Externality and common-pool resources: The case of artesian aquifers ☆

Abstract

This study examines a specific class of common-pool resources whereby rivalry is not characterized by competition for the resource stock. Artesian aquifers are a typical example of such resources since the stock never depletes, even when part of the resource is extracted. We first propose a dynamic model to account for the relevant features of such aquifers such as the water pressure and well yield and characterize the corresponding dynamics. We then compare the social optimum with the private exploitation of an open-access aquifer. The comparison of these two equilibria highlights the existence of a new source of inefficiency. In the long run, this so-called *pressure externality* results in an additional number of wells for the same water consumption, thereby raising costs. Finally, we characterize a specific stock-dependent tax to neutralize the pressure externality.

JEL Classification: H21, H23, Q15, Q25, C61

Keywords: common-pool resource, externality, optimal management, public regulation, dynamic optimization

1. Introduction

Common-pool resources (CPRs) have been studied extensively since the seminal papers by Gordon [16] and Hardin [17] in the 1950s and 1960s. Most studies have focused on how non-excludability and rivalry induce different forms of externalities. Specifically, the literature on aquifers mainly identifies *stock externalities*, which create two distinct effects (Burt and Provencher [7]): a reduction in extraction opportunities and an increase in future extraction costs. It also identifies *spatial externalities*, which consider the impact of the location of pumping activity on the extraction costs of neighboring wells (Brozovic et al. [6], Pfeiffer and Lin [31]). However, both these types of externalities rely on the idea that individual withdrawal reduces the water stock available for consumption and therefore the water table (perhaps with a geographical concern), which, in turn, increases the future extraction costs for everyone (perhaps in an asymmetric way).

However , these concerns do not apply to *artesian aquifers*, which have two particular properties. First, they are a kind of *confined aquifer*, which is an aquifer confined between an upper and a lower impermeable layer that recharges from a distant and more elevated aquifer that is often unconfined. Second, they have the so-called *artesian property*: water naturally flows from a well, without *any pumping*, and withdrawal from the aquifer is immediately compensated so that there is no dewatering at all. The absence of pumping and spatial externalities due to pumping therefore suggests that this resource does not suffer overexploitation under open access and needs no regulation. The main objective of our study is to show that this perception is wrong. As the artesian property of each well is linked to the global water pressure in a confined aquifer, this *pressure externality* leads, under an open-access regime, to an excessive number of wells and thereby additional costs. This result is obtained by contrasting the open-access outcome with the socially optimal equilibrium, which internalizes this externality. In other words, even through the groundwater stock within the confined aquifer does not decrease, we show that the behaviors of economic agents are not aligned with the socially optimal outcome.

Few studies on groundwater management consider the pressure externality. Since Gisser and Sanchez's 1980 paper [15], most researchers have continued to analyze the potential role of water management under different assumptions, but kept the focus on *unconfined aquifers with pumping cost externalities*. A number of studies have compared perfect competition with socially optimal management outcomes. For instance, Provencher [36] and Provencher and Burt [37] respectively show that property rights allow the recovery of a large proportion of welfare gains, but that these gains remain relatively low. Allen and Gisser [1] find a small difference between competition and social planning, while Brill and Burness [5] find an increased divergence between both scenarios under different hydrological and economic assumptions, including demand growth, declining well yields, and low social discount rates. Other studies contrast the social planner solution with strategic behaviors (Negri [29], Rubio and Casino [40]). Rubio and Casino [40], for instance, confirm Gisser and Sanchez's [15] result. Some studies expand on this in a number of directions, including examining uncertainty (Knapp and Olson [24], Tsur and Graham-Tomasi [49]), conjunctive surface water use (Azaiez [3], Stahn and Tomini [46],[47]), and water quality management (Roseta-Palma [39]). Nevertheless, all these works assume that an aquifer behaves as a "bathtub" with perfect hydraulic conductivity and is only subject to atmospheric pressure. The water level is therefore constant across the aquifer and equal to the potentiometric surface.[1] The dynamics of these "single cell" and unconfined aquifers can be summarized by changes in the water level, and since water disposal requires, in this case, pumping, their users share a common part in their current cost function: the cost of the water lift.

[1]The potentiometric line is an imaginary line where the water pressure is equal to the atmospheric pressure. Water will rise to this specific level. In an unconfined aquifer, the potentiometric line is equivalent to the water table level.

Spatial externalities are mainly based on the location of the pumping activity or final users. Chakravorty and Roumasset [8], Chakravorty and Umetsu [9], and Pongkijvorasin and Roumasset [34] stress final-user heterogeneity by linking the water conveyance lost/cost to the distance to the water source, while Pitafi and Roumasset [32] focus on users located at different elevations. Other studies look at the limitations of the bathtub assumption due to spatially differentiated hydrological components (Katic and Grafton [22]). They borrow elements from the hydrologic literature on imperfect transmissivity,[2] such as Darcy's law [10], which describes the water flow in a porous medium, and Theis' analysis [48] of the depression cone induced by pumping. Brozovic et al. [6] and Merrill and Guilfoos [28] build on Theis' approach and relate the marginal cost of the water lift at a given location to the pumping histories and distances of other wells. This modifies the strength of the externality and potential gain from regulation, but remains, nevertheless, a pumping externality. Other strands of the literature consider multi-aquifer (or multi-cell) systems with water exchanges between these aquifers (or cells). The porosity between cells is governed by Darcy's law. Exchanges mostly take place between lateral aquifers (or cells) and induce spatial externalities (e.g., Saak and Peterson [42], Peterson and Saak [30], Pfeiffer and Lin [31], Athanassoglou et al. [2], and Manning and Suter [27]. However, as long as there is pumping in each cell, it remains a pumping externality, although the magnitude may vary across space. Indeed, the implicit water price for a given cell (i.e., the co-state variable associated with this cell) is linked to the water prices of other cells.

Spatial externalities are different if we compare *unconfined with confined* aquifers. Theis ' argument [48] on depression cones is based on confined aquifers, even though it can also, to an extent, be applied to unconfined aquifers (see Appendix 2 of Brozovic

[2]Transmissivity refers to the horizontal movement of water through an aquifer, which may be considered as a slow (imperfect) or instantaneous process.

et al. [6]). Darcy's law, which describes the flow resulting from imperfect transmissivity between aquifers, is not restricted to lateral flows and therefore does not necessarily induce spatial externalities. For instance, Gisser and Mercado [14] consider a confined and an unconfined aquifer (the Pecos River Basin aquifer), the former being located below the latter, with vertical flows governed by Darcy's law. By contrast, Worthington et al. [52] consider a two-part aquifer (the Crow Creek Valley aquifer), one part confined and the other unconfined, but nevertheless with perfect transmissivity. The primary argument of these authors is that confined aquifers are under pressure, which explains why, for instance, water can flow upward, according to Gisser and Mercado [14]. In fact, the main difference between confined and unconfined aquifers is that the water table is different from the potentiometric line. With confined aquifer systems, users share not only a common stock but also a common pressure, both being affected by individual withdrawals. Similarly, Reinelt [38] considers an aquifer system composed of an unconfined recharge area, a confined aquifer that obtains water from the recharge area, and a discharge area for the confined aquifer, at sea level, with possible saline intrusions. Modeling the change in the potentiometric surface due to pumping in a spatio-dynamical setting, he derives an optimal water extraction strategy. In doing so, he captures both the standard marginal pumping cost externality in the unconfined aquifer and the spatial externalities within the confined aquifer and between the confined and unconfined aquifers, as well as the externalities induced by saline intrusion. However, he observes that the global marginal pumping cost externalities are different in the unconfined and confined aquifers because the latter consumes the water pressure and water volume.

Our study starts with a different rationale, that is, we isolate this pressure externality. The best option is to consider aquifers for which water consumption requires no pumping at all. This is typically the case for artesian aquifers—confined aquifers that obtain their recharge from an unconfined area but whose potentiometric surface is above ground level.

In this case, the pressure is sufficiently strong to ensure that water naturally flows over the top of a drilling well without the need for pumping. Water yields are, nevertheless, dependent on the pressure of the confined aquifer, and when a new well is drilled, the pressure of the whole aquifer decreases, which consequently exerts a negative externality on the yields of existing wells. We refer to this as the "pressure externality," which requires managing the overall number of wells.

Artesian aquifers can be found in many countries. The Great Artesian Basin in Australia is considered as the largest and deepest in the world, underlying 22% of the continent. Other important systems are the Edwards Aquifer in Texas, USA and the Northern Sahara Aquifer System in northern Africa. There are also artesian wells south of Vancouver, Canada and in several places in India, providing water for millions of people. Nevertheless, this resource is subject to the "rule of capture"and is thus threatened by human pressure. Moreover, in many of the wells, the flow is still uncontrolled. As such, optimizing the exploitation of this resource and production of groundwater reservoirs is a difficult challenge. Regulations have already been implemented in some areas to limit adverse impacts. For instance, the St. Johns River Water Management District in Florida encourages well owners to control the flow and even abandon problematic wells by plugging them.

In this paper, we present a simple hydro-economic dynamic model of a confined artesian aquifer. To emphasize the pressure externality, we restrict our analysis to free-flowing artesian wells:[3] water is obtained by drilling a hole without any pumping and therefore without any pumping externalities. We also assume perfect transmissivity between the recharge area and confined aquifer to avoid any spatial externalities. Water yields are therefore driven only by the evolution of the water pressure in the aquifer over time.

[3]Even if few artesian aquifers are exploited with free-flowing wells nowadays, we adopt this theoretical assumption to highlight the pressure externality. An extension to pumping is discussed in Section 7.

Specifically, we use fluid dynamics to describe the relationship between the water stock, water pressure, and water discharge. On this basis, we first assume that this resource operates under an open-access regime. Second, we introduce an optimal management problem for the number of wells that takes into account this pressure externality. For both management regimes, we characterize the long-run steady-state values and their properties with respect to changes in the economic and hydrological parameters of the model. We then compare these two management regimes by introducing two rates that capture the main differences. The first rate measures the yield losses per well compared with the optimal management case, while the second rate measures the rate of increase in the long-run water cost. These rates act as proxies for the loss of pressure and social cost of overextraction, respectively. This approach allows us to challenge the Gisser–Sanchez effect since our welfare gain can be large. We also analyze the sensitivity of these rates to the economic and hydrological parameters of the model and propose a state-dependent tax that aligns the open-access steady state with the optimal management. We finally discuss potential extensions concerning farmer behavior and technology choices, especially the richer water policy induced by non-myopic behavior, the limitations of our entry mechanism, and the possibility to supplement free water harvesting by pumping.

The remainder of this paper is organized as follows. The specific features of an artesian aquifer, particularly the consequences of the water pressure model, are presented in Section 2. Section 3 presents the main economic characteristics of our system and analyzes the short- and long-run properties of this hydro-economic model under open access. Section 4 analyzes the centralized water management problem and Section 5 describes the long-run properties of this optimal management. Section 6 contrasts these two regimes in terms of pressure losses and social costs. Section 7 discusses further extensions and Section 8 concludes. The proofs and computations can be found in Appendices A–D.

2. Aquifers with flowing artesian wells

Artesian aquifers are overlain by a relatively impermeable layer, such that water is under a pressure greater than atmospheric pressure. Consequently, water may rise above the top of the aquifer and even, when the pressure is sufficiently large, above the land surface when a drill hole penetrates an artesian aquifer. In particular, we can observe wells that naturally flow to (or above) the land surface, without the need for pumping, when the potentiometric line is above the surface. This level represents the pressure exerted by water, given the force of gravity. Consequently, the confined layer remains saturated, even with exploitation. However, the abstraction of groundwater resources implies variations in the pressure, which in turn trigger falls in the water flow at the surface.

These artesian aquifers never dewater since they are fed by water that moves from a distant *outcrop* area, also called the recharge zone. This area is usually located at a high elevation (e.g., near mountains), exposed at the ground level, and a considerable distance away from the artesian aquifer, which is at a lower elevation beneath an impermeable layer. This difference in elevation generates a water column, h, the weight of which exerts a pressure on the artesian aquifer. More precisely, we define the *water column* as the vertical distance between the top of the water table in the outcrop area and upper layer of the artesian aquifer. For simplicity, the elevation of this upper layer is normalized to 0 so that $h \in (0, h_{\max})$, with $h_{\max}$ the upper level of the outcrop area. If this area behaves as a bathtub unconfined aquifer with a flat bottom of area A_r perpendicular sides and storativity rate s,[4] the weight of the water column that exerts this pressure is sA_rh. The water column may rise and fall according to the recharge and discharge toward the artesian aquifer. We denote R as the *potential* recharge, that is, all the water available at the surface, which may or may not reach the ground. The part of the potential recharge

[4]The storativity rate is defined as the volume of water released from storage per unit surface area of the aquifer per unit decline in the hydraulic head.

that effectively soaks into the soil represents the *actual* recharge, while the remaining water runs off over land.[5] In other words, only the proportion ρR recharges the outcrop area, where ρ is the infiltration rate.

This net recharge moves down toward the confining layer, replacing the water flowing out of the artesian wells because of the pressure exerted by the weight of the water column. We even assume perfect transmissivity between and inside these two parts of the aquifer to avoid any spatial externalities as well as that no economic activity takes place in the recharge area. Exploitation only occurs over the impermeable layer (i.e., no return flow); moreover, since water flows naturally through a well to the land surface, pumping is not necessarily required. To avoid any pumping externalities, we even assume that pumping is not available.[6]. It is therefore important to know how much water flows out of a well under pressure. From fluid dynamics, we can characterize the relationship between the water pressure and water column and then deduce the *maximum water yield* flowing out of a well using Torricelli's law.[7] Let us assume that k is the size of the drill hole (i.e., a technological characteristic)[8] and g is the acceleration due to gravity; then, the maximum water yield, r, is approximately given by

$$r(h) = k\sqrt{2gh} \tag{1}$$

We can easily observe that the lower the water column, the lower is the maximum water yield ($r'(h) > 0$). The water yield approximatively captures the pressure and a change in

[5]This distinction is commonly used in the hydrogeological literature (e.g., Rushton [41]).

[6]This assumption can seem rather extreme. However, for instance, it simply means that the marginal pumping cost exceeds the marginal cost of a unit of water obtained from an additional well. The possibility of pumping is discussed in Section 7.

[7]Under perfect conductivity, this law is an application of Bernoulli's principle relating pressure, velocity, and elevation for a steady-flow system. Torricelli's law describes the fluid flowing out of an orifice.

[8]By size, we mean the surface of a section of the drill hole. It can therefore be related to the diameter of the bore hole.

the water yield is equivalent to a change in the artesian pressure. Moreover, if n active wells are operating at capacity, the total water discharge is $w(h) = r(h)n$. Hereafter, we consider n to be a real number. This means, for instance, that the last well is only partially active. Figure 1 illustrates this specific structure.

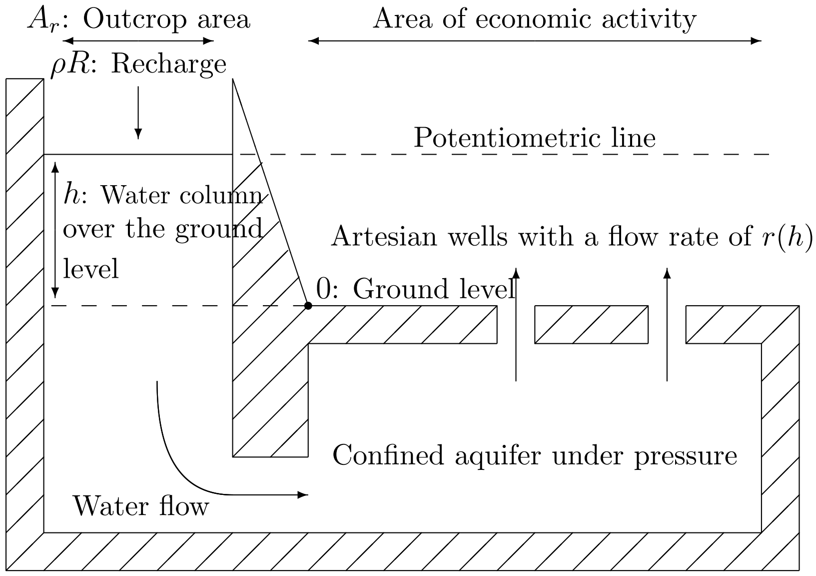

Figure 1: Schematic representation of a confined aquifer

Hence, we can characterize the hydrodynamics of artesian aquifers by two equivalent formulations: (i) a law of motion based on changes in the water column, $h(t)$, in the outcrop area and (ii) a law of motion based on changes in the maximum water yield, $r(t)$, of a well over the artesian aquifer. The dynamics of the water column in the recharge area result from a physical water balance between inflows and outflows. The actual recharge ρR is the source of incoming water, while outflows are the sum of the water discharge flowing at the surface in different wells, $w(h(t)) = r(h(t))n(t)$. Within this context, the time evolution of the water column (as long as $h \geq 0$) in the outcrop area is associated with changes in the potentiometric water head of the artesian aquifer and is described as

$$\dot{h}(t) = \frac{\rho R - w\left(h(t)\right)}{sA_r} \tag{2}$$

The relevant area A_r is now the size of the recharge area as opposed to the whole area of the aquifer in contrast to the Gisser–Sanchez [15] model of an unconfined aquifer, which explains why the gains from public management are not negligible.

Using Torricelli's law (1), we can characterize the time evolution of the maximum water yield:

$$\dot{r}(t) = \frac{kg\dot{h}(t)}{\sqrt{2gh(t)}} = \frac{k^2 g\dot{h}(t)}{r(t)} \tag{3}$$

Using the dynamics of Eq. (2), it follows that the dynamics of the yield of an artesian well are given by

$$\dot{r}(t) = \frac{k^2 g}{r(t)sA_r}\left[\rho R - r(t)n(t)\right] \tag{4}$$

These second dynamics sound more economically oriented since outflows result from the economic decision to build new wells, but they are unusual because they are based on the accumulation of resources flowing out of a reservoir. However, more water out of the aquifer means that the pressure correspondingly changes. Interestingly, this formulation captures the idea that the pressure of the water behaves as a CPR. Consequently, we may expect the existence of a new externality different from the pumping cost commonly observed in analyses of unconfined aquifers. This *pressure externality* results from the effect of an additional well on future yields and not on the future costs as is usual with pumping externalities.

It remains to set the initial condition of this system. For simplicity, we assume for Eq. (2) that the initial level of the water column is $h(0) = h_{\max}$ and for Eq. (4) that the initial yield is $r(0) = k\sqrt{2gh_{\max}}$.

3. Artesian aquifer as an open-access resource

As recharge occurs at elevated outcrop areas, we assume that water is only extracted above the artesian aquifer, such as in the distant lower areas (e.g., plains and valleys) in which the economic activity takes place. Water demand is described by decreasing inverse water demand with the overall production of water, $P(w)$, with $P'(w) < 0$, and providing a social benefit $\int_0^w P(\omega)d\omega$. Moreover, we ignore possible conveyance losses and thus assume that all the water flow at the surface is the actual volume of water used by consumers.

Individual wells are drilled to fulfill this demand, such that the number of wells corresponds to the number of well owners. A well owner faces an instantaneous profit function:

$$\Pi(t) = p(t)r(t) - c \tag{5}$$

with $p(t)$ being the market price the owner receives for the yield $r(t) = k\sqrt{2gh(t)}$ of his/her well and $c > 0$ the annual exploitation cost of the well.[9] Now, consider that an artesian aquifer is exploited by myopic well owners, who decide to exploit the aquifer when there is a positive profit. As the resource is open access, this also means that new wells are drilled as long as the observed profit remains positive. This free-entry condition associated with the competitive water market-clearing condition therefore induces the following dynamics of the number of wells:[10]

$$\dot{n}(t) = \alpha \left(P\left(n(t)r(t)\right) r(t) - c\right) \tag{6}$$

[9] Following the literature on irrigation networks (e.g., Chakravorty and Roumasset [8], Jandoc et al. [20]), this cost includes the per-period equivalent of the construction cost, operating and maintenance cost, and water provision because of irrigation networks, conveyance structures linking all the wells, and distribution to consumers.

[10] We adopt the standard free-entry condition based on the average rent earned from the exploitation resource. See, among others, Weitzamn [50] for a static characterization of open access and Gordon [16], who characterizes the bioeconomic equilibrium of a fishery.

with $\alpha > 0$ being an adjustment parameter or a behavioral constant given the market-clearing price $p(t) = P\left(n(t)r(t)\right)$.

Although there is no dewatering of the artesian aquifer, the water flows from any additional well reduce the water column ($h(t)$) in the outcrop area. A lower water column in this area consequently reduces the pressure in the confined aquifer and lowers the water yield of each existing well. This means that a complete description of the dynamics of the number of wells must include both the free-entry dynamics (Eq. (6)) and the dynamics of the water column in the outcrop area (Eq. (2)). With the help of Torricelli's law, which links the yields to the water column, we obtain

$$\begin{cases} \dot{n}(t) = \alpha\left(P\left(n(t)k\sqrt{2gh(t)}\right)k\sqrt{2gh(t)} - c\right) \\ sA_r\dot{h}(t) = \rho R - n(t)k\sqrt{2gh(t)} \end{cases} \tag{7}$$

The open-access hydro-economic equilibrium is thus obtained by setting the time variation in the number of wells and that of the water column in the outcrop area equal to 0: $\dot{n} = \dot{h} = 0$. We can easily deduce that the number of wells under open access is

$$n^m(c, \rho, R) = \tfrac{P(\rho R)}{c}\rho R \tag{8}$$

This number depends on the actual recharge, weighted somehow by a profitability rate. We then derive the steady-state value for the water column:

$$h^m\left(c, k, \rho, P\right) = \frac{1}{2g}\left(\frac{c}{kP\left(\rho R\right)}\right)^2 \tag{9}$$

At this point, we need a consistency assumption for the parameters of the model, which simply states that $h_{\max} > h^m$. Finally, using Torricelli's law (Eq. (1)), we can derive the long-run maximum water yield:

$$r^m(c, \rho, R) = \frac{c}{P(\rho R)} \tag{10}$$

and the global water consumption

$$w^m(\rho, R) = \rho R \tag{11}$$

We can even prove that the open-access hydro-economic equilibrium is stable, as summarized in the following proposition.

Proposition 1. *The dynamical system given by Eq. (7) has a unique steady state given by Eqs. (8) and (9) that is locally asymptotically stable. Table 1 presents the effects of any change in the economic parameters, that is, the unit cost c and size k of a well, and hydraulic parameters, that is, the recharge R and infiltration rate ρ . The notation +/− means that an effect can be either positive or negative depending on whether the elasticity $\varepsilon_P(w)$ of inverse demand evaluated at the steady-state consumption ρR is higher or lower than −1.*

Table 1: Effects of the economic and hydrological parameters on the open-access equilibrium

Parameters	Effects on water column	Effects on number of wells	Effects on water consumption	Effects on water yield
Cost (c)	+	−	0	+
size of the well (k)	−	0	0	0
Recharge (R)	+	+/−	+	+
Infiltration rate (ρ)	+	+/−	+	+

The impact of an increase in the unit cost per well is clear. It provides less incentive to drill additional wells, which typically constrains water in the outcrop area, the consequence of which is an increase in the pressure in the artesian aquifer and therefore of the yield. To understand how an increase in the size of the well affects the steady state, it is important to note that the steady-state yield of a well is, under free entry, independent of k (see

Eq. (10)). This means that any increase in k induces transitory dynamics toward a new steady state with an unchanged yield. However, during this transitory adjustment, the yield first increases, meaning that the new steady state can be reached only at a lower potentiometric level, h. Moreover, bearing in mind that the water consumption at the steady state is equal to the recharge, the number of wells must be unchanged. An increase in the infiltration rate or recharge increases the amount of water in the aquifer. At the steady state, this increases the water column, yield, and water consumption. The effect on the number of wells depends on the elasticity of inverse demand evaluated at the net recharge, that is, $\varepsilon_P(\rho R) = \frac{P'(\rho R)\rho R}{P(\rho R)}$. If $\varepsilon_P(\rho R) > -1$ ($\varepsilon_P(\rho R) < -1$), the number of wells increases (decreases).

4. Optimal management of an artesian aquifer

The social planner aims to choose the number of active wells within the artesian aquifer in each time period t, accounting for the water flow dynamics (Eq. (4)). If $\delta > 0$ denotes the social discount rate, he/she maximizes the discounted sum of social benefits net of exploitation costs:

$$\begin{aligned} &\max_{n(t)} \int_0^{+\infty} \left(\int_0^{r(t)n(t)} P(\omega)d\omega - cn(t) \right) e^{-\delta t} dt \\ &\text{s.t.} \quad \dot{r}(t) = \frac{k^2 g}{r(t) s A_r} \left[\rho R - r(t)n(t) \right] \end{aligned} \tag{12}$$

In this formulation, the objective of the social planner is to control the amount of water flowing out of the wells given the time variation in artesian well yields. Considering the total water discharge, we can write

$$\int_0^{r(t)n(t)} P(\omega)d\omega = \int_0^{w(t)} P(\omega)d\omega$$

and using the hydrological relationship between the artesian aquifer and outcrop area given by Torricelli's law (Eq. (1)), the social planner can take a more standard approach:

$$\max_{w(t)} \int_0^{+\infty} \left(\int_0^{w(t)} P(\omega) d\omega - \frac{cw(t)}{k\sqrt{2gh(t)}} \right) e^{-\delta t} dt$$

$$\text{s.t.} \quad sA_r \dot{h}(t) = \rho R - w(t) \tag{13}$$

This formulation is convenient to analyze the optimal exploitation of an artesian aquifer since the optimization program is defined only over water quantity, rather than water and the number of wells. Moreover, the dynamics of the resource are now independent of the state variable. This formulation enables us to derive (i) the optimal number of wells from the social planner's choice of water supply and (ii) the water pressure in the artesian aquifer resulting from its optimal exploitation. More precisely, we know that the potentiometric line of the aquifer corresponds to the level of the water column in the recharge area, h, and is the point to which water naturally flows from a single well. Given this level, we can first deduce the well yield and number of wells to obtain the total water discharge, w. Second, the potentiometric line directly allows us to capture the water pressure. More relevantly, this formulation associates the potentiometric line with the constant marginal cost of drilling an additional well, c, which enables us to capture the pressure externality, but not the pumping cost externality. For instance, a decrease in the potentiometric line reduces the water yield. This requires, for a given water production level, $w(t)$, additional wells to be drilled, thereby increasing the overall well exploitation cost.

However, this new setting has one problem. The co-state variable, $\lambda(t)$, associated with the water column dynamics in the program (Eq. (13)) now stands for the shadow price of a one-unit decrease in the water column in the recharge area. This variable only partially captures the shadow price of a unit of water naturally rising at the surface from an artesian well. This value is actually given by the co-state variable, $\Gamma(t)$, associated with the dynamics used in the optimization problem (Eq. (12)). This variable measures the monetary consequences of a decrease in the yield of an artesian well. However, let us denote the future values at t of the two programs (Eqs. (12) and (13)) along the optimal path by, respectively, $V_1^*(t) = V_1((n^*(t)), r^*(t), \Gamma^*(t))$ and $V_2^*(t) = V_2((w^*(t)), h^*(t), \lambda^*(t))$. We know from the equivalence of both programs that $\forall t$, $V_1^*(t) = V_2^*(t)$. It follows from Torricelli's law (Eq. (1)) and the usual property of the co-states that

$$\Gamma^*(t) = \frac{\partial V_1^*}{\partial r} = \frac{\partial V_2^*}{\partial h}\frac{\partial h}{\partial r} = \lambda^*(t)\left(\frac{r^*(t)}{k^2 g}\right) = \lambda^*(t)\left(\frac{\sqrt{2gh^*(t)}}{kg}\right) \tag{14}$$

In other words, we can obtain the shadow price of a unit of water naturally rising at the surface from an artesian well using the solution to the program given by Eq. (13).

The current-value Hamiltonian becomes

$$\mathcal{H}(w(t), h(t), \lambda(t)) = \int_0^{w(t)} P(\omega)d\omega - \frac{cw(t)}{k\sqrt{2gh(t)}} + \frac{\lambda(t)}{sA_r}\left[\rho R - w(t)\right] \tag{15}$$

where $\lambda(t)$ is the current-value shadow price associated with the water column of the recharge area. We derive the following first-order conditions:

$$P(w(t)) = \frac{c}{k\sqrt{2gh(t)}} + \frac{\lambda(t)}{sA_r} \tag{16}$$

$$\dot{\lambda}(t) = \delta\lambda(t) - \frac{cgw(t)}{k\sqrt{(2gh(t))^3}} \tag{17}$$

Eq. (16) represents the usual optimality condition, which yields a marginal benefit in each period equal to the total marginal costs, the sum of the marginal exploitation cost (of an additional unit of water) and the water shadow price, $\frac{\lambda(t)}{sA_r}$.[11]

[11]Recall that λ measures the shadow price of one additional drop of water in the ground and $\frac{\lambda}{sA_r}$ is

Eq. (17) describes the behavior of the shadow value. This equation shows that the time evolution of the shadow price depends on the discount factor, which following the literature on unconfined aquifers, depends on the marginal exploitation cost. In fact, a stock-dependent recharge impacts the value of the resource since the current exploitation of the artesian aquifer affects the water column in the outcrop area, which in turn influences the pressure and thus future flow.

5. Sustainable artesian water exploitation

We can now investigate the sustainable management of an artesian aquifer by characterizing the optimal steady state, that is, by setting $\dot{h} = \dot{\lambda} = 0$. Indeed, in the steady state, we characterize the highest rate of exploitation without depleting the aquifer, or rather without reducing to zero the water pressure in the long run. From Eq. (2), we can directly derive the steady-state level of The global water consumption:

$$w^o = \rho R \tag{18}$$

Regarding the standard model of an unconfined aquifer, the global water consumption corresponds to the actual recharge, but now in the distant outcrop area. Using Eq. (17) and the stationary state for the water discharge (Eq. (18)), we also obtain the steady state for the shadow value of water:

$$\lambda(h^o; c, k, \delta, \rho, R) = \frac{cg\rho R}{\delta k \sqrt{(2gh^o)^3}} \tag{19}$$

Then, using Torricelli's theorem (Eq. (1)), we can derive the long-run maximum water

consequently the shadow value of a marginal water column elevation in the outcrop area.

yield of a well

$$r(h^o;k) = k\sqrt{2gh^o} \tag{20}$$

the shadow value of the pressure

$$\Gamma(h^o;c,k,\delta,\rho,R) = \frac{c\rho R}{2\delta g k^2 h^o} \tag{21}$$

and the number of wells

$$n(h^o;k,\rho,R) = \frac{\rho R}{k\sqrt{2gh^o}} \tag{22}$$

All these steady-state values depend on the long-run level of the water column in the distant outcrop area, h^o. On the basis of previous observations and using Eq. (16), we obtain the condition required for the level of the water column at the steady state. This equates the long-term water price to the long-term marginal cost of an additional unit of water:

$$P\left(\rho R\right) = \frac{c}{k\sqrt{2gh^o}}\left(1+\frac{\rho R}{2\delta h^o s A_r}\right) \tag{23}$$

Using the long-term pressure (Eq. (20)) and its shadow value (Eq. (21)), it can also be written as

$$r^o P\left(\rho R\right) = c + \tfrac{k^2 g}{s A_R}\Gamma^o \tag{24}$$

In other words, the long-term marginal gain from an additional well must be equal to its cost c augmented by the long-term marginal cost of the pressure externality.

Hence, we can investigate the existence of a steady state based on this single condition and examine the stability properties. We again need a consistency assumption for the parameters to ensure that $h^o < h_{\max}$.[12] All the results are introduced in the proposition

[12] In Appendix B, we explicitly compute the solution to Eq. (23) and provide this consistency assumption.

below.

Proposition 2. *Any optimal water consumption path* $w^o(t)$ *and water column path* $h^o(t)$ *that satisfies the optimal first-order conditions (16) and (17), with the dynamics (2), admit a unique steady state, which is a local saddle point.*

We can now analyze the long-run impact of variations in the parameters of the model on the water column, water consumption, and water yield of a well as well as the number of wells. More precisely, we study the effect of the hydrological parameters $\{A_r, R, \rho\}$ and economic parameters $\{c, k, \delta\}$.[13]

Regarding **the long-run water column**, Eq. (23) shows that all the parameters, except the natural recharge R and ρ, affect the long-run costs only. This specifically modifies the incentive to exploit artesian water by affecting the full marginal cost of a unit of water, that is, the sum of the marginal exploitation cost and marginal user cost. Typically, the two parameters associated with exploitation, $\{c, k\}$, affect both parts of the full marginal cost. First, an increase in the cost of drilling a well raises the full marginal cost and thus the water column for the optimality condition (23) to hold in the long run. Conversely, a larger drill hole size, k, decreases the full marginal cost of a unit of water and leads to a decline in the long-run water column.

Second, the hydrogeological parameter A_r and discount rate δ both affect the time variation in the shadow value of the resource (Eq. (17)), and thus they impact its long-run value. A larger recharge area, A_r, provides a greater incentive to exploit water because the resource has a lower future value. Likewise, the higher the discount rate, the lower is the present value of the future benefit. Both parameters therefore lead to a lower water column in the long run.

Finally, an increased potential recharge R or a larger infiltration rate ρ simultaneously impacts both water demand and the long-run social cost. An increase in R or ρ allows

[13]Detailed computations can be found in Section Appendix C.

more water to infiltrate the outcrop area, which consequently decreases the water price. Intuitively, at the steady state, the higher inflows, the higher are the outflows of the system. This drives up marginal costs. The overall effect on demand and costs decreases the incentive to exploit artesian groundwater, thus leading to a higher water column in the long run. Table 2 summarizes these comparative statics results.

Table 2: Effects of the economic and hydrological parameters on the optimal long-run water column

Parameter	Effect
Cost of a well (c)	+
Size of the well (k)	−
Discount rate (δ)	−
Recharge area (A_r)	−
Recharge (R)	+
Infiltration rate (ρ)	+

Let us now analyze the effect of the variations in each parameter on **the long-run water consumption**, w^o. Eq. (18) shows that only the recharge and infiltration rate matter. In fact, a higher potential recharge, R, or a larger infiltration rate, ρ, increases the net recharge and thus the water consumption.

Concerning **the long-run water yield of a well**, Eq. (20) shows that all the parameters except k indirectly modify the yield after a change in the water column, which is positively related to the yield. It follows that for the parameters $\{c, \delta, A_r, R, \rho\}$, the signs of the effects are those found in Table 2. Finally, a single parameter has an *a priori* ambiguous impact: a larger drill hole results in a positive direct impact on yields and a negative indirect impact because of an abatement of the water column level. However, both effects alter the cost structure and the impacts of these opposite variations offset such that we observe a positive overall effect. Consequently, an increased drill hole size raises the long-run water yield.

The analysis of the impacts on **the long-run number of wells** is trickier. Recall that $n(h^o) = \frac{w^o}{r(h^o)}$. The variations in the number of wells thus depend on the simultaneous changes in the water consumption and water yield. For the parameters $\theta = \{c, \delta, A_r\}$, this is fairly straightforward since they only depend on the effects on the water column. More precisely, we observe a decline in the number of wells in the long run. Recall from our previous result that changes in the size of the drill hole, k, positively affect the water yield and thus decrease the number of wells. The effect of an increase in the recharge R or in the infiltration rate ρ is more ambiguous. Both increase the long-run water consumption and yield of a well. It is therefore difficult to estimate which effect dominates the other. To illustrate this ambiguity, we provide in Section Appendix C a class of examples in which this effect can be either positive or negative.

Table 3 summarizes the results.

Table 3: Variations in the water consumption, water yield, and number of wells

Parameter	Impact on water consumption (w)	Impact on water yield (r)	Impact on the number of wells (n)
Marginal cost (c)	0	+	-
Size of the drill hole (k)	0	+	−
Recharge area (A_r)	0	-	+
Discount rate (δ)	0	-	+
Recharge (R)	+	+	?
Infiltration rate (ρ)	+	+	?

6. Pressure externality

We now show that the behaviors of economic agents under open access are not aligned with the socially optimal outcome. Well owners are not rivals in water exploitation in the artesian aquifer since this aquifer never dewaters. However, this activity does reduce the water column in the outcrop area, which decreases the pressure in the artesian aquifer and reduces the water yields for other well owners. The extra decline in pressure that arises under an open-access regime is the pressure externality. This externality has a fairly intuitive interpretation. As the owners do not take this pressure externality into account, they have an incentive to drill more wells than the number required by the social optimum. However, at the steady state, the water consumption is the same in both cases. With more active wells for the same consumption, yields must be lower in the open-access case. Moreover, according to Torricelli's theorem (Eq. (1)), which links yields to the water column, this water column is lower. Therefore, under open access, there is less pressure on an artesian aquifer.

The question is now how to measure the impact of this externality. We can measure either the hydrological consequences or the economic consequences. First, it would be interesting to know the yield reduction rate of a well at the steady state with respect to optimal management. Using Eq. (10), which describes the steady-state yield under open access, and Eq. (20) for the optimal management regime (h_0 being the solution to Eq. (23)), this rate is given by

$$\Delta^{-}_{Yield}\left(c, k, \delta, A_r, R, \rho\right) = 1 - \frac{r^m\left(c, R, \rho\right)}{r^o\left(c, k, \delta, A_r, R, \rho\right)} \tag{25}$$

Second, we know that under open access, there are more wells in the long run for the same water consumption. It would thus be interesting to know the rate at which the cost of one unit of water rises with respect to optimal management. Without a pumping cost due to artesianism, this rate is

$$\Delta^{+}_{Cost} = \left(\frac{cn^m}{w^m}\right) \diagup \left(\frac{cn^o}{w^o}\right) - 1 \tag{26}$$

with n^m, w^m, n^o, and w^o given by, respectively, Eqs. (8), (11), (22), and (23). If we now recall that the water consumption is the same at both steady states, this rate of increase in the water cost coincides with the proportion of additional wells, that is,

$$\Delta^{+}_{Cost}\left(c,k,\delta,A_r,R,\rho\right)=\frac{n^m\left(c,R,\rho\right)}{n^o\left(c,k,\delta,A_r,R,\rho\right)}-1 \tag{27}$$

We can go a step further by examining the impact of the different parameters on these two rates. Concerning the size of the well, k, discount rate, δ, and size of the outcrop area, A_r, the results are fairly straightforward since these parameters only modify the steady-state yield and number of wells under optimal management. Since Δ^{-}_{Yield} and Δ^{+}_{Cost} increase and decrease r^0, respectively, we can immediately assert, using Table 3, that the rate of yield losses, Δ^{-}_{Yield}, and rate of additional water costs, Δ^{+}_{Cost}, both increase with the size of the well, k, and decrease with the discount rate, δ, and size of the outcrop area, A_r. However, the effect of the cost per well, c, or that of the net recharge, ρR, is less obvious since these quantities modify the yield and the number of wells in both steady states. Nevertheless, we can show that an increase in c reduces both rates, while the sign of the effect of an increase in the net recharge depends again on the elasticity of inverse demand assessed at the net recharge, $\varepsilon(\rho R)=\frac{P'(\rho R)\rho R}{P(\rho R)}$.

The following proposition summarizes our discussion.

Proposition 3. *Comparing the open-access steady-state values with those of the socially optimal one, there are more wells, $n^m > n^0$, a lower yield, $r^m < r^o$, and a lower water column, $h^m < h^0$, in the outcrop area. Table 4 describes the effects of the parameters on the rate of yield losses and the rate of increase in the water cost. The notation $+/-$ means that the effect can be either positive or negative depending on whether the elasticity, $\varepsilon_P(w)$, of inverse demand evaluated at the steady-state consumption, ρR, is higher or lower than $-1/2$.*

Table 4 also suggests that the Gisser–Sanchez effect is again at work since the rate of yield losses and rate of additional social costs both decrease with the area, A_r. However, contrary to the Gisser–Sanchez effect, this quantity is not the size of the artesian aquifer, but rather the size of only the recharge area, which, in the case of artesian systems, is

Table 4: Impact of the parameters on the rate of yield losses and additional social cost rate

Parameter	Yield losses, Δ^{-}_{Yield}	Additional water cost, Δ^{+}_{Cost}
Marginal cost (c)	$-$	$-$
Size of the drill hole (k)	$+$	$+$
Recharge area (A_r)	$-$	$-$
Discount rate (δ)	$-$	$-$
Recharge (R)	$+/-$	$+/-$
Runoff (ρ)	$+/-$	$+/-$

often relatively narrow with respect to the area covered by the artesian aquifer. This last observation suggests that the two equilibria would be different, thus requiring public regulation.

If a policy aims to limit this additional water cost, it should, on the basis of Eq. (27), refrain from providing an incentive to drill new wells and instead ensure that their number does not exceed that recommended by the steady state of an optimal management policy. This means that under open access, either well extension is limited by a standard command-and-control policy or the profitability of an addition well is reduced (see Eq. (5)) by imposing a fee to be paid for each new well. However, this last incentive-based policy must provide some signals related to pressure losses. Since pressure is inversely proportional to the height of the water column in the outcrop area, $h(t)$, this tax rate must decrease with $h(t)$. More precisely, we can claim the following.

Proposition 4. *A state-dependent fee per new well given by* $\tau\left(h(t)\right) = \frac{c\rho R}{2\delta h(t) s A_r}$ *ensures that the steady state of the open-access management of an aquifer coincides with the steady state under optimal management.*

7. Further discussions

To sharpen our analysis of the pressure externality, we make some simplifications, especially about the behavior of well owners and technology choices. Here, we discuss alternative water policies, particularly when agents do not behave myopically. We also discuss our entry mechanism when agents are heterogeneous. Finally, we provide some insights into the significance to our model for the introduction of pumping.

7.1. Alternative water policies

In this study, we have proposed a resource with open access (i.e., free and unregulated exploitation) and with myopic decisions based on current profits. In this case, only simple regulation tools can be considered such as the direct control of the number of wells and a fee for each new well. However, in practice, few resources are open access and most are collectively owned. Consequently, resource access is restricted to some users, who account for the evolution of stock over time; however, there may be no rule limiting extraction. In our case, this would mean a finite number of landowners, $i = 1...M$, who can choose the number of wells, $n_i(t)$, they drill on their own land by maximizing their discounted sum of profits:

$$\max_{n_i(t)} \int_0^{+\infty} \left(\int_0^{r_i(t)n_i(t)} P(\omega)d\omega - cn_i(t) \right) e^{-\delta t} dt \tag{28}$$

$$\text{s.t.} \quad \dot{r}(t) = \tfrac{k^2 g}{r(t)sA_r} \left[\rho R - \sum_{i=1}^{M} r_i(t)n_i(t) \right] \tag{29}$$

Let us assume, for simplicity, that farmers follow open-loop strategies since we know they account for their impact on the resource, but not on others, such that externalities arise. On this basis, we may consider alternative rules to manage such unregulated common-property resources . First, we can assume that a local institution exists that grants permits or licenses subject to fees such that the total amount of water collectively exploited is defined by the optimal level. The fees should recover the social costs of marginal reductions in pressure. Other measures can be introduced to control the number of wells, including taxes, quotas, property rights over the resource, and limits on demand.

Aside from such institutional regulation, voluntary cooperation between landowners may overcome overexploitation. A large part of the literature examines cooperation and contracting between oil and gas users (e.g., Libecap and Wiggins [25], [26], Smith [43]). This literature offers a new array of voluntary solutions. For instance, Libecap and Wiggins ([25]) analyze the use of prorationing contracts to control output production. Some examples of profit-sharing systems in fisheries demonstrate that cooperation can coordinate resource exploitation. For instance, Kaffine and Costello [21] propose a system under which users contribute a part of their profits to a pool, which is then redistributed among them.

All these alternatives are, nevertheless, implemented to internalize a range of externalities, including stock effects, strategic interactions, and spatial externalities if they exist. Our model allows us to design a simple instrument to address the pressure externality.

7.2. The entry mechanism

In line with Smith [44] [45], we have assumed that well owners decide to exploit the resource based only on the profitability condition of Eq. (6): a condition under which only the number of entrants matters. This means that we can examine entry using a simple counting device and that future entry conditions are influenced only by the number of pre-existing firms. This ideally requires that all the entrants are symmetric, which is the case in our mechanism, or at least that the water price and dynamics of artesian yields change after entry in proportion to the new number of installed well owners. Our model, for instance, does not readily extend to the situation in which well owners have the ability to choose, before entry, the size of their borehole for a cost that increases with borehole size. This choice, which is dictated by observations of current yields, creates heterogeneity between the installed well owners, which then affects both the water price and the dynamics of the yield, and hence future entrance conditions.

A radical solution is to forget the entry issue by moving to the "landowner" problem (Eq. (28)), allowing the choice of borehole size k for an additional cost $c(k)$, and replacing the dynamics of the yields (Eq. (29)) with the dynamics of the water column of the recharge area (Eq. (2)) exerting pressure on the artesian aquifer. In fact, maintaining the entry assumption becomes a real challenge, requiring a dynamic model for a competitive industry with heterogeneous firms that endogenously determine the processes for entry and exit. This approach has been used in real business-cycle models to show how entry and exit propagate the effects of aggregate shocks in line with Hopenhayn [18] [19]. There are also several applications of fishery management to understand fleet evolutions (see Weninger and Just [51], Da-Rocha and Sempere [11], and Da-Rocha et al. [12]). However, to the best of our knowledge, no equivalent modeling has been developed for aquifer management.

We are, nevertheless, confident that this does not change our basic result. In fact, we might expect a social planner to reach, in a steady state, a borehole size allocation that is globally cost-minimizing with regard to the water consumption. By contrast, the steady-state allocation of borehole size using an entry mechanism would inherit the history of the different entries and would not necessarily converge toward a cost-minimizing allocation. The misalignment with a socially optimal scenario will only increase in situations in which there is no choice of borehole size.

7.3. Pumping opportunities

To focus on the pressure externality, we do not introduce pumping. This makes sense in our study because with the artesian property, water flows naturally out of a well in contrast to the general perception that pumping is the only way to harvest groundwater. This, however, raises the question of what happens when the potentiometric line reaches ground level. In other words, the issue of the exploitation of the aquifer arises when water no longer flows when there is no pumping. This clear-cut extension is challenging.

First, in our model without pumping, we show that the potentiometric line never reaches ground level because although the long-run water price is a finite number, $P(\rho R)$, the cost of a drop of water may tend to infinity for very low water column levels, regardless of the management regime.[14] This implies that pumping must start before the artesian property disappears. However, we need to understand how to model pumping in an artesian aquifer as well as the optimal time to start pumping.

Second, if pumping drives the potentiometric line to below ground level, we must be able to define when this line drops below the upper confining layer. The first shift has no real consequences on the dynamics of the model since the aquifer is still a confined type whose water is under pressure. However, this is not the case for the second shift when we move from a confined to an unconfined aquifer. The dynamics of the water column in the recharge area are consequently replaced with the standard dynamics of the water height in the whole aquifer. In other words, the dynamics of the system should incorporate a potential regime shift (e.g., Polasky et al. [33], Boucekkine et al. [4], Prieur et al. [35], and De Zeeuw [13]).

Although the second problem is comparatively more technical and well documented, the first one induces global changes in the modeling. Introducing pumping at the well level is relatively easy. On the one hand, each pump has a gauge pressure that can be expressed as an additional water head, h_p, and therefore additional yields ($r = k\sqrt{2g(h+h_p)}$ in our formulation). On the other hand, pumping requires energy proportional to each unit of water lifted to this additional head as well as investment and maintenance costs (e.g., in the Gisser–Sanchez [15] formulation). However, the central question is to determine, especially in the optimal management problem, the time when the pumps are activated and which existing wells are involved. For artesian aquifers, this raises the more general question of moving from extensive water harvesting based on the development of new wells to a more intensive use based on pumping, which brings with it the risk of altering

[14]Recall that under open access, the unit cost is $\frac{c}{r^m(h)}$ from the simple transformation of Eq. (10), while under socially optimal management, it is $\frac{c}{k\sqrt{2gh^o}}\left(1+\frac{\rho R}{2\delta h^o s A_r}\right)$ using Eq. (23).

ecological systems due to regime shifting. This important issue could be addressed in further work.

8. Concluding remarks

This study contributes to the literature on CPRs and the externalities arising from the exploitation of these resources as well as to the literature on groundwater management. We analyzed the role of public policy when CPRs exhibit a low degree of rivalry in terms of consumption. We introduced a specific type of CPR, artesian aquifers, whose stock remains fully saturated even when the resource is exploited. Moreover, if such an aquifer is tapped by a well, the water naturally rises above the land surface since it is stored under pressure. A loss of pressure consequently alters the outflow at the surface. Economic agents therefore do not compete to appropriate part of the resource stock, but do suffer from a reduction in artesian pressure as the number of wells increases. The tragedy of the commons results here from the existence of a pressure externality. We evaluated the impact of this externality as the difference between the open-access outcome and socially optimal solution. We used fluid mechanics, particularly Torricelli's theorem, to introduce the water pressure and water yields flowing out at the surface in a dynamic optimization model. We showed that the number of wells depends on the level to which water will rise, which corresponds to the water column in the recharge area. In the long run, the height of this water column is therefore lower under open access than under socially optimal management. This externality results in the long run in additional wells for the same water consumption, and hence in additional costs, thus requiring public regulation. We even provide a state-dependent tax scheme that corrects this externality at the steady state.

We considered a simple setting in which the pressure externality is isolated and our analysis could be expanded in a number of directions by future research. First, it would be interesting to extend this analysis to the case of other management regimes (e.g., including rational agents and strategic externality) to analyze, for instance, how water users compete for a resource that never depletes. Second, we could also consider more complex dynamics of such aquifers by introducing an endogenous infiltration rate or leakages. Finally, we only characterized situations in which artesian properties hold in the long run. It may thus be interesting to analyze situations in which these properties disappear, that is, when an artesian aquifer becomes an unconfined aquifer in the long run, implying the need for pumping.

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Appendix A. Proof of Proposition 1

(i) *Local stability*

From Eq. (7), observe that the linear approximation of these dynamics around the steady state is given by

$$\begin{pmatrix} \dot{n} \\ \dot{h} \end{pmatrix} = \underbrace{\begin{bmatrix} \alpha P'\left(w^m\right) k^2 2gh^m & \alpha \left(P'\left(w^m\right) n^m k^2 g + P\left(w^m\right) k \sqrt{g} \left(2h^m\right)^{-1/2} \right) \\ -\frac{k\sqrt{2gh^m}}{sA_r} & -\frac{1}{sA_r}\left(n^m k \sqrt{g} \left(2h^m\right)^{-1/2} \right) \end{bmatrix}}_{=D_m} \begin{pmatrix} \dot{n} - n^m \\ \dot{h} - h^m \end{pmatrix} \tag{A.1}$$

Under our assumption, it is immediate that

$$trace(D_m) = \alpha P'\left(w^m\right) k^2 2gh^m - \frac{1}{sA_r}\left(n^m k \sqrt{g} \left(2h^m\right)^{-1/2} \right) < 0 \tag{A.2}$$

Moreover, after some computation

$$\det(D_m) = \frac{\alpha P\left(w^m\right) k^2 g}{sA_r} > 0 \tag{A.3}$$

We can therefore conclude that our dynamical system is locally asymptotically stable.

(ii) *Comparative statics at the steady state*

From Eqs. (8), (9), (10), and (11), we immediately obtain

	∂c	∂k	∂R	$\partial \rho$
∂h^m	$\frac{c}{g}\left(\frac{1}{kP(w^m)}\right)^2 > 0$	$-\frac{1}{gk^3}\left(\frac{c}{P(w^m)}\right)^2 < 0$	$-\frac{\left(\frac{c}{k}\right)^2 P'(w^m)\rho}{g(P((w^m))^3} > 0$	$-\frac{\left(\frac{c}{k}\right)^2 RP'(w^m)}{g(P(w^m))^3} > 0$
∂w^m	0	0	$\rho > 0$	$R > 0$
∂r^m	$\frac{1}{P(w^m)} > 0$	0	$-\frac{c\rho P'(w^m)}{(P(\rho R))^2} > 0$	$-\frac{cRP'(\rho R)}{(P(w^m))^2} > 0$
∂n^m	$-\frac{w^m P(w^m)}{c^2} < 0$	0	$\frac{\rho}{c}\left(P\left(w^m\right) + w^m P'\left(w^m\right)\right)$	$\frac{R}{c}\left(P\left(w^m\right) + w^m P'\left(w^m\right)\right)$

Finally, remark that $\frac{\partial n^m}{\partial R} \lesseqgtr 0 \Leftrightarrow \varepsilon_P(w^m) := \frac{w^m P'(w^m)}{P(w^m)} \lesseqgtr -1$ and $\frac{\partial n^m}{\partial \rho} \lesseqgtr 0 \Leftrightarrow \varepsilon_P(w^m) \lesseqgtr -1$.

Appendix B. Proof of Proposition 2

(i) Existence and uniqueness of the steady state

From our discussion, there exists a unique state if and only if Eq. (23), given by

$$P\left(\rho R\right) - \frac{c}{k\sqrt{2gh}}\left(1 + \frac{\rho R}{2h\delta s A_r}\right) = 0 \tag{B.1}$$

admits a unique solution in h. Setting $x = \frac{1}{\sqrt{2h}}$, this equation becomes

$$P\left(\rho R\right) - \left(\frac{c}{k\sqrt{g}}\right)x - \left(\frac{c\rho R}{k\delta s A_r\sqrt{g}}\right)x^3 = 0 \Leftrightarrow x^3 + \underbrace{\frac{\delta s A_r}{\rho R}}_{=p>0}x + \underbrace{\left(-\frac{k\delta s A_r\sqrt{g}P(\rho R)}{c\rho R}\right)}_{=q<0} = 0 \tag{B.2}$$

Since $4p^3 + 27q^2 > 0$, we know from the Cardano formula that this equation admits a unique solution in the reals and since $p > 0$ and $q < 0$, this solution, x^o, is strictly positive and is given by

$$x^o = \sqrt[3]{-\frac{q^2}{2} + \sqrt{\frac{q^2}{4} + \frac{p^3}{27}}} + \sqrt[3]{-\frac{q^2}{2} - \sqrt{\frac{q^2}{4} + \frac{p^3}{27}}} \tag{B.3}$$

As a consequence, $h^o = \frac{1}{2(x^o)^2}$ and the consistency assumption on our parameters now becomes $\frac{1}{2(x^o)^2} < h^{\max}$.

(ii) Local saddle point stability

Let us first remember that the dynamics are given by

$$\begin{cases} \dot{\lambda} = \delta\lambda - \partial_h \mathcal{H}\left(w,h,\lambda\right)\big|_{w=w(h,\lambda)} \\ \dot{h} = \partial_\lambda \mathcal{H}\left(w,h,\lambda\right)\big|_{w=w(h,\lambda)} \end{cases} \text{with } w(h,\lambda) \text{ solution to } \partial_w \mathcal{H}\left(w,h,\lambda\right) = 0 \tag{B.4}$$

It follows that the dynamics around the steady state can be approximated by

$$\begin{pmatrix} \dot{\lambda} \\ \dot{h} \end{pmatrix} = \underbrace{\begin{bmatrix} \delta - \left(\partial^2_{h,\lambda}\mathcal{H} - \partial^2_{h,w}\mathcal{H}\frac{\partial^2_{w,\lambda}\mathcal{H}}{\partial^2_{w,w}\mathcal{H}}\right) & -\left(\partial^2_{h,h}\mathcal{H} - \partial^2_{h,w}\mathcal{H}\frac{\partial^2_{w,h}\mathcal{H}}{\partial^2_{w,w}\mathcal{H}}\right) \\ -\partial^2_{\lambda,w}\mathcal{H}\frac{\partial^2_{w,\lambda}\mathcal{H}}{\partial^2_{w,w}\mathcal{H}} & \partial^2_{\lambda,h}\mathcal{H} - \partial^2_{\lambda,w}\mathcal{H}\frac{\partial^2_{w,h}\mathcal{H}}{\partial^2_{w,w}\mathcal{H}} \end{bmatrix}\Bigg|_{(w^o,h^o,\lambda^o)}}_{=D_o} \begin{pmatrix} \dot{\lambda} - \lambda^o \\ \dot{h} - h^o \end{pmatrix} \tag{B.5}$$

with

$$\begin{cases} \partial^2_{h,\lambda}\mathcal{H} = 0 & \partial^2_{h,w}\mathcal{H} = \frac{c}{k\sqrt{g}}(2h)^{-\frac{3}{2}} & \partial^2_{h,h}\mathcal{H} = -\frac{3cw}{k\sqrt{g}}(2h)^{-\frac{5}{2}} \\ \partial^2_{\lambda,w}\mathcal{H} = -\frac{1}{sA_r} & \partial^2_{w,w}\mathcal{H} = P'(w) & \partial^2_{\lambda,\lambda}\mathcal{H} = 0 \end{cases} \tag{B.6}$$

By substitution,

$$D_o = \begin{bmatrix} \delta - \frac{c(2h)^{-\frac{3}{2}}}{sA_rk\sqrt{g}P'(w)} & \frac{3cw(2h)^{-\frac{5}{2}}}{k\sqrt{g}} + \left(\frac{c(2h)^{-\frac{3}{2}}}{k\sqrt{g}}\right)^2 \frac{1}{P'(w)} \\ -\left(\frac{1}{sA_r}\right)^2 \frac{1}{P'(w)} & \frac{c(2h)^{-\frac{3}{2}}}{sA_rk\sqrt{g}P'(w)} \end{bmatrix} \tag{B.7}$$

It follows that $trace(D_o) = \delta > 0$ and, after some simplifications, that

$$\det(D_o) = \left(\frac{c(2h)^{-\frac{3}{2}}}{sA_rk\sqrt{g}}\right)\left(\delta + \frac{3w}{2sA_rh}\right)\frac{1}{P'(w)} < 0 \tag{B.8}$$

Finally, since $trace(D_o) > 0$ and $\det(D_o) < 0$, we know that our steady state is a locally stable saddle point.

Appendix C. Proof of the sensitivity analysis of Section 5

Let us introduce

$$\phi(h, c, k, \delta, A_r, R, \rho) = P(\rho R) - \frac{c}{k\sqrt{2gh}}\left(1 + \frac{\rho R}{2h\delta sA_r}\right) \tag{C.1}$$

Impact on the water column level

Let us first observe that

$$\frac{\partial\phi}{\partial h} = \frac{c}{k\sqrt{g}}(2h)^{-3/2}\left(1 + \frac{3\rho R}{2h\delta sA_r}\right) > 0 \tag{C.2}$$

This means that for all $\theta \in \{c, k, \delta, A_r, R, \rho\}$, $sign\left(\frac{\partial h}{\partial\theta}\right) = sign\left(-\frac{\partial\phi}{\partial\theta}\Big/\frac{\partial\phi}{\partial h}\right) = -sign\{\frac{\partial\phi}{\partial\theta}\}$. Moreover, by computation,

$$\begin{cases} \frac{\partial\phi}{\partial c} = -\frac{1}{k\sqrt{2gh}}\left(1 + \frac{\rho R}{2h\delta sA_r}\right) < 0 & \frac{\partial\phi}{\partial A_r} = \frac{c\rho R}{(2h)^{3/2}k\sqrt{g}\delta s(A_r)^2} > 0 \\ \frac{\partial\phi}{\partial k} = \frac{c}{k^2\sqrt{2gh}}\left(1 + \frac{\rho R}{2h\delta sA_r}\right) > 0 & \frac{\partial\phi}{\partial R} = \rho P'(w) - \frac{c}{k\sqrt{2gh}}\left(\frac{\rho}{2h\delta sA_r}\right) < 0 \\ \frac{\partial\phi}{\partial\delta} = \frac{c}{k\sqrt{2gh}}\left(\frac{\rho R}{2h\delta^2 sA_r}\right) > 0 & \frac{\partial\phi}{\partial\rho} = RP'(w) - \frac{c}{k\sqrt{2gh}}\left(\frac{R}{2h\delta sA_r}\right) < 0 \end{cases} \tag{C.3}$$

Hence,

$$\left\{ \frac{\partial h^o}{\partial c} > 0 \quad \frac{\partial h^o}{\partial k} < 0 \quad \frac{\partial h^o}{\partial \delta} < 0 \quad \frac{\partial h^o}{\partial A_r} < 0 \quad \frac{\partial h^o}{\partial R} > 0 \quad \frac{\partial h^o}{\partial \rho} > 0 \right. \tag{C.4}$$

Impact on the water consumption

At the steady state, $w^o = \rho R$; thus, all $\theta \neq R, \rho$ $\frac{\partial w^o}{\partial \theta} = 0$. Moreover, $\frac{\partial w^o}{\partial R} = \rho > 0$ and $\frac{\partial w^o}{\partial \rho} = R > 0$.

Impact on the yield of an artesian well

At the steady state, $r^o = k\sqrt{2gh^o}$. It follows that for all $\theta \neq k$, $\frac{\partial r^o}{\partial \theta} = \frac{kg}{\sqrt{2gh^o}} \cdot \frac{\partial h^o}{\partial \theta}$. From Eq. (C.4), we immediately obtain

$$\left\{ \frac{\partial h^o}{\partial c} > 0 \quad \frac{\partial h^o}{\partial \delta} < 0 \quad \frac{\partial h^o}{\partial A_r} < 0 \quad \frac{\partial h^o}{\partial R} > 0 \quad \frac{\partial h^o}{\partial \rho} > 0 \right. \tag{C.5}$$

It remains to study $\frac{\partial r^o}{\partial k} = \frac{kg}{\sqrt{2gh^o}} \cdot \frac{\partial h^o}{\partial k} + \sqrt{2gh^o}$ with $\frac{\partial h^o}{\partial k} = -\frac{\partial \phi}{\partial k} \Big/ \frac{\partial \phi}{\partial h}$ and $\frac{\partial \phi}{\partial h} > 0$ (see Eq. (C.2)). We can therefore say that

$$sign\left\{\frac{\partial r^o}{\partial k}\right\} = signe\left\{\underbrace{\sqrt{\frac{g}{2h^o}}}_{>0}\right\} \times sign\left\{k\frac{\partial h^o}{\partial k} + 2h^o\right\} = sign\left\{\underbrace{-k\frac{\partial \phi}{\partial k} + 2h^o\frac{\partial \phi}{\partial h}}_{=D}\right\} \tag{C.6}$$

Using Eqs. (C.2) and (C.3), we obtain

$$D = -\frac{c}{k\sqrt{2gh^o}}\left(1 + \frac{\rho R}{2h^o \delta s A_r}\right) + \frac{c}{k\sqrt{2gh^o}}\left(1 + \frac{3\rho R}{2h^o \delta s A_r}\right) = \frac{c}{k\sqrt{2gh^o}} \frac{\rho R}{h^o \delta s A_r} > 0 \tag{C.7}$$

and conclude that $\frac{\partial r^o}{\partial k} > 0$.

Impact on the number of artesian wells

At the steady state, $n^o = \frac{\rho R}{k\sqrt{2gh^o}}$. It follows that $\forall \theta \neq R, \rho, k$, $\frac{\partial n^o}{\partial \theta} = -\frac{\rho R}{k\sqrt{g}} (2h^o)^{-3/2} \frac{\partial h^o}{\partial \theta}$. From Eq. (C.4), we immediately obtain

$$\left\{ \frac{\partial n^o}{\partial c} < 0 \quad \frac{\partial n^o}{\partial \delta} > 0 \quad \frac{\partial n^o}{\partial A_r} > 0 \right. \tag{C.8}$$

Let us now concentrate on $\frac{\partial n^o}{\partial k}$. By computation,

$$\frac{\partial n^o}{\partial k} = -\frac{\rho R}{k\sqrt{g}} (2h^o)^{-3/2} \frac{\partial h^o}{\partial k} - \frac{\rho R}{k^2\sqrt{2gh^o}} \tag{C.9}$$

It follows that

$$sign\left(\frac{\partial n^o}{\partial k}\right) = -sign\left(\frac{\partial h^o}{\partial k} + \frac{2h^o}{k}\right) = -sign\left(-\frac{\partial \phi}{\partial k} + \frac{2h^o}{k}\frac{\partial \phi}{\partial h}\right) \tag{C.10}$$

Moreover, from Eqs. (C.2) and (C.3),

$$-\frac{\partial\phi}{\partial k}+\frac{2h^o}{k}\frac{\partial\phi}{\partial h}=-\frac{c}{k^2\sqrt{2gh^o}}\left(1+\frac{\rho R}{2h^o\delta sA_r}\right)+\frac{c}{k^2\sqrt{2gh^o}}\left(1+\frac{3\rho R}{2h^o\delta sA_r}\right)=\frac{c}{k^2\sqrt{2gh^o}}\left(\frac{\rho R}{h^o\delta sA_r}\right)>0 \quad \text{(C.11)}$$

We can therefore say that $\frac{\partial n^0}{\partial k}<0$.

To show that $\frac{\partial n^0}{\partial R}$ and $\frac{\partial n^0}{\partial \rho}$ are unsigned, let us introduce the following example. We set $P(w)=w^{-a}$ with $\alpha>1$ and choose the parameters $\{c,k,\delta,A_r\}$ such that $\frac{c}{k\sqrt{g}}=\frac{1}{2}$ and $\delta sA_r=1$. Under these assumptions, we know from Eq. (C.1) that h^o solves

$$\phi_1(h;\rho,R)=(\rho R)^{-\alpha}-\frac{1}{2}(2h)^{-\frac{1}{2}}-\frac{\rho R}{2}(2h)^{-\frac{3}{2}}=0 \quad \text{(C.12)}$$

Moreover, if we take a particular configuration of ρ and R such that $\rho R=1$, it is easy to check that $h^o=\frac{1}{2}$ solves Eq. (C.12). If we now change R, the change in h^0, in the neighborhood of this solution, is given by

$$\frac{\partial h^o}{\partial R}=-\frac{\partial\phi_1}{\partial R}\Big/\frac{\partial\phi_1}{\partial h}\Big|_{h=\frac{1}{2},\rho R=1}=-\frac{-\alpha\rho-\frac{\rho}{2}}{\frac{1}{2}+\frac{3}{2}}=\rho\frac{2\alpha+1}{4} \quad \text{(C.13)}$$

If we now look at the change in the number of wells (see Eq. (22)), we obtain

$$\frac{\partial n^o}{\partial R}=\left(\frac{\rho}{k\sqrt{2gh}}-\frac{\rho R}{k\sqrt{g}}(2h)^{-3/2}\frac{\partial h^o}{\partial R}\right)\Big|_{h=\frac{1}{2},\rho R=1}=\left(\frac{\rho}{k\sqrt{g}}-\frac{1}{k\sqrt{g}}\rho\frac{2\alpha+1}{4}\right)=\frac{\rho}{4k\sqrt{g}}(3-2\alpha) \quad \text{(C.14)}$$

This clearly shows that $\frac{\partial n^o}{\partial R}$ can be either positive or negative depending, in this example, on the elasticity of demand $\alpha\lesseqgtr\frac{3}{2}$. Finally, observe that the result is identical for $\frac{\partial n^0}{\partial R}$ as long as we replace ρ by R in Eq. (C.14).

Appendix D. Proof of Proposition 3

The ranking of the solution

To verify that $h^0>h^m$, let us remember that (i) h^0 solves $\phi(h^0)=0$ (see Eq. (C.1)) with $\phi'(h)>0$ (see Eq. (C.2)) and (ii) $h^m=\frac{1}{2g}\left(\frac{c}{kP(\rho R)}\right)^2$ (see Eq. (9)). Now, observe that

$$\phi(h^m)=P(\rho R)-\frac{c}{k\sqrt{2gh^m}}\left(1+\frac{\rho R}{2h^m\delta sA_r}\right)=-\frac{gk^2\rho R(P(\rho R))^3}{c^2\delta sA_r}<0 \quad \text{(D.1)}$$

Since $\phi'(h)>0$, we deduce that $h^0>h^m$. From Torricelli's formula (see Eq. (1)), we also know that $r=k\sqrt{2gh}$. Therefore, if $h^0>h^m$, we must also have $r^0>r^m$. Finally, recall that at the steady state, the number of wells is given by $n = \frac{\rho R}{r}$. Hence, if $r^0 > r^m$, we must have $n^0 < n^m$.

The rate of yield losses

Let us now concentrate on $\frac{\partial \Delta^-_{Yield}}{\partial \theta}$ with $\theta \in \{c, k, A_r, \delta, R, \rho\}$. This computation requires some preliminary observations. First, from Torricelli's formula (see Eq. (1)), Eq. (B.1) can be written as

$$P(\rho R) - \frac{c}{r^0} - \frac{c}{(r^0)^2} \frac{k^2 g \rho R}{\delta s A_r} = 0 \tag{D.2}$$

Moreover, we know from Eq. (10) that $r^m = \frac{c}{P(\rho R)}$. Inserting this quantity into Eq. (D.2), we obtain

$$P(\rho R) - c\left(\frac{P(\rho R)}{c}\right)\left(\frac{r^m}{r^0}\right) - \frac{ck^2 g\rho R}{\delta s A_r}\left(\frac{P(\rho R)}{c}\right)^3\left(\frac{r^m}{r^0}\right)^3 = 0 \tag{D.3}$$

or after some simplifications:

$$\psi\left(\left(\frac{r^m}{r^0}\right), X\right) = 1 - \left(\frac{r^m}{r^0}\right) - \underbrace{\frac{k^2 g\rho R (P(\rho R))^2}{c^2 \delta s A_r}}_{=X>0}\left(\frac{r^m}{r^0}\right)^3 \tag{D.4}$$

Moreover, we observe that $\frac{\partial \psi}{\partial (r^m/r^0)} = -1 - 2X\left(\frac{r^m}{r^0}\right)^2 < 0$ and $\frac{\partial \psi}{\partial X} = -\left(\frac{r^m}{r^0}\right)^3 < 0$.

Now, let us return to $\frac{\partial \Delta^-_{Yield}}{\partial \theta}$. From our previous results and the definition of $\Delta^-_{Yield} = 1 - \left(\frac{r^m}{r^0}\right)$, we deduce that

$$sign\left(\frac{\partial \Delta^-_{Yield}}{\partial \theta}\right) = -sign\left(\frac{\partial\left(r^m/r^0\right)}{\partial X}\right) sign\left(\frac{\partial X}{\partial \theta}\right) \tag{D.5}$$

$$= -sign\left(-\frac{\partial \psi}{\partial (r^m/r^0)} \Big/ \frac{\partial \psi}{\partial X}\right) sign\left(\frac{\partial X}{\partial \theta}\right) = sign\left(\frac{\partial X}{\partial \theta}\right) \tag{D.6}$$

and by computation, we easily check that

$$\begin{cases} \frac{\partial X}{\partial c} = -\frac{2k^2 g\rho R(P(\rho R))^2}{c^3 \delta s A_r} < 0 & \frac{\partial X}{\partial k} = \frac{2kg\rho R(P(\rho R))^2}{c^2 \delta s A_r} > 0 \\ \frac{\partial X}{\partial A_r} = -\frac{k^2 g\rho R(P(\rho R))^2}{c^2 \delta s A_r^2} < 0 & \frac{\partial X}{\partial \delta} = -\frac{k^2 g\rho R(P(\rho R))^2}{c^2 \delta^2 s A_r} < 0 \end{cases} \tag{D.7}$$

Until now, it follows that

$$\left\{ \frac{\partial \Delta^-_{Yield}}{\partial c} < 0 \quad \frac{\partial \Delta^-_{Yield}}{\partial k} > 0 \quad \frac{\partial \Delta^-_{Yield}}{\partial A_r} < 0 \quad \frac{\partial \Delta^-_{Yield}}{\partial \delta} < 0 \right. \tag{D.8}$$

Concerning $\frac{\partial X}{\partial R}$ and $\frac{\partial X}{\partial \rho}$, let us set $w = \rho R$. It follows that

$$\frac{\partial X}{\partial w} = \frac{k^2 g}{c^2 \delta s A_r} \left(P(w)^2 + 2wP(w)P'(w) \right) = \frac{k^2 g P(w)^2}{c^2 \delta s A_r} \left(1 + 2\varepsilon_P(w)\right) \tag{D.9}$$

We can therefore conclude that $\frac{\partial X}{\partial R} = \rho \left.\frac{\partial X}{\partial w}\right|_{w=\rho R} \gtreqless 0$ and $\frac{\partial X}{\partial \rho} = R \left.\frac{\partial X}{\partial w}\right|_{w=\rho R} \gtreqless 0$ if and only if $\varepsilon_P(w) = \frac{wP'(w)}{P(w)} \gtreqless -\frac{1}{2}$ evaluated at $w = \rho R$ or equivalently that

$$\frac{\partial \Delta^-_{Yield}}{\partial R} \gtreqless 0 \text{ and } \frac{\partial \Delta^-_{Yield}}{\partial \rho} \gtreqless 0 \text{ if and only if } \varepsilon_P(w) \gtreqless -\frac{1}{2} \tag{D.10}$$

The rate of increase in the water disposal cost

Since the number of wells is in both cases given by the net recharge ρR divided by the yield and since $\Delta^-_{Yield} = 1 - \left(\frac{r^m}{r^0}\right)$, we can say that

$$\Delta^+_{Cost} = \frac{n^m}{n^o} - 1 = \frac{\frac{\rho R}{r^m}}{\frac{\rho R}{r^0}} - 1 = \frac{r^o}{r^m} - 1 = \frac{1}{1 - \Delta^-_{Yield}} - 1 = \frac{\Delta^-_{Yield}}{1 - \Delta^-_{Yield}} \tag{D.11}$$

If we now observe that $\frac{\partial \Delta^+_{Cost}}{\partial \Delta^-_{Yield}} > 0$, we can immediately say that $sign\left(\frac{\partial \Delta^+_{Cost}}{\partial \theta}\right) = sign\left(\frac{\partial \Delta^-_{Yield}}{\partial \theta}\right)$ for all $\theta \in \{c, k, A_r, \delta, R, \rho\}$

Appendix E. Proof of Proposition 4

Applying a tax of $\tau(h(t)) = \frac{c\rho R}{2\delta h(t) s A_r}$ per well, the instantaneous profit of a well owner under free access (see Eq. (5)) becomes

$$\Pi = p(t)r(t) - c - \tau(h(t)) = p(t)r(t) - c\left(1 + \frac{\rho R}{2\delta h(t) s A_r}\right) \tag{E.1}$$

It follows that the dynamics of the open-access system (see Eq. (7)) are

$$\begin{cases} \dot{n}(t) = \alpha \left(P\left(n(t)k\sqrt{2gh(t)}\right) k\sqrt{2gh(t)} - c\left(1 + \frac{\rho R}{2\delta h(t) s A_r}\right)\right) \\ sA_r \dot{h}(t) = \rho R - n(t)k\sqrt{2gh(t)} \end{cases} \tag{E.2}$$

If we now concentrate on the new open-access steady state, we find from the second equation that n^m is given by

$$n^m = \frac{\rho R}{k\sqrt{2gh^m}} \tag{E.3}$$

with h^m, which solves

$$P\left(n^m k\sqrt{2gh^m}\right)k\sqrt{2gh^m} - c\left(1+\frac{\rho R}{2\delta h^m s A_r}\right) = 0$$
$$\iff P\left(\rho R\right) - \frac{c}{k\sqrt{2gh^m}}\left(1+\frac{\rho R}{2\delta h^m s A_r}\right) = 0 \tag{E.4}$$

However, this last equation in the same as that defined under the optimal management water column (see Eq. (23)). It then follows that the open-access steady state coincides with the optimal management steady state under this tax rate.