4 Things Nobody tells you about Online News:

a Model with Social Networks and Competition

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Abstract:

Social media create a new type of incentives for news producers. Consumers share content, influence the visibility of articles and determine the advertisement revenues ensuing. I study the new incentives created by sharing and evaluate the potential quality of ad-funded online news. Producers rely on a subset of rational and unbiased consumers to spread news articles. The resulting news has low precision and ambiguous welfare effects. Producers' incentive to invest in news quality increases with the private knowledge of the topic; hence, when information is most needed, the generated news tends to be of lesser quality. Competition does not necessarily improve news quality – it does so only if the sharing network is *sufficiently dense*. While ad-funded online news occasionally helps consumers take better decisions, it creates welfare mostly through entertainment. Some interventions, such as flagging wrong articles, substantially improve the outcome; other approaches, such as quality certification, do not.

JEL codes: D80, D85, L11, L14

Keywords: Online News, News Quality, Social Media, Competition, Networks

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1 Introduction

The media landscape has evolved throughout history. From the press to radio, television and the rise of the internet age, many past revolutions gave rise to concerns about news quality. Nowadays, social media are under the spotlight. The idea that the online news market may be worse than traditional media is puzzling as it arises in a highly competitive environment. Standard theory would predict that competition enhances news precision. Has social media metamorphosed the news market in a way that makes standard theory unapplicable?

While advertisement revenues and producers' reduced cost of entry date back to more than a century ago, online outlets brought something new: sharing. With social media, consumers play an active role in spreading news' article, raising their visibility, thereby producers' advertisement revenues. Hence, news producers behind ad-funded online outlets respond to new incentives. Because of advertisement revenues, their articles now need to be shared online. In this sense, the very presence of a news sharing network changes the effects of the previously existing market environment. In this paper, I evaluate the performance of such ad-funded online outlets, focusing on the incentives linked to sharing behaviors. Three dimensions of the market environment are explored: the amount of private knowledge, the connectivity of the communication network and the presence of competition. After studying the effects of the environment on the provision of qualitative information, I question whether such outlets are welfare enhancing and propose possible interventions.

I explore this question by introducing a general setup to represent the online news market. The market is populated by consumers on one side and producers on the other. The agents communicate about some state of the world. In addition to private signals, some consumers, called *influencers*, come across news' articles directly and can decide to share it on an exogenous network to other consumers, called *followers*. Influencers care about sharing true news ; followers read articles that influencers share and are not strategic. Given influencers' sharing behavior, producers decide on the quality of their outlet, i.e. the probability for an article from their outlet to be true. Each producer publishes only one article. They do not chose articles' content. They only care about how many consumers view their article. While the number of influencers reading a producer's outlet is exogenous, the number of followers seeing their article is endogenous. The producers' incentive to invest thus results from the additional views a true article would bring about as compared to a false piece of news, which would be shared less. When several producers co-exist in the market, they compete *through influencers* to reach other consumers, as each consumer is restricted to see only one article.

This model brings interesting insights. Even when consumers are not behavioral, the market fails to deliver precise news. Therefore, incentives created by social media do not suffice to induce qualitative online news, even in a market populated by rational and unbiased agents. The market environment has counter intuitive effects on news quality: lack of private knowledge is not substituted by better articles and the performance of a competitive market is tied to its connectivity. Furthermore, the presence of news outlets has ambiguous welfare consequences, that not all interventions can overcome. These four results can be further discussed.

First, ad-funded online outlets tend to fail when informative news would be the most beneficial. News quality is less valuable for a producer in an environment with low private knowledge: either because the consumers are not well-informed by their signals, or because the state of the world is *ex ante* very uncertain. The former effect results from consumers having difficulties distinguishing true news from false information, leading them to treat any news article very similarly. The latter result follows from the producers' benefits being greater when the most likely state of the world realizes, pushing up the *ex ante* investment.

Second, competition can be detrimental; its effects depends on the network connectivity. For any market structure, high connectivity negatively affects news quality. A monopolist's incentive to invest vanishes as the network gets very dense: one node sharing would reach almost all other consumers. The monopolist can thus create false content and rely on a few influencers receiving an erroneous private signal to reach many followers. This does not follow through in competitive markets. Producers cannot rely on these few influencers anymore; they need to be sufficiently shared in order to survive in the network. The tension between coexisting articles in the network indeed forces producers to compete inside the network to reach followers *through* influencers.

Yet, the effect of competition is ambiguous. Indeed, splitting the market might be detrimental to investment, since the cost of investment does not depend on the size of the market served. For instance, in a network arranged in pair, a monopolist could potentially reach twice as many followers has a duopolist who would serve half as many influencers. As the connectivity increases, competition of information inside the network is more biting and pushes the producers to increase their investments. Therefore, below a connectivity threshold, competition is detrimental.

Third, the welfare value of ad-funded online news is ambiguous. Any equilibrium is Pareto inefficient. I consider different measures of consumers' welfare to further the analysis. *Entertainment* – the utility derived from sharing – increases with news quality. To capture the value of information, I introduce an additional action, a bet, in which consumers must match the true state of the world. For symmetric priors, the market fails to let influencers take better decisions. This does not rely on the presence of competition or the timing of the game. A producer has no incentive to outperform a consumers' private signal. Hence, the influencers are always as well off following their private signal. Followers, however, might take better decisions if the market is competitive as the network tends to filter out false articles.

Introducing a cost to the bet allows to analyze whether online news pushes consumers towards action. As expected, there exists a range of costs for which online news indeed helps agents betting when it is beneficial. More suprisingly, there also exists a range of costs for which news outlets are detrimental. When an article contradicts agents' private signal, news outlets can discourage consumers to take the bet. Because articles are not more precise than private signals, more agents are wrong than right to opt out.

Fourth, I propose several interventions. The first one relates to flagging false articles before influencers decide to share it. This reduces the value of producing false information and removes the bound placed on the news outlet's precision. With flagging, news outlets let agents take better decisions. Interestingly, competition dilutes the effect of flagging. Actually, for any environment, there exists a level of flagging that makes competition detrimental. The second intervention relates to certifying a news outlet's quality. While this allows producers to internalize the effects of their investment on the influencers' sharing strategy, news quality is still bounded by the consumers' private information. Finally, moving to subscription-based news outlets has ambiguous welfare effects. The efficient level of investment in quality can be reached, but consumers now need to pay for news that was previously financed by ads.

Related literature

I contribute to several strands of the literature. I particularly relate to theoretical works on news markets, media economics and the spread of news in networks.

First, as to **news markets**, the existing theoretical literature accounts for the existence of bad quality news in a competitive but unconnected world. Allcott and Gentzkow [2017] find that uninformative news can survive if news quality is costly and if consumers cannot perfectly infer accuracy or if they enjoy partisan news. My setup is similar in that quality is costly and consumers cannot perfectly distinguish true from false articles. However, my mechanism does not fundamentally rely on outlets' quality being hidden. Furthermore, I introduce to such models an explicit network of information sharing to catalyze the spread of information.

In such unconnected news markets, the ambiguous effects of competition between news providers has been widely explored. Namely, Gentzkow and Shapiro [2008] find that competition is effective at reducing supply-driven biases, while its effects when demand-driven biases are ambiguous. Consistently with this conclusion, other authors find that competition has ambiguous effects when news consumers lack sophistication. For instance, Levy et al. [2017] study how media companies can exploit consumers' correlation neglect. They find that competition reduces the producers' ability to bias readers' beliefs, but that diversity has a cost in terms of optimal consumers' responses. Hu and Li [2018] and Perego and Yuksel [2018] study how rational inattention biases the provision of political information. Both find that competition inflates disagreement. Chen and Suen [2016] also find that competition is detrimental to the accuracy and clarity of news when readers endogenously allocate attention between outlets whose editors are biased. Interestingly, my results on competition is not motivated by biases of either side of the market.

Second, as to **media economics**, this paper relates in particular, to the influence of digitalization on media. Representative of this literature are the following papers. AndersonAnderson [2012] combines empirical and theoretical insights to offer an overview of the ad-financed business model in the internet age. Wilbur [2015] documents trends following digitalization for the mass media and how their business models has evolved. Finally, Peitz and Reisinger [2015] review various novel features resulting from new Internet media. I contribute to this literature by explicitly modelling one such new feature of online news market: shared content. I study its effects on producers incentives and equilibrium outcomes.

Note that Peitz and Reisinger [2015] briefly discuss how sharing decision might affect available content and link it to more general media biases. In this perspective, Hu [2021] studies the impact of media regulation in the digital age and finds that government regulation is rendered less effective by media biases inherent to the digital age. Because their model does not take into account any communication network, their analysis does not study interventions targeting the sharing behavior of consumers. My intervention evaluations, in contrast, only accounts for such incentives resulting from consumers' sharing decisions.

Third, as to **news in networks**, a connected world has rarely been the setup for news market models in the literature. To the best of my knowledge, only Kranton and McAdams [2020] study the effect of communication networks on the quality of information provided on the news market. My model is largely inspired by the setup they propose. While Kranton and McAdams [2020] give a compelling argument on how a network of consumer can change a producer's investment incentives, their mechanism abstracts from the role of competition. Furthermore, they do not address welfare effects of market outcomes. A key contribution of this paper is the introduction of competition and welfare considerations to the model.

Following the cascade literature[1], the recent working paper Hsu et al. [2019] provide optimal conditions on a signal's precision for a cascade to occur when sharing is endogenous and strategic. This could, in term, relate to a producer's objective, although no producer is featured in their setup. However, just as Kranton and McAdams [2020], Hsu et al. [2019] is set in an uncompetitive world. Finally, recent works explore the particular setup of learning on social media. Bowen et al. [2021] study learning via shared news and find that polarization emerges when agents hold misconceptions about their friends' sharing behavior. They find that news aggregators help curb polarization. Neither of these papers addresses the effects of competition between news providers in a connected world.

The remainder of the paper is organized as follow. The general model is presented in Section 2. Section 3 analyzes the equilibrium resulting from a monopoly and a duopoly respectively. Section 4 proposes a framework to assess welfare. Section 5 evaluates possible interventions. Section 6 concludes. Further extensions are provided in the Appendices A,B, D and E. Appendix C presents the proofs omitted in the main text.

2 Model

2.1 Environment

The market is populated by news' producers on one side and news' consumers on the other. Consumers learn about an unknown state of the world $\omega \in \{0,1\}$ through news articles and

[1] This literature is studies learning in networks when agents learn from actions. See Bikhchandani et al. [1992], Banerjee [1992] for their seminal work.

private signals.[2]. There is a common prior across all agents, $\Pr(\omega = 0) = w_0$. All agents are Bayesian.

I denote the set of news' consumers N. All consumers receive an informative private signal $\varsigma = s$ about ω.These signals are i.i.d. among consumers with $\Pr(\varsigma = w|w) = \gamma$ for $w = 0, 1$. I further impose $\gamma \geq w_0$, so that consumers trust their private signal more than their prior.

In addition to private signals, the consumers can come across news articles in the following ways: they can be exposed to it directly – in such a case they are called *influencers* and denoted i; or they can read such news because an influencer shared it. If consumers are not influencers, they are called followers and denoted f. All consumers are exposed to at most one article, but some followers might be exposed to none. When I do not want to explicitly distinguish influencers from followers, I denote the news' consumers j.

The consumers are arranged on a regular network of degree d. Consumers are randomly drawn to be influencers with probability b. Hence, all consumers, regardless of their role, have the same number of *random* neighbors d. The network allows followers to see articles shared by neighboring influencers. A follower is not exposed to any news if none of its neighbors shared content – either because the neighbors are not influencers or because they decided not to share. If several neighbors shared content from different sources, the article that a followers ends up seeing is determined stochastically. The probability with which the follower sees a given source is proportional to the number of neighbors sharing this source relative to the number of neighbors having shared any article. Thus, the probability that f sees a given article u is:

$$Pr(f \text{ sees } u|A \text{ neighbors shared } u, B \text{ neighbors shared}) = \frac{A}{B}$$

For instance, say four of f's neighbors shared a piece of information, but only one of them shared u, then, f sees u with probability one fourth, although f does see *some* piece of news with probability one.

On the other side of the market, I consider a finite set of producers U. Each producer, denoted by u, publishes exactly one article.[3] Each producer reaches an influencer with the same exogenous probability $\frac{b}{|U|}$. The producer chooses the overall quality of the news that is published. However, he does not choose the article's content, which is randomly determined. The content of producer u's article is denoted $\eta_u = n$.

Example. As a leading example throughout the exposition, I will consider vaccination. Vaccines can either be safe ($w = 0$) or unsafe ($w = 1$). News' consumers receive private signals, for instance through the number of their friends having suffered side effects after a shot. In addition, agents can read articles about the risks linked to vaccines. Producers all report on vaccines risk.

[2] w denotes the outcome of ω. For the remainder of the paper, random variables are denoted by greek letters, while the outcomes of stochastic processes are denoted by latin letters.

[3] Therefore, I can abuse notation by also denoting articles by u.

2.2 Timing, Objectives and Equilibrium Concept

All strategic interactions are assumed to be simultaneous.[4] The only strategic agents in this setup are influencers and producers.

The producers choose the quality of their outlet to maximize their profits. The quality of outlet u is defined as the probability of documenting the true state of the world, $x_u := \Pr(\eta_u = w|w)$ for $w = 0, 1$. Producers derive revenue from advertisement, hence from the visibility of their outlet. Their revenue is thus defined as the share of the network that sees their article [5]. Their (total) cost is determined by cost function C. C is common to all producers. I denote c the marginal cost function. I assume C increasing and strictly convex, i.e. $c(x) > 0$ and $c'(x) > 0$. Finally, I assume that without any investment in quality, the outlet produces uninformative content, that is, $x_u = 1/2$. Furthermore, $c(1/2) = 0$.

Influencers like sharing true information and to dislike sharing false information. Accordingly, they choose the probability with which they share an article. This can depend on the content of the article they read and the private signal they received. The probability with which the influencers share an article u whose content is n after having received private signal s is denoted by $z_{u|n,s}$. Therefore, the influencers' strategy is a vector: $(z_{u|n,s})_{(u,n,s)\in U\times\{0,1\}^2} \in [0,1]^{4|U|}$. As influencers want to share an article only when its content is truthful, they are assumed to receive a positive payoff from sharing true information and a negative payoff when sharing false information.[6] Influencers have the following payoff from sharing:[7]

$$u(\text{sharing article with content } n|\omega = w) = \begin{cases} 1 & \text{if } n = w \\ -1 & \text{otherwise} \end{cases}$$

They receive payoff 0 if they do not share.

I focus on Nash Equilibria. As will be clearly indicated, only some particular subset of all NE will be discussed.

Following this game, we assume that a further non-strategic action takes place. Once the strategic interactions are played out, consumers can take an action $a \in \{0, 1\}$ to match the state of the world. I think of this as a financial bet, but it can capture a wider range of utility derived from information. This action can depend on the content of the article they read (if any) and their private signal. I assume that this bet has price r. I represent the case in which the consumers cannot opt out of this bet with $r = 0$. The benefits from matching the state of the

[4]Comparisons between equilibria for simultaneous and sequential games when $w_0 = 1/2$ are provided in the Appendix E

[5]Intuitively, the revenues are scaled for size population because they relate to advertisement revenues. One might expect advertisers to be interested in the *portion* of the population a given news outlet is able to reach. Furthermore, with this representation, the model becomes scale-free. Finally, it allows their profits to be bounded below 1

[6]This assumption can represent the interests of truth-seeking consumers. Implicitly, it also accounts for wider concerns such as reputation or attention. In fact, Appendix B assumes that influencers seek attention for themselves. The results are qualitatively similar.

[7]In Appendix A, we consider a more general payoffs. Most of the results follow through, but additional equilibria might appear.

world is assumed to be the same as their loss from a mismatch:

$$u_j(a_j|\omega = w) = \begin{cases} 1 & \text{if } a_j = w \\ -1 & \text{otherwise} \end{cases}$$

Example. Producers reporting on vaccination risks only chose how many journalists to hire for their outlets, but not what these journalists would report. The more journalists, the higher the likelihood of reporting the true risks of vaccines. Furthermore, after sharing decision have been made, and all uncertainty has resolved, all consumers take a bet about vaccines risk. We can consider two bets: one from which consumers cannot opt out, for instance, whether or not to vaccinate; and one from which consumers can opt out, e.g. whether to invest in a pharmaceutical group, a homeopathy company, or not to invest at all.

2.3 Best Responses

2.3.1 Influencers' Problem

Take an influencer i who received private signal s and read a news article from producer u having content n. Let this influencer attribute prior probability x_u to an article from u being true. Then, the influencers expected utility from sharing is:

$$p(n, s; x_u, w_0) + \Big(1 - p(n, s; x_u, w_0)\Big)(-1)$$

where $p(n, s; x_u, w_0)$ is i's posterior on the probability that u published a true piece of information.

A piece of news is true if it matches the state of the world. Hence, the posterior is the probability that the state of the world is the one prescribed by the news, given what was written in the news and what the consumers themselves experienced from the world. That is, $p(n, s; x_u, w_0) := Pr(\omega = n|\eta = n, \varsigma = s)$. Using Bayes' rule, we find:

$$Pr(\omega = n|\eta = n, \varsigma = s) = \frac{Pr(\eta=n,\varsigma=s|\omega=n)Pr(\omega=n)}{Pr(\eta=n,\varsigma=s)} = \frac{Pr(\omega=n)Pr(\varsigma=s|\omega=n)x_u}{\sum_w Pr(\omega=w)Pr(\varsigma=s|\omega=w)Pr(\eta=n|\omega=w)}$$

Therefore:

$$p(0,0;x_u,w_0) = \frac{w_0\gamma x_u}{w_0\gamma x_u + (1-w_0)(1-\gamma)(1-x_u)} \quad \text{and} \quad p(0,1;x_u,w_0) = \frac{w_0(1-\gamma)x_u}{w_0(1-\gamma)x_u + (1-w_0)\gamma(1-x_u)}$$

$$p(1,0;x_u,w_0) = \frac{(1-w_0)(1-\gamma)x_u}{(1-w_0)(1-\gamma)x_u + w_0\gamma(1-x_u)} \quad \text{and} \quad p(1,1;x_u,w_0) = \frac{(1-w_0)\gamma x_u}{w_0(1-\gamma)(1-x_u) + (1-w_0)\gamma x_u}$$

As one would expect, all posteriors are increasing in x_u. Furthermore, the probability for a news to be true is higher if the influencer got a private signal that corresponds to what the article reports. Finally, an article is believed more easily if it reports the most likely state of the world: $p(0,0;x_u,w_0) > p(1,1;x_u,w_0)$ for $w_0 > \frac{1}{2}$.

An influencer prefers sharing an article when its expected utility from doing so is greater than the outside option 0. Therefore, i shares news n from producer u upon receiving signal s when:

$$p(n, s; x_u, w_0) \geq 1/2$$

It follows that the best response to news' content $n = 0$ is:

$$(z^*_{u|0,0}(x_u), z^*_{u|0,1}(x_u)) = \begin{cases} (0,0) & \text{if } x_u < \underline{t}_0 \\ (b,0) & \text{if } x_u = \underline{t}_0 \\ (1,0) & \text{if } x_u \in (\underline{t}_0, \bar{t}_0) \\ (1,b) & \text{if } x_u = \bar{t}_0 \\ (1,1) & \text{if } x_u > \bar{t}_0 \end{cases}$$

for any $b \in [0,1]$, where $\underline{t}_0 = \frac{(1-\gamma)(1-w_0)}{(1-\gamma)(1-w_0)+\gamma w_0}$ and $\bar{t}_0 = \frac{\gamma(1-w_0)}{\gamma(1-w_0)+(1-\gamma)w_0}$.

And the best response to news' content $n = 1$ is:

$$(z^*_{u|0,0}(x_u), z^*_{u|0,1}(x_u)) = \begin{cases} (0,0) & \text{if } x_u < \underline{t}_1 \\ (b,0) & \text{if } x_u = \underline{t}_1 \\ (1,0) & \text{if } x_u \in (\underline{t}_1, \bar{t}_1) \\ (1,b) & \text{if } x_u = \bar{t}_1 \\ (1,1) & \text{if } x_u > \bar{t}_1 \end{cases}$$

for any $b \in [0,1]$, where $\underline{t}_1 = \frac{(1-\gamma)w_0}{(1-\gamma)w_0+\gamma(1-w_0)}$ and $\bar{t}_1 = \frac{\gamma w_0}{\gamma w_0+(1-\gamma)(1-w_0)}$.

For $w_0 \in (1/2, \gamma)$, $\underline{t}_0 > \underline{t}_1 > \bar{t}_0 > \bar{t}_1$: the influencers' best response are weakly monotonic in x_u. Therefore, although $z_u = (z_{u|0,0}, z_{u|1,1}, z_{u|0,1}, z_{u|1,0})$ is a four-dimensional object, its set of undominated strategies can be represented on a line.

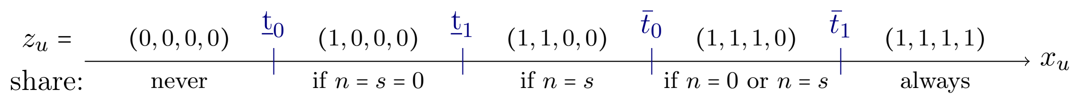

Figure 1: Sharing Decisions of Influencers for Different Quality of News

In particular, $z^*_{0,1} \geq 0$ if and only if $z^*_{0,0} = 1$ and $z^*_{1,0} \geq 0$ if and only if $z^*_{1,1} = 1$. That is, one shares an article reporting the opposite of their private signal only if one would be ready to share this article, were it to report the same as their private signal. Furthermore, for $w_0 \neq 1/2$, $z^*_{1,1} \geq 0$ if and only if $z^*_{0,0} = 1$ and $z^*_{1,0} \geq 0$ if and only if $z^*_{0,1} = 1$. That is, one shares an article reporting the least likely state of the world only if one would be ready to share this article, were it to report the most likely state of the world, given the same (dis)agreement with private signals. It follows that the best response to an article published by producer u, $z^*_u(x)$, can be represented on the line connecting $(0,0,0,0)$ to $(1,0,0,0)$ to $(1,1,0,0)$ to $(1,1,1,0)$ to $(1,1,1,1)$. Figure 1 represents how sharing decisions is affected by different news quality, and the monotone aspect of it; Figure 2 displays influencers' best response. The same applies for each producer u.

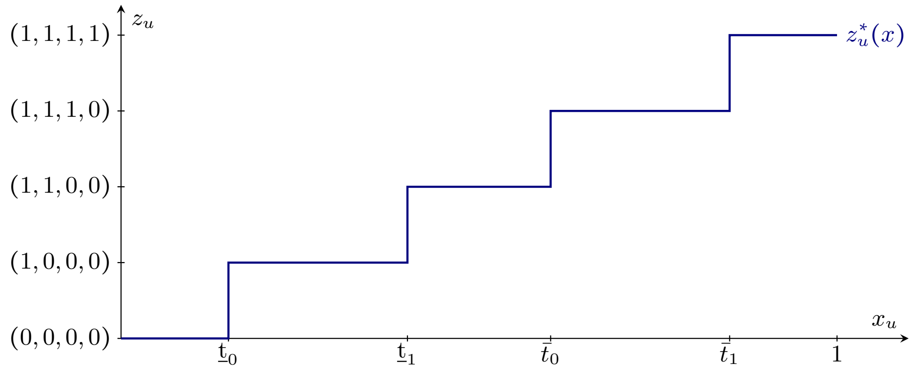

Figure 2: Best Response of Influencers as a Function of x_u

2.3.2 Producers' Problem

Consider a producer u. Let R_{ju} take value 1 if consumer j sees producer u's article. Assume that u is facing influencers who have strategy $\mathbf{z}$, while the other producers are investing $\mathbf{x_{-u}}$. Then, the expected profits for producer u who invests to reach quality x_u is:

$$\frac{\mathbb{E}(\sum_j R_{ju}|x_u;\mathbf{z},\mathbf{x_{-u}})}{|N|} - C(x_u) = \mathbb{E}(R_{ju}|x_u;\mathbf{z},\mathbf{x_{-u}}) - C(x_u)$$

where the last equality follows from all consumers being *ex ante* identical.

The expected share of reader as a function of u's investment in quality is found as follows. For a random node to share the article from producer u requires: the consumer to be an influencer – with probability b –, to come across u's article – with probability $|U|^{-1}$ – and to share. The probability to share z depends on the news content n and the private signal s. A random influencer coming across news n will receive private signal $s = w$ with probability γ and $s = -w$ with probability $1 - \gamma$. Thus the probability for a random node to share information content n from producer u given state of the world w is:

$$p_{u|w,n} = \frac{b}{|U|}(\gamma z_{u|n,w} + (1-\gamma)z_{u|n,-w})$$

The *ex ante* probability that a consumer reads u's article, whose content is n in state of the world w represents the value of such article and is denoted $V_{u|w,n}$. If the producer was alone, this would simply be:

$$V_{u|w,n}(z) := \Pr(j \text{ influencer}) + \Pr(j \text{ follower} \wedge \geq 1\ j\text{'s neigh. shared}) = b + (1-b)\big(1 - (1-p_{w,n})^d\big)$$

However, when the news producer u is not alone in the market, it is not enough that a followers' neighbor shared u's article; this followers also needs to see u against all other producers

$-u$'s articles. Therefore:

$$V_{u|w,n}(z) := \Pr(j \text{ influencer}) + \Pr(j \text{ follower}) \Pr(\geq 1\ j\text{'s neigh. shared}) \Pr(j \text{ sees } u \text{ against } -u)$$

$\Pr(j$ sees u against $-u)$ depends on the number of j's neighbors having shared u against $-u$. The number of j's neighbors having shared $-u$ depends on the content produced by other producers, which we denote $m := (m_v)_{v \neq u}$. The *ex ante* probability that a consumer reads u's article, whose content is n in state of the world w, $V_{u|w,n}(z)$ is then:

$$V_{u|w,n}(z) = \mathbb{E}(R_{ju}|w,n) = \sum_m \mathbb{E}(R_{ju}|w,n,m) \Pr(m|w).^8$$

Where (see Appendix C):

$$\mathbb{E}(R_{ju}|w,n,m) = \frac{b}{|U|} + (1-b)\frac{p_{u|w,n}}{p_{u|w,n} + p_{-u|w,m}}\left(1 - (1 - p_{u|w,n} - p_{-u|w,m})^d\right)$$

denoting $p_{-u|w,m} = \sum_{v \neq u} p_{v|w,m_v}$.

The probability for a follower to read information a has two factors. The former, $\frac{p_{u|w,n}}{p_{u|w,n}+p_{-u|w,m}}$ represents the expected share of followers u would get, conditional on them being reached by any news, whereas the latter factor $1-(1-p_{u|w,n}-p_{-u|w,m})^d$ represents the probability of news reaching followers. It means that sharing affects the producer's revenue through two channels: the size of the total readership and the portion of readers viewing a given producers. For instance, if the influencers of a producers' competitor start sharing more often, the total readership increases but the portion of the readership viewing said producer decreases. The relative strength of this two effects depends on the connectivity of the network d. Both factors are however increasing in $p_{u|w,n}$. Hence, as long as true articles are shared more than false articles, i.e. $z_{u|n,n} \geq z_{u|n,-n}$, true information is more visible, no matter the outcome of the competitor.

Finally, the expected portion fo the network reached given an investment x_u is:

$$\mathbb{E}(R_{jv}|x) = w_0[x_u V_{u|0,0}(z) + (1-x_u)V_{u|0,1}(z)] + (1-w_0)[x_u V_{u|1,1}(z) + (1-x_u)V_{u|1,0}(z)]$$

Because the profits are $\mathbb{E}(R_{jv}|x_u) - C(x_u)$, the maximization of profits implies:

$$x^*(z) = c^{-1}\left(w_0[V_{u|0,0} - V_{u|0,1}] + (1-w_0)[V_{u|1,1} - V_{u|1,0}]\right) := c^{-1}\left(\Delta V_u(z; x_{-u})\right)$$

Because $c'(x) \geq 0$, the equilibrium investment $x^*(z)$ is (weakly) increasing in $\Delta V_u(z; x_{-u})$. Thus, $\Delta V_u(z)$ denotes producer u's incentive to invest. Section 3 analyzes the function shape under more restrictive assumptions.

[8] $\Pr(m|w) = \prod_{v:m_v=w} x_v \cdot \prod_{v:m_v \neq w}(1-x_v)$. For instance, with two other producers in addition to u, $\Pr((0,1)|\omega = 0) = x_1(1-x_2)$.

3 Equilibrium

In this section, I restrict my attention to some specific assumptions about the agents setup so to characterize possible equilibria. In particular, I consider a non-competitive environment with asymmetric priors; and a competitive market with symmetric priors. I say that an environment is competitive when followers are restricted to see less articles than the amount available on the market. Indeed, in such case, producers are forced to compete through influencers to capture followers' views. Because in this setup, I restrict consumers to receive only one piece of information, I analyze the effects of competition using a duopoly.

3.1 Equilibrium without Competition

Let us assume only one producer on the market. For clarity purposes, I omit the u indice in this section.

We can rewrite explicitly the producer's best response:

$$\begin{aligned}\Delta V(z)/(1-b) &= w_0\left[(1-p_{0,1})^d-(1-p_{0,0})^d\right]+(1-w_0)\left[(1-p_{1,0})^d-(1-p_{1,1})^d\right]\\ &= w_0\big(1-b(\gamma z_{1,0}+(1-\gamma)z_{1,1})\big)^d-(1-w_0)\big(1-b(\gamma z_{1,1}+(1-\gamma)z_{1,0})\big)^d\\ &\quad+(1-w_0)\big(1-b(\gamma z_{0,1}+(1-\gamma)z_{0,0})\big)^d-w_0\big(1-b(\gamma z_{0,0}+(1-\gamma)z_{0,1})\big)^d\end{aligned}$$

Let us now analyze the shape of such best-response:

Lemma 1. *Shape of the Monopolist's Best Response to Sharing*

(i) $x^*(z)$ *is strictly decreasing in* $z_{0,1}$ *and* $z_{1,0}$ *.*

(ii) $x^*(z)$ *is single-peaked in* $z_{0,0}$ *(respectively,* $z_{1,1}$*), with local maxima* $\bar{z_{0,0}}\bar{z_{1,1}} \in (0;1]$.

(iii) There exists a non-empty set of priors W *such that* $\arg\max_z x^*(z) = \{(1,\bar{z_{1,1}},0,0)\}\ \forall w_0 \in W$ *and* $\arg\max_z x^*(z) = \{(\bar{z_{0,0}},0,0,0)\}\ \forall w_0 \in [0,1]-W$

(iv) $x^*(z)$ *is continuous in* z*.*

Figure 3 and 4 illustrate the shape of the producer's best response. Because the influencers' strategy is not a unidimensional object, I illustrate the shape of the producer's best-response on the set of influencers' undominated strategy. As before, I represent the influencers' strategy on a line and map the corresponding image as if the argument was unidimensional. The resulting function is non-monotonic. Each hump shaped segment is explained by the effect of the network. At first, when few agents are sharing, every additional share reaches an almost constant number of additional followers; because the probability that this share occurs after having issued a true article is higher, true information gains much more followers than false information – the best-response is increasing. But when *enough* shares have occurred, any additional share is likely to reach followers that would have been reached anyways; hence the marginal value of the share

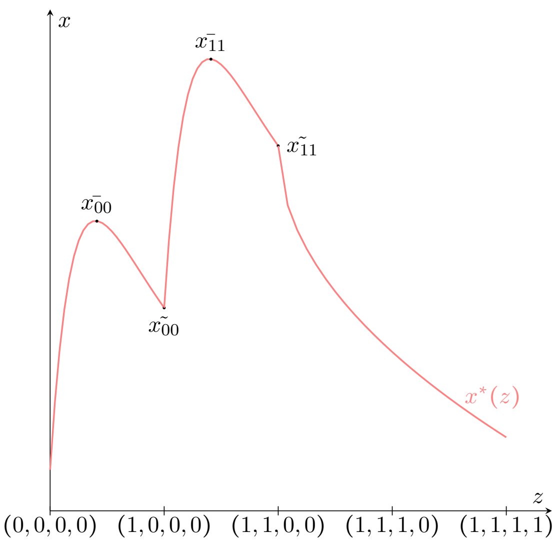

Figure 3: Producer's Best Response, $\bar{x}_{00} < \bar{x}_{11}$

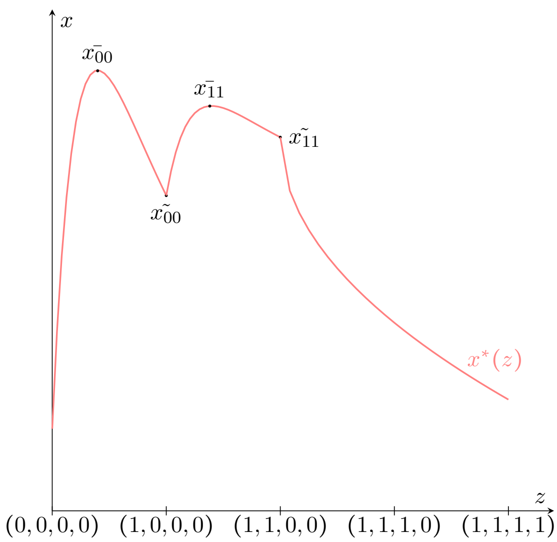

Figure 4: Producer's Best Response, $\bar{x}_{00} > \bar{x}_{11}$

is decreasing, because of redundant path to followers in the network. Therefore, the number of followers reached with a false article is increasing faster than those reached with true news; and the best-response is decreasing. The two humps follow from the same mechanism applying in two different cases: when news 0 is produced first, and then when news 1 is published. Subsequent to this, the best response is decreasing. Agents there start sharing news that does not correspond to their private signals. Therefore, the probability this concerns a false article is higher than the probability that is applies to true information. It follows that false information accumulates views faster than true news. The difference between the value of true and false information thus decreases, making the best-response decreasing.

Now, recall that $x^* \geq 1/2$, as zero investment would lead to $x = 1/2$. Furthermore, $\underline{t}_0 < \underline{t}_1 < 1/2$. Therefore, we can characterize the NE of the monopoly:

Proposition 1. *There exists a unique Nash equilibrium. It displays positive investment and is characterized by news' quality* $x^M = \max\{\min\{x^*(1,1,0,0), \bar{t}_0\}, \min\{x^*(1,1,1,0), \bar{t}_1\}\}$

Proof. First, notice that any positive equilibrium investment has to achieve $x \leq \bar{t}_1$. Indeed, $x = \bar{t}_1$ is enough to insure that the producer's news is always share, so that any additional investment would increase costs without increasing benefits. Furthermore, note that even if no investment occurs, sharing can occur. Indeed, faced to completely uninformative news' outlet, agents will still share an article whose content matches their private signal, because the private signal is informative. Therefore, any equilibrium displays $z_{0,0} = z_{1,1} = 1$; and the equlibrium will occur on the decreasing part of the producers' best response. Furthermore, note that $x^*(1,1,1,1) = 1/2$. Indeed, if news gets systematically shared, the producer has no incentive to invest since true news is treated as false news. Because the relevant portion of $x^*(z)$ is strictly decreasing, while $z^*(x)$ is weakly increasing, any intersection has to be unique. Because both best responses are continuous and that in $z = (0,0,0,0)$ the producer's best response is above the value ensuring

some sharing, while in $z = (1,1,1,1)$, the producer's best response is below the value ensuring full sharing, the intersection must be unique. Therefore, a NE must exist and is unique.
Because the cost function will determine different levels for $x^*(1,1,0,0)$ and $x^*(1,1,1,0)$, we need to understand how these values compare to $\bar{t}_0 < \bar{t}_1$. If $x^*(1,1,0,0) < \bar{t}_0$, from the shapes of the best responses, we have $x^*(1,1,1,0) < x^*(1,1,0,0) < \bar{t}_0 < \bar{t}_1$ so that $x^M = x^*(1,1,0,0)$. Indeed, in such a case, because $x^M < \bar{t}_0$, the influencers will share an article only if its content matches their private signal: $z^*(x^M) = (1,1,0,0)$. This is also optimal for the producer, as, by definition $c(x^M) = c(\Delta V(1,1,0,0))$. Furthermore, no other investment is optimal as c is strictly increasing. The same reasoning applies for $\bar{t}_0 < x^*(1,1,1,0) < \bar{t}_1$. Now, consider $\bar{t}_0 < x^*(1,1,0,0)$ but $x^*(1,1,1,0) < \bar{t}_0$. Then, $x^M = \bar{t}_0$. Indeed, as $x^*(1,1,1,0) < \bar{t}_0 < x^*(1,1,0,0)$, and because $x^*(z)$ continuous, there must exist some $z^*_{0,1}$ such that $c^{-1}(\Delta V(1,1,z^*_{0,1},0)) = \bar{t}_0$. It is easy to verify that this constitutes a NE. The same reasoning applies for $\bar{t}_1 < x^*(1,1,1,0)$. □

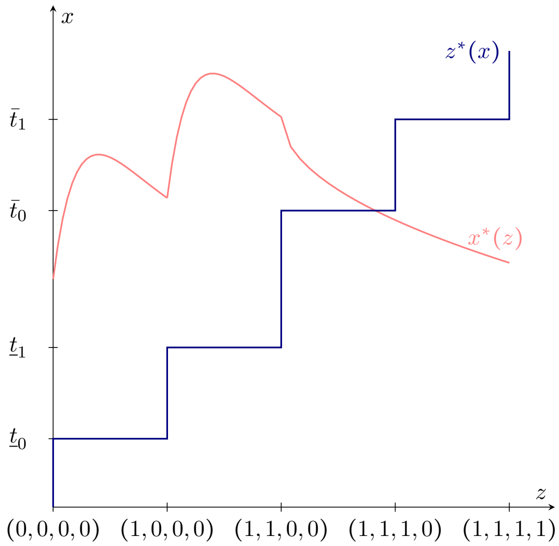

Figure 5: Investment Eq. with $x^M = \bar{t}_0$

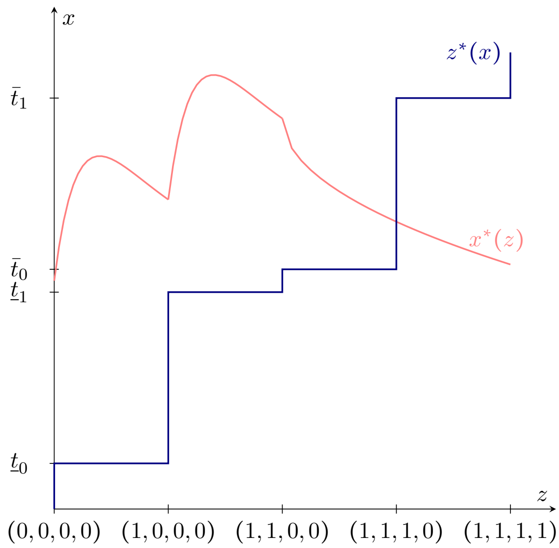

Figure 6: Investment Eq. with $x^M = x^*(1,1,1,0)$

Figure 5 and 6 illustrate two cases. Figure 5 shows the equilibrium with $x^*(1,1,0,0) > \bar{t}_0 > x^*(1,1,1,0)$. Figure 6 shows the equilibrium with $\bar{t}_1 > x^*(1,1,1,0) > \bar{t}_0$.

There are several important points to note on these results. First, $x^M < \max_z\{x^*(z)\}$, so that maximal veracity is never achievable. Furthermore, very high connectivity is generally bad for investment. In fact:

Proposition 2. *$\Delta V(z;d)$ is single peaked in d, i.e.: $\exists \bar{d} : \Delta V(z;d) \geq \Delta V(z;d') \geq \Delta V(z;\bar{d}) \forall d > d' > \bar{d}$ and $\Delta V(z;d) \leq \Delta V(z;d') \leq \Delta V(z;\bar{d}) \forall d > d' > \bar{d}$*

In particular notice that $\Delta V^M(z) \to (2w_0 - 1)\mathbb{1}_{z_{00}>0, z_{11}=0}$ when $d \to \infty$. This means that as the network grows more connected, the producer's incentive to invest vanishes. Take a complete network for instance, that is a network in which every node is connected with every node. In such a context, a monopolist would need to convince only a single influencer to share in order to reach every single consumers.

3.1.1 The role of the prior about the state of the world

An increase in the prior probability of witnessing a 0 state of the world has ambiguous effects, although it generally leads to an increase in equilibrium investment for both marginal and substantial increases in w_0.

First, we notice the following effects of w_0 on the agents' best responses:

Lemma 2.

(i) *w_0 as an ambiguous role on the influencers' best response. The thresholds $\underline{t}_0$ and $\bar{t}_0$ decrease when w_0 increase while $\underline{t}_1$ and $\bar{t}_1$ increase with w_0. In fact, $\underline{t}_0 \in [\frac{(1-\gamma)^2}{(1-\gamma)^2+\gamma^2}, 1-\gamma]$ and $\bar{t}_0 \in [1/2, \gamma]$ while $\underline{t}_1 \in [1-\gamma, 1/2]$ and $\bar{t}_1 \in [\gamma, \frac{\gamma^2}{\gamma^2+(1-\gamma)^2}]$*

(ii) *w_0 has a (weakly) positive effect on the producer's incentive to invest. In fact, it has the biggest effect in $z = (1,0,0,0)$ while it has no strict effect in $z = (0,0,0,0)$, $z = (1,1,0,0)$ and $z = (1,1,1,1)$.*

Proof. See Appendix C □

Notice that the producer's incentive to invest increases with w_0. Indeed, because influencers are easier to convince if the article content corresponds to the most likely state of the world, true news is always more beneficial to the producer when the state of the world is 0. Therefore, his expected profits from a given investment increases when the most beneficial state becomes more likely.

Because the requirement in order for influencers to share has an ambiguous reaction to a change in w_0, the overall effect is ambiguous. However, we can assess the condition under which the overall equilibrium investment decreases in more uncertain situation.

Corollary 1. *For any increase in w_0, the inequalities detailed in Proposition 1 do not change, so that the maximal equilibrium investment x^M increases iff $x^M \neq \bar{t}_0$.*

Proof. See Appendix C □

Interestingly, a more certain state of the world generally leads to better provision of information. It also means that the inefficiencies linked to monopolist revenues from ads are the highest when quality information is the most useful: for very uncertain state of the world. A proper setup to formally study such inefficiencies is introduced in Section 4.

3.2 Equilibrium with Competition

I now assume that two producers coexist on the market. However, for tractability purposes, I restrict my attention to $w_0 = 1/2$. In this case, $\underline{t}_0 = \underline{t}_1$, that I can now denote $\underline{t}$; likewise, $\bar{t}_0 = \bar{t}_1$, now denoted $\bar{t}$. Therefore, the set of undominated strategy cannot necessarily be projected on a line. I thus select the undominated strategies z_u such that $z_{u|0,0} = z_{u|1,1}$ and $z_{u|0,1} = z_{u|1,0}$. Because the both types of content are *ex ante* as likely, it is intuitive that both types of news' content would be treated similarly. I denote the strategy upon receiving a news' article congruent with one's private message by z_{uT}; while the probability of sharing an article whose content is opposite to one's private signal is denoted z_{uF}. Likewise, rather than $p_{u|0,0} = p_{u|1,1}$, I denote the probability with which any article matching the state of the world is shared by p_{uT}; p_{uF} becomes the probability with which a false information is shared.

Let me denote the two competitors by u and v. We rewrite producer u's best response given v's investment and sharing strategy z:

$$\Delta V_u(z; x_v^*) = (1-b)\Big[x_v^*(z) S_{x_v} + (1 - x_v^*(z)) S_{1-x_v}\Big]$$

Where:

$$S_{x_v} = \frac{p_{uT}}{p_{uT}+p_{vT}}\Big(1-(1-p_{uT}-p_{vT})^d\Big) - \frac{p_{uF}}{p_{uF}+p_{vT}}\Big(1-(1-p_{uF}-p_{vT})^d\Big)$$

$$S_{1-x_v} = \frac{p_{uT}}{p_{uT}+p_{vF}}\Big(1-(1-p_{uT}-p_{vF})^d\Big) - \frac{p_{uF}}{p_{uF}+p_{vF}}\Big(1-(1-p_{uF}-p_{vF})^d\Big)$$

Because $\mathbb{E}(R_{ju}|\omega = w, \eta_u = w, \eta_v = m) \geq \mathbb{E}(R_{ju} = 1|\omega = w, \eta_u = -w, \eta_v = m)$, we know that $S_{x_v} \geq 0, S_{1-x_v} \geq 0$, so that $\Delta V_u(z; x_v^*) \geq 0$.

The shape of $\Delta V_u(z; x_v^*)$ in z_u is similar to the monopoly case:

Lemma 3. *Shape of the duopolists' best response to sharing*

(i) $x_u^*(z; x_v)$ *is strictly decreasing in* z_{uF}.

(ii) $x_u^*(z; x_v)$ *is single-peaked in* z_{uT}; *the function is maximized for some* $\bar{z_{uT}} \in (0; 1]$.

(iii) For any x_v, *investment occurs as long as* $z_{uT} > z_{uF}$, *while* $x_u^*(z; x_v) = 1/2$ *for* $z_{uT} = z_{uF}$

(iv) $x_u^*(z; x_v)$ *is continuous in* z_u.

Proof. See Appendix C □

Furthermore, $\Delta V_u(z; x_v^*)$ also depends on z_v and x_v. In fact:

Lemma 4. *(i)* $\Delta V_u(z_u, z_v; x_v)$ *relation with* z_v *depends on* d. *For small* d, *it is decreasing for any* z_u, z_v. *For large* d, *it is decreasing for* z_v *such that* $p_{vF}^2 > p_{uT}p_{uF}$. *(ii)* $\Delta V_u(z_u, z_v; x_v)$ *is decreasing in* x_v *for any* $z_{uX} \leq z_{vX}$. *(iii)* $\Delta V_u(z_u, z_v; x_v)$ *is continuous in* z_v *and* x_v.

Proof. See Appendix C □

I can now characterize the NE. I call symmetric equilibria any equilibrium in which $z_u = z_v$ and $x_u = x_v$. Note that in this case, $\Delta V_u = \Delta V_v$. I denote this common function $\Delta V^D((z_T, z_F), x)$.

Proposition 3. *The only symmetric equilibrium features positive investment and is characterized by news precision* $x^D = \arg\min_{x\in[1/2,\gamma]} \left|\Delta V^D\big((1,0);x\big) - c(x)\right|$.

Proof. First note that any equilibrium news' quality lies in $[1/2, \bar{t}]$. Indeed, recall that $\Delta V^D((0,0),x) = \Delta V^D((1,1),x) = 0$. Clearly, for any $x > \bar{t}$, $c(x) > 0 = \Delta V^D(z^*(x), x)$, which would be suboptimal for the producer.

First I prove that a symmetric equilibrium exists. Then, I show it is unique.

Consider two cases:

1. If $c(\bar{t}) > \Delta V^D((1,0),\bar{t})$, then $\exists \tilde{x} \in [1/2, \bar{t}]$: $c(\tilde{x}) = \Delta V((1,0),\tilde{x})$. Indeed, recall that c is weakly increasing in x and $\Delta V^D((1,0),x)$ strictly decreasing in x. We also notice that $c(\tilde{x}) = \Delta V((1,0),\tilde{x})$ while $c(\tilde{x}) = \Delta V((1,0),\tilde{x})$. Because both c and ΔV are continuous in x, they must intersect on $[1/2,\bar{t}]$. Clearly, $(\tilde{x},(1,0))$ is a NE.
   This equilibrium is unique. First notice that for $z = (1,0)$, the intersection must be unique given the shape of the respective best responses. Let us show that no other undominated sharing rule can be consistent with an equilibrium in this case. A sharing rule $(z,0)$ with $z < 1$ would require $x < 1/2$, which is impossible. A sharing rule $(1,z)$ with $z > 0$ would require $x \geq \bar{t}$. This cannot occur in equilibrium since, for any $z \in [0,1]$, $\Delta V^D((1,z),\bar{t}) < \Delta V^D((1,z),x^D) = c(x^D) < c(\bar{t})$. Hence, $c(\bar{t}) > \Delta V^D((1,z),\bar{t})$, so that $x^*((1,z),\bar{t}) < \bar{t}$ for any z.

2. If $c(\bar{t}) < \Delta V^D((1,0),\bar{t})$, then $\exists \tilde{z}_F \in [0,1]$: $c(\bar{t}) = \Delta V((1,\tilde{z}_F),\bar{t})$. Indeed, by assumption $c(\bar{t}) < \Delta V^D((1,0);\bar{t})$ and we know that $c(\bar{t}) > 0 = \Delta V^D((1,1);\bar{t})$. Because $\Delta V^D(z;x)$ is continuous in z_F, there must exist such $\tilde{z}_F$. Because $V^D(z;x)$ is strictly decreasing in z_F, this equilibrium is unique.

□

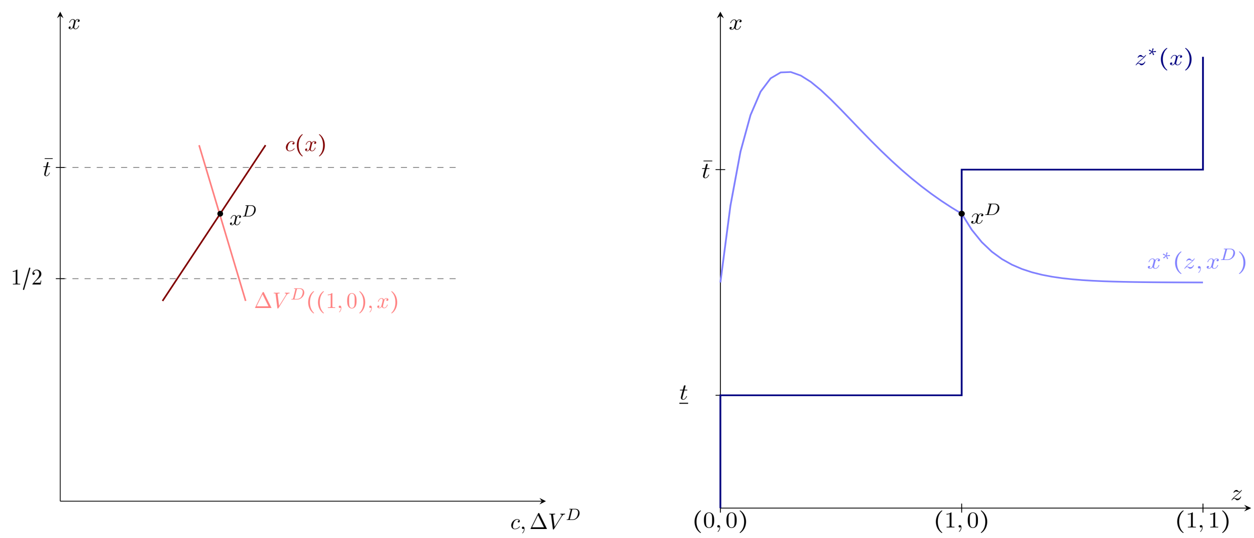

Figure 7: Illustration of a case for which $x^D \in (1/2, \bar{t})$

Figure 7 illustrates the first case of the proof, i.e. when $c(\bar{t}) > \Delta V^D((1,0),\bar{t})$. It means that there exists an intersection between $c(x)$ and $\Delta V^D((1,0),x)$ in the interval $(1/2,\bar{t})$ (left panel). Now, given that $-v$ invests x^D, $v's$ best response crosses the influencers' best response in $((1,0),x^D))$. Therefore, it is a NE.

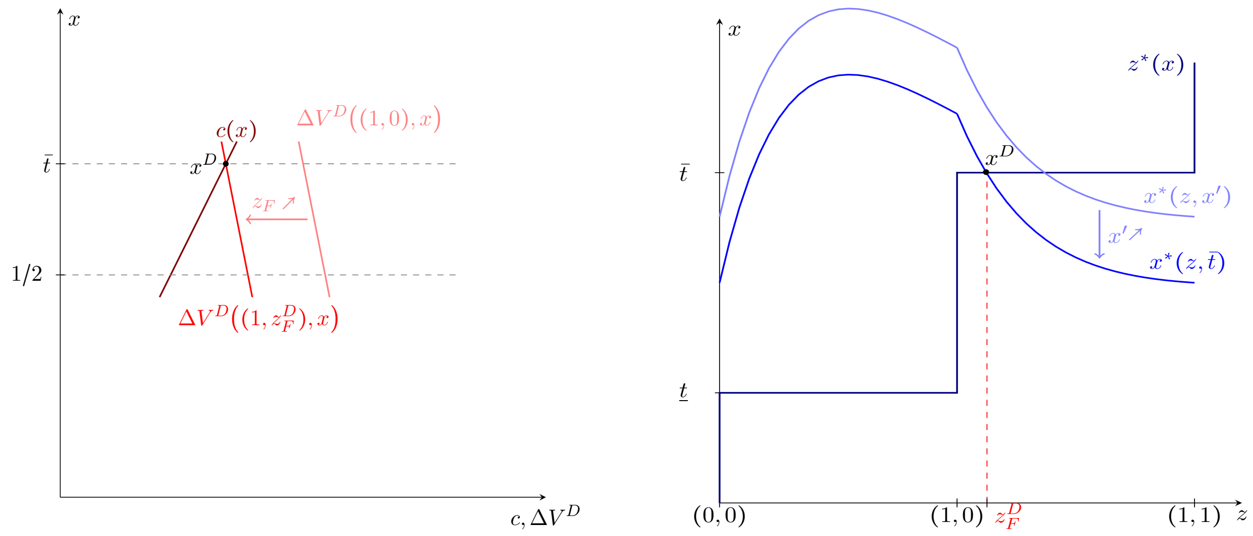

Figure 8: Illustration of a case for which $x^D = \bar{t}$

Figure 8 represents the second case of the proof, i.e. when $c(\bar{t}) < \Delta V^D((1,0),\bar{t})$. In this case, $c(x)$ lies completely on the left of $\Delta V^D((1,0),x)$, without ever intersecting in the interval $x \in (1/2,\bar{t})$ (left panel). Equivalently, u's best response to z given that v invests $x < \bar{t}$ does not intersect the influencers' best response z below $\bar{t}$ (right panel). This means that there does not exist a NE in which $z^* = (1,0)$. However, if $z_F > 0$, $\Delta V^D(z,x)$ is shifted to the left in the space $(\Delta V^D,x)$, so that it now crosses $c(x)$ (left panel). Furthermore, because x' increases to $\bar{t}$, the curve $x^*(z,x)$ is shifted downwards in the space (z,x) (right panel). Still, note that $((1,0),\bar{t})$ is not an equilibrium as $\Delta V^D((1,0),\bar{t}) > c(\bar{t})$. However, for $z_F^D > 0$, $\Delta V^D(1,z_F^D),\bar{t}) = c(\bar{t})$; we found the NE.

While the symmetric equilibrium is unique, asymmetric equilibria still exist and are not generally unique.

Remark 1. If the cost function is linear, there are no equilibrium with $x_u \neq x_v$ and $(x_u,x_v) \in (1/2,\gamma)$ as long as $c(x)$'s slope is different from S.

Proof. Assume $x^D \in (1/2,\gamma)$. Assume that there exists an $x_u > x_v$, with $(x_u,x_v) \in (1/2,\gamma)$. Then, $c(x_u) = \Delta V_u((1,0),(1,0),x_v)$ and $c(x_v) = \Delta V_v((1,0),(1,0),x_u)$, so that $c(x_u) - c(x_v) = S(x_u - x_v)$, which is impossible if c has a slope different from S. □

3.2.1 Effects of Competition

For cases to be comparable, we restrict our attention to the monopoly equilibrium with $w_0 = 1/2$.

We can now compare the symmetric equilibrium x^D with x^M. Depending on the parameters

d, b, γ, $x^D > x^M$ or $x^D < x^M$. We compare the two types of markets under the lights of connectivity and signal precision in particular.

The Role of Connectivity

Because $x^M(d) \geq x^D(d)$ if and only if $\Delta V^M(z;d) > \Delta V^D(z,x^D;d)$, we mainly focus on $\Delta V^M(z;d)$ and $\Delta V^D(z,x^D;d)$ in this section.

I begin by presenting two representative cases, $d = 1$ and $d \to \infty$, to illustrate the mechanism at stake before formally proving the principal result. Take $d = 1$. Then, there is no real competition of information inside the network. The network can be represented by isolated pairs of nodes, and a producer reaches portion of followers exactly proportional to the number of influencers who shares. Therefore, producers' incentive to invest is proportional to the number of influencers they would convince to share, as they would get exactly one more view from the one follower connected to each "convinced influencer". Because the monopolist has initially access to more influencers than the duopolists – 2 against 1 – his incentive to invest is higher by a factor of two. We indeed have:

$$\Delta V^M(z;1) = (1-b)(2\gamma-1)(z_T - z_F) > (1-b)\tfrac{1}{2}(2\gamma-1)(z_T - z_F) = \Delta V^D(z,x;1) \quad \forall z_T > z_F$$

Now take $d \to \infty$. This requires the network to be infinite. Because the finiteness of the network is not essential to our specification, let us assume, for convenience, that $|N| \to \infty$, and that d grows as fast as $|N|$. Having $d \to \infty$ means that all nodes are connected to each other, so that all followers are sure to be reached by at least one news as soon as the probability that sharing occurs is positive, however arbitrarily small. For a monopolist, the incentive to invest then vanishes, since having only one influencer (out of an infinity) sharing is enough for the entire network to be reached. It is quite the opposite in a duopoly. Because in this case, the competition of two coexisting information is all that matters for the producers, the incentive for the producer is proportional to the ratio of his influencers sharing, to all sharing influencers, which is positive and does not depend on d. We indeed have;

$$\Delta V^M(z;\infty) = 0 < (1-b)\frac{(2\gamma-1)(z_T-z_F)}{z_T+z_F} = \Delta V^D(z,x;\infty) \quad \text{as } (1-\varepsilon)^d \to 0 \ \forall \varepsilon > 0$$

The comparison between monopoly and duopoly investments' thus relies on a tradeoff between the two effect described above: on the one hand, as seen when d=1, because the quantity of influencer a producer reaches is exogenous, competition gives less "seeds" (influencer) to each producer; then the share of the network each producer could reach, even in the best case scenario, is lower, and so becomes his incentive to invest. On the other hand, as seen when $d \to \infty$, coexistence of competing news on the network itself pushes the producer to convince as many influencers as possible, and not only a few seeds who can freely spread the news. Which of these two forces dominates is linked to d.

Theorem 1. *There exists a unique threshold $\bar{d}$ such that $x^M(d) \geq x^D(d)$ for all $d < \bar{d}$ and*

$x^M(d) \leq x^D(d)$ for all $d > \bar{d}$

Proof. Define $DV(d) := \frac{\Delta V^M(z;d) - \Delta V^D(z,x;d)}{1-b}$. First, notice it that for $d = 1$, $DV(d) > 0$; however for $d \to \infty$, $DV(d) < 0$. Therefore, there must exist some d_0 such that $DV(d_0) \geq 0 > DV(d_0 + 1)$. All that is left to do is to show that such d_0 is unique. This is the case because if $DV(d_1) > DV(d_1 + 1)$ for some d_1, then DV is decreasing for all subsequent $d > d_1$. (See Appendix C for the details). □

Note that the threshold $\bar{d}$ depends on the parameters b, γ. As a rule of thumb, higher b and lower γ pushes $\bar{d}$ downward.

Remark 2. (Preliminary) A further increase in competition, beyond two producers, is either always detrimental, or detrimental for sparser networks.

Proof. Take any competition with $|U|$ competitors and a symmetric incentive to invest. Add one producer u'. The difference in the incentive to invest between the two can be proportional to:

$$\begin{aligned}\sum_m \Pr(m)\Bigg\{&\Big[\tfrac{p_{uT}}{p_{uT}+p_{-uM}}\big(1-(1-\tfrac{p_{uT}+p_{-uM}}{|U|})^d\big) - \tfrac{p_{uF}}{p_{uF}+p_{-uM}}\big(1-(1-\tfrac{p_{uT}+p_{-uM}}{|U|})^d\big)\Big] \\ &-x\Big[x\tfrac{p_{uT}}{p_{uT}+p_{u'T}+p_{-uM}}\big(1-(1-\tfrac{p_{uT}+p_{u'T}+p_{-uM}}{|U|+1})^d\big) - \tfrac{p_{uF}}{p_{uF}+p_{u'T}+p_{-uM}}\big(1-(1-\tfrac{p_{uT}+p_{u'T}+p_{-uM}}{|U|+1})^d\big)\Big] \\ &-(1-x)\Big[\tfrac{p_{uT}}{p_{uT}+p_{u'F}+p_{-uM}}\big(1-(1-\tfrac{p_{uT}+p_{u'F}+p_{-uM}}{|U|+1})^d\big) - \tfrac{p_{uF}}{p_{uF}+p_{u'F}+p_{-uM}}\big(1-(1-\tfrac{p_{uT}+p_{u'F}+p_{-uM}}{|U|+1})^d\big)\Big]\Bigg\}\end{aligned}$$

Whose sign is positive in $d = 1$, meaning that the incentive to invest with $|U| + 1$ producer is smaller in the symmetric case than the incentive to invest with $|U|$ producers. The expression's sign depends on the parameters for $d \to \infty$. □

The Role of Signal Precision

Proposition 4. *(i) When the influencers get perfectly informative private signal, monopoly yields higher investments than duopoly.*

(ii) When the influencers get perfectly uninformative private signal, neither a monopoly nor a duopoly would feature any investment.

Proof. (i) When $\gamma \to 1$, note that the set of the influencers' best response reduces to $\{(1,0)\}$. Then, we have:

$$\Delta V^M((1,0);1) = (1-b)(1-(1-b)^d) > (1-b)\tfrac{1}{2}x(1-(1-b)^d) + (1-x)(1-(1-\tfrac{1}{2}b)^d) = \Delta V_v((1,0),x;1)$$

Because $\frac{1-(1-b)^d}{1-(1-\frac{1}{b})^d} > \frac{1-x}{1-\frac{1}{2}x}$ $\forall x \in [0,1]$.

(ii) When $\gamma \to \frac{1}{2}$, $p_T = p_F$ for any z, so that the incentive to invest vanishes on both types of market: $\Delta V^M(z;\frac{1}{2}) = 0 = \Delta V_v(z,x;\frac{1}{2})$

□

When the signal is perfectly informative, influencers only share true information. Then, the monopolist has the highest possible incentive to invest: false information is worthless; with true information, he reaches all the followers the network allows him to reach. For the duopolist, false information is also worthless, but true information is less beneficial. Indeed, if the competitor released true information, they together reach the same portion of followers as the monopolist, but they split this audience in two; if the competitor released false information, the duopolist gets the whole share of followers he reaches, but he reaches less followers than the competitor, given that he is read by less influencers.

When the signal is perfectly uninformative, the result is very intuitive: as the private signal is noisy, the agents are not able to tell true from false information, so that they treat both type of news without accounting for their private signal. Because the game is simultaneous, the producer does not internalize the effect of his investment on the consumers' prior, so that no investment is featured in equilibrium.

Numerically, it seems that if b or d are high, there exists a threshold $\gamma_t(b,d)$ such that duopoly is yielding a higher investment than monopoly for any private signal with precision $\gamma \in [\frac{1}{2}, \bar{\gamma}_t]$. Furthermore, as a rule of thumb, it appears that $\gamma_t(b,d)$ exists as soon as $bd \approx 4$, roughly. Further analysis is required.

4 Welfare

So far, I have only been interested in the market outcomes, as measured by investment. Welfare has not been addressed. While the welfare criteria are introduced below, let us start by noting that the market outcome is inefficient.

Proposition 5. *Any equilibrium outcome on the news market with revenues derived from ads is Pareto inefficient*

Proof. Take the case of a monopoly, with equilibrium $e^* = (x^*, z^*)$. Define $x^c(z; e^*)$ as the level of news quality that makes a consumer whose sharing decision is z indifferent between (x^c, z) and e^*. Likewise, define $x^p(z; e^*)$ as the level of news quality that insures to the producer faced with sharing decision z the same revenue as e^*. If $\frac{\partial x^c}{\partial z} < \frac{\partial x^p}{\partial z}$ [9], there is room for Pareto improvement since the consumer requires less investment to marginally increase their sharing than the producer is ready to offer for the same marginal increase in sharing. Now, using the FOC of equilibrium, we know that $0 \leq \frac{\partial x^c}{\partial z} < \infty$ while $\frac{\partial x^p}{\partial z} \to \infty$. The same reasoning applies to duopolists. □

[9] We abuse notation here in order to keep the intuition as clear as possible. While z is a vector, recall that, when x increases, the consumers would first share the most likely congruent news, then any congruent news, then the most likely news anyways, and then any news. Therefore, with ∂z, we mean to designate a marginal change in the sharing probability **in the relevant dimension**. So for instance if z=(1,0,0,0), ∂z is actually $\partial z_{1,1}$; if $z = (0.5, 0, 0, 0)$, then we mean $\partial z_{0,0}$.

To analyze the welfare resulting from this game, I propose two approaches. The first one relates to the *entertainment* purposes of possible sharing behavior. In this sense, only influencers and producers are part of the analysis. The influencers' decision to share an article depends on the utility of sharing as defined above. However, this does not capture how *informative* the article was. In particular, it does not allow us to judge whether agents are making, on average, better choices. To address this question, I use the expected benefits from the financial bet introduced above. This allows us to analyze whether, on average, agents are able to take better decision; as well as whether the information contained in articles published in online outlets that derive revenues from advertisement can motivate agents to take actions they would have opted out from, were they informed only privately. Furthermore, this measure of welfare can reflect followers' well-being as well.

4.1 Framework of Analysis

4.1.1 Sharing utility

As developed above, the expect utility from sharing news' content n after private signal s is $2p(n, s; x_u, w_0) - 1$. Therefore, the expected utility of an influencer reading an article from u and who plays strategy z_u is:

$$\begin{aligned}\mathbb{E}(u(z_u)) &= \sum_{(n,s)} z_{u|n,s}(2p(n, s; x_u, w_0) - 1)\Pr(n, s)\\ &= \sum_{(n,s)} z_{u|n,s}\Big[x_u \Pr(s|\omega = n)\Pr(\omega = n) - (1 - x_u)\Pr(s|\omega \neq n)\Pr(\omega \neq n)\Big]\end{aligned}$$

For any strategy for which the influencers shares at least sometimes, this expected utility is strictly increasing in x_u.

It naturally follows that the utility of a random influencer is thus $\sum_u \frac{1}{|U|}\mathbb{E}(u(z_u))$; while the *ex ante* utility of a random consumer is: $\sum_u \frac{b}{|U|}\mathbb{E}(u(z_u))$

4.1.2 Consumers' Bet

Conditional on participating to the bet, the consumers' payoff structure of resembles that of sharing. In particular, for influencers, the matching decision will follow the same threshold rule as the sharing decision: the influencer will bet what is expressed in the article if the probability for the true state of the world to correspond to the news is greater than 1/2. When $p(n, s; x_u, w_0) = 1/2$, the influencers are indifferent between betting the news' content or its opposite. In order to keep consistency, we assume the following tie rule[10]: $a_i(n, s) = n$ is played with probability $z_{u|n,s}$.[11] By a slight abuse of notation, we refer to this strategy as $z_{u|n,s}$

[10]Because when indifferent between several strategies, by definition, their utility is equal among all strategies, this assumption does not influence the welfare analysis.

[11]Formally, we define $\mathbb{E}(\mathbb{1}_{a_i=n|\eta=n,\varsigma=s}) = z_{u|n,s}$, so that $\mathbb{E}(\mathbb{1}_{a_i\neq n|\eta=n,\varsigma=s}) = 1 - z_{u|n,s}$.

Despite the similarity in strategies, there is a key difference with respect to sharing decision: the influencer's expected payoff with the bet is zero if and only if $p(n,s;x_u,w_0) = 1/2$, while it is null for any $p(n,s;x_u,w_0) \leq 1/2$ when the influencer is bounded to choose between sharing or the outside option 0. This difference influences the respective expected utilities and the role of x_u on it. Indeed, the expected utility from this betting strategy is:

$$\mathbb{E}(u_i(z_u)) \quad = \sum_{n,s}(2z_{u|n,s} - 1)\Big[x_u \Pr(s|\omega = n)\Pr(\omega = n) - (1 - x_u)\Pr(s|\omega \neq n)\Pr(\omega \neq n)\Big]$$

We now notice that this utility is not necessarily increasing in x_u. To understand this, we can notice that an outlet that would systematically report an erroneous content, i.e. $x_u = 0$, would lead to no sharing, and so no utility from it; while it would be perfectly informative as betting the opposite than the article argues would always ensure to correctly match the state of the world.

In addition, we need to define the betting decision of followers. All consumers share the same preferences and priors; however, in competitive markets, the precision of articles received by followers is higher than that of the outlets that issues them. Indeed, as true articles are shared more, they have a higher probability to reach a follower than false articles. Under the assumption that followers are sufficiently sophisticated to internalize the sharing behavior of influencers, they start betting what is expressed by an article they would read for lower precision than influencers. The decision rule can be defined implicitly as $z^f_{u|n,s} = 1$ when the expected utility from betting the news content is higher than that from betting the private signal.[12] Consistently with the action of influencers, we assume that followers play $a_f(n,s) = n$ with probability $z^f_{u|n,s}$.

Hence, conditional on receiving some news' article, the expected utility from the follower's strategy is:

$$\sum_{(u,n,m,s)} \left(2z^f_{u|n,s} - 1\right)\left[x_u \Pr(m,s,\omega = n)\frac{p_{u|n,n}}{p_{u|n,n}+p_{-u|n,m}} - (1 - x_u)\Pr(m,s,\omega \neq n)\frac{p_{u|-n,n}}{p_{u|-n,n}+p_{-u|-n,m}}\right]$$

where, as before, n is the content created by the producer seen u, while m denotes the outcome for all other producers $-u$.

When receiving no news' article, a follower, who trusts their private signal more than their prior by assumption, will simply bet according to their private signal. Hence, conditional on not receiving any news' article, the expected utility from the follower's strategy is simple $2\gamma - 1$. This happens with probability:

$$\Pr(f \text{ sees } \varnothing) = \sum_{w}\sum_{n,m}(1 - p_{u|w,n} - p_{-u|w,m})^d Pr(n|w)Pr(m|w)\Pr(w)$$

If agents cannot opt out of the bet, or if the bet has a free cost of entry $r = 0$, the expected

[12]That is:

$z^f_{u|n,s} = 1 \Leftrightarrow \sum_m x_u \Pr(m,s|\omega = n)\frac{p_{u|n,n}}{p_{u|n,n}+p_{-u|n,m}}\Pr(\omega = n) - (1 - x_u)\Pr(m,s|\omega \neq n)\frac{p_{u|-n,n}}{p_{u|-n,n}+p_{-u|-n,m}}\Pr(\omega \neq n) > 0$

benefits from the bet is defined for a random consumer as:

$$\mathbb{E}(u(a)) = \frac{b}{|U|}\sum_u \mathbb{E}(u_i(z_u)) + (1-b)(2\gamma-1)\Pr(f \text{ sees } \varnothing)$$
$$+ (1-b)\sum_{u,n,m,s}\left(2z^f_{u|n,s}-1\right)\Bigg[x_u \Pr(m,s|\omega=n)\tfrac{p_{u|n,n}}{p_{u|n,n}+p_{u|n,m}}\left[1-(1-p_{u|n,n}-p_{-u|n,m})^d\right]\Pr(\omega=n)$$
$$-(1-x_u)\Pr(m,s|\omega\neq n)\tfrac{p_{u|-n,n}}{p_{u|-n,n}+p_{-u|-n,m}}\left[1-(1-p_{u|-n,n}-p_{-u|-n,m})^d\right]\Pr(\omega\neq n)\Bigg]$$

To understand whether news drives agent to take a bet they would have otherwise opted out from, consider a cost of entry $r > 0$. Agents participate to the bet if $\mathbb{E}(u_j(a)) \geq r$, where $\mathbb{E}(u_j(a))$ incorporate a consumer's role, and whether they encounter an article. The net benefits are $\mathbb{E}(u_j(a)) - r$.

4.2 Welfare for symmetric priors

The framework proposed above makes welfare quantifiable for all agents. By design, the equilibrium outcome for any type of actor is bounded between 0 and 1. Therefore, these values are comparable without any need to resort to exogenous weighting factors.

In this section, I evaluate the welfare benefits from news production when $w_0 = 1/2$. and compare different market structure. Furthermore, I discuss the potential benefits from competition.

First, I study whether the presence of news articles brings welfare to consumers by increasing their expected gains from betting. This would allow consumers to take better decisions.

Proposition 6. *When there is no state of the world ex ante more likely:*

(i) Influencers are not brought to better decisions by news articles.

(ii) Followers take better decisions if and only if the market is competitive and $x^D > \frac{\gamma-\sqrt{\gamma(1-\gamma)}}{2\gamma-1}$. *The maximal gain from the bet is still bounded by the precision of their private signal.*

Proof. See Appendix C □

With $w_0 = 1/2$, the news quality in equilibrium is such that $x^* \leq \bar{t}$. Influencers are thus either better off betting their private signal, or indifferent between betting their signal or the news content. Therefore, news' outlets are not improving on their choice. In a competitive market however, followers might be better off following the news rather than their private signal. With competition, true news is more visible as the network filters out false articles. This raises the quality of the articles read by followers; if the news quality is sufficiently close to $\bar{t}$, followers take better decisions by trusting news' articles.

The welfare consequences of competition can now be assessed more carefully, by comparing consumers' expected gains from betting in a monopoly and a duopoly.

Theorem 2. *(Preliminary)* $x^D > x^M$ *is neither sufficient nor necessary for competition to be welfare improving.*

Proof. To appear in further versions □

When considering the total welfare effect of competition, both sides of the markets have to be considered. Influencers are not made better off and their quantity does not change in expectation. Followers might be better off if x^D is close enough to $\bar{t}$; however, the number of followers encountering an article might be affected by competition. As influencers share more types of news, more followers come across possibly informative news, but the least informative becomes the news, as the network fails to filter out wrong articles. Producers split their readership while the total production cost doubles. Therefore, the effect of competition on welfare depends on the level of news quality, the connectivity of the network and the ratio of influencers in the population.

Finally, I study whether news' outlets help consumers better decide whether to participate to the bet.

Proposition 7. *When there is no state of the world ex ante more likely, news' outlets have ambiguous welfare effects on the agents' capacity to decide to enter the bet.*

(i) if the entry cost for the bet is high, that is $r \in (r_s, \bar{r}]$*, news outlets help influencers decide whether to enter the bet;*

(ii) if the entry cost for the bet is low, that is $r \in [\underline{r}, r_s]$*, news outlets reduce influencers' capacity to decide to enter the bet.*

where $\underline{r} = 2\frac{\gamma(1-x)}{\gamma(1-x)+(1-\gamma)x} - 1; r_s = 2\gamma - 1; \bar{r} = 2\frac{\gamma x}{\gamma x+(1-\gamma)(1-x)} - 1.$
Similar thresholds exist for followers if the market is competitive and $x^D < \frac{\gamma-\sqrt{\gamma(1-\gamma)}}{2\gamma-1}$.

Proof. Again $w_0 = 1/2$. Let us compare the decision to enter the bet with and without news. Without news, all consumers take the same action: they opt out of the bet if $r > r_s$ and enter the bet for $r \leq r_s$. With news, influencers would opt out for $r > \bar{r}$, enter following any news with $r \leq \underline{r}$ and enter only for $n = s$ with $\underline{r} < r \leq \bar{r}$. Their behavior changes only in the interval $[\underline{r}, \bar{r}]$.

- For $r \in (r_s, \bar{r}]$, news articles push agents to enter the bet. All agents with $n = s$ place a bet. Given any state of the world, there are γx agents receiving $n = s$ corresponding to the right state of the world, i.e. who win the bet; and $(1-\gamma)(1-x)$ influencers who lose the bet. As $\gamma x > (1-\gamma)(1-x)$, there are more winners than losers.
- For $r \in [\underline{r}, r_s]$, news articles discourage agents to enter the bet. All agents with $n \neq s$ opt out. Given any state of the world, there are $(1-\gamma)x$ agents receiving $n \neq s$ who had the wrong private signal so who are better off opting out; and $\gamma(1-x)$ influencers who are worse off. As $\gamma(1-x) > (1-\gamma)x$, there are more losers than winners.

□

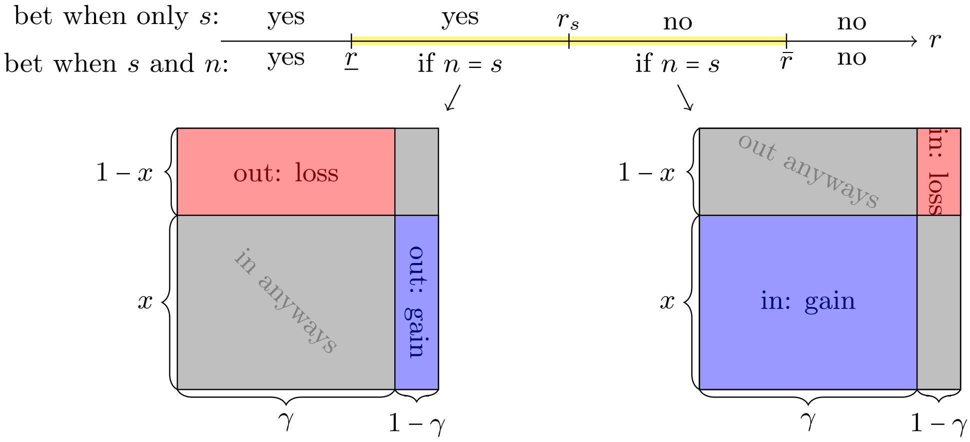

Figure 9: Illustration of Proposition 7's proof

4.3 Welfare for asymmetric priors

Corollary 2. *In uncompetitive markets, for any prior on the state of the world:*

(i) Consumers are brought to better decisions if $x^* > \gamma$.

(ii) The betting gains are still bounded by the precision of the private signal. In particular, $\mathbb{E}(u_j(a)) \in \left[2\gamma - 1; \frac{2\gamma-1}{1-2\gamma(1-\gamma)}\right]$

5 Evaluation of Interventions

In this section, I study three feasible interventions and their potential welfare benefits. In particular, I wonder about the effects of flagging false information before it is shared; and in particular its differential results in non-competitive and competitive markets. Furthermore, I question how much can quality certification improve the outlets' investment. Finally, I introduce a different business model in which producers rely exclusively on subscriptions for their revenues.

5.1 Flagging

I wonder here how flagging false information helps the provision of information on the market. In particular, let us assume that with some probability q, an information that does not correspond to the state of the world would be flagged by the platform on which influencers share before they decide whether to share. Because they care about truth only, such flagged information will never be shared. Hence, we can see flagging as perfectly informative signals, substituting the need for private signal. Therefore, one would expect this intervention to improve the outcome by decreasing the value of false information.

Remark 3. The presence of flagging removes the bound placed on news quality from the precision of private information.

Another interesting feature of flagging is that the marginal benefit of increasing the probability of being flagged depends on the market structure; in particular, when there is competition, there are strategic consideration to take into accounts. On one hand, an increased q makes false information relatively less valuable than true information; on the other hand, an increased q might make one's competitor more prone to being flagged, which in term decreases one's incentive to invest, as false information might be enough to survived faced to a flagged competitor.

To see this, let us rewrite the producers' best responses in a monopoly and duopoly when facing a probability q that false information is flagged. For the monopolist it is proportional to:

$$\Delta V^M(z;q) = V_T - (1-q)V_F$$

with $V_T = 1-(1-p_T)^d$ and $V_F = 1-(1-p_F)^d$.

For duopolists that behave symmetrically, it is proportional to:

$$\Delta V^D(z,x;q) = xV_{TT} + (1-x)\big[(1-q)V_{TF} + qV_{T\varnothing}\big] - x(1-q)V_{FT} - (1-x)(1-q)\big[(1-q)V_{FF} + qV_{F\varnothing}\big]$$

with $V_{TT} = \frac{p_T}{p_T+p_T}\big(1-(1-p_T)^d\big)$, $V_{TF} = \frac{p_T}{p_T+p_F}\big(1-(1-\frac{p_T+p_F}{2})^d\big)$, and $V_{T\varnothing} = 1-(1-\frac{p_T}{2})^d$; likewise $V_{FT} = \frac{p_F}{p_F+p_T}\big(1-(1-\frac{p_F+p_T}{2})^d\big)$, $V_{FF} = \frac{p_F}{p_F+p_F}\big(1-(1-p_F)^d\big)$, and $V_{T\varnothing} = 1-(1-\frac{p_F}{2})^d$

To analyze the tradeoff described above, we study how $\Delta V^M(z,x;q) - \Delta V^D(z,x;q)$ evolves with q. As expected, we find that flagging is more efficient in a monopoly.

Proposition 8. *(i) Flagging has a stronger effect in a monopoly than a duopoly*

(ii) When flagging never occurs, $q=0$, the tradeoff between monopoly and duopoly is modulated by d as described above. When flagging systematically occurs, $q=1$, monopoly yields higher investments than duopoly.

Proof. (i) In Appendix C we show that $\frac{\partial\left(\Delta V^M(z;q) - \Delta V^D(z,x;q)\right)}{\partial q} > 0$

(ii) It is enough to replace in the above expressions q with 0 and 1 respectively. With $q=1$, we find: $\Delta V^M(z;1) - \Delta V^D(z,x;1) = V_T - xV_{TT} - (1-x)V_{T\varnothing} > 0$.

□

These results echo that on the role of signal precision. In fact, when $q=1$ just as when $\gamma = 1$, false information is useless. Now, recall that competition can be positive in that it forces producers to survive in the network by convincing *enough* influencers to share, and this worsen the value of false information. But if false information is useless anyways, this effect vanishes. Only the negative effect of competition, that of reducing the number of viewers that can be captured by the duopolists in the best case scenario, remains.

However, beyond this similarity, flagging information allows to substitute consumers (often noisy) private signal. In this sense, it could force for news' outlets to provide news that goes beyond consumers private information, and thus create informative content. Of course, this conclusions ignore any type of partisanship or other distrust of flagging institutions.

5.2 Quality Certification

I now wonder how welfare could be improved upon if the consumers were observing the actual quality of information. In terms of policy, this could for instance correspond to the role of a third party institution in charge of certifying the average quality of a news source, or an average *fact checking* score to be displayed on the online outlet.

To understand the implication of such a policy, we need to assume a sequential move game. Appendix E presents the SPE of the monopoly when $w_0 = 1/2$. Because the outcome depends on the shape of the total cost function $C(x)$, our analysis in this section requires some stricter specification of the costs. Our strategy is to provide insights for $C(x)$ approximated by a third degree polynomial.

However, in a sequential move game, the influencers' best-responses would not change. Therefore, the threshold on news' quality for which they would share any type of information, regardless of their private signal, does not change. This threshold is also the maximum achievable quality in a sequential game, which is set by the consumers' private information.

Remark 4. Observable news' quality imposes the same bounds on outlets' informativeness.

Note however that, if news' quality improves, utility derived from the entertainment of sharing might improve as well.

It is interesting to notice that while both flagging and quality certification seem to rely on the same type of policy, and both relate to fact checking, their implications seem diametrically opposed.

5.3 Subscription-Based Revenues

Given the financial bet specified above, one could see the expected utility that consumers gain from more precise information as their willingness to pay for information. Therefore, a question naturally arises: can we do better if the revenues were not extracted from advertisement? In particular, would a business model that allows the producer to internalize the value of information for the consumer decrease inefficiencies? The obvious candidate is to allow the producers to charge consumers as a function of the news quality. I call this transfer *subscription*.

I have not yet fully characterize the conditions under which a news market with subscription based-revenue outperforms the case in which the producer profits are derived from the number of views. However, I propose the following setup. We move to a sequential game in which x is first decided by the producers, before being observed by influencers; furthermore, the producer proposes a price $t(x)$ to access to the news. Upon observing news quality x, influencers decide whether to access to the information at price $t(x)$. If they do, as before, they decide whether to share the news given their private signal.

In equilibrium, the marginal cost of investment for the producer equates the marginal value for the consumer; therefore, we can reach a Pareto optimal outcome in this framework. This comes at the cost of losing advertisement revenues. Assuming that such advertisement creates a surplus for the society, it is not clear which business model should be preferred. In particular, could the loss in advertisement revenue be compensated by an increased in efficiency? I thus wonder whether the total welfare in a news market working on subscriptions is greater than welfare with the ad-based outcome. This would imply that, under some redistribution between producers and consumers, the news market with subscription-based revenues is a Pareto improvement from the market with advertisement revenues. My preliminary results – to be presented here soon – seem to indicate that the total welfare in a monopoly with ad-based revenue can be achieved through a business model with subscriptions.

6 Conclusion

In this paper, I evaluate how good ad-based online news market can get. I find that, without any interventions, they tend to be highly inefficient. First, news quality on a topic is bounded by the amount of external knowledge existing on this topic. In particular, high news quality is achieved only when the topic documented is already well known: either because the outcome about this topic is rather certain; or because consumers are privately informed about it. This result from the incentive created by sharing behaviors. Indeed, all that matters to producer is the amount of sharing. But influencers rely on their knowledge to judge whether a content is worth sharing. Hence, convincing influencers to share is very easy when they are ill-informed. This further implies that producers' investment in news quality is more valuable when the more likely state of the world is realized, as influencers are more ready to share news documenting an expected state of the world then. Thus, a more uncertain topic would lead to lower investment that more certain issues for which news is hardly needed.

I additionally show that competition does not necessarily lead to better news quality. By comparing the outcomes of a monopoly and a duopoly, we conclude that neither market structure dominates the other. Rather, they complete each other: monopoly does well where duopoly fails. Overall, monopoly is preferable in less dense networks composed of agents with a good capacity to discern true from false information. This result puts into light important forces appearing with competition. In essence, the competition operates inside the network: by constraining the free spread of news through the network, the producers needs to convince enough, and not only a few, influencers. This reduces the value of false information, as it would barely survive in the network while competing with true information. On the one hand, competition reduces the value of true information, as it would not reach as many consumers even if it is shared enough. Because, for news, the cost of production is independent on the size of the market served, competition might be detrimental to news quality. This shows the limits of competition as a mean towards efficiency.

Finally, I show that the news market based on advertisement revenue is inefficient in the Pareto sense. I provide a framework to study welfare considerations. I find that online news market might create value from entertainment but are generally bad at being informative. I evaluate three possible interventions to improve upon informativeness. Flagging false information helps by reducing the value of false information. However, this intervention is less efficient in competitive markets, where strategic interactions between competitors have to be taken into account. Finally, I propose an alternative business model in which the producers' unique source of revenue is consumers' subscriptions. Although I do not characterize the conditions under which subscription-based outlets outperform markets in which the revenues are derived from ads, I believe this business model comparison to be tractable and compelling enough to open ways to more complete analysis.

Note that I introduce in the appendix an alternative objective for the influencers, and study its consequences. In particular, I assume that influencers only seek attention, not truth, which creates a trade-off between visibility and trustworthiness of the news they want to share. Because, qualitatively, the results in the monopoly are similar to those obtained with the naive influencers' objective function, we keep this extension in the appendix. However, the inherent tradeoff between attention and truthfulness is an interesting feature, which promises exciting ventures for this project.

I think of this analysis as attractive because it gives consumers an endogenous control over information flow but not on news content; furthermore, distortions that are inherent to a social network should be essential in underlining the differences between social media and other historical instances of ad-based business models for news. The central role of competition in this paper is reflected by its predominance in online outlets, as well as online networks. This allows my analysis to put in perspective the limits of ad-based online news market as a reliable source of information.

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Appendix

A. Asymmetric Loss From Sharing

A.1 Best Response

In this section of the appendix, we generalize the results derived in the main text for more general payoffs from sharing. In particular, while we restrict the benefit from sharing true news to 1, we consider a loss ℓ when false news is shared. The influencers' payoff thus becomes:

$$u(\text{sharing article with content } n|\omega = w) = \begin{cases} 1 & \text{if } n = w \\ -\ell & \text{otherwise} \end{cases}$$

This changes the influencers' best-response. In particular, it modifies the thresholds according to which they start sharing different news content after their private signal. We can redefine:

$$\underline{t}_0^\ell = \frac{\ell(1-\gamma)(1-w_0)}{\ell(1-\gamma)(1-w_0)+\gamma w_0} \qquad \bar{t}_0^\ell = \frac{\ell\gamma(1-w_0)}{\ell\gamma(1-w_0)+(1-\gamma)w_0}$$
$$\underline{t}_1^\ell = \frac{\ell(1-\gamma)w_0}{\ell(1-\gamma)w_0+\gamma(1-w_0)} \qquad \bar{t}_1^\ell = \frac{\ell\gamma w_0}{\ell\gamma w_0+(1-\gamma)(1-w_0)}$$

The producers' best response does not change.

A.2 Equilibrium without Competition

The Nash equilibria might change. In particular, because the thresholds can now be all above or all below the no investment quality 1/2, the two best responses might cross in many ways. We define $x_{\bar{0}0} := x^*(z_{\bar{0},0}, 0, 0, 0)$; $x_{\bar{1}1} = x^*(1, z_{\bar{1},1}, 0, 0)\}$ and $x_{\tilde{0}0} := x^*(1, 0, 0, 0)$; $x_{\tilde{1}1} := x^*(1, 1, 0, 0)$.

. ***Proposition 1.* A** *If either $1/2 \geq \bar{t}_1^\ell$; or both $x_{\bar{0}0} < \underline{t}_0^\ell$ and $x_{\bar{1}1} < \underline{t}_1^\ell$, then there is a unique set of equilibria with zero investment and $x^M = 1/2$. Otherwise, an equilibrium exists with positive investment, which is determined as follows:*

- $x^M = \max\{x_{\tilde{0}0}, \underline{t}_0^\ell\}$ *if* $x_{\bar{1}1} < \underline{t}_1^\ell$,
- $x^M = \max\{x_{\tilde{1}1}, \underline{t}_1^\ell\}\}$ *if* $\underline{t}_1^\ell \leq x_{\bar{1}1}$ *and* $x_{\tilde{1}1} \leq \bar{t}_0^\ell$
- $x^M = \max\{\bar{t}_0^\ell, \min\{x^*(1, 1, 1, 0), \bar{t}_1^\ell\}\}$ *otherwise.*

Proof. First notice that any positive equilibrium investment has to lie within $[\underline{t}_0^\ell, \bar{t}_1^\ell]$. Indeed, it is easy to see that for any $x < \underline{t}_0^\ell$, no news is ever shared so that the producer has no incentive to invest; likewise, $x = \bar{t}_1^\ell$ is enough to insure that the producer's news is always share, so that investing more than this does not increase the producer's benefit.
It follows that, if $1/2 \geq \bar{t}_1^\ell$, the producer will never want to invest more than 1/2 – intuitively, the

producer's best response lies above the influencers' best response. If $\bar{x_{00}} < \underline{t}^\ell_0$ and $\bar{x_{11}} < \underline{t}\ell_1$, it is too costly for the producer to invest more than 1/2, as for any sharing strategy z, the marginal benefit from investing $x > 1/2$ is lower than its marginal cost – intuitively, the producer's best response lies below the influencers' best response. Indeed, we know that $x < \underline{t}^\ell_0$ cannot be an equilibrium. For any $x \in [\underline{t}^\ell_0, \underline{t}\ell_1)$, by definition $\bar{x_{00}} = c^{-1}(\Delta V(\bar{z_{0,0}}, 0, 0, 0)) \geq c(\Delta V(z^*(x))$ so that $c(x) \geq c(\underline{t}^\ell_0) > c(\bar{x_{00}}) \geq c(\Delta V(z^*(x))$. Likewise, for $x \in [\underline{t}\ell_1, \bar{t}^\ell_0)$, as $\bar{x_{11}} < \underline{t}\ell_1$, we have $c(x) \geq c(\underline{t}\ell_1) > c(\bar{x_{11}}) \geq c(\Delta V(z^*(x))$. For any $x \geq \bar{t}^\ell_0$, $c(x) \geq c(\bar{t}^\ell_0) > c(\max\{\bar{x_{00}}, \bar{x_{11}}\}) \geq c(\Delta V(z^*(x))$
Now, let us understand what happens if positive investment is possible. If $\bar{x_{11}} < \underline{t}\ell_1$, as argued above, the investment has to be such that $x \in [\underline{t}^\ell_0, \underline{t}\ell_1)$. Because $\bar{x_{00}} > \underline{t}^\ell_0$, and $x^*(z)$ continuous, there there must exist some z^*_{00} such that $c^{-1}(\Delta V(z^*_{00}, 0, 0, 0)) = \underline{t}^\ell_0$. If $\tilde{x_{00}} < \underline{t}^\ell_0$, the maximal investment equilibrium is thus $\underline{t}^\ell_0$; otherwise, $\tilde{x_{00}}$ is an equilibrium as $\tilde{x_{00}} \in [\underline{t}^\ell_0, \underline{t}\ell_1)$ and by definition, $c(\tilde{x_{00}}) = \Delta V(1, 0, 0, 0)$, and leads to more investment. A similar reasoning applies to $\bar{x_{11}} \geq \underline{t}\ell_1$ and $\tilde{x_{11}} \leq \bar{t}^\ell_0$.
Finally, if $\bar{x_{11}} \geq \underline{t}\ell_1$ and $\tilde{x_{11}} > \bar{t}^\ell_0$, because $x^*(z)$ is decreasing in z_{01} and z_{10}, and contiunous, there must exist a $x' \geq \bar{t}^\ell_0$ and a $z' = (1, 1, z_{01}, z_{10})$ such that $c(x') = \Delta V(z')$. It is easy to verify that $\max\{\bar{t}^\ell_0, \min\{x^*(1, 1, 1, 0), \bar{t}^\ell_1\}\}$ yield the highest x on $[\underline{t}^\ell_0, \underline{t}\ell_1]$ such that $c(x') = \Delta V(z')$. □

The consequences on the comparative statistics are overall the same. We can note:

. ***Corollary 1.A*** *Take any increase in w_0.*

- *For a marginal increase, the inequalities detailed in Proposition 1.A do not change, so that the maximal equilibrium investment x^M increases iff $x^M \neq t_{00}$ and $x^M \neq t_{01}$*
- *For bigger increases, the maximal equilibrium investment x^M increases iff $x^M \neq t_{00}$, $x^M \neq t_{01}$ and c^{-1} is steep enough, i.e. c^{-1} is such that, for any $w'_0 > w_0$, $\bar{x_{11}} > t_{11}$ implies $\bar{x_{11}}' > t'_{11}$.*

Proof. See Appendix C □

A.2 Equilibrium with Competition

We now must define more possible cases for the equilibrium with competition. We define $\bar{x}^m_v := \max_{z_{vT}} x^*_v((z_{vT}, 0),)$ and $\tilde{x}^m = x^*((1, 0), (0, 0); 0)$

. Additional Equilibria *(i) There always exist a set of equilibria with zero investment $x^*_v \in [0, 1/2] \forall v \in K$.*
(ii) If $1/2 < \bar{t}^\ell$ and $\min_v \bar{x}_v \geq (\underline{t}^\ell)$, there exists a set of equilibria in which exactly one producer invests $x^m = \max\{\underline{t}^\ell), \min\{\tilde{x}^m, \bar{t}^\ell\}$, and the other does not invest.

Proof.

(i) $x^*_v(((0, 0), z_{-v}), x_{-v}) \in [0, \min\{1/2, \underline{t}^\ell\}]$ and $z_{vT}(x_v) = z_{vF}(x_v) = 0$ for $x_v \in [0, \min\{1/2, \underline{t}^\ell\}]$.

(ii) Notice that for $p_{-vX} = x_{-v} = 0$, $\Delta V_v(z, x_{-v}) = (1-b)\left[(1-1/2b\underline{t}^\ell)^d - (1-1/2b\bar{t}^\ell)^d\right]$, which corresponds to the monopoly case up to 1/2, which is accounted for when defining $\bar{x}_v$. Furthermore, it is a best response for $-v$ to not invest if $z_{-v} = (0,0)$, which is a best response if $x_v \in [0, \min\{1/2, \underline{t}^\ell\}]$.

□

Define $\bar{z}_T^D := \arg\max_{z_T} \left\{\Delta V\big((z_T, 0), \underline{t}^\ell\big)\right\}$ and $\bar{x}^D = \Delta V(\bar{z}_T^D, \underline{t}^\ell)$.

. *Proposition 3. A* *If* $1/2 < \bar{t}^\ell$ *and* $\underline{t}^\ell \le \bar{x}^D$*, there exists a symmetric equilibrium that features positive investment and* $x^D = \arg\min_{x \in [\underline{t}^\ell, \bar{t}^\ell]} \left|\Delta V^D\big((1,0); x\big) - c(x)\right|$.

Proof. First note that any equilibrium investment bigger than 1/2 has to lie in $[\underline{t}^\ell, \bar{t}^\ell]$. Indeed, recall that $\Delta V^D((0,0), x) = \Delta V^D((1,1), x) = 0$. Hence, clearly, for any $x < \min\{\underline{t}^\ell, \bar{x}\}$ or $x > \max\{\bar{t}^\ell, 1/2\}$, $c(x) > 0 = \Delta V^D(z^*(x), x)$, which would be suboptimal for the producer.

Given $1/2 < \bar{t}^\ell$ and $\underline{t}^\ell \le \bar{x}^D$, different parameters allow for three cases:

1. If $c(\underline{t}^\ell) < \Delta V^D((1,0), \underline{t}^\ell)$ and $\Delta V^D((1,0), \bar{t}^\ell) < c(\bar{t}^\ell)$, then $\exists \tilde{x} \in [\underline{t}^\ell, \bar{t}^\ell]$: $c(\tilde{x}) = \Delta V((1,0), \tilde{x})$. Indeed, recall that c is weakly increasing in x and $\Delta V^D((1,0), x)$ strictly decreasing in x. Clearly, $(\tilde{x}, (1,0))$ is a NE.
   It is the symmetric NE which leads to the highest investment. Indeed, assume there exists another symmetric equilibrium with investment $x' > x^D$. As argued above, $x' \in \{\underline{t}^\ell, \bar{t}^\ell\}$.
   For $x' = \bar{t}^\ell > x^D$ to be part of an equilibrium, there must exist a $z' = (1, z'_F)$ with $z'_F > 0$ such that $V^D(z', x') = c(\bar{t}^\ell)$. It is impossible, because $c(\bar{t}^\ell) > c(x^D) = \Delta V^D((1,0), x^D) > \Delta V^D((1,0), \bar{t}^\ell) > \Delta V^D((1, z_F), \bar{t}^\ell)$ $\forall z_F > 0$, where the last inequality uses that $\Delta V^D(z; x)$ is decreasing in z_F.

2. If $c(\underline{t}^\ell) > \Delta V^D((1,0), \underline{t}^\ell)$, then $\exists \tilde{z}_T \in [\bar{z}_T^D, 1]$: $c(\underline{t}^\ell) = \Delta V(\tilde{z}_T, \underline{t}^\ell)$. Indeed, by assumption $\Delta V^D((\bar{z}_T^D, 0); \underline{t}^\ell) > c(\underline{t}^\ell) > \Delta V^D((1,0); \underline{t}^\ell)$, and $\Delta V^D(z; x)$ is continuous and decreasing on $[\bar{z}_T^D, 1]$. Clearly, $(\underline{t}^\ell, (\tilde{z}_T, 0))$ is a NE.

3. If $c(\bar{t}^\ell) < \Delta V^D((1,0), \bar{t}^\ell)$, then $\exists \tilde{z}_F \in [0,1]$: $c(\bar{t}^\ell) = \Delta V((1, \tilde{z}_F), \underline{t}^\ell)$. Indeed, by assumption $\Delta V^D((0,0); \bar{t}^\ell) = 0 < c(\bar{t}^\ell) < \Delta V^D((1,0); \underline{t}^\ell)$, and $\Delta V^D(z; x)$ is continuous and decreasing in z_F. Clearly, $(\bar{t}^\ell, (1, \tilde{z}_F))$ is a NE.

□

Furthermore, we can distinguish equilibria with respect to their stability.

. *Additional Corollary* x^D *is the only stable equilibrium with symmetric positive investment.*

Proof. First if $1/2 > \underline{t}^\ell$, $\Delta V^D(z, \underline{t}^\ell) > \underline{t}^\ell$ for any z. Because $\Delta V^D(\bar{z}_T, x)$ is continuous and increasing on $[0, \bar{z}_T]$, we know that, given any x, $c^{-1}(\Delta V^D(z, \underline{t}^\ell))$ crosses $z_v^*(x)$ only once. So for any x_0, there is a unique $x', z^*(x')$. We pick the x_0 that leads to equilibrium $x_0, z^*(x_0)$, which must be unique.

If $\bar{x}^D \geq \underline{t}^\ell > 1/2$, then given any $x = \underline{t}^\ell$, $c^{-1}(\Delta V^D(z,x))$ crosses $z_v^*(x)$ twice: once for some $z'_T < \bar{z}_T$ with $\Delta V^D(z'_T, x)) = c(\underline{t}^\ell)$; and once afterwards. The slope of $\Delta V^D(z,x))$ in $z'_T < \bar{z}_T < 1$ is strictly increasing. The investment required for influencers to share upon receiving congruent private signal with probability z'_T is equal to $\underline{t}^\ell$ with slope 0. Therefore, the equilibrium $(\underline{t}^\ell, (z'_T, 0))$ cannot be stable. In particular, any stable equilibrium must have $z_u, z_v > z'_T$.

Finally, we prove that x^D is the only stable equilibrium with symmetric investment by noting that $x_u = x_v = x^*$ implies $z_u = z_v > z'_T$. Indeed, any equilibrium investment x^* requires $\Delta V_u(z_u, z_v; x^*) = \Delta V_v(z_v, z_u; x^*)$. Now, because $\frac{\partial \Delta V_u}{\partial z_u} \neq -\frac{\partial \Delta V_u}{\partial z_v}$ for every $z_u, z_v > z'_T$, the unique z_u, z_v supporting x^D must be defined by $\Delta V(z,x)$; therefore, $z_u = z_v$. □

Finally, we wonder about other asymmetric equilibria and find:

. ***Remark 1.A*** *(i) If $\bar{x}^m < \underline{t}^\ell$, the unique equilibrium is that featuring no investment. (ii) If $\bar{x}^D < \underline{t}^\ell \leq \bar{x}^m$, the only equilibria with positive investment have one producer investing x^m while the other does not invest. (iii) If $\bar{x}^D \geq \underline{t}^\ell = x^m$, the only equilibria with positive investment for both producers feature $x^D = \underline{t}^\ell$. (iv) If the cost function is linear, there are no equilibrium with $x_u \neq x_v$ and $(x_u, x_v) \in (\underline{t}^\ell, \bar{t}^\ell)$ as long as $c(x)$'s slope is different from S.*

Proof. (i) $\bar{x}^m < \underline{t}^\ell$ means that, even if the network is free of competition, there is no sharing rule that could convince a producer to invest. Therefore, no investment can occur.

(ii) $\bar{x}^D < \underline{t}^\ell$ means that there does not exist a symmetric equilibrium with positive investment, i.e. $\forall z_u, \underline{t}^\ell > \Delta V_u(z_u, z_u; \underline{t}^\ell)$. Furthermore, no other equilibrium with positive investment can exist. By using Corollary **??**'s proof, $z_u = z_v$ if $x_u = x_v$, so $z_u \neq z_v$ is inconsistent with $x_u < x_v$. Finally, $z_u < z_v$ and $x_u = \underline{t}^\ell < x_v$ cannot be an equilibrium as $\underline{t}^\ell > \Delta V_u(z_u, z_u; \underline{t}^\ell) > \Delta V_u(z_u, z_v; x_v)$ for $z_v > z_u$ $x_v > \bar{t}^\ell$.

(iii) $\underline{t}^\ell = x^m$ implies that $\Delta V_u(z_u, z_v; x_v) < \Delta V_u(z_u, (0,0); 0) < \underline{t}^\ell$ for any z_u. Hence x_u cannot exceed $\underline{t}^\ell$. As the same applies to x_v, both producers must be investing the minimum $\underline{t}^\ell$ if they do invest. Furthermore, $x_u = x_v$ implies $z_u = z_v$. (iv) Assume $x^D \in (\underline{t}^\ell, \bar{t}^\ell)$. Assume that there exists an $x_u > x_v$, with $(x_u, x_v) \in (\underline{t}^\ell, \bar{t}^\ell)$. Then, $c(x_u) = \Delta V_u((1,0),(1,0),x_v)$ and $c(x_v) = \Delta V_v((1,0),(1,0),x_u)$, so that $c(x_u) - c(x_v) = S(x_u - x_v)$, which is impossible if c has a slope different from S.

If $x^D \in \{\underline{t}^\ell, \bar{t}^\ell\}$, then for any x_v, $c(x_u) \neq \Delta V_u((1,0),(1,0),x_v)$ so there cannot be any equilibrium where both producer invest away from the minimum. □

Because the following comparison between monopoly and duopoly focuses on the producers' best-responses, all results follow through.

B. Attention-Seeking Influencers

In this part of the appendix, we explore an extension of the benchmark monopolisitic model with w_0, $z_{0,0} = z_{1,1} = z_T$ and $z_{0,1} = z_{1,0} = z_F$. We try to understand the effect of attention-seeking influencers on the producer incentives. In particular, we assume that influencers do not intrinsically care whether the news they share is true or false; but they do care about receiving good feedback about it, e.g. a lot of *likes*. Hence, there is a potential trade-off between visibility and veracity of a news. For the rest of this section, we assume that all influencers are attention-seekers.

B.1 The Attention-Seeker Problem

We assume that influencers, contrary to producers, cannot observe the actual number of followers they reach; however, they can observe how many followers reacted to their shared post, as, typically, social media feature some sort of feedbacks, be it comments, likes, or reshares. We focus on positive reactions, that we call *likes*, and assume that followers like a post if they receive a private signal consistent with it. In the context exposed previously, it means that followers receive a binary signal indicating whether the information is true or false, and like only if they receive a positive signal – regardless of the prior probability for news to be true.

As before, influencers simultaneously choose whether to share the piece of news issued by v, given their private signal s_i. Influencers decide to share if the amount of likes they expect to collect with their post exceeds a threshold $\tau \leq d$. It can be interpreted as the value of an outside option – e.g. posting another type of article would yield τ likes – or, simply, the cost of sharing.

For consistency, we still denote R_{fi} the random variable which is one if f sees the post from i. As before, a follower sees only one post. If more than one neighbor shared a post, the follower sees the post from one random sharing neighbor, with uniform probability, that is:

$$Pr(R_{fi} = 1 |\ s \text{ neighbors of } f \text{ shared}) = \frac{1}{s}$$

where s is the outcome of the random variable S counting the number of f's neighbors who shared.

Define the random variable L_{fi} which is one if f likes the post shared by i. Recall that s_f is the random private signal that a follower receives. Then:

$$Pr(L_{fi} = 1) = Pr(L_{fi} = 1 | R_{fi} = 1) Pr(R_{fi} = 1) = Pr(s_f = T) Pr(R_{fi} = 1)$$

An influencer expects a different amount of likes for true and false information because, if read, true news gets more likes than false information. The expected number of likes also depends on the visibility of the news, which in turn depends on the sharing decisions of all neighbors of each followers. Define n as the random variable counting the number of shares from f's neighbors, excluding i. The expected number of likes i gets from sharing a piece of information which is $X \in \{T, F\}$ is thus:

$$\mathbb{E}\left(\sum_{f \in \mathcal{N}_i} L_{fi} = 1|X\right) = dPr(f \text{ is a follower})Pr(s_f = T|X)\mathbb{E}\left(\frac{1}{S+1}|X\right)$$

Now recall, upon reading a piece of news, influencer i, too, gets a private signal about the truthfulness of the news, whose precision is γ. As before, all influencers have a common prior x_v about the probability for producer v to release true information. Let $p_v(s_i; x_v)$ denote i's posterior upon receiving signal s_i. Then, an influencer decides to share a piece of information if and only if:

$$p_v(s_i; x_v)d(1-b)\gamma\mathbb{E}\left(\frac{1}{S+1}|T\right) + (1 - p_v(s_i; x_v))d(1-b)(1-\gamma)\mathbb{E}\left(\frac{1}{S+1}|F\right) \geq \tau$$

Notice that the influencers' utility now depends on more than the producers' investment; it also depends on the behavior of other influencers. In particular, because influencers compete for likes, which occur only upon being seen, they would prefer a situation in which they are the only sharer. If true information is shared more, then this coordination concern would make them less prone to share true news; however, true information also brings more likes. Thus, there is a trade-off between visibility and veracity.

B.2 Influencer Best Response in a Monopoly

In this section, we focus on symmetric strategies $z_i = z \ \forall i$ and, by a slight misuse of language, we call best-response the pair of functions $(z_T^*(x), z_F^*(x))$ which maps x into $[0,1]$ such that $z^*(x, \mathbf{z}^*(\mathbf{x})) = z^*(x)$.[13] Hence, given any investment x, we look at the subset of strategies which can be consistent with a symmetric equilibrium on the influencers' side.

As usual, p_X denotes the probability that a news that is X gets shared. Then, $n \sim \mathcal{B}(p_X, d-1)$ We can rewrite:

$$\mathbb{E}\left(\sum_{f \in \mathcal{N}_i} L_{fi} = 1|X\right) = d(1-b)Pr(s_f = T|X)\frac{1}{dp_X}\left(1 - (1-p_X)^d\right)$$

Thus, the expected number of like is:

$$p_v(s_i; x_v)\gamma\frac{1-b}{p_T}\left(1 - (1-p_T)^d\right) + (1 - p_v(s_i; x_v))(1-\gamma)\frac{1-b}{p_F}\left(1 - (1-p_F)^d\right)$$

Proposition 9. *For any x, $z_T^*(x) \geq z_F^*(x)$.*

Proof. By contradiction, suppose that $z_F^*(x) > z_T^*(x)$, so that $p_F > p_T$. For this to be sustainable,

[13] Technically, each influencer's best response would be a pair of $(I+1)$-dimensional function, that each maps x and $\mathbf{z_{-i}}$ into $[0,1]$, with I the random variable counting the number of influencers, and whose expectation is bN.

we need $\mathbb{E}(\#\text{likes}|s_i = T) \leq \tau \leq \mathbb{E}(\#\text{likes}|s_i = F)$. However, this happens only when:

$$\frac{\gamma}{1-\gamma} < \frac{p_T}{(1-(1-p_T)^d)} \frac{(1-(1-p_F)^d)}{p_F}$$

Because $\frac{\gamma}{1-\gamma} > 1$, we need $\frac{p_T}{(1-(1-p_T)^d)} \frac{(1-(1-p_F)^d)}{p_F} > 1$. Because $f(x) = \frac{x}{(1-(1-x)^d)}$ is an increasing function, it means $p_T > p_F$, a contradiction. □

Corollary 3. *$\mathbb{E}(\ \#\ likes\ |X)$ is increasing in $p_v(s_i;x)$, for any $s_i \in \{T,F\}, X \in \{T,F\}$.*

Proof. It is enough to notice that, since $p_T > p_F$:

$$\frac{p_T}{(1-(1-p_T)^d)} \frac{(1-(1-p_F)^d)}{p_F} > 1$$

so that the coefficient of $p_v(s_v;x)$ is positive. □

Theorem 3.

(i) For any $\tau \leq \gamma\delta$, $z_T^*(x;\tau) = z_F^*(x;\tau) = 1$ if and only if $x \geq \hat{x}(\tau)$.

(ii) For any $\tau \geq (1-\gamma)d(1-b)$, $z_T^*(x;\tau) = z_F^*(x;\tau) = 0$ if and only if $x \leq \underset{\sim}{x}(\tau)$.

(iii) For any $\tau \in [\tau_1,\tau_2]$, $z_T^*(x;\tau) = 1, z_F^*(x;\tau) = 0$ if only if $x \in [x_1(\tau), x_2(\tau)]$.

Where:

$$\delta(b) = \frac{1-b}{b}[1-(1-b)^d], \quad \tau_1(b) = \frac{1-b}{b}[1-(1-b(1-\gamma))^d], \quad \tau_2(b) = \frac{1-b}{b}[1-(1-b\gamma)^d]$$

And, given $T = \frac{\frac{b\tau}{1-b} - 1 + (1-b(1-\gamma))^d}{(1-b(1-\gamma))^d - (1-b\gamma)^d}$,

$$\hat{x}(\tau) = \frac{\gamma}{2\gamma-1}\frac{\tau-(1-\gamma)\delta}{\tau}, \ \underset{\sim}{x}(\tau) = \frac{1-\gamma}{2\gamma-1}\frac{\tau-(1-\gamma)d(1-b)}{d(1-b)-\tau}, \ x_1(\tau) = \frac{(1-\gamma)T}{(1-\gamma)T+\gamma(1-T)}, \ x_2(\tau) = \frac{\gamma T}{\gamma T+(1-\gamma)(1-T)}$$

Proof. The details can be found in Appendix C.

(i) Given $\tau \leq \gamma\delta$, if $x \geq \hat{x}(\tau)$, it is easy to verify that always sharing is a best response, i.e. $\mathbb{E}(\#\text{ likes }|T, \mathbf{z_{-i}} = (\mathbf{1},\mathbf{1})) > \mathbb{E}(\#\text{ likes }|F, \mathbf{z_{-i}} = (\mathbf{1},\mathbf{1})) \geq \tau$. For proving the converse, recall that $\frac{1-(1-p)^d}{p}$ is decreasing in p. Suppose there exists another $p' < b$ that is sustained in equilibrium. Then, $\mathbb{E}(\#\text{ likes }|F, p' < b) > \mathbb{E}(\#\text{ likes }|F, p = b) \geq \tau$ so that i would have an incentive to deviate towards $p_i = 1$.

(ii) Likewise, given $\tau \geq (1-\gamma)d(1-b)$, if $x \leq \underset{\sim}{x}(\tau)$, then even $d(1-b)[p(T,x)\gamma + (1-p(T,x))(1-\gamma)]$ likes are not enough for anyone to share, so that $(0,0)$ is a the best response to any p given x and τ.

(iii) Again, we can simply verify that, given $\tau \in [\tau_1,\tau_2]$, if $x \in [x_1(\tau), x_2(\tau)]$ and every $-i$ influencer is sharing only when they receive a positive signal, $\mathbb{E}(\#\text{ likes }|T) \geq \tau \geq \mathbb{E}(\#\text{ likes }|F)$. Any $z_{-i,F} > 0$ would lower the $\mathbb{E}(\#\text{ likes }|F)$ further away from τ, making i set $z_{i,F} = 0$; any $z_{-i,T} < 1$ would increase the $\mathbb{E}(\#\text{ likes }|T)$ further away from τ, making i set $z_{i,T} = 1$.

□

Corollary 4.

(i) For any $\tau \leq \gamma\delta$, if $x \geq \hat{x}(\tau)$, $z_T(x, \mathbf{z_{-i}}; \tau) = z_F(x, \mathbf{z_{-i}}; \tau) = 1$ is the only best response for any (non symmetric) vector of influencers $-i \neq i$'s actions.

(ii) For any $\tau \geq (1-\gamma)d$, if $x \leq \underset{\sim}{x}(\tau)$, $z_T(x, \mathbf{z_{-i}}; \tau) = z_F(x, \mathbf{z_{-i}}; \tau) = 0$ is the only best response for any (non symmetric) vector of influencers $-i \neq i$'s actions.

Proof. Again, it is enough to recall that the number of likes is decreasing in the probability for another influencer to share □

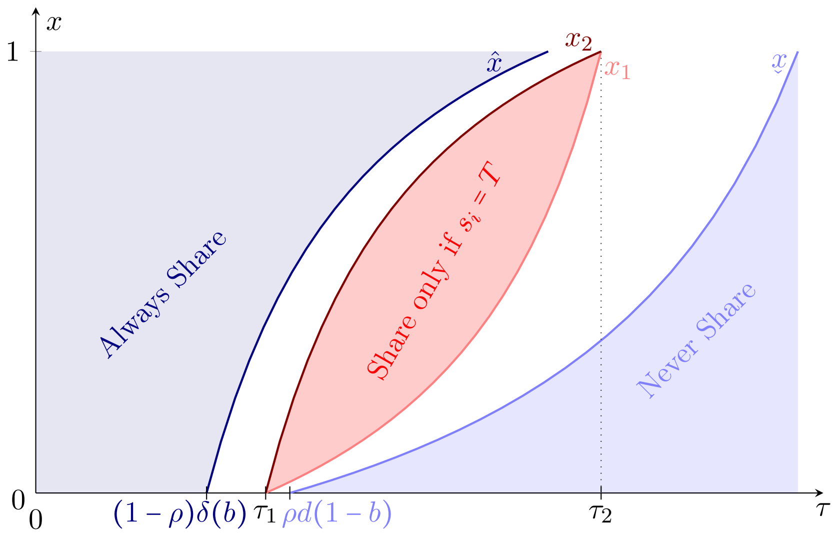

Figure 10: Illustration of $z^*_{ps}(x;\tau)$ with $b = 0.2, \gamma = 0.75, d = 5$

Corollary 5. *Define z_{ps} as the restriction of z to pure strategies. For any (x, τ), $z^*_{ps}(x;\tau)$ either does not exist or is unique.*

Proof. Consider the parameter space (τ, x). Theorem 3 describes three subsets of best-responses that do not intersect. No other pure strategies is sustainable, as, by proposition 9, $(0, 1)$ is never a best-response. □

Figure 10 illustrates the different region of pure strategy best-responses in space (τ, x). First, one can notice that for some values of τ, the investment of the producer has no effect on the sharing decision of influencers. If τ is *too* low, influencers are not very demanding in terms of likes, so that they are always willing to share. If τ is *too* high, influencers are too demanding in terms of likes, and they never share any information.

For intermediate values of τ, however, the symmetric best-response of influencers is fairly similar to that studied in the benchmark model. To understand so, let us fix a particular value for τ; we want to understand z^* as a function of x. This means fixing one value of τ on Figure 10 and translating the different areas in term of z. This results in Figure 11, which illustrates the

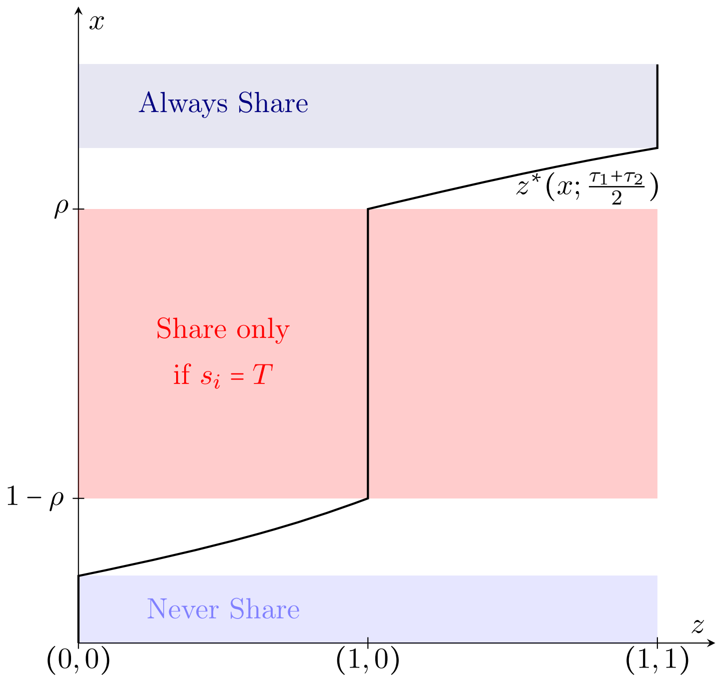

Figure 11: Illustration of $z^*(x;\tau)$ in $\tau = \frac{\tau_1+\tau_2}{2}$ with $b = 0.2, \gamma = 0.75, d = 5$

symmetric best-response $z^*(x)$ for $\tau = \frac{\tau_1+\tau_2}{2}$. Notice that, for this particular τ, $x_1 = 1-\gamma$ and $x_2 = \gamma$. It means that, for x between $1-\gamma$ and γ, the symmetric best-response of attention seeking influencers exactly corresponds to that of naive influencers in the benchmark model.

However, for $x \notin [1-\gamma, \gamma]$, attention-seekers' best response changes. Say the producer invests exactly $1-\gamma$. In the benchmark model, upon receiving a positive private signal, influencers were indifferent between sharing or not, as the probability the news was true in such a case was exactly one half. But now, attention seekers' strategies are substitutes; therefore, upon receiving a positive private signal, they can be indifferent between sharing or not only for one particular sharing strategies of the other influencers. This latter strategy is the unique only symmetric best-response to x. For $\underset{\sim}{x}\left(\frac{\tau_1+\tau_2}{2}\right) < x < 1-\gamma$, z_T^* is strictly increasing in x[14]; for $\hat{x}\left(\frac{\tau_1+\tau_2}{2}\right) > x > \gamma$, z_F^* is strictly increasing in x [15]

Because, in a monopoly, the best-response of attention-seeking influencer is fairly similar to that of naive influencers for the right value of τ, similar equilibrium results are to be expecting. Of course, studying attention-seeking objective in a duopoly should prove more insightful.

[14] z_T^* is implicitly determined by:

$$\frac{\gamma x}{(1-\gamma)(1-x)} = -\frac{\frac{\tau_1+\tau_2}{2}\frac{b}{1-b} - \frac{1-(1-b(1-\gamma)z_T)^d}{z_T}}{\frac{\tau_1+\tau_2}{2}\frac{b}{1-b} - \frac{1-(1-b\gamma z_T)^d}{z_T}}$$

[15] z_F^* is implicitly determined by:

$$\frac{(1-\gamma)x}{\gamma(1-x)} = -\frac{\frac{\tau_1+\tau_2}{2}\frac{b}{1-b} - \frac{1-(1-b(1-\gamma)-b\gamma z_F)^d}{1+\frac{\gamma}{1-\gamma}z_F}}{\frac{\tau_1+\tau_2}{2}\frac{b}{1-b} - \frac{1-(1-b\gamma-b(1-\gamma)z_F)^d}{1+\frac{1-\gamma}{\gamma}z_F}}$$

C. Proofs and computations

2.3.2 The Producers' Best Responses

Multinomial: the Distribution of an Outcome conditional on the Sum of a Subset of Outcomes

Consider a random vector $X \sim Multi(n,p)$ of dimension k. By definition, we have:

$$Pr(X_1 = a, X_2 = b) = {p_1}^a {p_2}^b (1 - p_1 - p_2)^{n-a-b} \frac{n!}{a!b!(n-a-b)!}$$

Now, because each trial is independent, we have that $X_1 + X_2 \sim \mathcal{B}(p_1 + p_2, n)$. Hence:

$$Pr(X_1 + X_2 = s) = (p_1 + p_2)^s (1 - p_1 - p_2)^{n-s} \frac{n!}{s!(n-s)!}$$

Therefore, we find the following conditional distribution:

$$\begin{aligned} Pr(X_1 = a | X_1 + X_2 = s) &= \frac{Pr(X_1 = a, X_2 = s - a)}{Pr(X_1 + X_2 = s)} \\ &= \frac{{p_1}^a {p_2}^{s-a} (1 - p_1 - p_2)^{n-s} \frac{n!}{a!(s-a)!(n-s)!}}{(p_1 + p_2)^s (1 - p_1 - p_2)^{n-s} \frac{n!}{s!(n-s)!}} = \frac{{p_1}^a {p_2}^{s-a}}{(p_1 + p_2)^s} \frac{s!}{a!(s-a)!} \end{aligned}$$

Note that it can be rewritten as:

$$Pr(X_1 = a | X_1 + X_2 = s) = \frac{{p_1}^a {p_2}^{s-a}}{(p_1 + p_2)^{s-a+a}} \frac{s!}{a!(s-a)!} = \left(\frac{p_1}{p_1 + p_2}\right)^a \left(\frac{p_2}{p_1 + p_2}\right)^{s-a} \frac{s!}{a!(s-a)!}$$

Hence, the conditional random variable $X_1|_{X_1+X_2} \sim \mathcal{B}\left(n, \frac{p_1}{p_1+p_2}\right)$.

The Probability of Being Read by a Follower (as a Producer)

First, note that:

$$\begin{aligned} Pr(\text{follower sees } v) &= \sum_{s=0}^{d} Pr(\text{follower sees } v \text{ and } s \text{ neighbors shared}) \\ &= \sum_{s=0}^{d} Pr(\text{follower sees } v \mid s \text{ shares}) Pr(s \text{ shares}) \end{aligned}$$

Now, we also have:

$$
\begin{aligned}
Pr(\text{follower sees } v \mid s \text{ shares}) &= \sum_{\nu=s}^{d} Pr(\text{follower sees } v \text{ and } \nu \text{ neighbors shared } v \mid s \text{ shares}) \\
&= \sum_{u=s}^{d} Pr(\text{follower sees } v \mid \nu \text{ and } s) Pr(\nu \text{ shares of } v \mid s \text{ shares})
\end{aligned}
$$

Finally, using the conditional probability derived above, we rewrite the probability for a follower to see a piece of news n from producer a, given b produced m and the state of the world is w, as:

$$
\sum_{s=1}^{d} \sum_{\nu=0}^{s} \frac{\nu}{s} \left(\frac{p_{u|w,n}}{p_{u|w,n} + p_{v|w,m}} \right)^{\nu} \left(\frac{p_{v|w,m}}{p_{u|w,n} + p_{v|w,m}} \right)^{s-\nu} \frac{s!}{\nu!(s-\nu)!} (p_{u|w,n}+p_{v|w,m})^{s} (1-p_{u|w,n}-p_{v|w,m})^{d-s} \frac{d!}{s!(d-s)!}
$$

where $p_{u|w,n}$ and $p_{v|w,m}$ are the probability that a neighbor shares a piece of news from a, b, given it is true. Defining $f(\nu)$ as the pmf of a $\mathcal{B}(s, \frac{p_{u|w,n}}{p_{u|w,n}+p_{v|w,m}})$, we simplify the latter expression by:

$$
\begin{aligned}
&\sum_{s=1}^{d} \frac{1}{s} (p_{u|w,n} + p_{v|w,m})^{s} (1 - p_{u|w,n} - p_{v|w,m})^{d-s} \frac{d!}{s!(d-s)!} \sum_{\nu=0}^{s} \nu f(\nu) \\
&= \sum_{s=1}^{d} \frac{1}{s} (p_{u|w,n} + p_{v|w,m})^{s} (1 - p_{u|w,n} - p_{v|w,m})^{d-s} \frac{d!}{s!(d-s)!} s \frac{p_{u|w,n}}{p_{u|w,n} + p_{v|w,m}} \\
&= \frac{p_{u|w,n}}{p_{u|w,n} + p_{v|w,m}} \sum_{s=1}^{d} (p_{u|w,n} + p_{v|w,m})^{s} (1 - p_{u|w,n} - p_{v|w,m})^{d-s} \frac{d!}{s!(d-s)!} \\
&= \frac{p_{u|w,n}}{p_{u|w,n} + p_{v|w,m}} \left(\sum_{s=0}^{d} (p_{u|w,n} + p_{v|w,m})^{s} (1 - p_{u|w,n} - p_{v|w,m})^{d-s} \frac{d!}{s!(d-s)!} \right. \\
&\qquad \left. - (p_{u|w,n} + p_{v|w,m})^{0} (1 - p_{u|w,n} - p_{v|w,m})^{d-0} \frac{d!}{0!(d-0)!} \right) \\
&= \frac{p_{u|w,n}}{p_{u|w,n} + p_{v|w,m}} \left(1 - (1 - p_{u|w,n} - p_{v|w,m})^{d} \right)
\end{aligned}
$$

We conclude by writing:

$$
Pr(R_{ja} = 1|w, n, m) = \alpha b + \frac{p_{u|w,n}}{p_{u|w,n} + p_{v|w,m}} \left(1 - (1 - p_{u|w,n} - p_{v|w,m})^{d} \right)
$$

3 Equilibrium

3.1 Equilibrium without Competition

Shape of Monopolist Best Response in z: Lemma 1

(i) Because $c^{-1}(x)$ is increasing in x by assumption, $x^*(z)$ is decreasing in $z_{0,1}$ and $z_{1,0}$ iff $\Delta V(z)$ is decreasing in $z_{0,1}$ and $z_{1,0}$. Now, we have:

$$\frac{\partial \Delta V(z)}{\partial z_{0,1}} \frac{1}{1-b} = -d\gamma(1-w_0)\big(1-b(\gamma z_{0,1}+(1-\gamma))\big)^{d-1} + dw_0\gamma\big(1-b(\gamma+(1-\gamma)z_{0,1})\big)^{d-1} < 0$$

Where the last inequality comes from $\frac{\gamma}{1-\gamma} > \frac{w_0}{1-w_0} \geq 1$. The derivation is similar for $z_{1,0}$.

(ii) Again, it suffices to show that $\Delta V(z)$ is single peaked in $z_{0,0}$ and $z_{1,1}$. We call single peaked a function which admits a single maximum point; therefore, any non-constant concave function $f(x)$ defined on a closed interval is single-peaked. Hence, we can simply show that $\Delta V(z)$'s first derivative is decreasing in $z_{0,0}$ and $z_{1,1}$.

For $z_{0,0}$, we have:

$$\frac{\partial \Delta V(z)}{\partial z_{0,0}} \frac{1}{1-b} = -d(1-\gamma)(1-w_0)\big(1-b(1-\gamma)z_{0,0})\big)^{d-1} + dw_0\gamma\big(1-b\gamma z_{0,0}\big)^{d-1}$$

Whose sign is ambiguous. It is positive for $z_{0,0} = 0$ and decreasing in $z_{0,0}$ since $\frac{1-b\gamma z_{0,0}}{1-b(1-\gamma)z_{0,0}} \leq 1$ and decreases with $z_{0,0}$

The derivation is similar for $z_{1,1}$.

The effects of w_0: Lemma 2 and Corollary 1

(i) It is easy to verify that $\frac{\partial \underline{t}_0}{\partial w_0} < 0$, $\frac{\partial \bar{t}_0}{\partial w_0} < 0, \frac{\partial \bar{t}_1}{\partial w_0} > 0, \frac{\partial \underline{t}_1}{\partial w_0} > 0$. The intervals are the values of t_{ns} for $w_0 = 1/2$ and $w_0 = \gamma$

(ii) Again, it is easy to verify that $\frac{\partial \Delta V(z)}{\partial w_0} > 0$ as $\big(1-b(\gamma z_{1,0}+(1-\gamma)z_{1,1})\big)^d - \big(1-b(\gamma z_{0,1}+(1-\gamma)z_{0,0})\big)^d > 0$ and $\big(1-b(\gamma z_{1,1}+(1-\gamma)z_{1,0})\big)^d - \big(1-b(\gamma z_{0,0}+(1-\gamma)z_{0,1})\big)^d > 0$.

Because from Lemma 2, $\tilde{x_{00}}$ and $\tilde{x_{11}}$, $\underline{t}_1$, $x^*(1,1,1,0)$and $\bar{t}_1$ are increasing in w_0, it is easy to see that x^M decreases with an increase in w_0 if and only if $x^M = \underline{t}_0$ or $x^M = \bar{t}_0$

3.2 Equilibrium with Competition

Shape of Duopolist Best Response in z_u: Lemma 3

(i) If $z_{uT} > z_{uF}$, $p_{uT} > p_{uF}$ so that $S_{x_v} > 0$ and $S_{1-x_v} > 0$. Therefore, $\Delta V_u(z;x_v) > 0$ and $x_u^*(z;x_v) = c^{-1}(\Delta V_u(z;x_v)) > 1/2$ as $c(x) >$ iff $x > 1/2$ by assumption. If $z_{uT} = z_{uF}$, $\Delta V_u(z;x_v) = 0$ so that $x_u^*(z;x_v) \in [0,1/2]$.

(ii) Because $c^{-1}(x)$ is increasing in x by assumption, $x_u^*(z;x_v)$ is decreasing in z_{uF} iff $\Delta V_u(z;x_v)$ is decreasing in z_{uF}. Now, we have:

$$\frac{\partial \Delta V_u(z;x_v)}{\partial z_{uF}} = (1-b)\left(x_v\frac{\partial S_{x_v}}{\partial z_{uF}} + (1-x_v)\frac{\partial S_{1-x_v}}{\partial z_{uF}}\right) < 0 \quad \text{because} \quad \frac{\partial S_{x_v}}{\partial z_{uF}} < 0, \frac{\partial S_{1-x_v}}{\partial z_{uF}} < 0$$

Indeed,

$$\begin{aligned}\frac{\partial S_{x_v}}{\partial z_{uF}} = \quad & \frac{1}{2}b\Bigg[p_{vT}\left(\frac{(1-\gamma)}{(p_{uT}+p_{vT})^2}\big(1-(1-p_{uT}-p_{vT})\big)^d - \frac{\gamma}{(p_{uF}+p_{vT})^2}\big(1-(1-p_{uF}-p_{vT})^d\big)\right) \\ & +d\frac{(1-\gamma)p_{uT}}{p_{uT}+p_{vT}}(1-p_{uT}-p_{vT})^{d-1} - d\frac{\gamma p_{uF}}{p_{uF}+p_{vT}}(1-p_{uF}-p_{vT})^{d-1})\Bigg]\end{aligned}$$

Which is a sum of negative terms. Indeed, the first term is negative because $\frac{1-(1-x)^d}{x^2}$ is decreasing in x and $p_{uT} \geq p_{uF}$. We know that $\frac{1-(1-x)^d}{x^2}$ is decreasing in x because:

$$\frac{\partial \frac{1-(1-x)^d}{x^2}}{\partial x}x^4 = d(1-x)^{d-1}x^2 - 2x(1-(1-x)^d) = (1-x)^{d-1}x\big(dx+2(1-x)\big) - 2x < x(-x+2) - 2x < 0$$

where the first inequality follows from $(1-x)^{d-1}\big((d-2)x+2\big)$ being decreasing in d so that among all d, $d=1$ maximizes the expression.

The second term is negative as $(1-p_{uT}-p_{vT})^{d-1} < (1-p_{uF}-p_{vT})^{d-1}$; and $\frac{(1-\gamma)p_{uT}}{p_{uT}+p_{vT}} < \frac{\gamma p_{uF}}{p_{uF}+p_{vT}}$. The last inequality holds because:

$$(1-\gamma)p_{uT}(p_{uF}+p_{vT}) - \gamma p_{uF}(p_{uT}+p_{vT}) = p_{uF}p_{uT}(1-2\gamma) + p_{vT}\big((1-\gamma)p_{uT} - \gamma p_{uF}\big)$$

is the sum of two negative terms; indeed: $1-2\gamma < 0$ and

$$(1-\gamma)p_{uT} - \gamma p_{uF} = \frac{1}{2}b\big[(1-\gamma)(\gamma+(1-\gamma)z_{uF}) - \gamma(1-\gamma+\gamma z_{uF})\big] = \frac{1}{2}b\big[(1-\gamma)^2-\gamma^2\big]z_{uF} < 0$$

Replacing p_{vT} by p_{vF} above, we can conclude that $\frac{\partial S_{1-x_v}}{\partial z_{uF}} < 0$

(iii) Again, it suffices to show that $\Delta V_u(z;x_v)$ is single-peaked in z_{uT}. In particular, we show that S_{x_v} and S_{1-x_v} are concave in z_{uT} for $z_{uT} \in [0,1]$. Consider S_{x_v}. We have:

$$\begin{aligned}S_{x_v} &= \frac{p_{uT}}{p_{uT}+p_{vT}}\big(1-(1-p_{uT}-p_{vT})^d\big) - \frac{p_{uF}}{p_{uF}+p_{vT}}\big(1-(1-p_{uF}-p_{vT})^d\big) \\ &= \frac{p_{uT}}{p_{uT}+p_{vT}}\big((1-p_{uF}-p_{vT})^d - (1-p_{uT}-p_{vT})^d\big) + \left(\frac{p_{uT}}{p_{uT}+p_{vT}} - \frac{p_{uF}}{p_{uF}+p_{vT}}\right)\big(1-(1-p_{uF}-p_{vT})^d\big)\end{aligned}$$

We know that $\frac{p_{uT}}{p_{uT}+p_{vT}}$ and $\big(1-(1-p_{uF}-p_{vT})^d\big)$ are both strictly increasing and weakly concave in z_{uT}. From the analysis of the monopolist's best response, we also know that $\big((1-p_{uF}-p_{vT})^d - (1-p_{uT}-p_{vT})^d\big)$ is single-peaked. As the product of weakly concave functions is weakly concave, all that is left to do is to show that $\frac{p_{uT}}{p_{uT}+p_{vT}} - \frac{p_{uF}}{p_{uF}+p_{vT}}$ is single peaked. We have:

$$\frac{\partial \frac{p_{uT}}{p_{uT}+p_{vT}} - \frac{p_{uF}}{p_{uF}+p_{vT}}}{\partial z_{uT}} = \frac{\partial \frac{p_{vT}(p_{uT}-p_{uF})}{(p_{uT}+p_{vT})(p_{uF}+p_{vT})}}{\partial z_{uT}} = \frac{\partial \frac{p_{vT}(\frac{1}{2}b(2\gamma-1)z_{uT})}{(\frac{1}{2}b\gamma z_{uT}+p_{vT})(\frac{1}{2}b(1-\gamma)z_{uT}+p_{vT})}}{\partial z_{uT}}$$

$$= \frac{p_{vT}\Big[\frac{1}{2}b(2\gamma-1)(\frac{1}{4}b^2\gamma(1-\gamma)z_{uT}{}^2 + \frac{1}{2}bp_{vT}z_{uT} + p_{vT}{}^2) - \big(\frac{1}{2}b^2\gamma(1-\gamma)z_{uT} + \frac{1}{2}bp_{vT}\big)\frac{1}{2}b(2\gamma-1)z_{uT}\Big]}{(p_{uT}+p_{vT})^2(p_{uF}+p_{vT})^2}$$

$$= \frac{p_{vT}\frac{1}{2}b(2\gamma-1)\Big[p_{vT} - \frac{1}{4}b^2\gamma(1-\gamma)z_{uT}{}^2\Big]}{2(p_{uT}+p_{vT})^2(p_{uF}+p_{vT})^2}$$

Which is positive for $z_{uT} = 0$ and decreases with z_{uT}.

The same applies to $\frac{\partial S_{1-x_v}}{\partial z_{uT}}$

Shape of Duopolist Best Response in z_v and x_v: Lemma 4

(i) For $d = 2$, $\frac{\partial S_{x_v}}{\partial z_{vX}}$ and $\frac{\partial S_{1-x_v}}{\partial z_{vX}}$ are decreasing in z_{vX} for $X = T, F$. We have:

$$\frac{\partial S_{x_v}}{\partial p_{vT}} = -\frac{p_{uT}}{(p_{uT}+p_{vT})^2}\Big[1-(1-p_{uT}-p_{vT})^{d-1}\big(1+(d-1)(p_{uT}+p_{vT})\big)\Big]$$
$$+\frac{p_{uF}}{(p_{uF}+p_{vT})^2}\Big[1-(1-p_{uF}-p_{vT})^{d-1}\big(1+(d-1)(p_{uF}+p_{vT})\big)\Big] < 0$$

If $-\frac{p_{uT}}{(p_{uT}+p_{vT})^2}\big(-(p_{uT}+p_{vT})^2\big) > \frac{p_{uF}}{(p_{uF}+p_{vF})^2}\big(-(p_{uF}+p_{vF})^2\big)$ which is ensured by $p_{uT} \geq p_{uF}$. The same applies for $\frac{\partial S_{1-x_v}}{\partial p_{vF}}$.[16]

When $d \to \infty$, the expression is determined by the sign of $-\frac{p_{uT}}{(p_{uT}+p_{vT})^2} + \frac{p_{uF}}{(p_{uF}+p_{vT})^2}$ which is negative for $p_{vT}^2 > p_{uF}p_{uT}$. Likewise, S_{1-x_v} is decreasing in z_{vX} for $p_{vF}^2 > p_{uF}p_{uT}$.

(ii) It is enough to prove that $S_{x_v} - S_{1-x_v} \leq 0$. We have:

$$S_{x_v} - S_{1-x_v} = \frac{p_{uT}}{p_{uT}+p_{vT}}\big(1-(1-p_{uT}-p_{vT})^d\big) - \frac{p_{uF}}{p_{uF}+p_{vT}}\big(1-(1-p_{uF}-p_{vT})^d\big)$$
$$-\frac{p_{uT}}{p_{uT}+p_{vF}}\big(1-(1-p_{uT}-p_{vF})^d\big) + \frac{p_{uF}}{p_{uF}+p_F}\big(1-(1-p_{uF}-p_{vF})^d\big)$$
$$= \frac{p_{uT}}{(p_{uT}+p_{vT})(p_{uT}+p_{vF})}\big(p_{vF}-p_{vT}-(1-p_{uT}-p_{vT})^d(p_{uT}+p_{vF})+(1-p_{uT}-p_{vF})^d(p_{uT}+p_{vT})\big)$$
$$-\frac{p_{uF}}{(p_{uF}+p_{vT})(p_{uF}+p_{vF})}\big(p_{vF}-p_{vT}-(1-p_{uF}-p_{vT})^d(p_{uF}+p_{vF})+(1-p_{uF}-p_{vF})^d(p_{uF}+p_{vT})\big)$$

Let us define α such that $p_{uT} + p_{vF} = \alpha(p_{uT}+p_{vT}) + (1-\alpha)(p_{uF}+p_{vF})$; therefore $p_{uF} + p_{vT} =$

[16] Notice that a similar inequality holds for $d = 3$. From numerical insights, the difference is expected to be increasing then decreasing in d.

$(1-\alpha)(p_{uT}+p_{vT})+\alpha(p_{uF}+p_{vF})$. Because $(1-x)^d$ is convex, we have:

$$\begin{aligned}
&(1-p_{uT}-p_{vF})^d(p_{uT}+p_{vT})+(1-p_{uF}-p_{vT})^d(p_{uF}+p_{vF})\\
&\quad-(1-p_{uT}-p_{vT})^d(p_{uT}+p_{vF})-(1-p_{uF}-p_{vF})^d(p_{uF}+p_{vT})\\
&<\alpha(1-p_{uT}-p_{vT})^d(p_{uT}+p_{vT})+(1-\alpha)(1-p_{uF}-p_{vF})^d(p_{uT}+p_{vT})\\
&\quad+(1-\alpha)(1-p_{uT}-p_{vT})^d(p_{uF}+p_{vF})+\alpha(1-p_{uF}-p_{vF})^d(p_{uF}+p_{vF})\\
&\quad-(1-p_{uT}-p_{vT})^d(p_{uT}+p_{vF})-(1-p_{uF}-p_{vF})^d(p_{uF}+p_{vT}) \qquad =0
\end{aligned}$$

Therefore the second factor of the first term of the sum is lower than the second factor of the second term of the sum, which is itself negative. If the first factor of the first term is greater than the first factor of the second term, we are done. And indeed, if $p_{uX}<p_{vX}$, we have:

$$\frac{p_{uT}}{p_{uT}^2+p_{uT}(p_{vT}+p_{vF})+p_{vT}p_{vF}}>\frac{p_{uF}}{p_{uF}^2+p_{uF}(p_{vT}+p_{vF})+p_{vT}p_{vF}}$$
$$p_{vT}p_{vF}(p_{uT}-p_{uF})>p_{uT}p_{uF}(p_{uT}-p_{uF})=p_{uT}^2p_{uF}-p_{uT}p_{uF}^2$$

3.2.1 The role of Connectivity

Shape of $\Delta V^M(z;d)-\Delta V^D(z,x;d)$ ***in*** d***: Theorem 1***

We want to show that $DV(d)>DV(d+1)\Rightarrow DV(d+1)>DV(d+2)$. For readability, let us define for this proof:

$$c_1=1-\frac{x}{2}\qquad c_2=\frac{1+x}{2}\qquad c_3=\frac{p_T}{p_T+p_F}-x$$

Note that $c_1>0$, $c_2>0$ and c_3's sign depends on z and x.

We begin by rewriting the assumption $DV(d)-DV(d+1)>0$ as:

$$-c_1\left((1-p_T)^d-(1-p_T)^{d+1}\right)+c_2\left((1-p_F)^d-(1-p_F)^{d+1}\right)+c_3\left(\left(1-\tfrac{p_T+p_F}{2}\right)^d-\left(1-\tfrac{p_T+p_F}{2}\right)^{d+1}\right)>0$$

$$-c_1\left((1-p_T)^dp_T\right)+c_2\left((1-p_F)^dp_F\right)+c_3\left(\left(1-\frac{p_T+p_F}{2}\right)^d\frac{p_T+p_F}{2}\right)>0$$

Therefore, defining for readability again:

$$A:=c_1\left((1-p_T)^dp_T\right)-\tfrac{1}{2}c_3\left(\left(1-\tfrac{p_T+p_F}{2}\right)^d\tfrac{p_T+p_F}{2}\right)$$

$$B:=c_2\left((1-p_F)^dp_F\right)+\tfrac{1}{2}c_3\left(\left(1-\tfrac{p_T+p_F}{2}\right)^d\tfrac{p_T+p_F}{2}\right)$$

$DV(d+1)-DV(d)<0$ is equivalent to $B>A$. Notice that $B>0$ because when $c_3>0$ makes B a sum of positive term, and when $c_3<0$ A is a sum of positive term so that $B>A>0$.

Likewise we develop $DV(d+2)-DV(d+1)$ as:

$$-c_1\left((1-p_T)^dp_T(1-p_T)\right)+c_2\left((1-p_F)^dp_F(1-p_F)\right)+c_3\left(\left(1-\frac{p_T+p_F}{2}\right)^d\frac{p_T+p_F}{2}\frac{1}{2}(1-p_T+1-p_F)\right)$$

Therefore:

$$DV(d+1) - DV(d+2) = -(1-p_T)A + (1-p_F)B > 0$$

where the last inequality follows from $p_T > p_F$

5 Evaluation of Intervention

5.1 Flagging

Differential effect of flagging with or without competition: Proposition 8

We define the difference of incentive to invest with flagging $FDV(z,x;q) := \Delta V^M(z;q) - \Delta V^D(z,x;q)$. Let us first rewrite:

$$\frac{\partial FDV(z,x;q)}{\partial q} = V_F + (1-x)V_{TF} - (1-x)V_{T\varnothing} - xV_{FT} - 2(1-x)(1-q)V_{FF} + (1-x)(1-2q)V_{F\varnothing}$$

To prove that this derivative is positive, we show that $\frac{\partial^2 FDV(z,x;q)}{\partial q \partial x} \geq 0$, so that $\frac{\partial FDV(z,x;q)}{\partial q} \geq \frac{\partial FDV(z,x;q)}{\partial q}\Big|_{x=1/2}$ We then move to show that $\frac{\partial FDV(z,x;q)}{\partial q}\Big|_{x=1/2} > 0$.

To show that $\frac{\partial^2 FDV(z,x;q)}{\partial q \partial x} \geq 0$, we rewrite:

$$\begin{aligned}
\frac{\partial^2 FDV(z,x;q)}{\partial q \partial x} &= -V_{TF} + V_{T\varnothing} - V_{FT} + 2(1-q)V_{FF} - (1-2q)V_{F\varnothing} \\
&= -\tfrac{p_T}{p_T+p_F}\Big(1-(1-\tfrac{p_T+p_F}{2})^d\Big) + \Big(1-(1-\tfrac{p_T}{2})^d\Big) - \tfrac{p_F}{p_T+p_F}\Big(1-(1-\tfrac{p_T+p_F}{2})^d\Big) \\
&\quad + (1-q)\Big(1-(1-p_F)^d\Big) - (1-2q)\Big(1-(1-\frac{p_F}{2})^d\Big) \\
&= q + (1-\tfrac{p_T+p_F}{2})^d - (1-\tfrac{p_T}{2})^d - (1-q)(1-p_F)^d + (1-2q)(1-\tfrac{p_F}{2})^d \\
&= q\Big[1+(1-p_F)^d - 2(1-\tfrac{p_F}{2})^d\Big] + \Big[(1-\tfrac{p_T+p_F}{2})^d - (1-\tfrac{p_T}{2})^d - (1-p_F)^d + (1-\tfrac{p_F}{2})^d\Big]
\end{aligned}$$

Now, this expression is the sum of two positive terms. Indeed:

- the first term is increasing in p_F so that $1+(1-p_F)^d-2(1-\frac{p_F}{2})^d \geq 1+(1-p_F)^d - 2(1-\frac{p_F}{2})^d\Big|_{p_F=0} = 0$
- the second term is increasing in p_T so that $(1-\frac{p_T+p_F}{2})^d - (1-\frac{p_T}{2})^d - (1-p_F)^d + (1-\frac{p_F}{2})^d \geq (1-\frac{p_T+p_F}{2})^d - (1-\frac{p_T}{2})^d - (1-p_F)^d + (1-\frac{p_F}{2})^d\Big|_{p_T=p_F} = 0$

We can thus conclude that $\frac{\partial^2\left(\Delta V^M(z;q) - \Delta V^D(z,x;q)\right)}{\partial q \partial x} \geq 0$

Let us now show that $\frac{\partial FDV(z,x;q)}{\partial q}\Big|_{x=1/2} > 0$. We can rewrite:

$$\frac{\partial FDV(z,x;q)}{\partial q}\Big|_{x=1/2} = V_F + \frac{1}{2}V_{TF} - \frac{1}{2}V_{T\varnothing} - \frac{1}{2}V_{FT} - (1-q)V_{FF} + \frac{1}{2}(1-2q)V_{F\varnothing}$$

Noting that $V_{FF} = \frac{1}{2}V_F$, we get:

$$\frac{\partial FDV(z,x;q)}{\partial q}\Big|_{x=1/2} = \left[\frac{(1+q)}{2}V_F - qV_{F\varnothing}\right] + \frac{1}{2}\left[V_{TF} - V_{FT} - V_{T\varnothing} + V_{F\varnothing}\right]$$
$$= \left[\tfrac{1+q}{2}V_F - qV_{F\varnothing}\right] + \tfrac{1}{2}\left[\tfrac{p_T-p_F}{p_T+p_F}\left(1-(1-\tfrac{p_T+p_F}{2})^d\right) + (1-\tfrac{p_T}{2})^d - (1-\tfrac{p_F}{2})^d\right]$$

Again, this is the sum of two positive terms.

- The first term is positive as $\frac{1+q}{2} > q$ and $V_F \geq V_{F\varnothing}$. Note that the term is strictly positive for $p_F > 0$.
- It is more cumbersome to show that the second term is positive. We show that it is non-decreasing in d and then show it is weakly positive for $d = 1$.To show that it is non-decreasing in d, we proceed by induction. For ease of notation, let us define for this proof:

$$E(d) := \tfrac{p_T-p_F}{p_T+p_F}\left(1-(1-\tfrac{p_T+p_F}{2})^d\right) + (1-\tfrac{p_T}{2})^d - (1-\tfrac{p_F}{2})^d$$

Then,

$$E(d) - E(d+1) = -\tfrac{p_T-p_F}{p_T+p_F}\tfrac{p_T+p_F}{2}(1-\tfrac{p_T+p_F}{2})^d + \tfrac{p_T}{2}(1-\tfrac{p_T}{2})^d - \tfrac{p_F}{2}(1-\tfrac{p_F}{2})^d$$
$$= \underbrace{\tfrac{p_T}{2}\left[(1-\tfrac{p_T}{2})^d - (1-\tfrac{p_T+p_F}{2})^d\right]}_{:=A} - \underbrace{\tfrac{p_F}{2}\left[(1-\tfrac{p_F}{2})^d - (1-\tfrac{p_T+p_F}{2})^d\right]}_{:=B}$$

Therefore $E(d) - E(d+1) < 0$ for $A < B$. We want to show that if $E(d)$ it is non-decreasing at some d', then it is non-decreasing for all subsequent $d > d'$. The inductive step requires us to show that for $A \leq B$, $E(d+1) - E(d+2) \leq 0$. This is indeed the case as:

$$E(d+1) - E(d+2) = \tfrac{p_T}{2}\left[(1-\tfrac{p_T}{2})^d(1-\tfrac{p_T}{2}) - (1-\tfrac{p_T+p_F}{2})^d(1-\tfrac{p_T}{2}) + (1-\tfrac{p_T+p_F}{2})^d(-\tfrac{p_F}{2})\right]$$
$$- \tfrac{p_F}{2}\left[(1-\tfrac{p_F}{2})^d(1-\tfrac{p_F}{2}) - (1-\tfrac{p_T+p_F}{2})^d(1-\tfrac{p_F}{2}) + (1-\tfrac{p_T+p_F}{2})^d(-\tfrac{p_T}{2})\right]$$
$$= (1-\tfrac{p_T}{2})A - (1-\tfrac{p_T}{2})B$$

Because $(1-\frac{p_T}{2}) \leq (1-\frac{p_F}{2})$ and $A \geq 0$, we do have: $A \leq B \Rightarrow (1-\frac{p_T}{2})A \leq (1-\frac{p_T}{2})B$. Finally, it is easy to verify that for $d = 1$, $A = B$, so that $E(1) - E(2) = 0$. [17]

We can thus conclude $\frac{\partial FDV(z,x;q)}{\partial q}\Big|_{x=1/2} \geq 0$ for any $p_F \geq 0$ and $\frac{\partial FDV(z,x;q)}{\partial q}\Big|_{x=1/2} > 0$ for any $p_F > 0$, which concludes our proof.

[17] Note that if $A > 0$ and $p_T > p_F$, $A \leq B \Rightarrow (1-\frac{p_T}{2})A > (1-\frac{p_T}{2})B$. Therefore, the term is strictly increasing for any $d \geq 2$, $p_T > p_F$.

B Attention-Seeking Influencers

B.1 Influencer Best Response in Monopoly

The Probability of Being Read by a Follower (as an Influencer)

Because we take the perspective of a given influencer i, we now define the random variable $S \sim \mathcal{B}(d-1, p_X)$ as the number of times i's followers' neighbors' have shared, **in addition to** i.

$$\begin{aligned}
\mathbb{E}\left(\frac{1}{S+1}\right) &= \sum_{s=0}^{d-1} \frac{1}{s+1}(p_X)^s(1-p_X)^{d-1-s}\frac{(d-1)!}{s!(d-1-s)!} \\
&= \frac{1}{dp_X}\sum_{s=0}^{d-1}(p_X)^{s+1}(1-p_X)^{d-s-1}\frac{d!}{(s+1)!(d-s-1)!} \\
&= \frac{1}{dp_X}\sum_{\tilde{s}=1}^{d}(p_X)^{\tilde{s}}(1-p_X)^{d-\tilde{s}}\frac{d!}{\tilde{s}!(d-\tilde{s})!} \\
&= \frac{1}{dp_X}\left[\sum_{\tilde{s}=0}^{d}(p_X)^{\tilde{s}}(1-p_X)^{d-\tilde{s}}\frac{d!}{\tilde{s}!(d-\tilde{s})!} - (p_X)^0(1-p_X)^{d-0}\frac{d!}{0!d!}\right] \\
&= \frac{1}{dp_X}\left[1-(1-p_X)^d\right]
\end{aligned}$$

where $\tilde{s} = s+1$.

Proof of Proposition 9

1. $\mathbb{E}(\ \#\text{likes})|s_i = T) \leq \tau \leq \mathbb{E}(\ \#\text{likes})|s_i = T) \Rightarrow \frac{\rho}{1-\rho} < \frac{p_T}{(1-(1-p_T)^d)}\frac{(1-(1-p_F)^d)}{p_F}$
   We have:

$$p_v(T;x_v)\rho\frac{1-b}{p_T}\left(1-(1-p_T)^d\right)+(1-p_v(T;x_v))(1-\rho)\frac{1-b}{p_F}\left(1-(1-p_F)^d\right) < p_v(F;x_v)\rho\frac{1}{p_T}\left(1-(1-p_T)^d\right)+(1-p_v(F$$

$$[p_v(T;x_v)-p_v(F;x_v)]\rho\frac{1}{p_T}\left(1-(1-p_T)^d\right) < [p_v(T;x_v)-p_v(F;x_v)](1-\rho)\frac{1}{p_F}\left(1-(1-p_F)^d\right)$$

$$\rho\frac{1}{p_T}\left(1-(1-p_T)^d\right) < (1-\rho)\frac{1}{p_F}\left(1-(1-p_F)^d\right)$$

2. $f(x) = \frac{x}{1-(1-x)^d}$ is an increasing function. We have:

$$\text{sign}\left(\left.\frac{\partial f}{\partial x}\right|_{x\in(0,1)}\right) = \text{sign}\left(\frac{1-(1-x)^d - xd(1-x)^{d-1}}{\left(1-(1-x)^d\right)^2}\right) = \text{sign}\left(1-(1-x)^d - xd(1-x)^{d-1}\right)$$

Now, $g(x) := (1-x)^d - xd(1-x)^{d-1} < 1$ over $x \in [0,1]$. Indeed, $g(0) = 1, g(1) = 0$, and g strictly decreasing in-between. Indeed,

$$\left.\frac{\partial g}{\partial x}\right|_{x\in[0,1]} = (d-1)(1-x)^{d-2}\Big[(1-x)-\big(1+x(d-1)\big)\Big] = (d-1)(1-x)^{d-2}[-xd] \leq 0$$

Finally note that f is continuous on $[0,1]$. Indeed: $f(0) = \frac{1}{d}$ and $f(1) = 1$.

Proof of Theorem 3

(i) If every other influencers always share, $p_T = p_F = b$ Then, the expected number of likes upon receiving a false signal is:

$$[p(F;x)\rho + (1-p(F,x))(1-\rho)]\frac{1-b}{b}(1-(1-b)^d)$$

Which is higher than τ iff:

$$p(F;x) \geq \frac{\frac{\tau}{\delta(b)} - (1-\rho)}{2\rho - 1}$$

Given that $p(F;x) = \frac{(1-\rho)x}{(1-\rho)x+\rho x}$, this happens iff:

$$x \geq \frac{\rho}{2\rho-1}\frac{\tau-(1-\rho)\delta}{\tau} = \hat{x}(\tau)$$

Note that because $\tau \leq \rho\delta$, $\hat{x}(\tau) \leq 1$; for $\tau < (1-\rho)\delta$, $\hat{x}(\tau) < 0$, so that the condition is always fulfilled.

(ii) If every other influencers never share, $p_T = p_F = 0$. Then, the expected number of likes upon receiving a true signal is:

$$[p(T;x)\rho + (1-p(T,x))(1-\rho)]d(1-b)$$

Which is lower than τ iff:

$$p(T;x) \leq \frac{\frac{\tau}{d(1-b)} - (1-\rho)}{2\rho - 1}$$

Given that $p(T;x) = \frac{\rho x}{\rho x+(1-\rho)x}$, this happens iff:

$$x \leq \frac{1-\rho}{2\rho-1}\frac{\tau-(1-\rho)d(1-b)}{d(1-b)-\tau} = \underset{\smile}{x}(\tau)$$

Note that because $\tau \geq (1-\rho)d(1-b)$, $\underset{\smile}{x}(\tau) \geq 0$; for $\tau > \rho d(1-b)$, $\hat{x}(\tau) > 1$, so that the condition is always fulfilled.

(iii) If every other influencers share only upon receiving a positive signal, $p_T = 1, p_F = 0$. Then, i also only shares upon receiving a positive signal iff:

$$p(T;x)\frac{1-(1-b\rho)^d}{b}+(1-p(T,x))\frac{1-(1-b(1-\rho))^d}{b}) > \tau > p(F;x)\frac{1-(1-b\rho)^d}{b}+(1-p(F,x))\frac{1-(1-b(1-\rho))^d}{b}$$

Which is possible only if $\tau \in [\tau_1, \tau_2]$. Note that if $\tau \in \{\tau_1, \tau_2\}$, $x_1 = x_2 \in \{0,1\}$.

Replace $p(T;x)$ and $p(F;x)$ by the adequate expression to find the range x_1, x_2.

C. Second Period Bet with $w_0 \neq 1/2$

Because the followers are not necessarily exposed to an article, we define $n \in \{\varnothing, 0, 1\}$. Furthermore, the identity of the news producer also matters. We define a random variable describing the identity of the producer whose news' article a follower is exposed to; to economize on notation, we denote both the r.V. and its outcome by e, with $e \in \{\varnothing, a, b\}$. Finally, we note $p(n, s, e; x_u, z, w_0)$ the followers' posterior probability with which the followers believe the news' content to be true.

Following the analysis of the influencer's problem, the follower's optimal action given her information set is $a(n, s) = |n - \mathbb{1}_{p(n,s,w;x_u,z,w_0)<1/2}|$.

If a follower sees some news article, he only sees one piece of information, drawn at random from their sharing neighborhood. This means that he can never observe how many neighbors shared, and thus cannot perfectly infer the distribution of their private signal. However, he accounts for the fact that one type of content might be more visible than another, and that one producer might be more shared than another.

Upon observing a piece of news $n \neq \varnothing$ from producer $e = u$, and receiving a private signal s, the followers' posteriors on the news' truthfulness are:

$$Pr(\omega = n|\eta_u = n, \varsigma = s, e = u) = \frac{Pr(\eta_u{=}n,\varsigma{=}s|\omega{=}n)Pr(e{=}u,\eta_u{\neq}\varnothing|\eta{=}n,\omega{=}n)Pr(\omega{=}n)}{\sum_w Pr(\eta_u{=}n,\varsigma{=}s|\omega{=}w)Pr(e{=}u,\eta_u{\neq}\varnothing|\eta{=}n,\omega{=}w)Pr(\omega{=}w)}$$

Now, $Pr(e = u, \eta_u \neq \varnothing|\omega = n)$ as to be expected over possible news' content for all others producers. As before, we denote m the vector outcome of $-u$'s news' content. We find:

$$Pr(e = u, \eta_u \neq \varnothing|\omega = w) = \sum_m \Pr(m) \frac{p_{u|w,n}}{p_{u|w,n} + p_{-u|w,m}} \Big(1 - (1 - p_{v|w,n} + p_{-u|w,m})^d\Big)$$

where $\Pr(m)$ and $p_{-u|w,m}$ are defined as in the main text.

We thus find:

$$p(n, s, w; x_u, z, w_0) = \frac{\Pr(\varsigma{=}s|\omega{=}n)\mathbb{E}(R_{fu}|\omega{=}n,\eta{=}n,\varsigma{=}s)x_u}{\sum_w \Pr(\varsigma{=}s|\omega{=}w)\mathbb{E}(R_{fu}|\omega{=}w,\eta{=}n,\varsigma{=}s)\Pr(\eta{=}n|\omega{=}w)}$$

Where $\mathbb{E}(R_{fu}|\omega = n, \eta = n, \varsigma = s) = 1 - (1 - p_{n,s})^d$ in the case of a monopoly.

Notice that even in the symmetric equilibrium, where $x_u = x_{-u}$ and $p_{u|n,s} = p_{-u|n,s}$, the above expression still needs to account for the presence of two potential sources of information. In particular, the follower accounts for the fact that there should be a higher probability for a true information to reach her in equilibrium.

When no news is shared in the follower's neighborhood, he does not know which content has been published. However, it can still be informative. Say for instance that z is such that agents never share with $\eta = 1$ and always share with $\eta = 0$. Then, not seeing any information means $\eta = 1$.

Her posterior about the state of the world upon not seeing any information, i.e. $\eta = \varnothing$ is:

$$Pr(\omega = 0| \text{ no neighbor shared}) = \frac{Pr(\text{ no shares } |\omega{=}0)Pr(\omega{=}0)}{\sum_w Pr(\text{no shares } |\omega{=}w)Pr(\omega{=}w)}$$

Now, noting m' the set of outcome describing all producers v, we have:

$$Pr(\text{no shares } |\omega = 0) = \sum_{m'} Pr(\text{no shares } |\eta = m', \omega = 0)Pr(\eta = m')$$

We find:

$$Pr(\omega = 0|\eta = \varnothing, \varsigma = s) = \frac{Pr(s|\omega{=}0)\left[\sum_m (1-p_{v|0,m})^d \Pr(m)\right] w_0}{\sum_w Pr(s|\omega{=}w)\left[\sum_m (1-p_{v|0,m})^d \Pr(m)\right] \Pr(\omega{=}w)}$$

Notice that if we assume z to be such that both types of information are treated the same by the influencers, then the follower's posterior is simply w_0, because $p_{v|0,0} = p_{v|1,1}$ and $p_{v|0,1} = p_{v|1,0}$. As in the main text, seeing no shares in such a case is uninformative.

The followers' strategy directly follows from this.

D. Equilibria with Sequential Moves

In this part, we solve the model presented in section ?? with $w_0 = 1/2$ and $z_{v|0,0} = z_{v|1,1} := z_T$, $z_{v|0,0} = z_{v|1,1} := z_F$, as a sequential game. In particular, we assume the following timing:

t=1 Producers v simultaneously choose their precision level $Pr(\text{news } v \text{ T}) = x_v$.

* Network is formed. One piece of news per producer is issued. Influencers receive a private signal about the piece of news' truthfulness.

t=2 Influencers i simultaneously choose

Note that, as the influencer plays last, their problem does not change. x is now the actual investment and not their prior about it; and the best-response is now their contigent strategy. Nothing else changes. Thus, we will only analyze the choice of the producer in the first period.

6.0.1 Monopoly

Now, the producer is internalizing his effect on influencers' action. Because their strategy is not smooth, the producer's consider different cases. Recall that the producer wants to maximize:

$$x\Delta V(z(x)) - V_F(z) - C(x)$$

- If restricted to $x < 1-\gamma$, any investment would be costly without yielding benefits, so he would pick $x = 0$ and get 0 benefits.
- If restricted to $x > \gamma$, any extra investment from γ would be fruitless, so $x \to \gamma$ and no $x > \gamma$ can be part of an SPE.
- If restricted to $x \in (1-\gamma, \gamma)$,
  - If there exists a $x^{M'}$ such that $\Delta V(1,0) = c(x^{M'})$, then the producer would pick $x^{M'}$ and have benefits $1-(1-b)[x^{M'}(1-b\gamma)^d + (1-x^{M'})(1-b(1-\gamma))^d] - C(x^{M'})$
  - Otherwise, no $x \in (1-\gamma, \gamma)$
- $x = 1-\gamma$ would be consistent with an equilibrium if $z_T^*(1-\gamma)$ is such that $1-(1-b)[x^{M'}(1-b\gamma)^d + (1-x^{M'})(1-b(1-\gamma))^d] - C(1-\gamma) \geq 0$, which would be the benefits
- $x = \gamma$ would be consistent with an equilibrium if $z_F^*(\gamma) = 1$; the benefits would be $1-(1-b)^{d+1} - C(\gamma)$.

Note that if there exists $x^{M'} > 1-\gamma$ such that $\Delta V(1,0) = c(x^{M'})$, $x = 1-\gamma$ cannot be part of a SPE, since in $1-\gamma$ there is a deviation to $x' > 1-\gamma$ – at $1-\gamma$ marginal benefits are higher than marginal cost. Furthermore, with such $x^{M'}$ existing, we know that $\Delta V(z^*(x^M)) - C(x^M) \geq 0$. Indeed, by assuming free entry and because c is increasing, we know that:

$$\Delta V(z^*(x^M)) - C(x^M) = \int_0^{x^M} \Delta V(z^*(x^M)) - c(x)dx \geq 0$$

We thus specify the possible SPE as follows:

- If there exists $x^{M'} \in (1-\gamma, \gamma)$ such that $\Delta V(1,0) = c(x^{M'})$
  - If $V(1,1) - V(1,0) > C(\gamma) - C(x^{M'})$, then the SPE are such that the producer invests γ and the influencers set $(z_T^*(x), z_F^*(x)) = \begin{cases} (0,0) & \text{if } x < 1-\gamma \\ (a,0) & \text{if } x = 1-\gamma \\ (1,0) & \text{if } x \in (1-\gamma,\gamma) \\ (1,1) & \text{if } x \geq \gamma \end{cases}$
  - If $V(1,1) - V(1,0) \leq C(\gamma) - C(x^{M'})$, then the SPE are such that the producer invests $x^{M'}$ and the influencers set $(z_T^*(x), z_F^*(x)) = \begin{cases} (0,0) & \text{if } x < 1-\gamma \\ (a,0) & \text{if } x = 1-\gamma \\ (1,0) & \text{if } x \in (1-\gamma,\gamma) \\ (1,a) & \text{if } x = \gamma \\ (1,1) & \text{if } x > \gamma \end{cases}$
  - If $V(1,1) - V(1,0) \leq C(\gamma) - C(x^{M'})$ then both set of SPE described above exist
- If there exists no such $x^{M'}$:
  - If $\Delta V(1,0) > c(x)\ \forall x \in (1-\gamma,\gamma)$, then the SPE are such that the producer invests γ and the influencers set $(z_T^*(x), z_F^*(x)) = \begin{cases} (0,0) & \text{if } x < 1-\gamma \\ (a,0) & \text{if } x = 1-\gamma \\ (1,0) & \text{if } x \in (1-\gamma,\gamma) \\ (1,1) & \text{if } x \geq \gamma \end{cases}$
  - If $\Delta V(1,0) < c(x)\ \forall x \in (1-\gamma,\gamma)$, there exists two set of SPE:
    * The producer invests $1-\gamma$ and the influencers set $(z_T^*(x), z_F^*(x)) = \begin{cases} (0,0) & \text{if } x < 1-\gamma \\ (a_1,0) & \text{if } x = 1-\gamma \\ (1,0) & \text{if } x \in (1-\gamma,\gamma) \\ (1,a_2) & \text{if } x = \gamma \\ (1,1) & \text{if } x > \gamma \end{cases}$

      for all a_2 and with a_1 such that
      $1 - (1-b)[x^{M'}(1-b\gamma)^d + (1-x^{M'})(1-b(1-\gamma))^d] - C(1-\gamma) \geq 0$

* The producer invests 0 and the influencers set $(z_T^*(x), z_F^*(x)) = \begin{cases} (0,0) & \text{if } x < 1-\gamma \\ (a_1, 0) & \text{if } x = 1-\gamma \\ (1,0) & \text{if } x \in (1-\gamma, \gamma) \\ (1, a_2) & \text{if } x = \gamma \\ (1,1) & \text{if } x > \gamma \end{cases}$

  for all a_2 and with a_1 such that
  $1-(1-b)[x^{M'}(1-b\gamma)^d + (1-x^{M'})(1-b(1-\gamma))^d] - C(1-\gamma) \leq 0$