Prices as signals of product quality in a duopoly

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Abstract

In a duopoly model of horizontal and vertical differentiation, where consumers are ex-ante unaware of product qualities, we study the firms' incentives to signal quality via prices. Consumers, after they observe prices, can evaluate a firm's product quality before purchase if they incur a search cost. We show that a complete information (undistorted) separating equilibrium and a unique pooling equilibrium (in pure strategies) exist. A lower search cost moves the market equilibrium from pooling to separating and induces a mean-preserving spread in the distribution of the equilibrium prices.

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JEL classifications: D8, L13.

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1 Introduction

In markets where consumers are ex-ante unaware of product qualities, we examine rival firms' incentives to signal quality through prices. Consumers form beliefs about product qualities after they observe prices and can learn a firm's product quality before purchase if they incur a search cost. For example, consumers can physically visit a firm's store and learn the quality of the product. The search cost involves the time spent and the effort exerted by a consumer to 'visit' another store in order to assess the quality of the product for sale in that store. Alternatively, a consumer may be shopping online and the search cost is the time and effort associated with ordering online a second product from a different online retailer if the first product is of lower than expected quality. For some products a careful inspection may be enough to assess quality, e.g., clothes, furniture. Many other products offer free trials to entice consumers to make a purchase. Free trials are costly for consumers, in terms of opportunity cost, but can facilitate quality discovery before a final purchase is made.[1]

Our main departure from the literature is the assumption that consumers can assess a firm's product quality before purchase, provided they incur a search cost. Most of the existing literature has assumed that consumers may discover the quality of the product only after purchase (see Section 1.1 for a literature review). This assumption is certainly appropriate for credence or experience goods, for example, but there are many markets where quality can be evaluated before purchase. Our paper fills this gap.

We develop a model of horizontal and vertical differentiation with two rival firms. The product quality of each firm is a random variable whose realization, independent across firms, can be either high or low. Hence, both firms can be of low quality, or of high quality, or one firm of high and the other of low quality. The product quality realizations are common knowledge between firms, but consumers are only aware of the distribution. Consumers observe prices, form beliefs about the product qualities and decide which firm to 'visit' first. After a visit, a consumer discovers the firm's product quality, updates his beliefs about the product quality of the rival firm and decides whether to visit the other firm or not. A visit to the other firm entails a fixed (search) cost.

Our model best fits markets where product innovations are frequent, not deterministic, and so it is not easy for a firm to communicate to consumers its realized product quality at any given time. Moreover, prices are easily observable by consumers and quality can be assessed before consumers make a final purchase if they exert a reasonable effort.[2]

[1] Software and premium network TV channels routinely offer free trials. More recently, Carvana, an online used car retailer, known for its multi-story Car Vending Machines, offers a "seven day test to own", giving buyers the option to return a vehicle within seven days if they are not satisfied with it.

[2] As an example, consider the market for cellular phones. Innovations are frequent, e.g., chips (CPU), cameras, screens, fingerprint, 5G, battery. The realization of the innovations is not deterministic as there are many new technologies that may not work as well as expected (for example, new chips may be faster, but also hotter and reduce the length of battery). It is not easy for a firm to communicate to consumers its realized product quality, because sometimes the innovation is not easy to be felt by consumers, for example, the CPU may be faster by 20%, but for an average consumer, he/she cannot feel that the cellphone is significantly faster. Moreover, quality can be assessed by consumers before they make a final purchase if they exert a reasonable effort. Usually, producers of cellphone have a preview before introducing a new product (for example, Apple always has a preview before a new iPhone is introduced), and many professionals may write "test reports" sharing their feeling

We show that firms can signal product quality with the complete information prices. Hence, no price distortions arise in a separating equilibrium. This observation is a well–known implication of the unprejudiced belief refinement (see Bagwell and Ramey (1991)). Given the multi-sender nature of the game in an oligopoly market, consumers anchor their beliefs about product qualities on the price of the non-deviating firm. Hence, it becomes much more difficult to sustain an equilibrium that does not involve the complete information prices. Our main focus in this paper is on the conditions that guarantee the existence of a separating (and a pooling) equilibrium, in pure strategies, and on how an equilibrium is affected by consumer search costs. Existence is more likely when: i) the difference between high and low quality is higher, ii) the market is more competitive (lower transportation cost), iii) the search cost is lower and iv) the probability of high quality is higher.

We also show that a unique pooling equilibrium exists, provided that search cost is high, relative to the quality differential. The prices in the pooling equilibrium are the expected prices in the separating equilibrium. A lower search cost moves the market equilibrium from pooling to separating and induces a mean-preserving spread of the equilibrium prices. Therefore, the only effect of a lower search cost is on price dispersion and not on average price. In fact, lower search cost increases price dispersion.

It would be useful to compare our main finding with traditional search models, where consumers are mainly searching for the lowest price. The relationship between search costs and price dispersion in these models can be positive, negative or even non-monotonic (see Baye et al. (2006) for a review of the literature). In Rob (1985), for example, lower search cost (in the form of a bigger mass of consumers with low search costs) decreases price dispersion. Chandra and Tappata (2011) present a theoretical search model where price dispersion can increase or decrease with search costs, but the empirical evidence from gasoline markets suggests that it is decreasing as search costs decrease. Consistent with this finding, Dahlby and West (1986) show that car insurance premiums are less dispersed for the class of drivers who are associated with lower search costs. In these models price dispersion is entirely due to search costs related to price discovery, while in our model consumers observe prices but not product qualities and price dispersion is due to both quality differences and search costs. When search costs are high, firms pool their prices together, quality differences are 'masked', and hence there is no price dispersion, but when search costs are low quality differences dominate and drive price dispersion.

Finally, dissipative advertising is an ineffective way to signal high quality (see our discussion in Section 6).

1.1 Literature review

Signaling product quality or cost through prices is an important issue in industrial organization. The fundamental question is what kind of prices can credibly signal that a firm has a high quality product or a low cost. The first classification is whether there is a single signal sender (monopoly) of multiple possible types or many signal senders (oligopoly), each of multiple possible types.

in using the new cellphone. So for consumers, if they make an effort to look at the preview or "test reports", they can assess the quality of the new good before buying it. In addition, consumers can spend time at a store inspecting a new cellphone.

In a monopoly market a separating equilibrium entails price (or advertising) distortions (high price signals high quality and low price signals low cost), e.g., Wilson (1980), Milgrom and Roberts (1982), Milgrom and Roberts (1986), Bagwell and Riordan (1991), Linnemer (2002), Orzach et al. (2002) and Pires and Catalão-Lopes (2011).

In an oligopoly market, which is our main focus, there are additional classifications:

- the private information can be common among signal senders (i.e., types are common knowledge among firms) or not
- the distribution of types across firms can exhibit some correlation or can be independent
- beliefs can be refined or not and if they are refined the type of refinement
- signal receivers (consumers) can learn the true quality after purchase, or some consumers are informed before purchase
- in addition to prices, advertising (usually dissipative) can be used to signal quality
- competition among signal senders (firms) can feature horizontal differentiation or it can be a pure vertical differentiation model
- signal receivers can or cannot observe individual (firm) behavior.

In an oligopoly market, separating prices may or may not be distorted, if distorted they can be upward or downward distorted, depending on the information firms have about the types of the rival firms, on the beliefs refinement and whether advertising is used to signal quality or not. A difference with monopoly is that the receivers of the signal can utilize the signals of the non-deviating players to draw credible inferences.

Wolinsky (1983) considers a model where competing firms can signal quality through prices, and consumers can observe the quality of a firm before purchase at a cost. Unlike our findings, he shows that, in a separating equilibrium, firms distort their prices upwards to signal quality. He assumes that each firm can observe only its own quality, whereas we consider common private information and considers small search costs, whereas our search cost ranges from zero to very high values. Finally, he does not search for a pooling equilibrium.

Bagwell and Ramey (1991) introduce a limit pricing model, with common private information among (incumbent) firms, and the unprejudiced belief refinement and find that only non-distorted separating equilibria exist. Further, under additional assumptions, the intuitive criterion of Cho and Kreps (1987) eliminates all equilibria with pooling. Schultz (1999) considers a variation of Bagwell and Ramey (1991) and, again, separating equilibrium prices are not distorted. Bester and Demuth (2015) in a common private information setting, with horizontal and vertical differentiation, assumes that some consumers are informed about an entrant's product quality (the rest learn the quality after purchase) and shows that a separating equilibrium exists when the fraction of informed consumers is high. The equilibrium must entail the complete information prices, a result of unprejudiced beliefs. They do not analyze pooling equilibria. In our paper, private information is common among firms and we use the unprejudiced belief refinement. Consumers, however, are all aware of a firm's product quality before a purchase provided they incur a search cost. Otherwise, a purchase is not possible. As in the above papers, in our model a separating equilibrium, if it exists, must entail the complete information prices. But also a unique pooling equilibrium exists.

Harrington (1987) and Orzach and Tauman (1996) consider a homogeneous good market with multiple incumbents and one (or more) potential entrant. In the first paper it is the high-cost firm who tries to prove its "weakness", while in the second, the signal sent by the low cost contestants, though costly, is also quite rewarding: it increases their payoff level. The difference with Bagwell and Ramey (1991) is that in Harrington (1987) and Orzach and Tauman (1996) the entrant does not observe individual behavior. In our model individual behavior is observable, since products are differentiated and consumers observe prices, which results in an undistorted separating equilibrium.

Hertzendorf and Overgaard (2001) study a duopoly model of vertical differentiation where consumers learn a firm's product quality after purchase, with perfectly negative correlated product qualities and common private information among firms. Advertising can also be used to signal product quality. They use a restricted unprejudiced beliefs refinement and show that a distorted (upward or downward) separating equilibrium exists. Complete information prices cannot be an equilibrium. In a similar to the Hertzendorf and Overgaard (2001) model, Hertzendorf and Overgaard (2000), show that fully revealing separating equilibria satisfying the unprejudiced belief refinement do not exist. Yehezkel (2008) extends the Hertzendorf and Overgaard (2001) model by allowing some consumers to be informed. When the fraction of informed consumers exceeds a threshold the separating equilibrium entails the complete information prices. Pooling prices are distorted upwards. Mirman and Santugini (2019) analyze a monopoly model with a competitive fringe and a fraction of uninformed buyers about the monopolist's product quality. The fringe is necessary for the existence of a unique, fully revealing equilibrium. In our model a low search cost is needed for the existence of a separating equilibrium. A low search cost increases the lost market share of a firm that tries to wrongly signal high quality, much like a high fraction of informed consumers does in Yehezkel (2008) and Bester and Demuth (2015) or a sizeable competitive fringe in Mirman and Santugini (2019). Moreover, our pooling prices are not distorted, they are actually the expectation of the separating equilibrium prices.

In Fluet and Garella (2002), consumers cannot observe quality before purchase and the ex ante distribution of the firms' qualities is such that either both firms offer low quality or one firm offers low and the other high quality. Private information is common among firms. The authors avoid the use of selection criteria and find multiple separating and pooling equilibria. For small quality differences separation can only be achieved with a combination of upward distorted prices and advertisement.

Daughety and Reinganum (2008) study a model of horizontal and vertical differentiation where quality is each firm's private information and consumers discover product quality after purchase. Firms signal high quality via higher prices, so a separating equilibrium entails a distortion. Janssen and Roy (2010), in a duopoly model where each firm's product quality can be high or low and information about it is private, show that even when there is no horizontal differentiation, there exist symmetric fully revealing equilibria, where the low quality firm randomizes over an interval of prices, while the high quality firm sets a high price. Both types of firms may exhibit considerable market power.

In sum, all relevant papers assume that consumers learn the product quality after purchase, except for Yehezkel (2008) and Bester and Demuth (2015) who assume that a fraction of consumers is informed about product quality even before observing any prices. Our assumption about product quality discovery before purchase if a cost is incurred makes the model, the markets to which our model applies, the comparative statics, the empirical and policy/managerial implications different (see our discussion in Section 6).

2 The model

There is a continuum of consumers uniformly distributed on the $[0, 1]$ Hotelling line and two firms, $i = a, b$, located at the two endpoints of the interval (a at 0 and b at 1). We denote the rival firm by j. Consumers incur a linear transportation cost and buy at most one unit of the good from one firm. The maximum each consumer is willing to pay depends on the quality of the product and is denoted by V_a and V_b. Hence, the indirect utility of the consumer located at x is $V_i - td_x - p_i$, where $t > 0$ is the per-unit of distance transportation cost, p_i is the price and d_x is consumer x's linear distance to the firm. We assume consumers know the products' prices, but are unaware of firms' product qualities. Following the literature, we assume that qualities are random variables whose realizations are only observed by the firms. Consumers form expectations about a firm's product upon observing the products' prices and they learn a firm's product quality once they visit a firm (store) and inspect the product. As is commonly assumed in consumer search models, and without loss of generality, a first visit to a firm's store is costless, but a visit to the second firm entails a cost $\kappa > 0$. The horizontal locations of the two firms on the Hotelling line are common knowledge. We assume that V_a, V_b are independently drawn from a two point distribution, V_H and V_L, where $\Pr(V_i = V_H) = q$. We assume that unit costs are the same across qualities and normalized to zero.

We make the following assumptions regarding the model parameters.

Assumption 1 *V_L is sufficiently high.*

Assumption 2 *$V_H - V_L < 3t$.*

The first assumption ensures that the market is covered as the marginal consumer has a nonnegative utility. The second assumption ensures that both firms, when product qualities are asymmetric, have positive market shares.

The timing of the game is as follows:

1. Nature draws, independently, the product qualities of both firms. Consumers do not observe the quality realizations, but these become common knowledge to the firms.

2. Firms choose their prices and consumers observe them.

3. Each consumer visits costlessly one store and decides whether to make a purchase or to incur search cost κ and visit the second store. If a consumer visits the second store, he can costlessly return and buy from the first.

We will search for a perfect Bayesian Nash equilibrium in pure strategies. In particular, we are interested in each firm's pricing decision and whether prices can serve as signals of product qualities. All proofs are in the Appendix.

2.1 Complete information prices

As a benchmark, we derive the complete information equilibrium. It can be easily shown that the complete information (CI) prices are given by

$$(p_a^{CI}, p_b^{CI}) = \left(\frac{V_a - V_b}{3} + t, -\frac{V_a - V_b}{3} + t\right). \tag{2.1}$$

3 Preliminaries

Since consumers are ex-ante unaware of product qualities, consumer beliefs play a key role. There are three different kind of beliefs: *ex-ante*, that is before consumers visit any firm (but after they have observed the prices), *interim*, that is after a consumer visits one firm, in which case he learns the product quality of the firm he visited and updates his belies about the product quality of the rival firm and *final*, that is after a consumer has visited both firms, in which case the consumer knows the true product qualities of both firms. In a separating equilibrium, the ex-ante beliefs are correct and so all three beliefs coincide, but in a pooling equilibrium, or out-of-equilibrium, this need not be true.

Let $\mu_a^e(p_a, p_b)$ be the consumers' ex-ante belief that firm a has a high quality product and $\mu_b^e(p_a, p_b)$ be the belief that firm b has a high quality product, as a function of the observed prices. Also, let $\mu_{ai}^{in}(p_a, p_b)$ be the interim (in) belief of the consumers who have visited firm i that firm a has a high quality product and $\mu_{bi}^{in}(p_a, p_b)$ be the interim belief of the consumers who have visited firm i that firm b has a high quality product, where $i = a, b$, as a function of the observed prices. The interim beliefs can differ between two consumers depending on which firm a consumer visited first. In other words, the belief, held by consumers who have visited firm a, that firm a has a high quality product, $\mu_{aa}^{in}(p_a, p_b)$, may not be equal to the same belief held by consumers who visited firm b first, $\mu_{ab}^{in}(p_a, p_b)$. Clearly, since consumers learn perfectly a firm's product quality after they visit the firm, $\mu_{ii}^{in}(p_a, p_b) = 1$ if $V_i = V_H$ and $\mu_{ii}^{in}(p_a, p_b) = 0$ if $V_i = V_L$. Also, let $m^e(p_a, p_b)$ be the probability that both firms are of high quality. Given the strategic interaction between the two firms, beliefs can be correlated, that is $m^e(p_a, p_b) \neq \mu_a^e(p_a, p_b)\mu_b^e(p_a, p_b)$.

Consumers observe two signals, that is, two prices. When consumers observe an out-of-equilibrium price pair, which is the case when a firm unilaterally deviates, we assume that they correctly believe that a firm has unilaterally deviated. These are the unprejudiced beliefs, which is a natural assumption in this multi-sender environment. The price of the non-deviating firm can, in many cases, provide useful information about the qualities of both firms. For example, suppose that when one firm has a high and the other a low quality product the equilibrium is separating $(\hat{p}_H, \hat{p}_L)$ and suppose consumers instead observe $(p', \hat{p}_L)$ with $p' \neq \hat{p}_H, \hat{p}_L$. Consumers assign probability one that one firm has deviated and will use $\hat{p}_L$ to eliminate product quality profiles that are inconsistent with this price and unilateral deviations. This line of reasoning will be used in the derivations of the equilibria.

Let $\overline{V} \equiv qV_H + (1-q)V_L$ be the expected quality of a firm's product before any prices are observed, $\Delta V \equiv V_a - V_b$ be the true difference between the firms' product qualities and $DV \equiv V_H - V_L$ the difference between high and low quality. Note that there are three possibilities in our model: i) firms have the same product qualities, $\Delta V = 0$, ii) firm a is the high and firm b the low quality firm, $\Delta V = DV$ and iii) firm b is the high and firm a the low quality firm, $\Delta V = -DV$. Also let $\Delta V^e(p_a, p_b) \equiv EV_a - EV_b = (\mu_a^e V_H + (1-\mu_a^e)V_L) - (\mu_b^e V_H + (1-\mu_b^e)V_L)$ be the ex-ante belief of consumers about the difference in the product qualities, after prices are observed.[3] So, after prices are observed, the marginal consumer is given by

$$x^e = \frac{p_b - p_a + \Delta V^e + t}{2t}. \tag{3.1}$$

Let $\Delta V_a^{in}(p_a, p_b) \equiv V_a - EV_b^{in} = V_a - (\mu_{ba}^{in} V_H + (1-\mu_{ba}^{in})V_L)$ and $\Delta V_b^{in}(p_a, p_b) \equiv (\mu_{ab}^{in} V_H + (1-\mu_{ab}^{in})) - V_b$ be the two different interim beliefs about the expected quality difference, where the subscript in ΔV_i^{in} indicates the firm a consumer visited first. Note that the initial belief about the firm's expected quality, EV_i, is not necessarily the same as the interim EV_i^{in}, $i = a, b$. Consumers can incur the switching cost κ and visit the other firm, if they believe their indirect utility will increase.

The (interim) marginal consumers, after prices have been observed and consumers have visited one firm, are

$$x_a^{in} = \frac{p_b - p_a + \Delta V_a^{in} + \kappa + t}{2t} \text{ and } x_b^{in} = \frac{p_b - p_a + \Delta V_b^{in} - \kappa + t}{2t}. \tag{3.2}$$

Note that one difference between the two marginal consumers is the different sign in front of κ. This is because x_a^{in} excludes the consumers who switch from a to b and so a high switching cost increases a's market share by making switching harder. For firm b higher κ makes switching from b to a harder, and so x_b^{in} decreases, implying higher market share for b.

There are three different possibilities, depending on the locations of the interim and the ex-ante marginal consumers.

- *Two-sided switching.* If, $\Delta V_a^{in} + \kappa < \Delta V^e < \Delta V_b^{in} - \kappa$, then $x_a^{in} < x^e < x_b^{in}$, in which case the consumers who initially visited firm a and are in $[x_a^{in}, x^e]$ also visit b and the consumers who initially visited b and are in $[x^e, x_b^{in}]$ also visit a. For this we need $\kappa < \Delta V^e - \Delta V_a^{in}$ and $\kappa < \Delta V_b^{in} - \Delta V^e$. Note that $\Delta V^e - \Delta V_a^{in}$ captures the change in the expected quality differential

[3] To save on space, when needed, we will be suppressing the dependence of beliefs and expected qualities on prices.

in favor of firm b, from the ex-ante to the interim stage, among the consumers who first visit a. Similarly, $\Delta V_b^{in} - \Delta V^e$ captures the same change, in favor of a, among those who first visit b.

- *One-sided switching from a to b.* If $\kappa < \Delta V^e - \Delta V_a^{in}$ and $\kappa > \Delta V_b^{in} - \Delta V^e$, then only consumers in $[x_a^{in}, x^e]$ switch to b.
- *One-sided switching from b to a.* If $\kappa > \Delta V^e - \Delta V_a^{in}$ and $\kappa < \Delta V_b^{in} - \Delta V^e$, then only consumers in $[x^e, x_b^{in}]$ switch to a.
- *No switching.* If $\kappa > \Delta V^e - \Delta V_a^{in}$ and $\kappa > \Delta V_b^{in} - \Delta V^e$, then no consumer switches firms at the interim stage. The marginal consumer in this case is given by (3.1).

To better understand the switching behavior, suppose $\Delta V^e > 0$. This means that consumers, after they have observed firms' prices, believe that firm a has a higher quality product. Then assume that $\Delta V_a^{in} < 0$, which means that consumers who visit a update their ex-ante belief and think that it is firm b that has a higher quality product. This can be because, in out-of-equilibrium, consumers believed b is the low and a the high quality firm but it was actually the other way around. Consumers who visit a realize it is a low quality firm. In this case $\Delta V^e - \Delta V_a^{in} > 0$ and so some consumers will also visit b if the search cost is low.

Since consumers can costlessly return and buy from the first firm they visited, the final market share, if some consumers switched firms at the interim stage, is given by

$$x^f = \frac{p_b - p_a + \Delta V + t}{2t}, \tag{3.3}$$

where ΔV is the true product quality difference, since those who have switched firms know both product qualities. There are three possibilities regarding the location of the final marginal consumer when there is switching at the interim stage.

- Suppose there is one-sided switching from b to a ($\kappa < \Delta V_b^{in} - \Delta V^e$). Then, the final marginal consumer is $x^f \in (x^e, x_b^{in})$ if $\Delta V_b^{in} - \kappa > \Delta V > \Delta V^e$. If one of the inequalities is not satisfied, the final marginal consumer is either x^e or x_b^{in}.
- Suppose there is one-sided switching from a to b ($\kappa < \Delta V^e - \Delta V_a^{in}$). Then, the final marginal consumer is $x^f \in (x_a^{in}, x^e)$ if $\Delta V_a^{in} + \kappa < \Delta V < \Delta V^e$. If one of the inequalities is not satisfied, the final marginal consumer is either x^e or x_a^{in}.
- Suppose there is two-sided switching case ($\kappa < \Delta V^e - \Delta V_a^{in}$ and $\kappa < \Delta V_b^{in} - \Delta V^e$). Then, the final marginal consumer is $x^f \in (x_a^{in}, x_b^{in})$ if $\Delta V_b^{in} - \kappa > \Delta V > \Delta V_a^{in} + \kappa$. If one of the inequalities is not satisfied, the final marginal consumer is either x_b^{in} or x_a^{in}.

Let $\pi_i(p_i, p_j, V_i, V_j, \mu_i^e(p_i, p_j), \mu_j^e(p_i, p_j), \mu_{ij}^{in}(p_i, p_j), \mu_{ji}^{in}(p_i, p_j))$, $i = a, b$ and $j = a, b$, be firm i's expected profit as a function of prices, product qualities and consumer beliefs, both ex-ante and interim. We begin with pooling equilibria and then turn to separating.

4 Uninformative prices (pooling equilibrium)

We search for an equilibrium where prices convey no information about the product qualities. Both firms choose the same price p^*. Ex-ante equilibrium beliefs are $\mu_i^e(p^*, p^*) = q$, for $i = a, b$. To find the set of p^* that can be supported by beliefs as a pooling equilibrium we need to ensure that both firms (regardless of their type) prefer to set p^*. If firm i prefers to set its best-response to p^*, $BR_i(p^*)$, even if by doing so is perceived by consumers as a low quality firm, then it is not possible to find out-of-equilibrium beliefs that make such a deviation unprofitable.

Suppose firm i is of low quality and firm j of high quality, that is (H, L) or (L, H). Furthermore, let's assume that $\mu_i^e(BR_i(p^*), p^*) = 0$ and $\mu_j^e(BR_i(p^*), p^*) = q$, that is the firm that deviates is perceived as a low quality, while the ex-ante belief about the firm that did not deviate that is of high quality is still q. Then the candidate pooling equilibrium must satisfy the following inequalities

$$\pi_i(p^*, p^*, V_L, V_H, q, q, q, q) \geq \pi_i(BR_i(p^*), p^*, V_L, V_H, 0, q, 0, q) \tag{4.1}$$

$$\pi_j(p^*, p^*, V_L, V_H, q, q, q, q) \geq \pi_j(p^*, BR_j(p^*), V_L, V_H, q, 0, q, 0). \tag{4.2}$$

When both firms set p^*, ex-ante and interim consumer beliefs are the same and equal to q. Inequality (4.1) says that a low quality firm would not want to play its best-response to p^* if it is perceived as a low quality firm, while the ex-ante and interim beliefs for the rival firm, who is of high quality, are not affected by such a deviation. Inequality (4.2) ensures that a similar deviation on part of the high quality firm is unprofitable. We will have similar inequalities in the (H, H) and (L, L) cases.

4.1 Pooling equilibria with unrefined beliefs

4.1.1 High search cost, $\kappa \geq qDV$

We summarize in the lemma below.

Lemma 1 *Suppose $\kappa \geq qDV$. Then any price*

$$p^* \in \left[t + qDV - 2\sqrt{tqDV}, t + qDV + 2\sqrt{tqDV}\right]$$

can be supported as a pooling equilibrium with out-of-equilibrium beliefs $\mu_i^e(p, p^*) = 0$, $i = a, b$, for any $p \neq p^*$.

The search cost is higher than $qDV = (qV_H + (1 - q)V_L) - V_L$, which is the expected product quality gain for a consumer who has visited a low quality firm and contemplates visiting the other firm. Moreover, the firm that deviates from (p^*, p^*) is viewed as of low quality. Then, there is a continuum of pooling equilibria.

4.1.2 Low search cost, $\kappa < qDV$

We summarize in the lemma below. Let Ω be a set of prices (it is defined in the proof of the Lemma in the Appendix).

Lemma 2 *Suppose $\kappa < qDV$. Then any price $p^* \in \Omega$ can be supported as a pooling equilibrium with out-of-equilibrium beliefs $\mu_i^e(p, p^*) = 0$, $i = a, b$, for any $p \neq p^*$. For κ's close to qDV, $\Omega \neq \varnothing$, so a pooling equilibrium exists, while for low κ's, $\Omega = \varnothing$, implying that there is no pooling equilibrium.*

The search cost is lower than the expected product quality gain for a consumer who has visited a low quality firm and contemplates visiting the other firm. As in the high search cost case, the deviating firm is perceived as low quality. For relatively high search costs, within that range, a continuum of pooling equilibria exist. For low search costs a pooling equilibrium does not exist. In particular, for low search costs, there are price intervals that prevent profitable deviations for each quality configuration, i.e., (H, L), (H, H) and (L, L). But the intersection of these intervals is empty.

4.2 Pooling equilibria with refined beliefs

To support a pooling equilibrium in Section 4.1, we assumed that a firm that sets an out-of-equilibrium price is perceived as low quality. These beliefs gave rise to a continuum of pooling equilibria for relatively high search costs. In this Section, we try to refine the pooling equilibria. Suppose a high quality firm can profitably deviate to a price if it is perceived by consumers as high quality, while the low quality firm is worse off, relative to the pooling equilibrium, if it deviates to that price, even if it is viewed as high quality. Then, reasonable beliefs should attach probability one that such a deviation is associated with a high quality firm. More precisely (see also Yehezkel (2008)):

Definition 1 *A pooling equilibrium $p^* \in \Omega$ is intuitive if $\mu_i^e(p, p^*) = 1$ for all p satisfying*

$$\pi_i(p, p^*, V_H, V_L, 1, q, 1, q) > \pi_i(p^*, p^*, V_H, V_L, q, q, q, q)$$

and

$$\pi_j(p^*, p, V_H, V_L, q, 1, q, 1) < \pi_j(p^*, p^*, V_H, V_L, q, q, q, q).$$

A pooling equilibrium survives Definition 1 if there exists no price p other than p^* such that a high quality firm benefits from deviating. Next, we examine the firms' incentives to unilaterally deviate from (p^*, p^*), with $p^* \in \Omega$, if a deviator is perceived as high quality.

Proposition 1 *Suppose $\kappa < DV$. There does not exist a pooling equilibrium with intuitive beliefs.*

All pooling equilibria identified in Lemma 2 and most of the ones in Lemma 1 do not survive the intuitive beliefs refinement. A high quality firm can profitably deviate to a price range when such a deviation is unprofitable for the low quality firm even if it is perceived as high quality. The low search cost is key for this result. A low quality firm that mimics the high quality, loses significant market share when consumers discover the true quality and search cost is low.

When, on the other hand, the search cost is high, $\kappa \geq DV$, the profit function of the low quality firm is the same as the profit function of the high quality firm, because consumers learn a firm's quality after they visit the firm but given the high search cost they are captive. In particular, the profit function of firm a is $p_a x^e = \frac{p_a(p_b - p_a + \Delta V^e + t)}{2t}$, where x^e is given by (3.1), which shows that the profit function is independent of whether a has a high or a low quality product. The profit function is affected by product quality only through the term ΔV^e, which depends on ex-ante consumer beliefs and not on the actual product quality of a firm. This implies that the high quality firm cannot charge a high price to credibly signal its superior product quality. Hence, either type of a firm is equally likely to deviate and the intuitive refinement has no bite.

Next, we introduce the concept of impartial beliefs, Hertzendorf and Overgaard (2001).

Definition 2 *Out-of-equilibrium beliefs are impartial at a pooling equilibrium p^* if identical payoffs are associated with out-of-equilibrium ex-ante beliefs $\mu_i^e(p, p^*) = q$, $i = a, b$.*

Consumers revert to their ex-ante beliefs following a unilateral deviation from the pooling equilibrium. We can now state the main result of this Section.

Theorem 1 *Suppose search cost is high, $\kappa \geq DV$. The only pooling equilibrium that is sustained by impartial out-of-equilibrium beliefs is $(p^*, p^*) = (t, t)$. If $\kappa < DV$, there does not exist a pooling equilibrium that satisfies the intuitive beliefs.*

Following Assumption 2, $t \in \left[t + qDV - 2\sqrt{tqDV}, t + qDV + 2\sqrt{tqDV}\right]$. Impartial beliefs eliminate all pooling equilibria stated in Lemma 1 but one. Firms in a pooling equilibrium are treated symmetrically and a deviation cannot credibly signal superior product quality. Moreover, given that the search cost is high, consumers are captive. A low cost firm can costlessly mimic the high cost. Therefore, (t, t) is the only price pair such that no firm wishes to deviate from.

5 Informative prices (separating equilibrium)

We search for an equilibrium where consumers can infer, from the advertised prices, whether the firms have the same quality (either high H or low L) or one firm has high and the other low quality. There are four cases: (H, L), (L, H), (H, H) and (L, L), where the first letter in each case refers to the product quality of firm a located at 0 and the second to the product quality of firm b located at 1.

Let $p_H^*(HL)$ and $p_L^*(HL)$ be the candidate equilibrium prices in the (H, L) case and $p^*(HH)$ and $p^*(LL)$ the candidate symmetric equilibrium prices in the (H, H) and (L, L) cases respectively. Clearly, the (L, H) case is symmetric to (H, L). Moreover, given the covered market assumption we look for an equilibrium where $p^*(HH) = p^*(LL) = p^*$. Also, $p_H^*(HL) \neq p_L^*(HL) \neq p^*$. Consumer ex-ante beliefs are as follows.

$$\mu_a^e(p_a,p_b)=\begin{cases}1, & \text{if } p_a=p_H^*(HL) \text{ and } p_b=p_L^*(HL), \text{ or } p_b\neq\{p_L^*(HL),p^*\}\\ \frac{1}{2}, & \text{if } p_a=p_b=p_H^*(HL)\\ \frac{q^2+q(1-q)}{q^2+(1-q)^2+q(1-q)}, & \text{if } p_a=p_H^*(HL) \text{ and } p_b=p^*\\ \frac{q^2}{q^2+(1-q)^2}, & \text{if } p_a=p_b=p^*\\ 1, & \text{if } p_a\neq\{p_H^*(HL),p^*\} \text{ and } p_b=p_L^*(HL)\\ \frac{q^2+q(1-q)}{q^2+(1-q)^2+q(1-q)}, & \text{if } p_a=p^* \text{ and } p_b=p_H^*(HL)\end{cases} \tag{5.1}$$

and

$$\mu_b^e(p_a,p_b)=\begin{cases}0, & \text{if } p_a=p_H^*(HL), \text{ or } p_a\neq\{p_H^*(HL),p^*\} \text{ and } p_b=p_L^*(HL)\\ \frac{1}{2}, & \text{if } p_a=p_b=p_H^*(HL)\\ \frac{q^2}{q^2+(1-q)^2+q(1-q)}, & \text{if } p_a=p_H^*(HL) \text{ and } p_b=p^*\\ \frac{q^2}{q^2+(1-q)^2}, & \text{if } p_a=p_b=p^*\\ 0, & \text{if } p_a=p_H^*(HL) \text{ and } p_b\neq\{p_L^*(HL),p^*\}\\ \frac{q^2}{q^2+(1-q)^2+q(1-q)}, & \text{if } p_a=p_H^*(HL) \text{ and } p_b=p^*.\end{cases} \tag{5.2}$$

In addition, beliefs are correlated, i.e., $m^e(p^*,p^*)=\frac{q^2}{q^2+(1-q)^2}$, the probability that both firms have high quality when prices are equal to p^* is not equal to the product of the marginal probabilities.

The beliefs given by (5.1) and (5.2) are reasonable given the multi-sender nature of the game and very close to the *unprejudiced beliefs* of Bagwell and Ramey (1991), where the receiver of the signals must take into account the number of price deviations that would be needed to generate a deviant price pair. In particular, unprejudiced beliefs satisfy a minimality rule, whereby the receiver (i.e., a consumer) infers a particular product quality pair if under that type the deviant price pair can be rationalized with the fewest number of deviations from the equilibrium strategies. Essentially, consumers can rely on the price of the non-deviating firm, whenever possible, to infer the qualities of both firms.

For example, when consumers observe $p_a=p_b=p_H^*(HL)$, they do not know whether it is (H,L) or (L,H) but can rule out (H,H) and (L,L), since if it was one in the latter two cases, and consistent with unilateral deviations, one price must have been p^*. Therefore, the ex-ante belief that firm a has a high quality product is $\frac{1}{2}$.

The following lemma is a direct consequence of the unprejudiced beliefs.

Lemma 3 *We assume that consumer beliefs are unprejudiced and satisfy* (5.1) *and* (5.2)*. Then, if a separating equilibrium exists it must entail the complete information prices* (2.1).

The question that arises next is under what conditions a separating equilibrium exists. First, we analyze each one of the three cases, i.e., (H, L), (H, H) and (L, L), separately and then we combine them.

5.1 Firm a has a high and firm b has a low quality product, (H, L)

Lemma 4 *Suppose firm a has a high and firm b has a low quality product and $q > \frac{2}{3}$. Then, if*

$$\kappa \leq \min\left\{\frac{DV(DV - 9t(1-q))}{9t}, \frac{q^3}{1-q+q^2}DV\right\}$$

the equilibrium prices are

$$p_H^*(HL) = \frac{DV}{3} + t \text{ and } p_L^*(HL) = -\frac{DV}{3} + t.$$

Consumers upon observing the equilibrium prices can infer the product quality of each firm.

The high quality firm has no incentive to deviate. The low quality firm may have an incentive to raise its price to make consumers think that its quality is high and it is the other firm that has deviated. If the search cost is high, few consumers who first visit the deviating firm, and realize that its quality is low, switch to the other one. In this case it pays for the low quality firm to deviate. Therefore, such a deviation is unprofitable for relatively low search costs.

We need $q > \frac{2}{3}$ to ensure that $\frac{DV(DV-9t(1-q))}{9t} > 0$ and Assumption 2 is satisfied.

5.2 Both firms have high quality products, (H, H)

Lemma 5 *Suppose both firms have high product quality products, $t \leq \frac{(1-2q)^2 DV}{9q(1-q)}$ and $q < \frac{1}{2} - \frac{\sqrt{21}}{14}$, or $q > \frac{1}{2} + \frac{\sqrt{21}}{14}$. Then the equilibrium prices are*

$$(p^*, p^*) = (t, t).$$

Consumers upon observing the equilibrium prices can infer that firms have the same quality products.

A firm may have an incentive to raise its price to $p_H^*(HL)$ to make consumers believe that they are in the (H, L) case and so the rival has a low quality. No consumer visits a second firm and that is why the search cost κ does not feature in the conditions of this case. This is because the consumers who visit the firm that deviated confirm their ex-ante belief about its high quality and the consumers who visit the non-deviating firm find out that instead of low it is high quality. A high t makes this deviation profitable because the deviating firm loses very little market share when it raises its price and t is high. That is why we need a low t to ensure no deviation. Also, from assumption 2 we need a high t to ensure strictly positive market shares for both firms. Both conditions about t are satisfied for extreme values of q.

5.3 Both firms have low quality products, (L, L)

Lemma 6 *Suppose both firms have low product quality products. Then if*

- $\kappa < \min\left\{\frac{(DV)^2}{3(DV+3t)}, \frac{q(1-q)DV}{1-q+q^2}\right\}$
- *or,* $\kappa > \frac{q(1-q)DV}{1-q+q^2}$, $t \leq \frac{(1-2q)^2 DV}{9q(1-q)}$ *and* $q < \frac{1}{2} - \frac{\sqrt{21}}{14}$, *or* $q > \frac{1}{2} + \frac{\sqrt{21}}{14}$

the equilibrium prices are

$$(p^*, p^*) = (t, t).$$

Consumers upon observing the equilibrium prices can infer that firms have product of the same quality.

A firm can increase its price to make consumers believe it is high quality. If the search cost is low this strategy is not profitable as consumers upon realizing that the firm's quality is actually low switch to the other firm. If the search cost is high, the firm that deviates can keep the consumers who first visit it. For this deviation to be unprofitable we need a low t so that a higher price implies significant loss of market share (as was the case in (H, H)).

5.4 Existence of a separating equilibrium

We now combine the three lemmas (Lemma 4, 5 and 6) to present the main result of this Section.

Theorem 2 *Suppose* $DV \in \left[\frac{9q(1-q)t}{(1-2q)^2}, 3t\right)$, $\kappa \leq \frac{DV(DV-9t(1-q))}{9t}$, $q > \frac{1}{2} + \frac{\sqrt{21}}{14} \approx 0.8273$ *and the unprejudiced beliefs satisfy* (5.1) *and* (5.2). *Then, a separating equilibrium exists and it coincides with the complete information prices given by* (2.1).

For a separating equilibrium to exist, we need a high quality differential DV relative to the competitiveness of the market captured by the transportation cost t (but lower than $3t$ so that the high quality firm does not drive the low quality out of the market), or a competitive market (low transportation cost) relative to DV, a relatively low search cost (low κ) and a relatively high probability of a high product quality (high q).

Example 1 *Suppose* $t = 1$ *and* $q = 85\%$. *Then, the permissible range for* DV *is* $DV \in [2.34, 3)$. *If firms have the same quality, which occurs with probability* $1 - 2q(1-q) = 74.5\%$, *the prices are* $(1, 1)$, *while if firm* a *has high and firm* b *has low quality the prices are* $\left(\frac{DV}{3} + 1, -\frac{DV}{3} + 1\right)$ *(the case where* a *is low quality and* b *high is symmetric). A pooling equilibrium exists for* $\kappa \geq DV$ *and the equilibrium prices are* $(1, 1)$. *A separating exists for* $\kappa \leq \frac{DV(DV-9t(1-q))}{9t}$ *and the equilibrium prices can be either* $(1, 1)$, *or* $\left(\frac{DV}{3} + 1, -\frac{DV}{3} + 1\right)$, *or* $\left(-\frac{DV}{3} + 1, \frac{DV}{3} + 1\right)$.

Given that there exists a parameter space, i.e., κ between $\frac{DV(DV-9t(1-q))}{9t}$ and DV, where a pure strategy equilibrium does not exist, in what follows we consider drastic reductions of consumer search cost, i.e., from $\kappa > DV$ to $\kappa < \frac{DV(DV-9t(1-q))}{9t}$. A drastic decrease of κ moves the market equilibrium from pooling to separating. It can be easily deduced from the equilibrium prices given by (2.1) that this generates a mean-preserving spread in the price distribution.

Corollary 1 *A drastic reduction in search cost κ induces a mean-preserving spread in the distribution of the equilibrium prices.*

Thus, in our model a technological improvement that results in a drastic reduction of search costs, has the potential to affect positively the dispersion of prices and leave the average price unaffected.

6 Conclusion

We study a duopoly model of horizontal and vertical differentiation where private information about firm quality is common among firms. Consumers are ex-ante unaware of firms' product qualities, they observe prices and decide which firm to 'visit' first. A visit to a firm allows consumers to assess product quality before purchase. After a visit to the first firm consumers decide whether to purchase the firm's product, or to incur the search cost and visit the second firm. If the search cost is high, a unique reasonable pooling equilibrium exists. A separating equilibrium with unprejudiced beliefs entails the complete information prices. Existence of a separating equilibrium requires a low search cost, a relatively competitive market (low transportation cost) and a high probability of a high product quality. A drastic decrease of the search cost, moves the market equilibrium from pooling to separating and the equilibrium prices exhibit a mean-preserving spread.

Dissipative advertising would not affect our results in any substantive way. When a separating equilibrium exists, allowing for advertising would have no effect as the high quality firm can signal its quality with price. When a separating equilibrium does not exist dissipative advertising cannot restore equilibrium. It is clear first that a low quality firm would not advertise. The high quality firm would cut on advertising expenses in a separating equilibrium with positive advertising because consumers can infer from the price of the non-deviating firm that it is of high quality, a direct consequence of the unprejudiced beliefs. Furthermore, when a pooling equilibrium exists, the payoff functions of both types of firms are the same (because due to a high search cost consumers do not visit the second firm) and hence both types have the same incentive to advertise to signal high quality. Hence, dissipative advertising expenditures are a non-credible signal of high quality and no firm will use it in equilibrium.

Building on our discussion about the ineffectiveness of dissipative advertising expenditures, an implication of our model is that other forms of advertising may be used when prices cannot signal quality. Informative advertising can be a credible way to signal high quality. Another managerial implication is about consumer search costs. For many products, consumer search costs are partially affected by managerial decisions, e.g., free trials, higher transparency regarding product characteristics. Managers of high quality firms should make these types of investments to ease consumer search. Finally, an empirical implication is a negative relationship between search costs and price dispersion, but no effect of search costs on average price.

Our focus in this paper was to determine parameter constellations under which separating and pooling equilibria in pure strategies exist. Future research can examine the type of equilibria in the parameter space where pure strategy equilibria do not exist.

A Appendix: Proofs of Lemmas and Propositions

A.1 Proof of Lemma 1

We assume that firm $j = a$ (located at 0) sets the pooling equilibrium price p^*, while firm $i = b$ (located at 1) deviates by setting $BR_i(p^*)$ and the deviator is perceived as low quality.

We are in the (H, L) case and the low quality firm deviates. Consumers who visit b first confirm their belief and do not switch, while consumers who visit a first realize that its quality is higher than the average quality and do not switch either. The deviation profit is given by $p^{dev}(1 - x^e)$, where x^e is given by (3.1) with $\Delta V^e = \overline{V} - V_L = qDV$. The maximum deviation profit is $\frac{(p^*+t-qDV)^2}{8t}$. On the equilibrium, consumers who visit the low quality firm, firm b, realize that its quality is lower than the average. So, $\Delta V_b^{in} - \Delta V^e = \overline{V} - V_L - 0 = qDV$, but since $\kappa \geq qDV$ no consumer switches from b to a. Furthermore, those who visit a realize its quality is high and they stay. Hence, the pooling equilibrium profits is $\frac{p^*}{2}$, which is higher than the deviation profit if and only if

$$p^* \in \left[t + qDV - 2\sqrt{tqDV}, t + qDV + 2\sqrt{tqDV}\right]. \tag{A.1}$$

Now we are in the (L, H) state and we assume that the high quality firm deviates. No consumer switches from b to a, since b is actually high quality. Those who visit a realize its quality is low but they believe b is also low, so $\Delta V_a^{in} = 0$. Given that $\Delta V^e = \overline{V} - V_L = qDV$, we have $\Delta V^e - \Delta V_a^{in} = qDV$ and since $\kappa \geq qDV$ no consumer switches from a to b. This implies that the maximum deviation profit is the same as when the low quality firm deviated. Hence, the no deviation pooling price range (A.1) does not change.

Now we assume that we are in the (L, L) case. For the consumers who first visit a we have $\Delta V^e - \Delta V_a^{in} = qDV + 0 = qDV$. For the consumers who first visit b we have $\Delta V_b^{in} - \Delta V^e = qDV - qDV = 0$. Since $\kappa \geq qDV$, no consumer switches firms. The profit of the deviating firm is the same as in the (H, L) case above. The same is true in the (H, H) case. Therefore, the no deviation price range is given by (A.1).

So, it is possible to support any p^* in (A.1) as a pooling equilibrium with beliefs $\mu_i(p, p^*) = 0$ for any $p \neq p^*$ and $i = a, b$.

A.2 Proof of Lemma 2

We assume that firm $j = a$ (located at 0) sets the pooling equilibrium price p^*, while firm $i = b$ (located at 1) deviates by setting $BR_i(p^*)$ and the deviator is perceived as low quality.

Let's start with the (L, H) case. We assume that the high quality firm deviates. Since the high quality firm is perceived as low the consumers who first visit b stay. For the consumers who visit a first we have $\Delta V^e - \Delta V_a^{in} = qDV$ and since $\kappa < qDV$ some switch to b. Because $\kappa > \Delta V - \Delta V_a^{in} = -DV$, the relevant marginal consumer is x_a^{in}, given in (3.2), with $\Delta V_a^{in} = 0$. The deviation profit is given by $p^{dev}(1 - x_a^{in})$. The maximum deviation profit for the high quality firm is $\frac{(p^*+t-\kappa)^2}{8t}$. The equilibrium profit of the high quality firm is given by $p^*(1-x_a^{in})=\frac{p^*(t-\kappa+qDV)}{2t}$, where x_a^{in} is given in (3.2) with prices equal to p^* and $\Delta V_a^{in}=V_L-\overline{V}=-qDV$, since some consumers who visit firm a, who is low quality, will also visit firm b, on the expectation of average quality, and since b is of high quality they buy from it. Such a deviation is not profitable if and only if

$$p^* \in \Omega_{LH}^H \equiv \left[t-\kappa+2qDV-2\sqrt{qDV(t-\kappa+qDV)}, t-\kappa+2qDV+2\sqrt{qDV(t-\kappa+qDV)}\right], \tag{A.2}$$

where the subscript of Ω indicates the state and the superscript the firm that deviates.

Now assume we are in the (H,L) state and the low quality firm deviates from p^*. The consumers who visit the high quality firm a realize that it has a higher quality than what they expected and those who visit b confirm their ex-ante belief. The search cost κ has no effect on the deviation profits, so the ex-ante and interim marginal consumers coincide. The deviation profit is $p^{dev}(1-x^e)$, where x^e is given by (3.1) with $\Delta V^e=\overline{V}-V_L=qDV$. The maximum deviation profit of the low quality firm is $\frac{(p^*+t-qDV)^2}{8t}$. The equilibrium profit of the low quality firm is $p^*(1-x_b^{in})=\frac{p^*(t+\kappa-qDV)}{2t}$, where x_b^{in} is the relevant marginal consumer, given by (3.2), with prices equal to p^* and $\Delta V_b^{in}=\overline{V}-V_L=qDV$. Some consumers on the equilibrium path who visit firm b realize that its quality is low and switch to a and stay (since its quality is high). Such a deviation is not profitable if and only if

$$p^* \in \Omega_{HL}^L \equiv \left[t+2\kappa-qDV-2\sqrt{\kappa(\kappa+t-qDV)}, t+2\kappa-qDV+2\sqrt{\kappa(\kappa+t-qDV)}\right]. \tag{A.3}$$

Now we assume that we are in the (L,L) case. We have $\Delta V^e=\overline{V}-V_L=qDV$. For the consumers who first visit a we have $\Delta V^e-\Delta V_a^{in}=qDV-0=qDV$. For the consumers who first visit b we have $\Delta V_b^{in}-\Delta V^e=qDV-qDV=0$. If $\kappa<q\Delta V$, only some consumers who visit a switch to b. Since $\kappa>\Delta V-\Delta V_a^{in}=0-0=0$ it must be that $x^f=x_a^{in}$. The deviation profit is $p^{dev}(1-x_a^{in})$. The maximum deviation profit is $\frac{(p^*+t-\kappa)^2}{8t}$. The equilibrium profit is $\frac{p^*}{2}$. Such a deviation is not profitable if and only if

$$p^* \in \Omega_{LL} \equiv \left[t+\kappa-2\sqrt{t\kappa}, t+\kappa+2\sqrt{t\kappa}\right]. \tag{A.4}$$

Finally, we assume that we are in the (H,H) case. The high quality firm deviates and is perceived as low quality. No consumer switches firms, since both firms are of high quality. The deviation profit is $p^{dev}(1-x^e)$, where x^e is given by (3.1) with $\Delta V^e=\overline{V}-V_L=qDV$. The maximum profit of the deviating firm is $\frac{(p^*+t-qDV)^2}{8t}$. The equilibrium profit is $\frac{p^*}{2}$. Such a deviation is not profitable if and only if

$$p^* \in \Omega_{HH} \equiv \left[t+qDV-2\sqrt{tqDV}, t+qDV+2\sqrt{tqDV}\right]. \tag{A.5}$$

Let $\Omega \equiv \Omega_{LH}^H \cap \Omega_{HL}^L \cap \Omega_{LL} \cap \Omega_{HH}$ be the intersection of the four sets, (A.2)–(A.5). If $p^* \in \Omega$, then no firm finds a deviation from (p^*,p^*) profitable if it is perceived as low quality, regardless of the quality of the rival. The set Ω is non-empty for high values of κ. When $\kappa=qDV$, $\Omega=\left[t+qDV-2\sqrt{tqDV}, t+qDV+2\sqrt{tqDV}\right]$. Note that in this case Ω coincides with the price range when $\kappa \geq qDV$, see Lemma 1. The higher the $tqDV$ the higher the price range. By continuity, Ω is non-empty for κ's less than (but close to) qDV. But when $\kappa = 0$, the Ω_{LL} collapses to t, while the Ω^L_{HL} collapses to $t - qDV$. Therefore, Ω is empty, implying that for κ's close to 0 there does not exist a pooling equilibrium.

A.3 Proof of Proposition 1

We assume that firm $j = a$ (located at 0) sets the pooling equilibrium price p^*, while firm $i = b$ (located at 1) deviates to p^{dev} and is perceived as high quality.

Let's assume that we are in the (L, H) case. The consumers who visit firm b confirm their belief and stay. The consumers who visit a have $\Delta V^{in}_a = V_L - V_H = -DV$. Also, $\Delta V^e = \overline{V} - V_H = -(1-q)DV$ and $\Delta V^e - \Delta V^{in}_a = qDV$ and when $\kappa < qDV$ some consumers who visit a switch to b. When $\kappa \geq qDV$ no consumer switches firms. Because $\Delta V^{in}_a + \kappa = -DV + \kappa > \Delta V = -DV$ the final marginal consumer is x^{in}_a. Thus, the deviation profits are given by $p^{dev}(1 - x^{in}_a)$ where x^{in}_a is given by (3.2), with $\Delta V^{in}_a = V_L - V_H = -DV$. In equilibrium, $\Delta V^{in}_a = V_L - \overline{V} = -qDV$. Also, $\Delta V^e = 0$ and so $\Delta V^e - \Delta V^{in}_a = qDV$. When $\kappa < qDV$ some consumers in equilibrium switch from a to b (no consumer switches from b to a since b has a high quality). Because $\Delta V^{in}_a + \kappa = -qDV + \kappa > \Delta V = -DV$ the final marginal consumer is x^{in}_a. The equilibrium profits are $p^*(1 - x^{in}_a) = \frac{p^*(t-\kappa+qDV)}{2t}$. Such a deviation, when $\kappa < qDV$, is profitable if and only if[4]

$$p^{dev} \in \left[\frac{p^* - \kappa + t + DV}{2} \pm \frac{\sqrt{(p^*)^2 + ((2-4q)DV - 2t + 2\kappa)p^* + (t - \kappa + DV)^2}}{2}\right]. \quad \text{(A.6)}$$

When $\kappa \geq qDV$, as we showed above no consumer switches firms both in equilibrium and out-of equilibrium. The deviation profit is $p^{dev}(1 - x^e)$, where x^e is given by (3.1) with $\Delta V^e = -(1-q)DV$. The equilibrium profit is $\frac{p^*}{2}$. A deviation is profitable if and only if

$$p^{dev} \in \left[\frac{p^* + t + (1-q)DV}{2} \pm \frac{\sqrt{(1-q)^2(DV)^2 + 2(1-q)DV(p^* + t) + (p^* - t)^2}}{2}\right]. \quad \text{(A.7)}$$

Next, we assume that we are in the (H, L) state and the low quality firm deviates. The consumers who first visit a stay since its quality is high. For the consumers who first visit b we have $\Delta V^{in}_b - \Delta V^e = qDV - (-(1-q)DV) = DV$. So, if $\kappa < DV$, some consumers who first visit b will switch to a. Because $\Delta V^{in}_b - \kappa = qDV - \kappa < \Delta V = DV$, the final marginal consumer is x^{in}_b. The deviation profits are given by $p^{dev}(1 - x^{in}_b)$ where x^{in}_b is given by (3.2), with $\Delta V^{in}_b = \overline{V} - V_L = qDV$. In equilibrium, $\Delta V^{in}_b = \overline{V} - V_L = qDV$ and $\Delta V^e = 0$. So, $\Delta V^{in}_b - \Delta V^e = qDV$. If $\kappa < qDV$, some consumers in equilibrium who first visit b switch to a (no consumer switches from a to b since a has a high quality).

[4]The market share of the deviating firm cannot exceed 1. This is guaranteed if and only if $p^{dev} \geq p^* + DV - \kappa - t$. Therefore, the lower bound in the interval below must be higher than $p^* + DV - \kappa - t$.

Because $\Delta V_b^{in} - \kappa = qDV - \kappa < \Delta V = DV$ the final marginal consumer is x_b^{in}. The equilibrium profits are $p^*(1 - x_b^{in}) = \frac{p^*(t+\kappa-qDV)}{2t}$. When $\kappa < qDV$, such a deviation is profitable if and only if

$$p^{dev} \in [\min\{\kappa + t - qDV, p^*\}, \max\{\kappa + t - qDV, p^*\}] . \tag{A.8}$$

When $\kappa \in [qDV, DV)$, the equilibrium profit is $\frac{p^*}{2}$ (since no consumer switches firms), but the deviation profit is still $p^{dev}(1 - x_b^{in})$, the same as when $\kappa < qDV$. A deviation is profitable if and only if

$$p^{dev} \in \left[\frac{p^* + t + \kappa - qDV}{2} \pm \frac{\sqrt{t^2 + 2t(\kappa - p^* - qDV) + (p^* + \kappa - qDV)^2}}{2}\right]. \tag{A.9}$$

We now assume that we are in the (L, L) case. Consumers who initially visit firm a have $\Delta V^e - \Delta V_a^{in} = -(1-q)DV + DV = qDV$, so when $\kappa < qDV$ some switch to b. Consumers who initially visit firm b have $\Delta V_b^{in} - \Delta V^e = qDV + (1-q)DV = DV$, so when $\kappa < DV$ some switch to a. When $\kappa \in [qDV, DV)$ consumers who visit a stay, those who visit b visit also a. We have $\Delta V_b^{in} - \kappa = qDV - \kappa < \Delta V = 0$ and $\Delta V = 0 > \Delta V^e = -(1-q)DV$. So, the final marginal consumer is x_b^{in}, given by (3.2) with $\Delta V_b^{in} = qDV$. The equilibrium profit is $\frac{p^*}{2}$. Deviation and equilibrium profits are the same as in the (H, L) case when the low quality firm deviates and $\kappa \in [qDV, DV)$. Hence, a deviation when $\kappa \in [qDV, DV)$ is profitable if and only if (A.9) is satisfied.

Now we assume that $\kappa < qDV$. There is now two-sided switching. Since $\Delta V_b^{in} - \kappa > \Delta V > \Delta V_a^{in} + \kappa$ holds (i.e., $qDV - \kappa > 0 > -DV + \kappa$), the final marginal consumer is x^f, given by (3.3), with $\Delta V = 0$. The deviation profit is $p^{dev}(1 - x^f)$. The equilibrium profit is $\frac{p^*}{2}$. A deviation is profitable if and only if

$$p^{dev} \in [\min\{t, p^*\}, \max\{t, p^*\}] . \tag{A.10}$$

It can be verified that when $\kappa = DV$, (A.7) is the same with (A.9). In this case no consumer switches firms, and hence high and low type firms have the same incentives to deviate, in the (H, L) and the (L, L) cases, when they are perceived as high types. The same of course is true for $\kappa > DV$. Moreover, it can be verified that the upper bound of (A.9) is decreasing as κ decreases, but (A.7) is not a function of κ. Therefore, for $\kappa < DV$ the high quality firm can always set a price in (A.7) and above the upper bound of (A.9), that cannot be mimicked by the low quality firm in the (H, L) and (L, L) cases, to signal its high quality.

A.4 Proof of Lemma 3

A candidate symmetric equilibrium when firms have the same quality, (H, H) or (L, L), is (t, t). Any other symmetric pair of prices when firms have the same product quality cannot be an equilibrium. To see this suppose $(p^*, p^*) \neq (t, t)$ is a candidate equilibrium in the (H, H) and (L, L) cases. Consumers, upon observing (p^*, p^*) in equilibrium, know that firms have the same quality products. If a firm can unilaterally deviate to p^{dev} without affecting consumer beliefs, then it is clearly better off, since p^* is not a best response to p^*. Any $p^{dev} \neq p_H^*(HL)$ or $p^{dev} \neq p_L^*(HL)$, leaves consumer beliefs unchanged because consumers observe (p^{dev}, p^*) and realize from the price of the non-deviating firm p^* that product qualities are the same. If it happens that the best deviating price is equal to $p_H^*(HL)$ or $p_L^*(HL)$, in which case consumers may not know for sure who is the deviating firm and what the product qualities are, the deviating firm can set a price ε away from $p_H^*(HL)$ or $p_L^*(HL)$, so that consumer beliefs are unchanged, and still be better off.

Next, assume that $(p_H^*(HL), p_L^*(HL))$ is not equal to the complete information prices given by (2.1). Then, given the beliefs in (5.1) and (5.2), firm a can deviate to its best response $BR_a(p_L^*(HL)) \neq p_H^*(HL)$ and become better off. Firms believe that the deviating firm is high quality, based on the price of the non-deviating firm. The unique equilibrium under the unprejudiced beliefs is the complete information equilibrium.

A.5 Proof of Lemma 4

Firm a has high and firm b has low product qualities, (H, L). The equilibrium prices are $(p_H^*(HL), p_L^*(HL))$. We begin with firm b's deviation, from $p_b = p_L^*(HL)$ to p_b^{dev}. Firm b can deviate to $p_H^*(HL)$, to $p^* = t$ or to any other price.

First, we assume that $p_b^{dev} = p_H^*(HL)$. Consumers observe $(p_H^*(HL), p_H^*(HL))$ and do not know which firm is the low quality firm, although they know that one firm must be of low quality (given the unprejudiced beliefs consumers know that it cannot be (H, H) or (L, L), because if that was the case one price must have been t). Therefore, from the beliefs (5.1) and (5.2) we have $\mu_a^e = \mu_b^e = \frac{1}{2}$ and hence $\Delta V^e = 0$ and the marginal consumer x^e, given by (3.1), is at $\frac{1}{2}$. Consumers then who visit either firm realize that firm a is the high quality firm and firm b has a low quality product and is the one that deviated, that is $\Delta V_a^{in} = \Delta V_b^{in} = DV$. Some may switch from b to a, if $\kappa < DV$, but no consumer from a will switch to b. The interim marginal consumer is x_b^{in} given by (3.2). Since $\Delta V = DV > \Delta_b^{in} - \kappa = DV - \kappa$ the final marginal consumer is x_b^{in}. This deviation is unprofitable if and only if

$$\begin{aligned} p_L^*(HL)(1 - x^e) &= p_L^*(HL)\left(1 - \frac{p_L^*(HL) - p_H^*(HL) + DV - \kappa + t}{2t}\right) \\ &\geq p_H^*(HL)\left(1 - x_b^{in}\right) = p_H^*(HL)\left(1 - \frac{DV - \kappa + t}{2t}\right). \end{aligned} \tag{A.11}$$

Second, we assume that $p_b^{dev} = p^* = t$. Consumers observe $(p_H^*(HL), t)$ and know that it cannot be (L, H), because the candidate equilibrium prices in this case are $(p_L^*(LH), p_H^*(LH))$ and both firms would have to have deviated, for which unprejudiced beliefs assign probability zero. But consumers do not know whether it is (H, H) or (L, L) and one firm raised its price, or (H, L) and the low quality firm deviated. From the ex-ante beliefs given by (5.1) and (5.2) we have $\mu_a^e = \frac{q^2+q(1-q)}{q^2+(1-q)^2+q(1-q)}$ and $\mu_b^e = \frac{q^2}{q^2+(1-q)^2+q(1-q)}$. So, the expected values for the two firms' products are

$$EV_a = \frac{(q^2+q(1-q))V_H + (1-q)^2 V_L}{q^2+(1-q)^2+q(1-q)} \text{ and } EV_b = \frac{q^2 V_H + (q(1-q)+(1-q)^2)V_L}{q^2+(1-q)^2+q(1-q)}, \quad \text{(A.12)}$$

with $EV_a > EV_b$. Consumers initially sort out according to (3.1) using the above expected qualities for each firm. The marginal consumer is given by

$$x^e = \frac{(DV + p_H^*(HL) - 2t)q(1-q) + 2t - p_H^*(HL)}{2t(1-q+q^2)}.$$

The consumers who visit firm b they realize that its quality is low and update their beliefs about the quality of firm a by ruling out (H, H). The interim belief of the consumers who visited b about the quality of a being high is $\mu_{ab}^{in} = \frac{q(1-q)}{(1-q)^2+q(1-q)}$. Using these beliefs, the expected quality of firm a for consumers who first visit b is $EV_a^{in} = \frac{q(1-q)V_H+(1-q)^2V_L}{(1-q)^2+q(1-q)}$. The interim marginal consumer, using (3.2), is given by

$$x_b^{in} = \frac{t - p_H^*(HL) + EV_a^{in} - V_L - \kappa + t}{2t} = \frac{qDV - p_H^*(HL) - \kappa + 2t}{2t}.$$

Some consumers will switch from b to a if and only if $x_b^{in} > x^e \Leftrightarrow \kappa < \kappa_2 \equiv \frac{q^3}{1-q+q^2}DV$.

Also it is clear that no consumer will switch from a to b, since a is high quality.

To summarize, when $p_b^{dev} = t$, if $\kappa < \kappa_2$, then x_b^{in} is the relevant marginal consumer, while if $\kappa \geq \kappa_2$ the relevant marginal consumer is x^e.

We first assume that $\kappa < \kappa_2$. Firm b will not find a deviation from $p_L^*(HL)$ to t profitable if and only if

$$\begin{aligned} p_L^*(HL)(1-x^e) &= p_L^*(HL)\left(1 - \frac{p_L^*(HL) - p_H^*(HL) + DV + t}{2t}\right) \\ &\geq t\left(1 - x_b^{in}\right) = t\left(1 - \frac{qDV - p_H^*(HL) - \kappa + 2t}{2t}\right). \end{aligned} \quad \text{(A.13)}$$

The RHS of (A.13) is higher than the RHS of (A.11) if and only if $\kappa < \frac{DV(p_H^*(HL)-qt)}{p_H^*(HL)-t}$, which holds in the case we are in, since $\kappa_2 < \frac{DV(p_H^*(HL)-qt)}{p_H^*(HL)-t}$. So, the relevant constraint is only (A.13).

Firm b can also deviate to $p_b^{dev} \neq t$ and $p_b^{dev} \neq p_H^*(HL)$. In this case consumers observe $(p_H^*(HL), p_b^{dev})$ and immediately realize from $p_H^*(HL)$ that firm a has high quality and firm b has low quality. Therefore, only the complete information prices given by (2.1) ensure that such a deviation is unprofitable.

Using (2.1), the no deviation constraint (A.13) is satisfied if and only if $\kappa \leq \kappa_1 \equiv \frac{DV(DV-9t(1-q))}{9t}$.[5]

[5] Also, $\kappa_1 < \kappa_2$ if and only if $t > \frac{(1-q+q^2)DV}{9(1-2q+2q^2)}$.

In what follows we show that there is no equilibrium when $\kappa \geq \kappa_2$. Let assume that $\kappa \geq \kappa_2$. Firm b will not find a deviation from $p^*_L(HL)$ to t profitable if and only if

$$p^*_L(HL)\left(1-\frac{p^*_L(HL)-p^*_H(HL)+DV+t}{2t}\right) \geq$$
$$t\,(1-x^e)=t\left(1-\frac{[DV+p^*_H(HL)-2t]q(1-q)+2t-p^*_H(HL)}{2t(1-q+q^2)}\right). \tag{A.14}$$

The RHS of (A.14) is higher than the RHS of (A.11) if and only if

$$\kappa < \frac{DV}{p^*_H(HL)(1-q+q^2)}\left[p^*_H(HL)-(p^*_H(HL)+t)q(1-q)\right].$$

After we substitute $p^*_H(HL)$ from (2.1) the above inequality becomes $\kappa < \kappa_3 \equiv \frac{DV[DV(1-q+q^2)+3t(1-2q+2q^2)]}{(1-q+q^2)(DV+3t)}$.

We have that $\kappa_3 > \kappa_2$. As a result, for $\kappa \in [\kappa_2, \kappa_3]$ the relevant constraint is (A.14), whereas for $\kappa > \kappa_3$ the relevant constraint is (A.11).

When the relevant constraint is (A.14), using (2.1), the no deviation constraint is satisfied for $t < \frac{(1-q+q^2)DV}{9(1-2q+2q^2)}$. However, this contradicts assumption 2.

When the relevant constraint is (A.11), using (2.1), the no deviation constraint is satisfied for $\kappa < \kappa_4 \equiv \frac{4DV^2}{3(3t+DV)}$. Also, $\kappa_4 > \kappa_3$ if and only if $t < \frac{(1-q+q^2)DV}{9(1-2q+2q^2)}$. However, this contradicts assumption 2.

Now let's turn to firm a's incentive to deviate. Equilibrium profits for the high quality firm are increasing in the quality difference $V_a - V_b$, which is the highest in equilibrium: any deviation on part of firm a, as we have demonstrated above, will decrease the expected quality of a and will increase that of b. Consumers will attach some probability that firm a is of low quality and firm b is of high quality. Thus, firm a who has high quality has no incentive to deviate.

A.6 Proof of Lemma 5

Both firms have high quality products, (H,H). The candidate equilibrium prices are $(p^*,p^*)=(t,t)$ and the equilibrium profits $\pi_a=\pi_b=\left(\frac{t}{2},\frac{t}{2}\right)$.

First, let's consider firm a's deviation to $p_a^{dev}=\frac{DV}{3}+t$. Consumers, upon observing $\left(\frac{DV}{3}+t,t\right)$, realize that one firm has deviated. So, it can be (H,H), or (L,L) and one firm raised its price to $\frac{DV}{3}+t$, or (H,L) and the low quality firm raised its price to t (they can, however, rule out (L,H), given the unprejudiced beliefs). The ex-ante consumer beliefs about expected product qualities are given by (A.12). Consumers initially sort out according to (3.1), using (A.12)

$$x^e=\frac{3t-DV-q(3t-4DV)+q^2(3t-4DV)}{6t(1-q+q^2)}. \tag{A.15}$$

After consumers visit firm a they realize that its product is of high quality and they update their beliefs about firm b being high quality, by eliminating (L,L), to $\mu^{in}_{ba}=\frac{q^2}{q^2+q(1-q)}$. Hence, firm b's expected product quality, for the consumers who visited a first, is $EV_b^{in} = \frac{q^2 V_H + q(1-q)V_L}{q^2+q(1-q)}$. The interim marginal consumer for a, using (3.2), must satisfy

$$x_a^{in} = \frac{p^* - p_a^{dev} + V_H - EV_b^{in} + \kappa + t}{2t} = \frac{(2-3q)DV + 3\kappa + 3t}{6t}.$$

Some consumers will switch from a to b if and only if $x_a^{in} < x^e \Leftrightarrow \kappa < -\frac{(1-q)^3 DV}{1-q+q^2}$. So, no such switching takes place. Also, no consumer will switch from b to a. This is because those who visited b first had a belief that a had a higher expected quality than b and after their visit to b they realize that both have the same quality. Therefore, the relevant marginal consumer for firm a is x^e.

Hence, a deviation is unprofitable if and only if

$$\frac{t}{2} \geq p_a^{dev} x^e = \left(\frac{DV}{3} + t\right)\left(\frac{3t - DV - q(3t-4DV) + q^2(3t-4DV)}{6t(1-q+q^2)}\right),$$

which holds if and only if

$$t \leq \frac{(1-2q)^2 DV}{9q(1-q)}.$$

Recall that we need $3t > DV$, assumption 2. From above we have $t \leq \frac{(1-2q)^2 DV}{9q(1-q)}$. The two conditions hold simultaneously if and only if $q < \frac{1}{2} - \frac{\sqrt{21}}{14}$, or $q > \frac{1}{2} + \frac{\sqrt{21}}{14}$.

Second, firm a can deviate to $p_a^{dev} \neq \frac{DV}{3} + t$. Consumers do not know whether it is (H,H) or (L,L), but they know that firms have the same quality and one firm has deviated. Therefore, such a deviation will not be profitable.

Finally, it is easy to see that if a firm does not want to deviate to $\frac{DV}{3} + t$, then it would not want to deviate to $-\frac{DV}{3} + t$. This is because in this case the initial consumer beliefs about the expected quality difference is tilted in favor of firm b and a firm's profit is increasing in the quality differential.

A.7 Proof of Lemma 6

Both firms have low quality products, (L,L). The candidate equilibrium prices are $(p^*, p^*) = (t,t)$ and the equilibrium profits $\pi_a = \pi_b = \left(\frac{t}{2}, \frac{t}{2}\right)$.

First, let's consider firm a's deviation to $p_a^{dev} = \frac{DV}{3} + t$. Consumers, upon observing $\left(\frac{DV}{3} + t, t\right)$, realize that one firm has deviated. So, it can be (H,H), or (L,L) and one firm raised its price to $\frac{DV}{3} + t$, or (H,L) and the low quality firm raised its price to t (it cannot be (L,H), since we assume unprejudiced beliefs). The initial consumer beliefs about expected product qualities are given by (A.12), where $EV_a \geq EV_b$. Consumers initially sort out according to (3.1), using (A.12), which yields the same x^e as in (A.15).

After consumers visit firm a they realize that its product is of low quality and they update their beliefs, by eliminating (H, H), and (H, L), so they also learn the quality of firm b, that is $EV_b^{in} = V_L$. The interim marginal consumer for a, using (3.2), must satisfy

$$x_a^{in} = \frac{p^* - p_a^{dev} + V_L - V_L + \kappa + t}{2t} = \frac{-DV + 3\kappa + 3t}{6t}.$$

Some consumers will switch from a to b if and only if $x_a^{in} < x^e \Leftrightarrow \kappa < \frac{q(1-q)DV}{1-q+q^2}$ (where x^e is given by (A.15)). Consumers who visit firm b first, eliminate (H, H), but do not know whether it is (L, L) or (H, L). So their updated belief about firm a being high quality is $\mu_{ab}^{in} = \frac{q(1-q)}{q(1-q)+(1-q)^2}$. The expected product quality of firm a becomes $EV_a^{in} = \frac{q(1-q)V_H+(1-q)^2V_L}{q(1-q)+(1-q)^2} > V_L$. The interim marginal consumer for a, using (3.2), must satisfy

$$x_b^{in} = \frac{p^* - p_a^{dev} + EV_a^{in} - V_L - \kappa + t}{2t} = \frac{(-1 + 3q)DV - 3\kappa + 3t}{6t}.$$

Some consumers will switch from b to a if and only if $x_b^{in} > x^e \Leftrightarrow \kappa < \frac{q^3 DV}{1-q+q^2}$ (where x^e is given by (A.15)). But if consumers who visited b first switch to a, they realize that a's product is of low quality. Given that they initially visited b with the expectation that a has higher quality, $EV_a \geq EV_b$, and now, after they have sunk the cost κ, they realize that both firms have low quality, they, as we have assumed, costlesly return to b. Therefore, no consumer will switch from b to a.

Thus, there are the following two different cases that we should examine.

Case 1: If $\kappa < \frac{q(1-q)DV}{1-q+q^2}$, then the market share of the deviating firm a is x_a^{in}.

Case 2: If $\kappa > \frac{q(1-q)DV}{1-q+q^2}$, then the market share of the deviating firm a is x^e.

We analyze each one of these two cases below.

When $\kappa \leq \frac{q(1-q)DV}{1-q+q^2}$, a deviation on part of firm a is unprofitable if and only if

$$\frac{t}{2} \geq p_a^{dev} x_a^{in} = \left(\frac{DV}{3} + t\right)\left(\frac{-DV + 3\kappa + 3t}{6t}\right),$$

which holds if and only if

$$\kappa \leq \frac{(DV)^2}{3(DV + 3t)}.$$

When $\kappa > \frac{q(1-q)DV}{1-q+q^2}$, a deviation on part of firm a is unprofitable if and only if

$$\frac{t}{2} \geq p_a^{dev} x^e = \left(\frac{DV}{3} + t\right)\left(\frac{3t - DV - q(3t - 4DV) + q^2(3t - 4DV)}{6t(1 - q + q^2)}\right),$$

which holds if and only if

$$t \leq \frac{(1 - 2q)^2 DV}{9q(1 - q)}.$$

Notice that we need $3t > DV$, assumption 2. From above we have $t \leq \frac{(1-2q)^2 DV}{9q(1-q)}$. The two conditions hold simultaneously if and only if $q < \frac{1}{2} - \frac{\sqrt{21}}{14}$, or $q > \frac{1}{2} + \frac{\sqrt{21}}{14}$.

The next two cases are the same as in the (H, H) case.

Firm a can deviate to $p_a^{dev} \neq \frac{DV}{3} + t$. Consumers do not know whether it is (H, H) or (L, L), but they know that firms have the same quality and one firm has deviated. Therefore, such a deviation will not be profitable.

Finally, it is easy to see that if a firm does not want to deviate to $\frac{DV}{3} + t$, then it would not want to deviate to $-\frac{DV}{3} + t$. This is because in this case the initial consumer beliefs about the expected quality difference is tilted in favor of firm b and a firm's profit is increasing in the quality differential.

A.8 Proof of Theorem 2

First, we need $t \leq \frac{(1-2q)^2 DV}{9q(1-q)}$ and $q > \frac{1}{2} + \frac{\sqrt{21}}{14} \approx 0.8273$ (recall from Lemma 4 that $q > \frac{2}{3}$, so $q < \frac{1}{2} - \frac{\sqrt{21}}{14}$ is eliminated). Combined with assumption 2, $DV \in \left[\frac{9q(1-q)t}{(1-2q)^2}, 3t\right)$. When we combine the conditions from Lemma 4, 5 and 6, we arrive at the following conditions that must be satisfied

- $\kappa < \min\left\{\frac{DV(DV-9t(1-q))}{9t}, \frac{q^3}{1-q+q^2}DV, \frac{(DV)^2}{3(DV+3t)}, \frac{q(1-q)DV}{1-q+q^2}\right\}$
- or, $\min\left\{\frac{DV(DV-9t(1-q))}{9t}, \frac{q^3}{1-q+q^2}DV\right\} > \kappa > \frac{q(1-q)DV}{1-q+q^2}$.

Note that $\frac{DV(DV-9t(1-q))}{9t} < \frac{q^3}{1-q+q^2}DV$, because even when $\frac{(DV-9t(1-q))}{9t}$ attains its maximum, which happens for $DV = 3t$ and $q = 1$ and $\frac{q^3}{1-q+q^2}$ attains its minimum which happens for $q = \frac{1}{2} + \frac{\sqrt{21}}{14}$, it is still the case that $\frac{DV(DV-9t(1-q))}{9t} < \frac{q^3}{1-q+q^2}DV$. Therefore, $\frac{q^3}{1-q+q^2}DV$ never binds and the constraints can be expressed as follows

- $\kappa < \min\left\{\frac{DV(DV-9t(1-q))}{9t}, \frac{(DV)^2}{3(DV+3t)}, \frac{q(1-q)DV}{1-q+q^2}\right\}$
- or, $\frac{DV(DV-9t(1-q))}{9t} > \kappa > \frac{q(1-q)DV}{1-q+q^2}$.

Next, note that $\frac{(DV)^2}{3(DV+3t)} \geq \frac{q(1-q)DV}{1-q+q^2}$ if and only if $t \leq \frac{(1-2q)^2 DV}{9q(1-q)}$. Therefore, $\frac{(DV)^2}{3(DV+3t)}$ is redundant. Hence, the constraints become

- $\kappa < \min\left\{\frac{DV(DV-9t(1-q))}{9t}, \frac{q(1-q)DV}{1-q+q^2}\right\}$
- or, $\frac{DV(DV-9t(1-q))}{9t} > \kappa > \frac{q(1-q)DV}{1-q+q^2}$,

which suggests that the only relevant constraint is $\kappa \leq \frac{DV(DV-9t(1-q))}{9t}$.

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