Monopolistic Duopoly*

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Abstract

We consider an unexplored potential of the Hotelling price competition framework. We relax the standard assumption of full market coverage, which requires the marginal transportation cost to be low relative to the consumer maximum willingness to pay and implies that the indifferent consumer gets strictly positive utility at equilibrium. We identify a parametric interval where the marginal transportation cost is relatively high and duopolists act as follows: they have an incentive to set relatively low prices, so as to fully cover the market, but not as low as to be able to steal consumers from the competitor, with the effect that the indifferent consumer gets zero equilibrium utility. We refer to this scenario as Monopolistic Duopoly, in that the cross-price elasticity of firm demands is positive when the competitor reduces its price, as in the standard Hotelling game, but zero when the competitor's price rises, as in a monopolistic setting.

Keywords Hotelling duopoly price competition; Cross-price elasticity; Monopolistic Duopoly.

JEL Codes: L13 (Oligopoly and Other Imperfect Markets); C72 (Noncooperative Games).

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1 Introduction

In this paper, we consider the celebrated Hotelling price competition game with the aim of unveiling the following unexplored potential. We relax the standard assumption of full market coverage, which requires the marginal transportation cost to be low relative to the consumer maximum willingness to pay and implies that the indifferent consumer gets strictly positive utility at equilibrium. We identify a parametric interval where the marginal transportation cost is relatively high and duopolists act as follows: they have an incentive to set relatively low prices, so as to fully cover the market, but not as low as to be able to steal consumers from the competitor, with the effect that the indifferent consumer gets zero equilibrium utility. We refer to this scenario as Monopolistic Duopoly, in that the cross-price elasticity of the firm demands is positive when the competitor reduces its price, as in the standard Hotelling game, but zero when the competitor's price rises, as in a monopolistic setting.

To the best of our knowledge, only the (unpublished) paper by Thepot (2007) has previously tackled this topic, but without characterizing the equilibrium analysis.

We believe the extension of the Hotelling price game analyzed by this paper might be useful to verify the robustness of the results proposed by a number of recent and relevant papers that rely on such framework to analyze up-to-date topics. An example in the context of two-sided markets is Armstrong (2006, Rand Journal of Economics); another in environmental economics is Deltas et al. (2013, Journal of Economics and Management Strategy).

2 The model

We consider a Hotelling segment of unit length with extremes 0 and 1. Two firms, indexed by $i = 0, 1$, are located at the extremes of the segment, firm 0 at 0 and firm 1 at 1; each firm supplies a variety of a differentiated product. A unit mass of consumers is uniformly distributed on the segment; the location of each consumer denotes her preferred variety. Consumers demand at most one unit of the product; their gross utility for the unit consumption is v and their disutility if not consuming the preferred variety is linear in the distance between the consumed variety and the preferred one. In Figure 1, we provide a graphical representation of firms and consumers in this Hotelling segment.

Figure 1 Here

If a consumer located at $x \in [0, 1]$ buys one unit of product from firm i at price p_i her net indirect utility is:

$$\mathcal{U}(x, p_i) = v - td_i(x) - p_i, \tag{1}$$

where $d_i(x)$ is the Euclidean distance between the location of the consumer and that of firm i and t is the marginal disutility of distance.

We solve equation $\mathcal{U}(x, p_0) = \mathcal{U}(x, p_1)$ for x and get the location of the consumer indifferent between purchasing from either firm,

$$x_I(\mathbf{p}) = \frac{1}{2} - \frac{(p_0 - p_1)}{2t} \tag{2}$$

where the boldfaced letter $\mathbf{p}$ represents the price vector. We also solve equations $\mathcal{U}(x, p_0) = 0$ and $\mathcal{U}(x, p_1) = 0$ for x and get the location of the consumer indifferent between purchasing from firm 0 and firm 1, respectively, or not purchasing,

$$x_0 = \frac{v - p_0}{t} \text{ and } x_1 = 1 - \frac{v - p_1}{t}. \tag{3}$$

We refer to x_i as the location of the (possibly) last consumer purchasing from firm i.

Firms supply their variety of the product by operating a constant-returns-to-scale technology characterized by a marginal cost c, which we normalize to zero. Firms play the following price competition game: each firm i chooses the price p_i simultaneously to maximize the revenue function

$$R_i(p_i) = p_i D_i,$$

where D_i is the demand of firm i.

2.1 Hotelling Duopoly

We first examine the case in which the indifferent consumer gets positive utility at the Nash equilibrium of the price competition game,

$$\mathcal{U}(x_I(\mathbf{p}^H), p_i^H) > 0, \tag{4}$$

where $\mathbf{p}^H$ denotes the equilibrium prices under condition (4). Such condition implies that all consumers buy from either firm 0 or firm 1, i.e., the market is fully covered, and that the indifferent consumer is also the last one purchasing from either firm; graphically, $x_1 < x_I(\mathbf{p}^H) < x_0$, as illustrated in Figure 2. In this case, the demand of firm 0, D_0, is given by the consumers located in $[0, x_I(\mathbf{p}^H)]$ and that of firm 1, D_1, by the consumers in $[x_I(\mathbf{p}^H), 1]$. Most importantly, the cross price elasticities of the two demands are positive in that the demand of firm $i = 0, 1$, D_i, is increasing in the competitor's price p_j, $j = 1, 0$; in other words, the two varieties of the product are substitute. This is the (standard) scenario of Hotelling competition.

We write the revenue function of firm i for a generic $\mathbf{p}$,

$$R_i = p_i \left(\frac{1}{2} - \frac{p_i - p_j}{2t} \right),$$

The Nash equilibrium prices of the competition game are symmetric and equal to

$$p_0^H = p_1^H = p^H = t.$$

Plugging these values into R_i gives the equilibrium revenues of both firms,

$$R^H = \frac{t}{2}.$$

Finally, at the equilibrium prices $p^H = t$ condition (4) becomes

$$\mathcal{U}(x_I \left(p^H\right), p^H) = v - \frac{t}{2} - t > 0 \Leftrightarrow t < \frac{2}{3}v. \tag{5}$$

Figure 2 Here

2.2 Local Monopoly

We turn our focus to the case in which the indifferent consumer gets negative utility at the Nash equilibrium,

$$\mathcal{U}(x_I \left(\mathbf{p}^L\right), p_i^L) < 0, \tag{6}$$

where $\mathbf{p}^L$ denotes the equilibrium prices under condition (6). Such condition implies the market is partially covered. Indeed, $x_0 < x_I \left(\mathbf{p}^L\right) < x_1$ as illustrated in Figure 3, meaning that consumers located in (x_0, x_1) prefer not to purchase; in other words, x_0 (x_1) is the location of the last consumer buying from firm 0 (1). This is the local monopoly scenario where $D_0 = x_0 \left(p_0^L\right)$ and $D_1 = 1 - x_1 \left(p_1^L\right)$ and the cross price elasticities between the two varieties are zero. Superscript L stands for Local Monopoly.

We write the revenue function of firm i for a generic $\mathbf{p}$,

$$R_i = p_i \frac{v - p_i}{t}, \tag{7}$$

The prices p_i that maximize the two revenue functions are symmetric and equal to

$$p_0^L = p_1^L = p^L = \frac{v}{2}$$

Plugging these values into R_i yields the equilibrium revenues of firms,

$$R^L = \frac{v^2}{4t}. \tag{8}$$

Finally, condition (6) at the equilibrium prices $p^L = \frac{v}{2}$ requires

$$\mathcal{U}(x_I\left(\mathbf{p}^L\right), p^L) = v - \frac{t}{2} - \frac{v}{2} < 0 \Leftrightarrow t > v. \tag{9}$$

FIGURE 3 HERE

2.3 Monopolistic Duopoly

We finally explore the case in which the indifferent consumer gets zero equilibrium utility,

$$\mathcal{U}(x_I\left(\mathbf{p}^M\right), p_i^M) = 0, \tag{10}$$

where $\mathbf{p}^M$ denotes the equilibrium prices under condition (10).

Inequalities (5) and (9) implies that $t \in \left[\frac{2}{3}v, v\right]$ is a necessary condition for (10) to be fulfilled. We therefore focus on this interval and observe that the market is still fully covered, but the location of the indifferent consumer and those of the last consumers are equivalent, $x_1 = x_I\left(\mathbf{p}^M\right) = x_0$, as illustrated in Figure 4. In this case, $D_0 = x_I\left(\mathbf{p}^M\right)$ and $D_1 = 1 - x_I\left(\mathbf{p}^M\right)$. Interestingly, the cross-price elasticities of the two demands, D_i, are positive when p_j drops, but zero when p_j rises. Since positive cross-price elasticities capture product substitutability, which is typical of (Hotelling) competition, while zero cross-price elasticities imply monopoly settings, this scenario is referred to as *Monopolistic Duopoly*; superscript M stands for Monopolistic Duopoly.

We rewrite (10) for a generic $\mathbf{p}$ and solve for $p_0 + p_1$

$$\mathcal{U}(x_I, p_i) = 0 \Leftrightarrow p_0 + p_1 = 2v - t. \tag{11}$$

This is the relationship the prices must abide in order to be in the Monopolistic Duopoly scenario. Hence there is a continuum of candidate equilibria.

In the appendix, we show that the (continuum of) Nash equilibrium prices are

$$p_i^M = p^M - \epsilon \text{ and } p_j^M = p^M + \epsilon, \tag{12}$$

where $p^M = v - \frac{t}{2}$ is the symmetric price that fulfills (11) and $\epsilon \in [0, \epsilon^*]$, with $\epsilon^* = \min\left\{\frac{t}{2} - \frac{v}{3}, \frac{v-t}{2}\right\}$.[1]

[1]To be more precise, there are two continua of equilibrium prices, which are equal up to a permutation in

Figure 4 Here

3 Results and Discussion

We summarize our findings in the following

Proposition 1. *Three alternative equilibrium scenarios arise depending on the level of t relative to v:*
(i) if $t < \frac{2}{3}v$, firms choose the symmetric price $p_0^H = p_1^H = p^H = t$ and act as Hotelling Duopolists;
(ii) if $t > v$, firms choose the symmetric price $p_0^L = p_1^L = p^L = \frac{v}{2}$ and act as Local Monopolists;
(iii) if $\frac{2}{3}v \le t \le v$, firms choose prices $p_i^M = v - \frac{t}{2} - \epsilon$ and $p_j^M = v - \frac{t}{2} + \epsilon$, with $\epsilon \in [0, \epsilon^*]$, and act as Monopolistic Duopolists.

Proof. See Appendix A. □

To discuss the results of Proposition 1, we remark that as t increases, consumers become less sensitive to price, i.e., the demand gets decreasingly elastic.

If $t < \frac{2}{3}v$, the demand is relatively elastic; as a result, a price cut triggers a large increase in demand, resulting in high competitive pressure. This explains why the indifferent consumer gets positive utility and the Hotelling Duopoly scenario arises at equilibrium.

If by contrast $t > v$, the demand is relatively inelastic. This induces firms to increase their prices up to the point where competition disappears altogether and firms are (local) monopolists. Here, the indifferent consumer gets negative equilibrium utility.

For intermediate values of t, the demand is neither inelastic enough to lead to Local Monopolies, nor elastic enough to result in full-fledged Hotelling Duopoly. The resulting equilibrium behavior is that of Monopolistic Duopoly, where firms have an incentive to set relatively low prices, so as to fully cover the market, but not as low as to be able to steal consumers from the competitor; as a consequence, the indifferent consumer gets zero equilibrium utility.

To understand the firms' strategic behavior, we use (11) and (12) to write the firms' reaction functions at the Monopolistic Duopoly equilibria,

$$p_i^M = 2v - t - p_j^M \cap 0 \le \epsilon \le \epsilon^*,$$

and observe that prices are strategic substitutes (Thepot, 2007).[2] If firm j increases its price, the market is no longer fully covered, because the farthest share of firm j's consumers obtain negative utility. The demand is elastic enough to make the rival firm i willing to

the labels of firms.

[2]Interestingly, this is in contrast with the Hotelling Duopoly scenario, where prices are strategic complements.

decrease p_i^M and serve these consumers; at the same time, the demand is inelastic enough to prevent firm i from further reducing p_i^M and compete on any consumer that patronize firm j. Symmetrically, if firm j pursues an aggressive strategy by decreasing its price, it steals the farthest share of firm i's consumers. The demand is inelastic enough for firm i to raise the price and compensate the loss in the extensive margin through a higher intensive one. At the same time, the demand is elastic enough to avoid a monopolistic behavior either: p_i^M is increased up to the point where the market is just fully covered.

At the Monopolistic Duopoly equilibrium price pair, p_i^M and p_j^M, the higher-price firm j earns lower profits than the lower-price firm i and the revenue gap is increasing in ϵ. We remark that the asymmetry of equilibrium prices is bounded from above by the minimum between $\frac{t}{2} - \frac{v}{3}$ and $\frac{v-t}{2}$. These thresholds have an interesting economic interpretation. If $t < \frac{5}{6}v$, the binding threshold is $\frac{t}{2} - \frac{v}{3}$. When ϵ exceeds that value, firm j setting the higher price wants to deviate to a lower price. In the light of the above, this is intuitive: here t is relatively low, which makes the demand relatively elastic and a price cut attractive. A mirror reasoning explains the case $t > \frac{5}{6}v$, where the binding cutoff is $\frac{v-t}{2}$. A relatively high t makes the demand more rigid. When ϵ exceeds $\frac{v-t}{2}$, the lower-price firm i has an incentive to deviate to a higher price and behave like a local monopolist. In either case, a pure-strategy Nash equilibrium fails to exist.

A Monopolistic Duopoly: equilibrium analysis

At the candidate equilibrium prices (12), the indifferent consumer is found by substituting p_i^M and p_j^M into (2), which gives

$$x_I(\mathbf{p}^M) = \frac{1}{2} \pm \frac{\epsilon}{t}. \tag{13}$$

Lower-price firm i profits are

$$\left(v - \frac{t}{2} - \epsilon\right)\left(\frac{1}{2} + \frac{\epsilon}{t}\right) = \frac{(2v - t - 2\epsilon)(t + 2\epsilon)}{4t}, \tag{14}$$

while higher-price firm j profits are

$$\left(v - \frac{t}{2} + \epsilon\right)\left(\frac{1}{2} - \frac{\epsilon}{t}\right) = \frac{(2v - t + 2\epsilon)(t - 2\epsilon)}{4t}. \tag{15}$$

In what follows, we check whether firms find it profitable to deviate from prices (12). Before proceeding, we remark that (14) $-$ (15) $= 2\epsilon\frac{v-t}{t}$, which is nonnegative in $t \in \left[\frac{2}{3}v, v\right]$. Since the higher-price firm j gets lower candidate equilibrium profits, we first consider deviations by this firm.

A.1 Higher-price firm j

As mentioned, a unilateral deviation by firm j violates (10) and must therefore lead to a configuration other than Monopolistic Duopoly. If the deviation price is lesser than p_j^M the deviation is to Hotelling Duopoly, whereas if the deviation price is larger than p_j^M, it is to Local Monopoly.

Downward deviation. Following a downward price deviation, firm j profit is maximized at

$$p_j^{D,H} = \frac{t + 2v - 2\epsilon}{4}.$$

Note that $p_j^{D,H} < p_j^M \Leftrightarrow \epsilon > \frac{t}{2} - \frac{v}{3}$. The deviation profit is equal to $\frac{(t+2v-2\epsilon)^2}{32t}$ and higher than (15). Accordingly, there exists a profitable downward deviation, provided that $\epsilon > \frac{t}{2} - \frac{v}{3}$.

Upward deviation. An upward price deviation is easily dealt with, since it must lead to Local Monopoly, under which the optimal price is $p^L = \frac{v}{2}$. Note, however, that $p^L > p_j^M \Leftrightarrow \epsilon < \frac{t-v}{2}$, which is not fulfilled in $t \in \left[\frac{2}{3}v, v\right]$. This implies that (i) an upward deviation is not feasible in the interval of interest; (ii) any feasible upward deviation, that is a price nonlower than p_j^M, is not profitable.

Overall, the higher-price firm j has no profitable deviation from the candidate Monopolistic Duopoly equilibrium (12) when $\epsilon \leq \frac{t}{2} - \frac{v}{3}$.

A.2 Lower-price firm i

On the above basis, prices (12) turn out to be a Monopolistic Duopoly equilibrium if the lower-price firm i has no profitable deviation either, when $\epsilon \leq \frac{t}{2} - \frac{v}{3}$. We therefore focus on this interval.

Downward deviation. Following a downward price deviation, firm i profit is maximized at

$$p_i^{D,H} = \frac{t + 2v + 2\epsilon}{4}.$$

Note, however, that $p_i^{D,H} > p_i^M \Leftrightarrow \epsilon > 0 > -\left(\frac{t}{2} - \frac{v}{3}\right)$, which is true. This implies that (i) a downward deviation is not feasible in the interval of interest; (ii) any feasible downward deviation, that is a price nonhigher than p_i^M, is not profitable.

Upward deviation. An upward price deviation leads to Local Monopoly, under which the optimal price is $p^L = \frac{v}{2}$. Note that $p^L > p_i^M \Leftrightarrow \epsilon > \frac{v-t}{2}$. The deviation profit is equal to $\frac{v^2}{4t}$ and higher than (14): there exists a profitable upward deviation, provided that $\epsilon > \frac{v-t}{2}$.

Overall, the lower price i firm has no profitable deviation from the candidate Monopolistic Duopoly equilibrium (12) when $\epsilon \leq \frac{v-t}{2}$.

We can conclude that no firm has profitable deviation from (12) when $\epsilon \leq \min\left\{\frac{t}{2} - \frac{v}{3}, \frac{v-t}{2}\right\}$. We highlight that at these values of ϵ the lower price $p_i^M = v - \frac{t}{2} - \epsilon$ is strictly positive.

FIGURE 1: Hotelling Segment

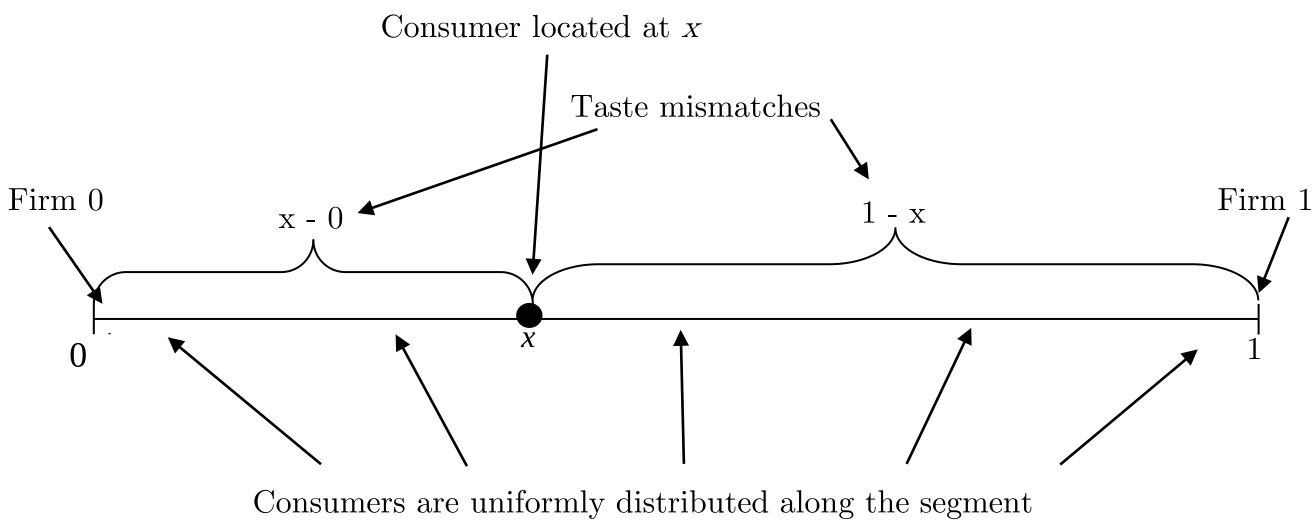

FIGURE 2: Hotelling Duopoly

Horizontal axis: unit segment denoting location of consumers
Left vertical axis: utility of consumers buying from firm 0 as a function of their location
Right vertical axis: utility of consumers buying from firm 1 as a function of their location

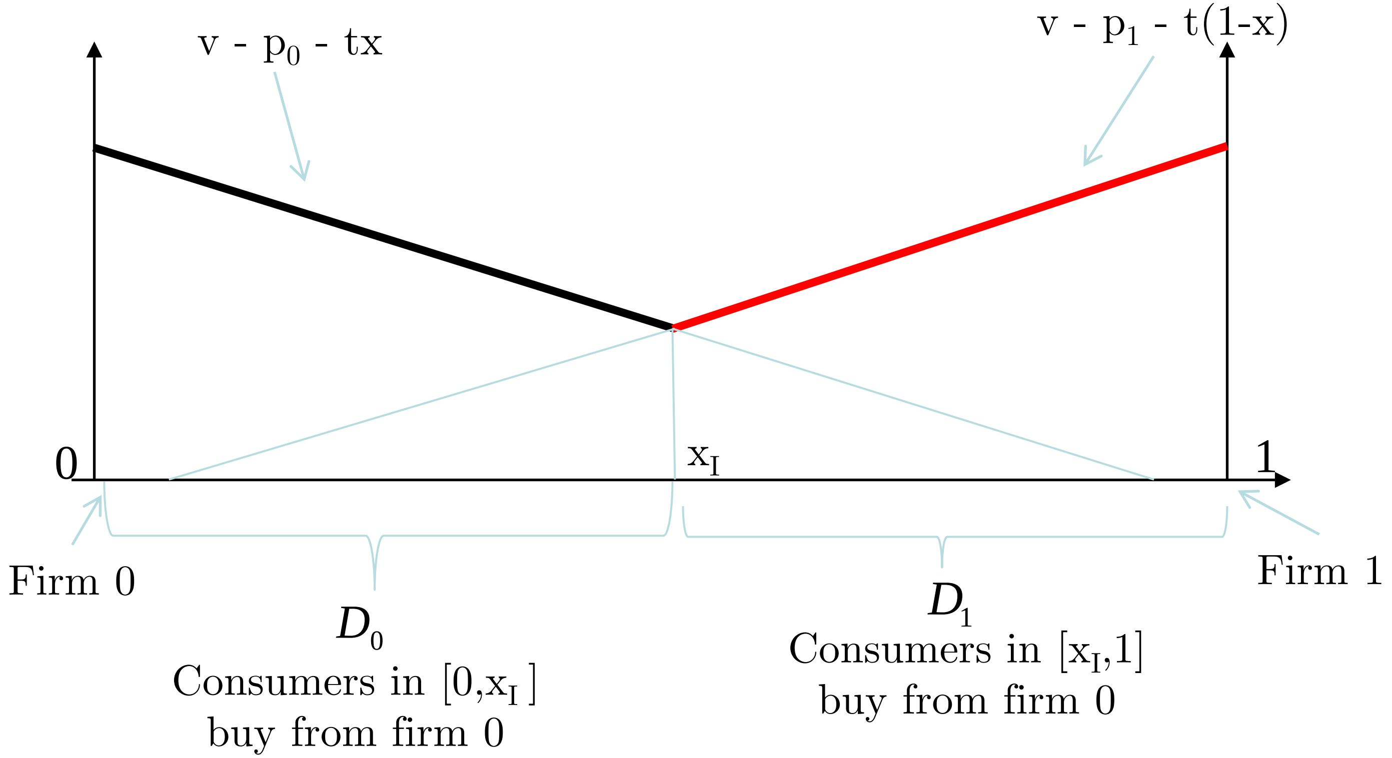

FIGURE 3: Local Monopoly

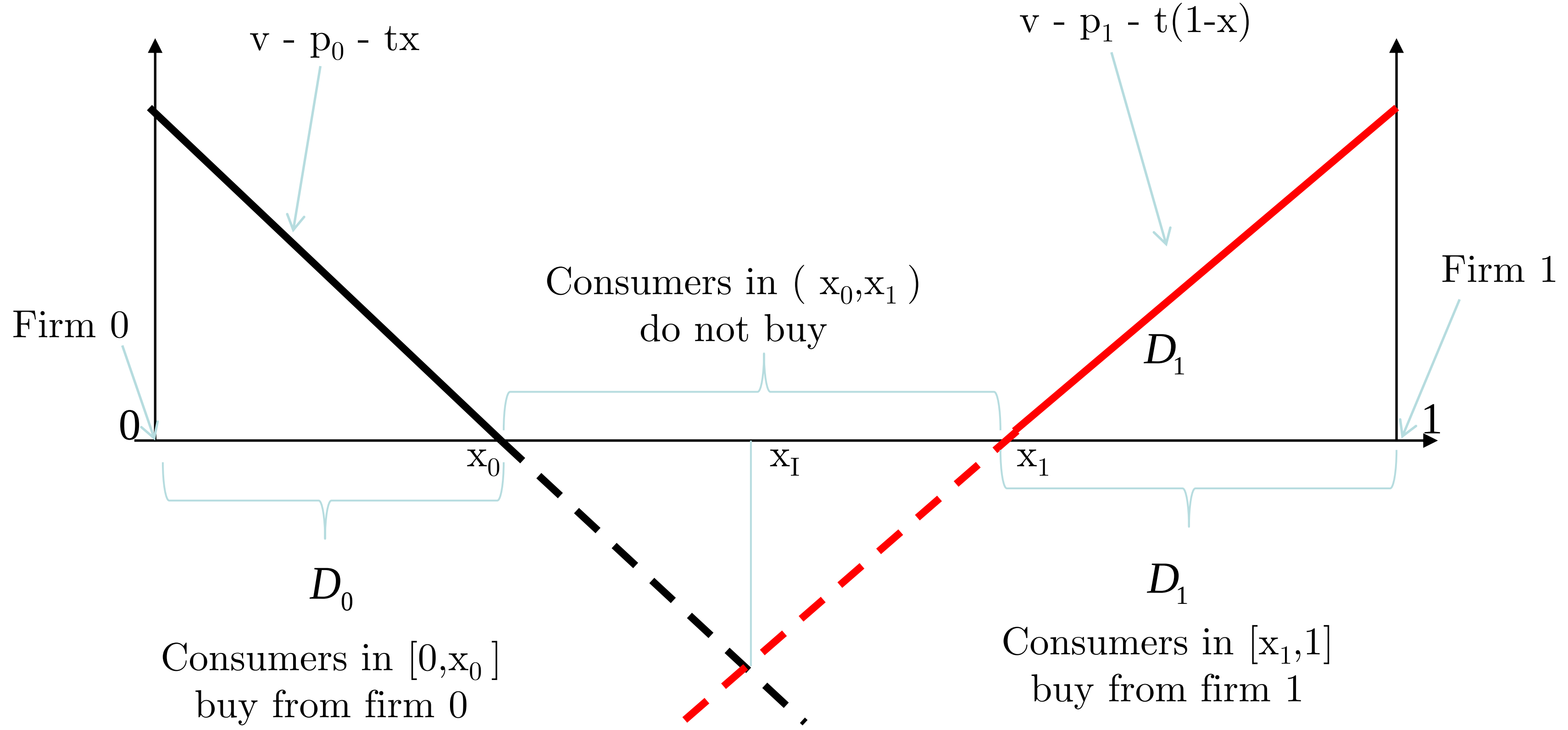

FIGURE 4: Monopolistic Duopoly