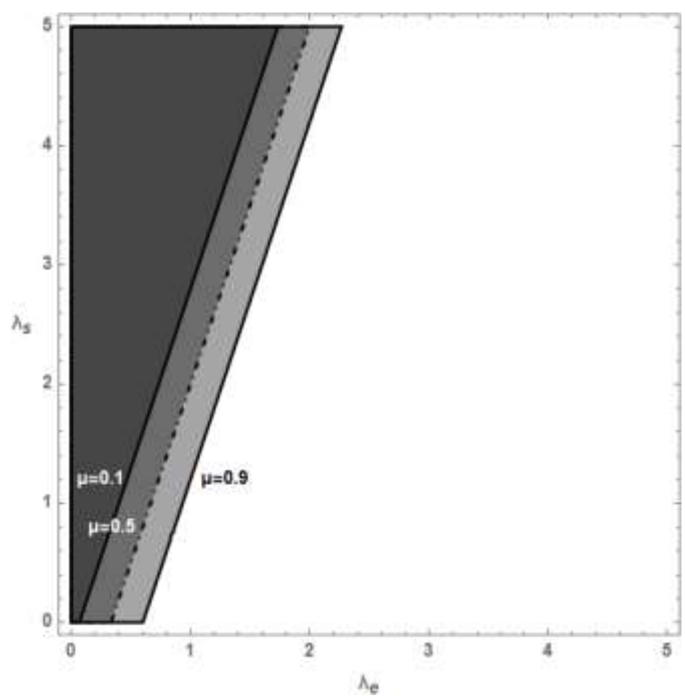

Figure 1: Grey areas are permissible, while the blank area is non-permissible for the duopoly, for various values of $\mu = \frac{\sigma_L}{\sigma_H} \in [0,1]$.

Since closed-form comparative statics are cumbersome, we rely on limit behavior to capture the effect of envy or status on demands/quantities and prices. In this case, limits are useful since, by definition, there must be a (non-infinite) real number $\varepsilon > 0$ such as for every $\lambda_i > \varepsilon, i = e, s$ limit converges. Therefore, we are entitled to claim that **Lemma 1**, as the other limit Lemmas that follow, holds for a large set of arbitrary (non-infinite) real numbers. The proof is omitted since it requires basic algebra.

Lemma 1. *(i)* $\lim_{\lambda_e \to +\infty} D_0^D = 1$, $\lim_{\lambda_e \to +\infty} D_i^D = 0$, while $\lim_{\lambda_e \to +\infty} p_L^D = \frac{\sigma_L + \lambda_s}{2}$, $\lim_{\lambda_e \to +\infty} p_H^D = \frac{1}{4}(2\,\sigma_H - \sigma_L + \lambda_s)$, and $\lim_{\lambda_e \to +\infty} \pi_i^D = 0, i = L, H$.

(ii) $\lim_{\lambda_s \to +\infty} D_0^D = \lim_{\lambda_s \to +\infty} D_L^D = \frac{1}{4}$, $\lim_{\lambda_s \to +\infty} D_H^D = \frac{1}{2}$, while $\lim_{\lambda_s \to +\infty} p_i^D = +\infty$, and $\lim_{\lambda_s \to +\infty} \pi_i^D = +\infty, i = L, H$.

Lemma 1 deals with two different extreme situations, where either the envy (case i) or the status effect (case *ii*) go to infinity (i.e., are *very* high compared to the other model parameters).

Consider case (*i*). The existence of a very strong envy effect denies both low- and high-quality consumers from buying any product at all, even if their prices are "relatively reasonable" (in the sense that they are proportional to their respective qualities). Firms have zero profits. What we experience here is a market failure due to the behavioral negative externality (the envy effect). People are afraid of the jealousness felt by the ones below them on the income ladder, so they decide not to buy at all. The very strong envy effect cancels out any positive utility enjoyed by the consumers due to the consumption of any product.

Finally, it is easy to show that: $\lim_{\lambda_e \to +\infty} \pi_H^D - \pi_L^D = 0$, and $\lim_{\lambda_s \to +\infty} \pi_H^D - \pi_L^D = +\infty$. So, when the duopolists exhibit a very high envy effect, firms' profits are equal, so they could not be ranked as their qualities. This contrasts the literature, e.g., Gabszewicz et al. (1986), where it shown that being located farther up in the quality range is more profitable since profitability rises with quality (Donnenfeld and Shlomo, 1992). But, when the status effect is high, the high-quality firm is ranked higher in profitability compared to the low-quality firm.

Case (*ii*) interpretation is based on real-world observations. A "simple" Patek Philippe Nautilus wristwatch could cost from \$20,000 to \$30,000, while some "special" editions could cost as high as \$200,000. On the other hand, the best-selling Casio Illuminator series wristwatch cost from \$10 to \$20. The intrinsic value of both watches could not be far since they both have comparable usability and functionality (e.g., are durables, wrist-worn, waterproof, and, above all, can tell the time). The premium paid for a Patek Philippe Nautilus is more than a thousand times higher (i.e., very high) of a Casio

Illuminator just to account for the status effect provided to the owner of the first. In other words, you pay a thousand times more expensive watch to get a thousand times more status. This explains $\lim_{\lambda_s \to +\infty} p_H^D = +\infty$, and $\lim_{\lambda_s \to +\infty} \pi_H^D = +\infty$.

Besides the high-income consumers, there is a status effect felt by the low-income consumers who bought the low-quality good over the non-buyers. In this case, the ones having a cheap car are obviously in the worst position from those who own a luxurious car but are in far better condition than those who don't have a car at all. So, the cheap car owners may have paid say $10,000 for a small city car, which is tenths of a thousand times more than zero (the price of the non-buyers). This explains $\lim_{\lambda_s \to +\infty} p_L^D = +\infty$, and $\lim_{\lambda_s \to +\infty} \pi_L^D = +\infty$.

4.2 Merge to monopoly

Consider the case of firms' merging to monopoly. In this case, the monopolist should have *net* profits equal to: $\pi^M(p_L, p_H) = p_L\ q_L(p_L, p_H) + p_H\ q_H(p_L, p_H)$. As in the duopoly case, the monopolist has a quadratic fixed (*sunk*) cost of quality improvement $\frac{\sigma_L^2+\sigma_H^2}{2}$ which is paid in a pre-stage. Maximizing π^M with respect to p_L and p_H we get the following system of reaction functions:

$$R_L^M(p_H) = \frac{p_H\ (2\ \sigma_L - \lambda_e + \lambda_s) - \lambda_e(\sigma_L + \lambda_s)}{2\ (\sigma_H + \lambda_s - \lambda_e)}$$

$$R_H^M(p_L) = \frac{1}{2}\left(p_L + \sigma_H - \sigma_L + \lambda_e + \lambda_s + \frac{p_L\ \sigma_L - \sigma_L\ \lambda_e + \lambda_e^2}{2\ (\sigma_L + \lambda_s - \lambda_e)}\right) \tag{10}$$

Solving the system of these two reaction functions, we get the equilibrium prices:

$$p_L^M = \frac{1}{\psi}\ ({\sigma_L}^2(2\sigma_H + \lambda_s - \lambda_e) - \sigma_L(\lambda_s - \lambda_e)(3\sigma_H - \lambda_s + 2\lambda_e) + (\lambda_s - \lambda_e)(\sigma_H(\lambda_s - \lambda_e) + (2\lambda_s - \lambda_e)\lambda_e) - 2{\sigma_L}^3)$$

$$p_H^M = \frac{1}{\psi}(2{\sigma_H}^2(\lambda_e - \lambda_s) - (\lambda_e - 2\lambda_s)(\lambda_e - \lambda_s)\lambda_s + 2\sigma_L^2(\sigma_H + \lambda_s) - 2\sigma_H({\lambda_e}^2 - 2\lambda_e\lambda_s + 2\lambda_s^2) + \sigma_L(-2\sigma_H^2 - 2\sigma_H\lambda_s + \lambda_e(\lambda_e + \lambda_s))) \tag{11}$$

where: $\psi = 4\sigma_L(\sigma_H - \sigma_L) + (\lambda_s - \lambda_e)\big(4\ \sigma_H - 3(\lambda_s - \lambda_e)\big)$.

As in the duopoly case, substituting (11) back to (1) and (6) we get the equilibrium results. Profits are stated in the Appendix. **Lemma 2** summarizes some limit results. The proof is straightforward therefore omitted.

Lemma 2. *(i)* $\lim_{\lambda_e \to +\infty} D_0^M = 1$, $\lim_{\lambda_e \to +\infty} D_i^M = 0$, *while* $\lim_{\lambda_e \to +\infty} p_L^M = \frac{1}{3}(\sigma_L + \sigma_H + 2\,\lambda_s)$, $\lim_{\lambda_e \to +\infty} p_H^M = \frac{1}{4}(2\,\sigma_H - \sigma_L + \lambda_s)$, *and* $\lim_{\lambda_e \to +\infty} \pi^M = 0$.

(ii) $\lim_{\lambda_s \to +\infty} D_0^M = \lim_{\lambda_s \to +\infty} D_i^M = \frac{1}{3}$, *while* $\lim_{\lambda_s \to +\infty} p_i^M = +\infty$, *and* $\lim_{\lambda_s \to +\infty} \pi^M = +\infty, i = L, H$.

As in the duopoly case, **Lemma 2** characterizes results close to real-world examples. When the envy effect is very strong (case i), the monopolist faces zero demand and has zero profits. Even if both prices are "relatively reasonable", there are no consumers buying anything at all. Note that these prices are higher compared to the ones in **Lemma 1** case (*i*), i.e., $\lim_{\lambda_e \to +\infty} p_i^D < \lim_{\lambda_e \to +\infty} p_i^M, i = L, H$.

On the other hand, when the status effect is very strong (case *ii*), consumers are segmented in thirds: one third does not buy anything, one third buys the low-quality product and the last third buys the high-quality product. Comparing with **Lemma 1** case (*ii*), we readily observe that, in monopoly, the non-buyers and the low-quality buyers increase at the expense of the high-quality buyers.

What we derive from both **Lemmas 1** and **2**, is that the effect of very high status or envy on market outcomes is not qualitatively affected by the market structure. In both monopoly and duopoly, firms qualitatively endure the same issues, while the envy effect may lead to market failure.

4.3 Main findings

Given the equilibrium results of sections **3.1** and **3.2**, our next steps are the following. First, we show that there are infinite uncountable sets of parameters $\{\sigma_L, \sigma_H, \lambda_e, \lambda_s\}$ where the monopolist finds it optimal to be multiproduct. In the same spirit, we show that a merge to monopoly is beneficial for both duopolists. And finally, for the same type of sets, we show that consumer surplus and total welfare are higher under monopoly. **Lemma 3** summarizes the first of them. Since the algebraic treatment of the equilibrium results is cumbersome, in **Lemma 3**'s proof we provide counterexamples and use numerical methods. The proof is relegated in the Appendix.

Lemma 3. *There exist an infinite uncountable set of parameters* $\{\sigma_L, \sigma_H, \lambda_e, \lambda_s\}$ *where the monopolist finds it optimal to be multiproduct.*

Lemma 3 highlights that behavioral considerations (as the positional ones in our model) may facilitate the arise of (an otherwise absent) equilibria. It is well documented that when the range of consumers' income is narrow compared to the products' quality range, the monopolist finds it optimal to bunch all consumers on the top-quality product (Gabszewicz et al., 1986).[12] Further, a merge to monopoly always finds it more profitable to prune some quality variants, especially the lower ones (Gabszewicz et al., 2017). What we show here is that when consumers have behavioral considerations as the ones in our model, then the anticipated behavior is not always realized. When consumers are subject to status or envy effect(s), there exists an infinite uncountable set of model's exogenous parameters where the monopolist may find it optimal to sell both qualities. Therefore, bunching is subject to behavioral considerations.

For $\lambda_e = \lambda_s = 0$ our results are in line with established literature, i.e., the monopolist always chooses to be a single product, serving only half of the market with the high-quality. But **Lemma 3** still holds for $\lambda_e > 0$ and $\lambda_s = 0$. So, it is not the status effect that drives this result. In the case of a single product monopolist, the enviousness felt by the no-buyers is so high that forces the monopolist to be multiproduct. By introducing an intermediate category of consumers (above the no-buyers and below the high-quality buyers), the monopolist relieves and effectively increases his profits.

Interestingly, for various constellations of the model's parameters, the monopolist may find it optimal to serve only the low- or high-quality or even both. Consider **Figure 2**. Note that $\sigma_S = 0.5$. The grey areas represent a constellation of parameters where the multi-product monopolist is more profitable than the single product counterpart (for non-negative demands and prices). As the quality difference shrinks (i.e., $\mu \to 1$) the monopolist finds it profitable to be multi-product for an even wider area of parameters. Note that grey areas are exclusively where $\lambda_s > \lambda_e$. Therefore, a strong envy effect hinders quality variation.

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[12]Two important notes are in line regarding this result. First, depends on the finiteness property. And second, it holds for strictly positive lower income.

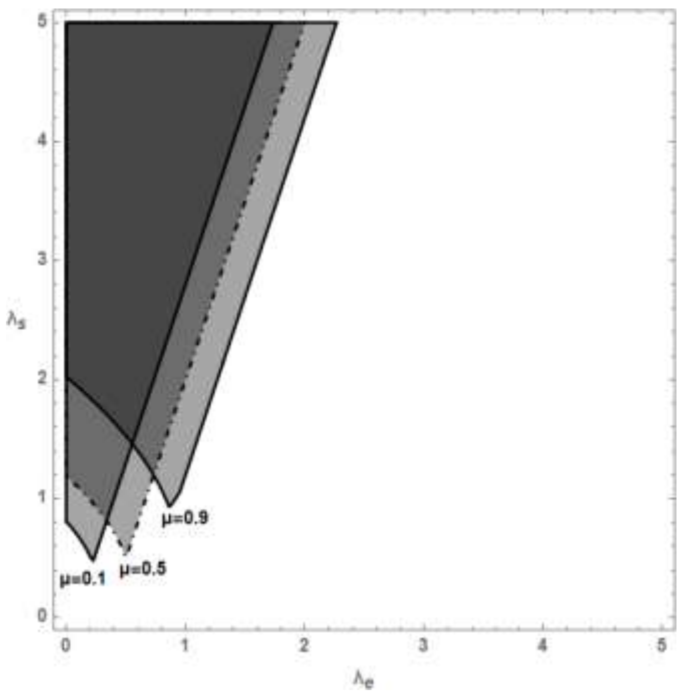

Figure 2: In grey areas, the multi-product monopolist is more profitable than the single product, for various values of $\mu = \frac{\sigma_L}{\sigma_H} \in [0,1]$.

Having these in mind, it is meaningful to move to our next step. Since a multiproduct monopolist may exist (for a wide range of parameters), **Lemma 4** characterizes the incentives of the duopolists to merge. Again, the proof is in the Appendix.

Lemma 4. *There exist an infinite uncountable set of parameters $\{\sigma_L, \sigma_H, \lambda_e, \lambda_s\}$ where a merge to a multiproduct monopolist is optimal for both duopolists.*

Several notes are in line regarding **Lemma 4**. As Barbot (2001) summarizes: (*i*) it is always profitable for the duopolists to merge to monopoly, and (*ii*) when this merge happens, the high quality always expels the low quality leading to a single product monopolist. In contrast, we show here that (*a*) it is not always profitable to merge to monopoly, and (*b*) even if this happens, it is not always optimal to expel the low quality. Again, for these remarks to hold, it is sufficient to have $\lambda_e > 0$ even for $\lambda_s = 0$, i.e., it is the envy effect that drives these results as well.

For the welfare analysis, set consumer surplus CS to be set equal to $\int_0^{\theta_L} \mathcal{U}_0 d\theta + \int_{\theta_L}^{\theta_H} \mathcal{U}_L d\theta + \int_{\theta_H}^{1} \mathcal{U}_H d\theta$. Social or total welfare SW is equal to the sum of consumer and producer surpluses. The equilibrium expressions are in the Appendix. The following **Proposition 1** wraps up previous Lemmas and delivers our main result.

Proposition 1. *(i) There exists an infinite uncountable set of parameters $\{\sigma_L, \sigma_H, \lambda_e, \lambda_s\}$ where a merge to a (multiproduct) monopoly leads to increased consumer surplus and social welfare.*

(ii) $\lim_{\lambda_e \to +\infty} \frac{CS^D}{CS^M} = \lim_{\lambda_e \to +\infty} \frac{SW^D}{SW^M} = \frac{3(\sigma_L + 2\sigma_H + 3\lambda_s)}{4(\sigma_L + \sigma_H + 2\,\lambda_e)} > 1$,

and $\lim_{\lambda_s \to +\infty} \frac{CS^D}{CS^M} = \frac{9(5\,\sigma_L + 4\,(\sigma_H - 6\,\lambda_e))}{16(3\,\sigma_L + \sigma_H - 13\,\lambda_e)}$, *while* $\lim_{\lambda_s \to +\infty} \frac{SW^D}{SW^M} = \frac{15}{16}$.

What **Proposition 1** states that for some constellations of the model parameters, there are instances where the multiproduct monopoly may produce higher consumer surplus compared to a duopoly, i.e., there are instances where the following inequality may hold $CS^M > CS^D$.

From an inspection of the equilibrium CS (and consequently, SW) values stated in the Appendix, it is obvious that no closed-form solution could be derived for inequality $CS^M > CS^D$. Once again, we must rely on numerical methods to export qualitative results. First, is easy to see that for $\lambda_e = \lambda_s = 0 \Rightarrow CS^M < CS^D$ always and for any pair of qualities $0 \le \sigma_L \le \sigma_H$. The same holds for social welfare SW. Moreover, for $\lambda_e = 0$ and $\lambda_s > 0$ the ratio $\frac{CS^D}{CS^M} > 1$ always, albeit this does not hold for SW. For $\lambda_e > 0$ and $\lambda_s = 0$ the ratio is unbounded (can take any real value).

Note that $\lim_{\lambda_e \to +\infty} \frac{CS^D}{CS^M} = \lim_{\lambda_e \to +\infty} \frac{SW^D}{SW^M} = \frac{3(\sigma_L + 2\sigma_H + 3\lambda_s)}{4(\sigma_L + \sigma_H + 2\lambda_e)}$ is bounded in $[\frac{9}{8}, \frac{3}{2}]$ therefore $\lim_{\lambda_s \to +\infty} \frac{CS^D}{CS^M} > 1$ and $\lim_{\lambda_s \to +\infty} \frac{SW^D}{SW^M} > 1$ for any arbitrary qualities. Therefore, enviousness favors duopoly in the sense that in a state of very high envy effect, the duopoly may produce more consumer surplus and social welfare compared to a multiproduct monopoly.

Further, $\lim_{\lambda_s \to +\infty} \frac{CS^D}{CS^M}$ is unbounded in the sense that this limit may take values above or below 1. To show that the set of values where $\lim_{\lambda_s \to +\infty} \frac{CS^D}{CS^M} < 1$ is non-empty, consider the following numerical example. Let $\sigma_L = 0.5$, $\sigma_H = 0.9$, $\lambda_e = 3$, and $\lambda_s = 22.78$. It is easy to show that $CS^M = -2.10783 >$

$-2.15653 = CS^D$ (demands and prices are strictly positive).[13] Further, for this set of values $SW^M = 6.8218 > 6.2959 = SW^D$. The same inequalities hold for various values of λ_s like, e.g., 39.82, 58.78, or 97.17. So, for a high status effect, a multiproduct monopoly may produce more consumer surplus and social welfare compared to a duopoly.

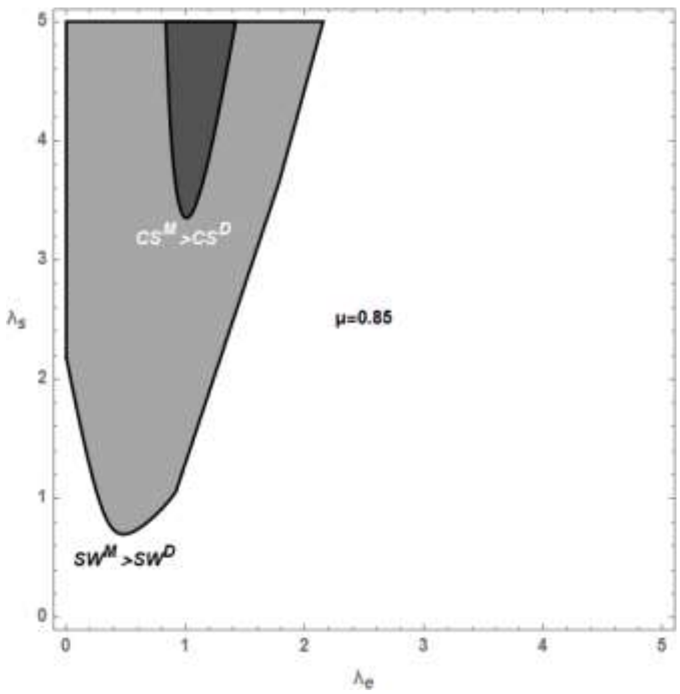

Figure 3: Area where the consumer surplus CS^M and social welfare SW^M produced by a multiproduct monopolist is higher compared to the one produced by a duopoly CS^D, SW^D respectively.

Figure 3 provides an illustrative example of **Proposition 1**. Consider the case of $\mu = 0.85$. For λ_e roughly in $(0.8, 1.4)$ and for $\lambda_s > 3.4$ we get the shaded area where the consumer surplus produced by the multiproduct monopolist is higher compared to the one produced by a duopoly. For this area, the multiproduct monopolist's profits are higher compared to the sum of the duopolists' profits, plus we get positive demands. As a numerical example, set: $\sigma_L = 0.85$, $\sigma_H = 1$, $\lambda_e = 1$, and $\lambda_s = 3.5$, to get: $CS^M = -0.5158 > -0.5168 = CS^D$, $SW^M = 1.1518 > 1.1029 = SW^D$, $D_i^M = \{0.1992, \; 0.3947, \; 0.4060\}$, $p_i^M = \{1.26, 3.13\}$, $p_i^D = \{0.82, 2.32\}$, and $D_i^D = \{0.1361, 0.2335, 0.6303\}$, for $i = 0, L, H$.[14]

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[13]Negative consumer surplus is just a matter of normalization. If we consider utilities to be $U_i = V + \mathcal{U}_i, i = 0, \; L,$ H where $V > 0$ selected accordingly, then $CS_{new}^j = V + CS^j, j = D, M$. So $CS_{new}^M = V - 2.10783 > V - 2.15653 = CS_{new}^D$.

[14]Note that there are instances (particularly when the envy effect is high) where consumer surplus turns out to be negative (keeping all other positive). This is mainly an issue of normalization, i.e., by setting utility equal to $U_i =$

5. Extensions

To check the robustness of our results, we consider the following extensions: (*i*) firms compete in quantities in the product market, and (*ii*)firms have variable costs. The other characteristics of the base model remain the same.

5.1 Quantity competition

By inverting the system of demands $D_i = D_i(p_L, p_H) = q_i, i = L, H$ we get the following prices as a function of quantities:

$$p_L = (1 - q_L - q_H)\,(\sigma_L + \lambda_s) + q_L\,\lambda_e$$

$$p_H = (\lambda_e - \lambda_s - \sigma_H)q_H + \lambda_s + \sigma_H + (\lambda_e - \sigma_L)q_L \qquad (12)$$

For known reasons, quantity competition will not affect the equilibrium results of the monopolist. Duopoly profits are: $\pi_i^Q = p_i\,q_i, i = L, H$. Again, firms endure a quadratic (sunk) fixed cost of quality improvement $-\frac{\sigma_i^2}{2}$, set in a pre-stage. Maximizing profits with respect to the quantity we get the following reaction functions: $R_L^Q = \frac{(1-q_H)(\sigma_L+\lambda_s)}{2\,(\sigma_L-\lambda_e+\lambda_s)}$, and $R_H^Q = \frac{\sigma_H+\lambda_s-q_L(\lambda_e-\sigma_L)}{2\,(\sigma_L-\lambda_e+\lambda_s)}$. Solving this system of equations, we get the equilibrium quantities:

$$q_L = \frac{(\sigma_L + \lambda_s)(\sigma_H - 2\,\lambda_e + \lambda_s)}{(\sigma_L - \lambda_e)(\sigma_L - 4\,\sigma_H + 4\,\lambda_e) - 4\,\lambda_s^2 - (3\,\sigma_L + 4\,\sigma_H - 7\,\lambda_e)\lambda_s}$$

$$q_H = \frac{\sigma_L^2 + 2\,\sigma_H(\lambda_e - \lambda_s) + (\lambda_e - 2\,\lambda_s)\lambda_s - \sigma_L(2\,\sigma_H + \lambda_e + \lambda_s)}{(\sigma_L-\lambda_e)(\sigma_L-4\,\sigma_H+4\,\lambda_e)-4\,\lambda_s^2-(3\,\sigma_L+4\,\sigma_H-7\,\lambda_e)\lambda_s} \qquad (13)$$

Substituting (13) back to (12) and to π_i^Q, we get the equilibrium expressions of duopoly's profits under quantity competition.[15] As in the base model, the extraction of closed-form solutions is cumbersome. Therefore, once again, we rely on numerical methods. Consider the following quadruplet: $\sigma_L = 0.5$, $\sigma_H = 0.9$, $\lambda_e = 3$, and $\lambda_s = 15.191$. Duopoly's demand and profits are: $q_L = 0.2249$, $q_H = 0.6360$, $\pi_L^Q = 0.5173$, and $\pi_H^Q = 4.8913$. So, the no-buyers are $1 - q_L - q_H = 0.1389$. Consumer

$V + \mathcal{U}_i, i = 0, L, H$. In our case, V is normalized to zero. But, for an appropriate (infinite) set of $V > 0$, consumer surplus is always positive.

[15]Consumer surplus (or any other) equilibrium expression is available by the authors upon request.

surplus is: $CS_Q^D = -2.31688 < -2.29743 = CS^M$, while a merge to monopoly may be profitable for the duopolists since: $\pi^M - \pi_L^Q - \pi_H^Q = 0.5408$. As in the base model, since $\pi^M > \pi_L^Q - \pi_H^Q$, by setting total welfare to be equal to the sum of consumer surplus and producer surplus, we get that for the same quadruplet of parameters a merge to monopoly may be welfare-enhancing as well. The same qualitative results also hold for, *at least*, the following values $\lambda_s = \{33, 48.4, 73.6, 99.2\}$. So, by providing few counterexamples, we have shown that even under quantity competition, there are instances where a merge to monopoly may increase welfare.

5.2 Variable costs

In this extension, consider the case where firms experience quadratic variable costs of quality improvement, e.g., $\pi_i^{DV} = \left(p_i^V - \frac{\sigma_i^2}{2}\right) q_i^V, i = L, H$. Monopolist's profits are: $\pi_V^M = \sum \left(p_i^V - \frac{\sigma_i^2}{2}\right) q_i^V$. Following a similar methodology as in the base model, we derive the equilibrium results (which are available upon request). Consider the quadruplet $\sigma_L = 0.5$, $\sigma_H = 0.9$, $\lambda_e = 3$, and $\lambda_s = 22.393$. Duopoly's demand and profits are: $DD_0^V = 0.2154$, $D_L^{DV} = 0.2438$, $D_H^{DV} = 0.5406$, $\pi_L^{DV} = 1.351$, and $\pi_H^{DV} = 6.772$. Monopolist's demand and profits are: $D_0^{MV} = 0.2462$, $D_L^{MV} = 0.3814$, $D_H^{MV} = 0.3722$, and $\pi_V^M = 8.6028$. Consumer surplus is: $CS_V^D = -2.1412 < -2.0914 = CS_V^M$, while a merge to monopoly may be profitable for the duopolists since: $\pi_V^M - \pi_L^{DV} - \pi_H^{DV} = 0.4793$. The total welfare argument holds. The same qualitative results also hold for, *at least*, the following values $\lambda_s = \{36.311, 54.188, 73.696, 97.393\}$. Once again, by providing a set of counterexamples, we have shown that even under quadratic variable costs of quality improvement, there are instances where a merge to monopoly may increase welfare.

6. Conclusions

We consider a market where consumers have behavioral considerations over two vertically differentiated products. We care for both cases of a duopoly and a multiproduct monopoly.

We show that, in contrast to existing literature, going multiproduct may be a profit-maximizing choice for the monopolist. Further, a merge to monopoly may or may not be optimal for the duopolists, according to the level of the positional effects. Finally, we show that, under some constellation of the model's parameters, the monopoly may produce more welfare (in terms of consumer surplus) compared to a duopoly.

To check the robustness of our results, we consider few extensions of our base model. First, we considered quantity competition in the product market. And second, we considered quadratic variable costs of quality improvement. In both cases, we were able to provide several counterexamples where a merge to monopoly may increase welfare.

The empirical literature backs our theoretical results. People do buy fancy-looking products just to show off (Bursztyn et al., 2018), while envy considerations seem to be more important than status considerations, at least in social media (Lin et al., 2018). All in all, empirical literature on the economic impact of behavioral considerations like envy or status is scarce. Therefore, as a suggestion for future research, we could argue the need for more data-driven arguments which we hope to further back our theoretical results.

Appendix

Proof of ***Lemma 3***. First, we need to characterize the equilibrium results of the single-product monopolist. Consider the case of a monopolist serving only one quality in the market, say $\sigma_S > 0$. All consumers are bunched in this single quality.

The no-buyers' utility is: $U_N^S = -\lambda_e\,(1-\theta_S)$. The buyers' utility is: $U_B^S = \theta_S\,\sigma_S - p_S + \lambda_s\,\theta_S$, where p_S is the price of the product. Marginal consumer θ_S satisfies $U_N^S = U_B^S \Rightarrow \theta_S = \frac{p_S-\lambda_e}{\sigma_S+\lambda_{s-\lambda_e}}$. So, monopolist's demand and profits are: $D_B^S = q_S = 1-\theta_S$ and $\pi_S = p_S\,q_S$ respectively. The single-product monopolist has a fixed (sunk) quadratic cost of quality improvement $\frac{\sigma_S^2}{2}$ determined in a pre-stage. Maximizing profits over price we get: $p_S = \frac{\sigma_S+\lambda_s}{2}$, $D_B^S = q_S = \frac{\sigma_S+\lambda_s}{2\,(\sigma_S+\lambda_s-\lambda_e)}$ and $\pi_S = \frac{(\sigma_S+\lambda_s)^2}{4\,(\sigma_S+\lambda_s-\lambda_e)}$. Note that for a non-negative demand we need the assumption $\sigma_S+\lambda_s > \lambda_e$.

Comparative statics lead to: $\frac{\partial p_S}{\partial \lambda_e} = 0$, $\frac{\partial \pi_S}{\partial \lambda_e} > 0$, and $\frac{\partial D_B^S}{\partial \lambda_e} > 0$. An increase in λ_e increases the disutility of those who do not purchase and makes consumers more willing to buy the product. As a result, demand, and profits increase. On the other hand, the monopoly price does not change as the envy effect increases, as those who buy are not affected by the envy effect.

On the other hand, $\frac{\partial p_S}{\partial \lambda_s} > 0$, and $\frac{\partial D_S}{\partial \lambda_s} < 0$ but $\frac{\partial \pi_S}{\partial \lambda_s} > 0$ if and only if $\lambda_s + \sigma_S > 2\,\lambda_e$. So, an increase in the status effect λ_s increases the price that consumers are willing to pay for the good. Since the buyers' willingness to pay increases with respect to the status effect, the monopolist finds it profitable to sell to fewer consumers as λ_s increases. The monopolist experiences higher profits when the utility of the intrinsic value and the status effect of those who buy are sufficiently higher than the envy effect.

Limit behavior leads to: $\lim_{\lambda_e \to +\infty} D_N^S = 1$, $\lim_{\lambda_e \to +\infty} D_B^S = 0$, $\lim_{\lambda_e \to +\infty} p_S = +\infty$, and $\lim_{\lambda_e \to +\infty} \pi_S = 0$. When the envy effect goes to "infinity" (i.e., becomes very high), the monopolist charges an "infinite" (very high) price to counteract the severe consumers' enviousness. This leads to zero sales and consequently to zero profits, given that the quality-related fixed costs are considered as sunk. As the envy effect diminishes so does price. So, there could be consumers buying the good and the monopolist starts enjoying positive profits.

Similarly, $\lim_{\lambda_s \to +\infty} D_N^S = \frac{1}{2}$, $\lim_{\lambda_s \to +\infty} D_B^S = \frac{1}{2}$, $\lim_{\lambda_s \to +\infty} p_S = +\infty$, and $\lim_{\lambda_s \to +\infty} \pi_S = +\infty$. As the status effect goes to "infinity", half the market is covered. Even though prices are very high ("infinite"), this does not deprive consumers of buying because they care for the status effect entitled to the good. This leads to extremely high ("infinite") profits for the monopolist.

We will now move on to proving Lemma 3. The equilibrium analysis of section 3.2 gives rise to the following equilibrium profits for the multiproduct monopolist.

$$\pi^M = \frac{\phi_0 + [\sigma_L^2 + (\sigma_L + \sigma_H)(\lambda_e - \sigma_H)]\,\lambda_s + [-2\,\sigma_H + \lambda_e]\,\lambda_s^2 - \lambda_s^3}{(\lambda_e - \lambda_s)(4\,\sigma_H - 3\lambda_e + 3\lambda_s) - 4\,\sigma_L\,(\sigma_H - \sigma_L)}$$

where: $\phi_0 = \sigma_H^2\,\lambda_e - \sigma_L(\sigma_H - \sigma_L)(\sigma_H + \lambda_e)$.

The extraction of closed-formed solutions is cumbersome due to the functional complexity of the equilibrium results. Therefore, we will restrict our attention to numerically showing the existence of a “wide” area of the model’s exogenous parameters where the monopolist may find it better (in terms of profitability) to sell both qualities. For example, consider the following constellation of parameters: $\sigma_L = 0.5$, $\sigma_H = 0.9$, $\lambda_e = 2$, and $\lambda_s = 5$. Note that for all $\sigma_S \in \mathbb{R}$ the following inequality holds: $\pi^M > \pi_S$, i.e., the multiproduct monopolist gains more profits compared to the single product monopolist. Moreover, it is easy to show that this result holds for all $\lambda_s > 5$, so there can be no injective function of the set $\{\sigma_L, \sigma_H, \lambda_H, \lambda_e\}$ to the set of natural numbers, therefore the former is infinite and uncountable.

This inequality does not hold for high envy effect $\lim_{\lambda_e \to +\infty} \pi^M - \pi_S = 0$, but it may hold for a high status effect $\lim_{\lambda_s \to +\infty} \pi^M - \pi_S = +\infty$. Note that for this constellation of parameters demands and prices are non-negative ($D_0^M = 0$, $D_1^M = D_2^M = \frac{1}{2}$, $p_1^M = 1$, $p_2^M = 4.7$, $D_N^S = 0.2058$, $D_B^S = 0.7942$, $p_S = 2.7$) ■

Proof of ***Lemma 4****.* The equilibrium profits of the multiproduct monopolist are stated in the Proof of Lemma 3. The equilibrium profits of the duopolists emerge from section 3.1.

$$\pi_L^D = \frac{1}{\omega_1}\left[(\sigma_L + \lambda_s)^2\,(\lambda_s + \sigma_H - \lambda_e)\big(\lambda_s^2 + (\sigma_H - 2\lambda_e)\lambda_s + (\lambda_e - \sigma_L)(\sigma_L + 2\,\lambda_e - \sigma_H)\big)^2\right]$$

$$\begin{aligned}\pi_H^D = \frac{1}{\omega_1}\,\big[(\lambda_e - \sigma_L \\ &+ \lambda_s)({\sigma_L}^2(-2\sigma_H + \lambda_e - 2\lambda_s) + 2\sigma_H^2(\lambda_e - \lambda_s)\ - 2\sigma_H\big({\lambda_e}^2 - 2\lambda_e\lambda_s + 2{\lambda_s}^2\big) \\ &- \lambda_s\big({\lambda_e}^2 - 2\lambda_e\lambda_s + 2{\lambda_s}^2\big) - \sigma_L(2{\sigma_H}^2 + 2\sigma_H\lambda_s + \lambda_e(\lambda_s - \lambda_e)))^2\big]\end{aligned}$$

where: $\omega_1 = \big((\sigma_L - \lambda_e)(\sigma_L - 4\,\sigma_H + 4\,\lambda_e) + \lambda_s\,(7\,\lambda_e - 3\,\sigma_L - 4\,\sigma_H) - 4\,\lambda_s^2\big)^2\,\big((\sigma_L - \lambda_e)\,(\sigma_L - \sigma_H + \lambda_e) + \lambda_s\,(\lambda_e - \sigma_H) - {\lambda_s}^2\big)$.

Due to its algebraic complexity, it is cumbersome to get a closed-form solution for the inequality $\pi^M > \pi_L^D + \pi_H^D$. But we could use counterexamples to show that there are instances where the multiproduct monopolist may have more profits compared to the sum of profits of the two duopolists. Respectively, there might be instances where the latter argument does not hold.

Using the same quadruplet as in the proof of Lemma 3 above, i.e., $\sigma_L = 0.5$, $\sigma_H = 0.9$, $\lambda_e = 2$, and $\lambda_s = 5$ (which leads to non-negative demands and prices), we get: $\pi^M - \pi_L^D - \pi_H^D = 0.1793 > 0$. It is easy to show that this result holds for every $\lambda_s > 2$. Therefore, there is an infinite uncountable set of parameters $\{\sigma_L, \sigma_H, \lambda_e, \lambda_s\}$ where $\pi^M > \pi_L^D + \pi_H^D$.

On the other hand, for $\sigma_L = 0.5$, $\sigma_H = 0.9$, $\lambda_e = 5$, and $\lambda_s = 2$ we get: $\pi^M - \pi_L^D - \pi_H^D = -0.2859 < 0$. Again, this holds for all λ_s in $(0,4]$. So, there is an infinite uncountable set of parameters $\{\sigma_L, \sigma_H, \lambda_e, \lambda_s\}$ where $\pi^M < \pi_L^D + \pi_H^D$.

Limit behavior is: $\lim_{\lambda_e \to +\infty} \pi^M - \pi_L^D - \pi_H^D = 0$, while $\lim_{\lambda_s \to +\infty} \pi^M - \pi_L^D - \pi_H^D = +\infty$. That is, when the envy effect is high, the difference between π^M and $\pi_L^D + \pi_H^D$ reaches zero, making the merge-to-monopoly decision indifferent to both duopolists. Note that it is sufficient to set $\lambda_e = 1{,}000$ (other parameters remain the same) to get $\pi^M - \pi_L^D - \pi_H^D = -0.00002$. But when the status effect is very high, duopolists may find it profitable to merge to monopoly, since the monopolist's profits are higher compared to the sum of the duopolists' profits ■

*Proof of **Proposition 1**.* The equilibrium values of the consumer surplus in cases of a multiproduct monopolist and a duopoly are the following.

$$
\begin{aligned}
CS_M = {} & (4\,\sigma_L^4\,(\sigma_H - 2\,\lambda_e) - 4\,\sigma_L^3\,(2\,(\sigma_H - \lambda_e)^2 + \lambda_s^2) + \sigma_L^2\,(4\,\sigma_H^3 - 8\,\sigma_H^2\,\lambda_s - \sigma_H\,(9\,\lambda_e^2 - 34\,\lambda_e\,\lambda_s \\
& + \lambda_s^2) + 2\,(\lambda_e - \lambda_s)\,(6\,\lambda_e^2 - 11\,\lambda_e\,\lambda_s + \lambda_s^2)) + (\lambda_e - \lambda_s)\,(4\,\sigma_H^3\,(\lambda_e - \lambda_s) \\
& + 12\,\lambda_e\,\lambda_s(\lambda_e - \lambda_s)^2 + \sigma_H\,(\lambda_e - \lambda_s)\,(6\,\lambda_e^2 - 34\,\lambda_e\,\lambda_s + \lambda_s^2) - 4\,\sigma_H^2\,(2\,\lambda_e^2 - 7\,\lambda_e\,\lambda_s \\
& + \lambda_s^2)) + \sigma_L\,(8\,\sigma_H^3\,(\lambda_s - \lambda_e) + 3\,(\lambda_e - \lambda_s)^2\,(2\,\lambda_e^2 + 2\,\lambda_e\,\lambda_s + \lambda_s^2) - 2\,\sigma_H\,(\lambda_e \\
& - \lambda_s)\,(10\,\lambda_e^2 - 9\,\lambda_e\,\lambda_s + 3\,\lambda_s^2) + \sigma_H^2\,(17\,\lambda_e^2 - 34\,\lambda_e\,\lambda_s + 5\,\lambda_s^2)))\;/\;(2\,(4\,\sigma_L^2 \\
& - 4\,\sigma_L\,\sigma_H + (\lambda_e - \lambda_s)(4\,\sigma_H - 3\,\lambda_e + 3\lambda_s)^2))
\end{aligned}
$$

$$
CS_D = \frac{-\lambda_e}{144}\left(9 - \frac{4\lambda_e^2}{-(\sigma_L-\lambda_e)(\sigma_L-\sigma_H+\lambda_e)+(\sigma_H-\lambda_e)\lambda_s+\lambda_s^2} - \frac{9\sigma_L^2+3\sigma_L(\lambda_e+3\lambda_s)+\lambda_e(12\sigma_H-20\lambda_e+15\lambda_s)}{(\sigma_L-\lambda_e)(\sigma_L-4\sigma_H+4\lambda_e)+(-3\sigma_L-4\sigma_H+7\lambda_e)\lambda_s-4\lambda_s^2}\right)
$$

$$
\left(3 + \frac{4\lambda_e^2}{(\sigma_L-\lambda_e)(\sigma_H-\sigma_L-\lambda_e)+(\sigma_H-\lambda_e)\lambda_s+\lambda_s^2} + \frac{9\sigma_L^2+3\sigma_L(\lambda_e+3\lambda_s)+\lambda_e(12\sigma_H-20\lambda_e+15\lambda_s)}{(\sigma_H-\lambda_e)(\sigma_L-4\sigma_H+4\lambda_e)+(-3\sigma_L-4\sigma_H+7\lambda_e)\lambda_s-4\lambda_s^2}\right)
$$

$$
+[(\sigma_L+\lambda_s)(\sigma_H-\lambda_e+\lambda_s)(\sigma_L^2-2\lambda_e^2+\sigma_L(-\sigma_H+\lambda_e)+\sigma_H(\lambda_e-\lambda_s)+2\lambda_e\,\lambda_s-\lambda_s^2)]
$$

$$
\frac{1}{(\sigma_L^2-4\lambda_e^2+\sigma_L(-4\sigma_H+3\lambda_e-3\lambda_s)+4\,\sigma_H(\lambda_e-\lambda_s)+7\lambda_e\lambda_s-4\lambda_s^2)^2\left(-\sigma_L^2+\sigma_L\,\sigma_H+\lambda_e^2-\lambda_e\lambda_s+\lambda_s^2+\sigma_H(\lambda_e-\lambda_s)\right)^2}
$$

$$
\begin{aligned}
&\Big[\Big(\sigma_L^5 + \sigma_L^4\,(\lambda_e - 2\sigma_H) \\
&\qquad + \sigma_L^3\left(\sigma_H^2 - 2\lambda_e^2 + \lambda_e\lambda_s - 2\lambda_s^2 - \sigma_H\,(\lambda_e + 2\lambda_s)\right)\sigma_L^2\left(2\sigma_H^2\,\lambda_s\lambda_e\,(-3\lambda_e^2 + 6\lambda_e\lambda_s - 4\,\lambda_s^2)\right. \\
&\qquad \left. + \sigma_H(6\lambda_e^2 - 5\,\lambda_e\,\lambda_s + 2\,\lambda_s^2)\right) \\
&\qquad + \sigma_L\left(3\,\lambda_e^4 - 4\lambda_e^3\,\lambda_s + 3\,\lambda_e^2\,\lambda_s^2 - \lambda_e\,\lambda_s^3 + \lambda_s^4 + \sigma_H^2\,(-3\,\lambda_e^2 + 3\,\lambda_e\,\lambda_s + \lambda_s^2)\right. \\
&\qquad \left. - \sigma_H(\lambda_e^3 + 2\,\lambda_e^2\,\lambda_s - 2\,\lambda_e\,\lambda_s^2 - 2\,\lambda_s^3)\right) \\
&\qquad + \lambda_e\left(\sigma_H^2\,(2\,\lambda_e^2 - 5\,\lambda_e\,\lambda_s + 3\,\lambda_s^2) + \lambda_s(-3\,\lambda_e^3 + 7\lambda_e^2\,\lambda_s - 7\lambda_e\,\lambda_s^2 + 3\,\lambda_s^3)\right. \\
&\qquad \left. + \sigma_H(-2\,\lambda_e^3 + 9\,\lambda_e^2\,\lambda_s - 12\,\lambda_e\,\lambda_s^2 + 6\,\lambda_s^3)\right)\Big)\Big]
\end{aligned}
$$

$$
+\frac{1}{6}\left(3 - \frac{2\lambda_e\,(\sigma_L-\lambda_e+\lambda_s)}{(\sigma_L-\lambda_e)(\sigma_L-\sigma_H+\lambda_e)+(\lambda_e-\sigma_H)\lambda_s-\lambda_s^2} - \frac{3\sigma_L^2+\lambda_e\,(\lambda_s-4\lambda_e)+\sigma_L(\lambda_e+3\,\lambda_s)}{(\sigma_L-\lambda_e)(\sigma_L-4\,\sigma_H+4\,\lambda_e)+(-3\,\sigma_L-4\,\sigma_H+7\,\lambda_e)\lambda_s-4\,\lambda_s^2}\right)
$$

$$
[\left(\frac{(-\sigma_L+\lambda_e)\left(2\,\sigma_H\,(\sigma_H-\lambda_e)+\sigma_L(\lambda_e-2\,\sigma_H)\right)-\left(-2\,\sigma_L^2+2\,\sigma_H^2-4\,\sigma_H\lambda_e+\lambda 1^2+\sigma_L(2\,\sigma_H+\lambda_e)\right)\lambda_s+2\,(\lambda_e-2\,\sigma_H)\lambda_s^2-2\,\lambda_s^3}{-(\sigma_L-\lambda_e)(\sigma_L-4\,\sigma_H+4\,\lambda_e)-(-3\,\sigma_L-4\,\sigma_H+7\,\lambda_e)\lambda_s+4\,\lambda_s^2}\right)
$$

$$
+\frac{\lambda_s}{6}\left(3 + \frac{2\,\lambda_e\,(\sigma_L-\lambda_e+\lambda_s)}{(\sigma_L-\lambda_e)(\sigma_L-\sigma_H+\lambda_e)+(-\sigma_H+\lambda_e)\lambda_s-\lambda_s^2} + \frac{3\,\sigma_L^2+\lambda_e\,(-4\lambda_e+\lambda_s)+\sigma_L(\lambda_e+3\lambda_s)}{(\sigma_L-\lambda_e)(\sigma_L-4\sigma_H+4\lambda_e)+(-3\sigma_L-4\sigma_H+7\lambda_e)\lambda_s-4\lambda_s^2}\right)]
$$

$$+\sigma_H\,[\frac{1}{2}-\frac{1}{72}\left(3+\frac{2\lambda_e(\sigma_L-\lambda_e+\lambda_s)}{(\sigma_L-\lambda_e)(\sigma_L-\sigma_H+\lambda_e)+(\lambda_e-\sigma_H)\lambda_s-\lambda_s^2}+\frac{3\sigma_L^2+\lambda_e(\lambda_s-4\lambda_e)+\sigma_L(\lambda_e+3\lambda_s)}{(\sigma_L-\lambda_e)(\sigma_L-4\sigma_H+4\lambda_e)+(-3\sigma_L-4\sigma_H+7\lambda_e)\lambda_s-4\lambda_s^2}\right)^2]$$

$$+\frac{\sigma_L}{288}[4\left(3+\frac{2\lambda_e(\sigma_L-\lambda_e+\lambda_s)}{(\sigma_L-\lambda_e)(\sigma_L-\sigma_H+\lambda_e)+(\lambda_e-\sigma_H)\lambda_s-\lambda_s^2}+\frac{\left(3\sigma_L^2+\lambda_e(\lambda_s-4\lambda_e)+\sigma_L(\lambda_e+3\lambda_s)\right)}{(\sigma_L-\lambda_e)(\sigma_L-4\sigma_H+4\lambda_e)+(-3\sigma_L-4\sigma_H+7\lambda_e)\lambda_s-4\lambda_s^2}\right)^2$$

$$-\left(3+\frac{4\lambda_e^2}{-(\sigma_L-\lambda_e)(\sigma_L-\sigma_H+\lambda_e)+(\sigma_H-\lambda_e)\lambda_s+\lambda_s^2}+\frac{9\sigma_L^2+3\sigma_L(\lambda_e+3\lambda_s)+\lambda_e(12\sigma_H-20\lambda_e+15\lambda_s)}{(\sigma_L-\lambda_e)(\sigma_L-4\sigma_H+4\lambda_e)+(-3\sigma_L-4\sigma_H+7\lambda_e)\lambda_s-4\,\lambda_s^2}\right)^2]$$

Moreover, we set total or social welfare to be $SW^D = CS^D + \pi_L^D + \pi_H^D$ under duopoly, and $SW^M = CS^M + \pi^M$ under monopoly. The algebraic complexity of the equilibrium values would not allow us to conclude in a closed-form solution. Therefore, we must rely on numerical methods to extract some qualitative results. The fact that the set of parameters $\{\sigma_L, \sigma_H, \lambda_e, \lambda_s\}$ where the following inequalities hold (not necessarily at the same time) $CS^M > CS^D$ and $SW^M > SW^D$ is infinite and uncountable comes from a simple inspection of **Figure 3**. Moreover, if we set $\sigma_L = 0.85$, $\sigma_H = 1$, and $\lambda_e = 1$ we get that these inequalities hold for all $\lambda_s > 3.3531$, i.e., an infinite and uncountable set of quadruplets $\{0.85, 1, 1, \lambda_s\}$.

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