Child Survival and the Decline in Hours Worked

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Abstract

This paper presents a two-periods OLG model in which families make decisions about their labor supply, quantity and quality of children taking into account that spending more per child improves their offspring's survival chances. Due to this last feature, the model can explain why higher earning individuals, despite raising fewer children, devote more time to child-rearing activities and work fewer hours. Moreover, these microeconomic patterns allow us to build a unified growth model that yields a downwards trend in hours worked consistent with recent findings.

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1 Introduction

Over the last century, many developed economies have experienced a secular decline in the number of hours worked per worker (see Boppart and Krusell (2020)). Contrary to conventional economic thinking, this trend could be easily explained if higher wages discourage the supply of labor. Actually, such hypothesis already has some empirical support (see Bick et al. (2018)). Moreover, it would be consistent with the fact that higher wage individuals devote more hours to child-rearing activities (see Guryan et al. (2008) or Doepke et al. (2019)). This paper presents a novel mechanism by which higher wages shift the use of time in the mentioned direction.

Our premise is that families internalize the positive effect of spending more per child on their offspring's survival chances when making decisions about quantity and quality of children. In other words, families are aware that their children will grow up healthier, and are thus more likely to reach adulthood, if they spend more per child in nutrition, clothes, medicines, and other forms of "consumption".[2] The key point about this assumption is that the internalization of child survival increases the opportunity cost associated with time use asymmetrically across the income distribution.

Obviously, if families associate their children's survival chances to their spending capacity, the marginal utility of income should be higher than in the case of an exogenous survival function. However, given that spending more per child must have progressively less impact on survival (decreasing elasticity), the effect generated by the internalization of child survival must dilute as wages grow.[3] Thus, the opportunity cost associated with the use of time should go down instead of up when wages grow, explaining why higher earning individuals work less hours and invest more time in child-rearing activities.

In order to illustrate our reasoning, we embed a survival function that depends on child consumption (spending) within a standard two-periods overlapping-generations (OLG) model, which is a well known framework in the literature (e.g. Kalemli-Ozcan (2003) or Galor (2011)). Essentially, the model addresses the optimal distribution of time between labor supply, quantity and quality of children. It also underscores the optimal distribution of family income between adult and child consumption (spending per child), which enter the utility function as separate sources of felicity.

The main prediction of the model is that higher wages indeed shift the use of time from labor to child-rearing. Nonetheless, we find that the reduction in hours worked is not always channelled towards child quality (more hours per child). If wages are too low, families can spend little per child regardless on the number of hours worked. Consequently, the opportunity cost of investing time in child quality may be too high to be optimal. This gives rise to a discontinuity in the optimal household allocations in the form of a wage threshold, which is what we call poverty effects.

When families are in poverty situation, i.e. wages below the threshold, they invest in each child the minimum number of hours needed to raise her: pregnancy, baby-care, etc. This is because families in such situation use higher wages to raise more children instead of

[2] The internalization of child mortality has been explored in the context of an inelastic labor supply, e.g. Blackburn and Cipriani (1998), Strulik (2004a), Strulik (2004b).

[3] Given that survival chances are bounded above, any continuous function that maps from spending to survival chances must exhibit a decreasing elasticity, at least above certain threshold.

promoting their skill formation. So, in line with the findings of Vogl (2015), the model predicts a positive gradient between family income and fertility among the poor.

This picture changes drastically when families are out of poverty. The reason is that families begin to complement the number of hours invested per child and their spending per child, that is, families that spend more per child also invest more hours per child. Because of this complementarity, families react to wage increases by investing more hours per child, promoting their quality. In turn, the optimal fertility level becomes a sort of inferior "good", meaning that parents out of poverty with higher wages raise fewer children.

At the macroeconomic scale, these outcomes are important because they provide a potential explanation for historical development dynamics. Our second main contribution is to describe how such microeconomic patterns can give rise to a virtuous cycle of economic development. More specifically, we exploit the discontinuity in the optimal household allocations to build a conditional dynamical system à la Galor (2011). The resulting theory predicts then an endogenous transition between three different regimes called Malthusian, Transitional and Modern.

During the Malthusian regime, wages are initially so low that fertility is a necessary good, whose demand increases with higher wages, while human capital remains stationary across generations. The sole source of productivity growth is innovation, which occurs at a very slow pace because of the low levels of human capital. Moreover, since wage growth promotes fertility, the positive effect of innovation on output production is partially offset and material living standards (consumption) change little across generations, hence the name Malthusian regime.

Once innovation brings wages above the poverty line, the fertility demand changes from necessary to inferior, and the number of hours invested per child starts to rise. As a result, both the accumulation of human capital and the pace of innovation take-off, boosting economic growth. During this transitional phase, the mortality decline offsets the fertility one, causing a demographic boom.

Finally, the economy converges to a modern regime in which human capital and ideas grow at a stable rate. This convergence is induced by the decreasing returns that ideas have on innovation (see Bloom et al. (2020)). Moreover, the demography stabilizes as fertility rates become practically flat, while further reductions in child mortality require more and more spending per child.

We shall stress out that, although the contributions of this paper are purely theoretic, it should be of interest for empirical researchers. First, because it highlights the importance of testing the hypothesis that families internalize the impact of their choices on their offspring's survival chances. Second, our microeconomic results suggest some potential econometric issues that may have been overlooked in the empirical literature. For instance, regarding potential attenuation bias or sensibility to sample selection.

The remainder of the paper is as follows. In the next section, we describe the model and provide the microeconomic results stated above. In the third section, we depict the resulting equilibrium dynamics. The last section concludes, making special emphasis on the empirical questions raised by the model.

2 The Model

Time is discrete and periods are ordered by $t \in \mathbb{N}_+$. Our unit of analysis is an isolated geographic area, meaning that there are no migration flows and the economy runs under autarchy. There is no public sector, consisting the economy of firms that hire labor and produce final output (supply side), and families that supply labor and purchase final output (demand side).

2.1 Demography

During each period, two overlapping generations called adults and children coexist. Individuals are homogeneous within generations and take the same decisions once they reach adulthood (children remain passive). Nonetheless, individuals may differ across generations, being the source of heterogeneity their human capital endowment h_t. Also, we assume that families are mono-parental, so the total number of adults $S_t > 0$ equals the number of families. Therefore, we speak indistinctly about adults, families or households.

Given our demographic assumptions, the micro fertility choice b_t coincides with the aggregate birth rate, which implies that the total census P_t can be expressed as

$$P_t = S_t(1 + b_t). \tag{1}$$

When a period ends, all adults die while a proportion $1 > m_{t+1} > 0$ of the children survive and reach adulthood. Again due to our assumptions, m_{t+1} measures both the survival chances of a child (micro) and the fraction of surviving children (macro). This mortality pattern implies that the number of adults living during a period is determined by the fertility and mortality rate of the preceding period,

$$S_{t+1} = S_t b_t m_{t+1}. \tag{2}$$

2.2 Supply Side

The economy produces final output Y_t combining units of efficient labor H_t and effective resources $A_t L$, where L denotes land and A_t indexes the endogenously determined efficiency with which land is employed. The production function exhibits constant returns to scale and reads as

$$Y_t = (A_t L)^{\beta} (H_t)^{1-\beta}, \quad 1 > \beta > 0. \tag{3}$$

Since all workers are identical, the aggregate efficient labor engaged in production H_t equals the total number of hours worked N_t multiplied by the average efficiency of labor or human capital h_t. In turn, the total number of hours hired N_t can be further decomposed as number of hours supplied per worker n_t multiplied by the total number of workers S_t. We then have that the following identities hold

$$H_t = N_t h_t = S_t n_t h_t.$$

Regarding the endogenous determination of the efficiency index A_t, we shall make some empirically oriented assumptions.

Most part of the empirical literature finds no population scale effects in output production, which in terms of this model would mean that $\partial(Y/N)/\partial N = 0$. However, it is generally accepted that human capital improves the productivity per hour worked, i.e. $\partial(Y/N)/\partial h > 0$. These empirical insights are consistent with the production function (3) only if the efficiency index A_t is proportional to the number of hours worked N_t. A "classical" way to justify this assumption is that hiring more hours of labor delivers efficiency gains through the division of labor.

More specifically, we consider that the efficiency index A_t grows via innovation and when the number of different tasks performed during production increases (specialization). We then approach the number of different tasks performed by the total number of hours engaged in production. Formally, we assume that

$$A_t = I_t N_t, \tag{4}$$

where I_t is the stock of ideas and N_t enters to proxy the efficiency gains derived from specializing labor. Including the division of labor in this way has no qualitative impact on our subsequent results, but allows for tractable and intuitive equilibrium dynamics.

A consequence of including the division of labor as a source of productive efficiency is that it compensates the decreasing returns to "raw" labor in output production, making the wage rate insensible to the number of hours hired. If it is assumed that there are no property rights over land as in Galor (2011), the wage rate compatible with zero profits is given by the ratio output per efficient labor

$$w_t = \frac{Y_t}{H_t} = \left(\frac{I_t L}{h_t}\right)^{\beta}. \tag{5}$$

It follows then that the wage rate grows through innovation and decays with human capital accumulation. Notice, however, that full wages or earnings per hour worked $w_t h_t$ still depend positively on the worker's human capital h_t.

Recent empirical studies find that modern economies innovate at a fairly stable rate despite devoting a growing share of resources to it. This suggests that innovation becomes harder the more ideas exist (see Bloom et al. (2020)). Following these insights, we assume that the rate of innovation depends negatively on the stock of ideas and positively on average human capital:

$$\gamma_t^I = \frac{I_{t+1} - I_t}{I_t} = \left(\frac{h_t}{I_t}\right)^{\varphi}, \quad 1 > \varphi > 0.$$

Rewriting the last expression, we derive the sequential equation governing innovation in this model

$$I_{t+1} = I_t + I_t^{1-\varphi} h_t^{\varphi}. \tag{6}$$

An appealing feature about this law of motion is that exponential innovation, which is what we call modern growth, can be achieved only if the ratio $x_t = h_t/I_t$ remains constant. The latter means that modern economic growth can be sustained only if human capital and ideas accumulate at the same speed, so the decreasing returns of ideas on innovation are countered by rising human capital.

2.3 Demand Side

Families in this economy spend all their income purchasing units of final output y_t that is consumed by the adult c_t and its children $b_t z_t$, being z_t the amount of income spend per child. In turn, the sole source of family income is labor $w_t h_t n_t$, which depends on the adult's human capital h_t, the number of hours she works n_t, and the wage rate paid in the labor market w_t. There is no savings market, so the household's budget constraint reads as

$$w_t h_t n_t = c_t + b_t z_t. \tag{7}$$

Besides supplying labor, families can devote time to rear children. Following the standard, we assume that raising a child requires a minimum time investment of $\bar{\epsilon}$ hours that must be devoted to baby-care and other necessary activities. In addition, families can invest ϵ_t hours per child to promote their quality. Denoting the total number of available hours by $T > 0$, the distribution of time between working and child-rearing activities must then satisfy the time constraint

$$T = n_t + (\bar{\epsilon} + \epsilon_t) b_t. \tag{8}$$

As advanced, the quality or human capital of a child h_{t+1} depends on the amount of parental attention received during childhood ϵ_t but also on the skill level of their parents h_t. Since the point of this paper is to identify a causal effect between full wages $w_t h_t$ and family composition, we consider a production function of human capital in which the parental skill level h_t enters in a multiplicative manner:

$$h_{t+1} = (\theta + \epsilon_t) h_t. \tag{9}$$

Under standard preferences, the optimal household allocations remain neutral to h_t entering the production function, but still allows for sustained accumulation across generations. In turn, the term $\theta > 0$ ensures that children reach adulthood endowed with a positive skill level $h_{t+1} > 0$ even if $\epsilon_t = 0$.[4]

The main feature of this model is that families take into account that their children's survival chances m_{t+1} vary with nutrition, clothing, medicines and other forms of consumption captured by z_t. Formally, we assume that families internalize the survival function

$$m_{t+1} = M[z_t], \tag{10}$$

which is strictly increasing $M_z[z_t] > 0$ and concave $M_{z,z}[z_t] < 0$. Given that survival chances are bounded above by definition, they must become less and less sensible to higher expenditures, that is, the survival function must exhibit a decreasing elasticity.[5] So, if the elasticity of the survival function is denoted as

$$\varepsilon_{M,z} = \frac{\partial M[z_t]}{\partial z_t} \frac{z_t}{M[z_t]},$$

the survival function satisfies the following two properties

$$i)\ \frac{\partial \varepsilon_{M,z}}{\partial z_t} < 0, \qquad ii)\ \lim_{z \to \infty} \varepsilon_{M,z} = 0.$$

[4] This assumption emphasises that children already acquire skills by their own through genetics, observation, imitation, social interaction and other channels, whereas teaching, reading or playing with them spurs the process of human capital accumulation.

[5] This must be true at least above certain spending level.

For simplicity, we consider that families have certainty about the proportion of children that will reach adulthood, but not about which specific child will do it. In consequence, they treat all their children equally. The utility function of an adult is then defined over its own consumption level, their children's consumption level and the quantity-quality mix of their surviving offspring:

$$U[c, m, b, h, z] = \alpha_c \ln c_t + \alpha_q \ln[m_{t+1} b_t h_{t+1}] + \alpha_z \ln z_t, \tag{11}$$

where parameters $\alpha_i > 0, \quad i = c, q, z$ weight the importance of each variable in the preferences.

Finally, in order to ensure the existence of an interior solution $\epsilon_t, b_t > 0$, we shall impose two conditions over the parameters. If the first condition, which reads as $\alpha_q > \alpha_z$, is not met, having a surviving offspring would generate too little utility and families would find it optimal to have no children, i.e. $b_t = 0$. The second one, $\theta > \bar{\epsilon}$, guarantees that the fixed time cost of fertility is not so high that investing time in child quality entails a suboptimal choice regardless of the family income.

Definition 1. *Given* $\{h_t, w_t\}$*, a household allocation* $\phi_t^* \equiv \{c_t^*, b_t^*, \epsilon_t^*, z_t^*\}$ *is said to be optimal if it maximizes* (11)*, subject to equations* (7)*,* (8)*,* (9)*,* (10)*.*

Recall that the opportunity cost of investing $b_t(\epsilon_t + \bar{\epsilon})$ hours in child-rearing activities depends on the marginal utility of income denoted by λ_t (shadow value). The main effect generated by the internalization of child survival is that it increases the marginal utility of income, hence opportunity cost of rearing children, compared to the case of an exogenous survival, i.e. $\varepsilon_{M,z} = 0$ (see Kalemli-Ozcan (2003)). This intuition is clearly reflected by the first order condition (FOC) of child consumption z_t:

$$\frac{\alpha_z + \alpha_q \varepsilon_{M,z}}{z_t b_t} = \lambda_t.$$

But even more importantly, the FOC indicates that the magnitude of this effect dilutes as families spend more per child because the elasticity of child survival falls. This means that the optimal number of hours devoted to child-rearing activities $b_t^*(\epsilon_t^* + \bar{\epsilon})$ should be positively associated with the per child expenditure through the elasticity of the survival function. Indeed, after doing some algebra with the FOCs (see Appendix 1), we can derive an equation that shows it:

$$b_t^*(\epsilon_t^* + \bar{\epsilon}) = \frac{\alpha_q T}{\alpha_c + \alpha_q} - \frac{(\alpha_z + \alpha_q \varepsilon_{M,z})T}{\alpha_c + \alpha_q}. \tag{12}$$

Thus, if higher earning families spend more per child, they should also devote more time to rear children.

In this model, there are two reasons why higher earning parents would spend more per child. First, because families derive utility from the material living standards enjoyed by their children (more consumption or spending). Second, and maybe more important, because the survival chances of their offspring's hinge on it. In any case, the model does predict that parents with higher wages spend more per child (see Lemma 1), and thus, they also devote more time to rear children (see Lemma 2).

Lemma 1. *The optimal spending level per child z_t^* is a strictly increasing function of the full wage $w_t h_t$.*

Proof: See Appendix 1. □

Recall that spending more per child has progressively less impact on child survival (properties *i*) and *ii*)). Therefore, equation (12) establishes that the causal association between earnings and the number of hours devoted to rear children is concave-like. This detail is empirically relevant because linear estimation methods may suffer from attenuation bias when trying to measure correlation or causation between variables that share a non-linear association.

Lemma 2. *The optimal number of hours invested in childrearing $b_t^*(\bar{\epsilon} + \epsilon_t^*)$ is a strictly increasing function of the full wage $w_t h_t$.*

Proof: See Appendix 1. □

Surveys on the use of time indicate that parents with higher wages devote more time to rear children, which is consistent with Lemma 2. The evidence also indicates that this pattern is not driven by rising fertility, it emerges instead because higher earning families invest more hours per child (see Guryan et al. (2008) or Doepke et al. (2019)). Consistent with the latter, this model predicts that higher wage individuals invest more hours per child, but once the full wage $w_t h_t$ has surpassed a certain poverty threshold (see Proposition 1).

Proposition 1. *When the full wage is below a certain poverty threshold $w_t h_t \leq k$, families do not invest time in promoting their children's quality, i.e. $\epsilon_t^* = 0$. By contrast, when the full wage surpasses the threshold $w_t h_t > k$, families invest*

$$\epsilon_t^* = (\theta - \bar{\epsilon}) \left(\frac{1}{\alpha_z + \alpha_q G[w_t h_t]} - \frac{1}{\alpha_q} \right) \tag{13}$$

hours per child, where $G[w_t h_t]$ is a strictly decreasing function of $w_t h_t$ bounded below by zero.

Proof: See Appendix 1. □

The intuition behind Proposition 1 can be easily described if the first order conditions (FOCs) with respect to the number of children b_t and child consumption z_t are combined, which yields the following equation:

$$\frac{z_t^*}{\alpha_z + \alpha_q \varepsilon_{M,z}} - \frac{z_t^*}{\alpha_q} = \frac{w_t h_t (\epsilon_t^* + \bar{\epsilon})}{\alpha_q}. \tag{14}$$

What this expression tells us is that families complement their spending per child z_t^* and the number of hours in quality ϵ_t^*, i.e. families that spend more per child also invest more hours per child. The reason is simple, spending more per child improves the child's survival chances, which reduces the opportunity cost of investing time in activities that promote the skill formation of children.

Corollary 1. *Families that spend more per child also invest more hours in child quality. Thus, higher skilled children exhibit better survival chances.*

Proof: The first claim boils down to $w_t h_t(\bar{\epsilon} + \epsilon_t^*)$ being an increasing function of z_t^*, and hence, $\partial \epsilon_t^* / \partial z_t^* > 0$. The second claim states correlation and it is established just by noticing that $\partial m_{t+1}^* / \partial z_t^* > 0$ and $\partial h_{t+1}^* / \partial \epsilon_t^* > 0$. $\square$

A brief note about Corollary 1 is in order. What it states is not causation but correlation between skills and "health" (survival chances) across individuals. For instance, children coming from richer families should be taller and speak more languages. From an econometric point of view, this result implies that part of the correlation between survival related and quality related measures across individuals, e.g. health and income or education, would be spurious. Thus, estimates that do not take into account the internalization of child survival may be overrated or even spurious.

Back to Proposition 1, it is reasonable to think that poverty constraints the capacity of families to invest time in child quality. Most models in the literature induce such poverty restrain by imposing a subsistence condition over consumption. In our case, the model generates it endogenously in the form of a threshold as stated in Proposition 2. The exact value of this poverty line depends on the parametrization of the survival function.[6]

Definition 2. *Households facing earnings below threshold $w_t h_t \leq k$ are called poor.*

The importance of poverty in the accumulation of human capital can be depicted if equation (9) is evaluated taking into account Proposition 1. We obtain then that children coming from poor families reach adulthood endowed with a skill level equal to

$$h_{t+1}^* = \theta h_t, \qquad w_t h_t < k, \tag{15}$$

whereas children coming from families out of poverty, which receive more parental attention, i.e. $\epsilon_t^* > 0$, reach adulthood endowed with

$$h_{t+1}^* = (\theta + \epsilon_t^*) h_t, \qquad w_t h_t \geq k. \tag{16}$$

If there were cross-sectional heterogeneity in human capital endowments, these equations could explain why the skill level of children is caused by their parent's income as some empirical studies find (e.g. Duncan et al. (2011) and Dahl and Lochner (2012)). Moreover, since ϵ_t^* is a concave increasing function of earnings, the model predicts that the association between family income and children's skills is concave too, which is in line with the findings of Løken et al. (2012).

In our case, rather than the cross-sectional implications, we are interested in the temporal evolution of human capital. When an economy is undeveloped or in poverty situation, lack of investment in child quality leads to a rate of human capital accumulation across generations equal to

$$\gamma_t^h = \frac{h_{t+1} - h_t}{h_t} = \theta - 1, \qquad w_t h_t < k. \tag{17}$$

[6] If the elasticity of the survival function is assumed to depend on some dynamic macroeconomic factors such as population density, innovation or public spending, this poverty threshold would be dynamic, which could give rise to poverty traps and other interesting dynamics.

If it is assumed that $\theta \approx 1$, the model would predict that human capital levels remain more or less constant in undeveloped economies, maybe rising slowly through natural selection and similar mechanism. But once an economy has managed it out of poverty, differences in human capital across generations emerge driven by increasing investment in child quality:

$$\gamma_t^h = \epsilon_t^* + \theta - 1 > 0, \qquad w_t h_t \geq k. \tag{18}$$

This discontinuity induced by poverty suggest that a potential trigger of the historical rise in human capital and demographic transition is innovation, like in Galor (2011). But in contrast to the latter, the link between human capital accumulation and innovation is not assumed as a property of the human capital production function, it emerges endogenously through the household's optimization process.

There is another interesting lesson that can be extracted regarding child consumption and the use of time. When families invest the minimum per child $\epsilon_t^* = 0$, equation (14) implies that the income/wage elasticity of child consumption is positive but below one (necessary good). By contrast, the income/wage elasticity of child consumption equals one for families out of poverty. The fact that child consumption is a necessary good for poor families indicates that the internalization of child mortality induces a certain hierarchy of needs in which survival precedes quality.

Corollary 2. *Child consumption is a necessary good for poor families.*

Proof: See Appendix 1. □

In any case, families below and above the poverty line increase their expenditure per child when they face higher earnings. The model thus predicts a positive causal association between family earnings and offspring's survival chances, which is consistent with the empirical evidence (see Currie (2009)). Moreover, from a macroeconomic point of view, this causal association can explain the well-known co-movement between income per capita and child mortality rates (see Steckel (2008)).

Corollary 3. *Higher earning parents spend more per child, and thus, their children grow up healthier and have better survival chances.*

Proof: It follows directly from evaluating equation (10) taking into account Lemma 1. □

Now, turning the focus to the optimal fertility rule, we find that poverty in the sense of Definition 2 gives rise to a non-monotonic gradient in line with the findings of Vogl (2015). Below the poverty threshold, families invest in each child the minimum number of hours needed to raise her (Proposition 1), using higher earnings to raise more children instead of promoting their quality. This is reflected by the fertility rule:

$$b_t^* = \frac{\alpha_q}{\alpha_c + \alpha_q} \frac{w_t h_t T}{w_t h_t \bar{\epsilon} + z_t^*}, \qquad w_t h_t \leq k, \tag{19}$$

where we recall that the wage elasticity of z_t^* is below one when $w_t h_t \leq k$. By contrast, families out of poverty prefer to improve the skill formation of their children and reduce their fertility levels when they face higher earnings. But notice that, since the effect induced by the internalization of child mortality dilutes as families spend more per child, the negative causal effect of higher wages on fertility weakens as well, resulting in a convex-decreasing gradient for families out of poverty:

$$b_t^* = \frac{\alpha_q}{\alpha_c + \alpha_q} \frac{T}{\theta + \epsilon_t^* + \bar{\epsilon}}, \qquad w_t h_t \geq k. \tag{20}$$

Corollary 4. *Families in poverty situation consider fertility as a necessary good. By contrast, fertility is an inferior good for families out of poverty.*

Proof: See Appendix 1 □

The econometric derivatives of Corollary 4 are maybe the strongest of all. Any econometric method based on monotonicity arguments, which are the most, would be heavily sensible to the sample selected. If the sample consists mainly of poor families/countries, the result would be the exact opposite to that obtained with a sample consisting mainly of families out of poverty. Moreover, if the poverty line is dynamic due to some macroeconomic factor affecting the elasticity of the survival function, the econometric issues would be even more.

The last point to be addressed regards the labor supply n_t^*. Since families use their available time either working or rearing children, Proposition 1 implies that the labor supply must be a decreasing function of earnings above and below the poverty threshold.

Corollary 5. *The optimal labor supply n_t^* is a strictly decreasing function of the full wage $w_t h_t$*

Proof: It follows directly from the fact that $n_t^* = b_t^*(\epsilon_t^* + \bar{\epsilon})$ and Lemma 2. □

Although conventional economic thinking postulates that higher wages should encourage the supply of labor, a decreasing labor supply is consistent with some recent empirical studies finding that, within and across countries, the number of hours worked per worker varies inversely to the wage or income (see Bick et al. (2018)). Moreover, since our model yields secular wage growth, hence decline in hours worked, Corollary 5 is also consistent with the cross-country time series evidence (see Boppart and Krusell (2020)).

The main lesson from this section is that the internalization of child mortality can explain why higher earning individuals work fewer hours, invest more time in child-rearing and raise fewer children. The reason is that the opportunity cost of devoting time to rear children varies inversely with wages when families internalize the positive effect of spending more on children survival. Put differently, the internalization of child mortality increases the concavity of the preferences over income, giving rise to income effects in the direction mentioned.

3 Equilibrium Dynamics

The model just described has flow- and state-like variables. The former are determined within each period by the state of economy and are the optimal household allocations ϕ_t and the wage rate w_t. State-like variables change between periods and are the stock of human capital h_t, the stock of ideas I_t, and the adult population size S_t. So as to provide a transparent discussion, we differentiate now between intra-generational equilibrium and equilibrium path.

Definition 3. *Given a state of the economy $\Omega_t = \{h_t, I_t, S_t\}$, an intra-generational equilibrium is a set $\Psi_t^* = \{w_t^*, \phi_t^*\}$ such that w_t^* clears the labor market and ϕ_t^* is an optimal household allocation.*

The term intra-generational equilibrium refers to the determination of the control-like variables. From our asserted hypothesis about wage rate formation, it follows that there are no firm benefits regardless on how much human capital is hired. This ensures that all the human capital supplied by families is hired and the labor market clears,

$$H_t^* = n_t^* h_t S_t. \tag{21}$$

Moreover, equation (5) implies that the equilibrium wage rate is also unique for a given state of the economy. Thus, the intra-generational equilibrium exists and is unique.

Lemma 3. *There is a unique inter-generational equilibrium denoted as $\Psi_t^* = F[\Omega_t]$.*

The temporal evolution of each state-like variable is governed by its own difference equation that depends on the current state of the economy and the control-like variables, so we can define the following system of recursive equations:

$$\Omega_{t+1} = G[\Psi_t, \Omega_t],$$

where G refers to the right hand-side of equations (2), (6) and (9). By plugging the intra-generational equilibrium into this dynamical system, we obtain that the equilibrium dynamics are governed by a first-order, autonomous recursive system:

$$\Omega_{t+1} = G[\Psi_t^*, \Omega_t] = G[F[\Omega_t], \Omega_t] = \bar{G}[\Omega_t]. \tag{22}$$

Definition 4. *An equilibrium path is an infinite sequence $\{\Omega_t^*\}_{t=0}^{\infty}$ that solves the dynamical system* (22) *for a given initial condition Ω_0.*

Notice that the domain of definition or feasibility set of each state variable is the set of positive real numbers, $h_t, S_t, I_t > 0$. The fact that the optimal household allocations are uniquely defined and take finite positive values for any feasible state of the economy implies that the dynamical system (22) maps from the feasibility set back to itself, i.e. $\bar{G} : \mathbb{R}_+^3 \to \mathbb{R}_+^3$. Thus, the existence and uniqueness of an equilibrium path is guaranteed for any initial condition belonging to the feasibility set $\Omega_0 \in \mathbb{R}_+^3$.

Lemma 4. *There is a unique equilibrium path.*

Our interest lies in studying unified growth dynamics. Therefore, we assume that the initial state of the economy is such that the full wage start out below the poverty threshold stated in Definition 2. In addition, we make certain parametric assumptions that will simplify the exposition and have no qualitative impact on the equilibrium dynamics.

Assumption 1. *It holds that $\bar{\epsilon} = 0$, $\theta = 1$ and $L = 1$. Moreover, the initial state of the economy is such that $h_0 = 1$ and $I_0 < k^{1/\beta}$.*

Due to the discontinuity in the optimal household allocations induced by poverty, the dynamical system describing the equilibrium dynamics is conditional in the sense that the right hand-side of (22) changes once earnings have surpassed the poverty line. That is why we split the remaining analysis in two parts.

3.1 Malthusian Regime

During this developmental stage, earnings are so low that both fertility and child consumption are necessary goods (Corollaries 3 and 4), and families do not devote family time to promote their children's human capital (Proposition 1). The lack of parental investment in child quality translates into low and stationary levels of human capital (equation (17)). Consequently, innovation cannot reach an exponential pace and evolves slowly across generations (see equation (6)).

Proposition 2. *The Malthusian Regime has a finite duration, ending in period $\bar{t} < \infty$. During this stage, the pace of innovation is slower than exponential and is partially offset by rising fertility rates, so output per person increases little across generations.*

Proof: See Appendix 2.

Although the little innovation happening during this regime suffices to elicit some modest wage growth, it does not fully translate into better material living standards (output per person). The reason is that families channel part of the wage increment into higher fertility levels (Corollary 3), hence the name of Malthusian regime. In fact, living standards change so little that child consumption remains practically flat. To see the latter, notice that child consumption becomes a unit of final output $z_t^* = 1$ under Assumption 1.

Regarding the demography, a practically stationary expenditure per child involves stagnant child mortality rates (equation (10)). Thus, population growth during the Malthusian Regime is driven by fertility (equation (2)). Given that the latter is an increasing function of earnings, the model predicts a positive causal link between economic and population growth during this developmental stage. In other words, prior to the Industrial Revolution, richer countries should also being more densely populated, consistent with conventional wisdom (see X, X or X).

3.2 Transitional Dynamics and Modern Regime

Once innovation has brought earnings above the poverty line, i.e. $w_t^* h_0 > k$, the equilibrium dynamics change. Fertility becomes inferior (Corollary 4), the income elasticity of child consumption increases up to one (Corollary 3) and families begin to invest more time per child (Proposition 1). Consequently, differences in human capital across generations take-off ((16)), stimulating innovation and economic growth (equation (6)).

Proposition 3. *Once the Malthusian Regime ends, human capital accumulation and innovation accelerate, converging asymptotically to an exponential rate of growth.*

Proof: See Appendix 3.

The sharp acceleration initiated with the end of the Malthusian era is only transitional, and the economy ends stabilizing in a Modern regime. This final developmental stage is characterized by a common rate of innovation and human capital accumulation, which is induced by the decreasing returns that ideas have on innovation. To see the latter, let us define the ratio between human capital and ideas as $x_t^* = h_t^*/I_t^*$, whose law of movement is

$$\gamma_t^x = e_t^* - (x_t^*)^{\varphi} \,. \tag{23}$$

The fact that e_t^* converges asymptotically to a constant e^{ss} as $w_t h_t$ tends to infinite (Proposition 1) implies that this law of motion exhibits a unique, asymptotic steady state:

$$x^{ss} = (e^{ss})^{\frac{1}{\varphi}},$$

which is globally stable. The asymptotic converge towards this steady state means that human capital and ideas tend to grow at the same positive rate over the long-run. As a result, the virtuous cycle between innovation, wage growth and human capital accumulation ends freezing, making wage rates stationary. Thus, in contrast to the Malthusian Regime, economic growth is mainly driven by human capital accumulation during the Modern Regime.

As for the demography during this regime, the decreasing elasticity of the survival function makes the child survival rate less and less sensible to economic development (increments in z_t^*), placing again fertility as the engine of population growth. In turn, because fertility is an inferior good, the model predicts that the fertility rate converges asymptotically to

$$b_t^{ss} = \frac{\alpha_q}{\alpha_c + \alpha_q} \frac{T}{\theta + \epsilon^{ss}}.$$

Consequently, the rate of population growth slows down, heading asymptotically towards a stable rate equal to

$$\gamma_S^{ss} = \frac{\alpha_q}{\alpha_c + \alpha_q} \frac{T}{\theta + \epsilon^{ss}} - 1.$$

Whether the gradient between population and economic growth remains positive along the whole transition or turns negative is not determinate and depends on the parameterization. In fact, there could be the case that it turns from positive to negative, and back to positive, depending on the elasticity of the survival function and other parameters.

4 Conclusions

The bottom line of this paper is to present a novel mechanism by which economic growth shifts the use of time from labor towards child-rearing activities despite falling fertility rates. Our explanation is based on the hypothesis that families internalize their children's mortality rate as a function of their children's consumption level, which captures the positive effect of a better nutrition, clothing, medicines, health-care, etc.

A key point about this hypothesis is that, by spending more per child, families also reduce the risk of investing time in children that may not reach adulthood. This implies that economic growth in the form of higher wages gives families the option to reduce this risk by rising their spending per child, encouraging a change in the use of time from quantity towards quality of children. In turn, the latter loops back as further economic growth in the future by boosting innovation rates.

Although our paper is purely theoretic, it advances some econometric issues that may have been overlooked in the empirical literature. A first question refers to the correlation between health related and skill related measures across individuals such as height and income. The model predicts that any change encouraging time investment in child quality also leads to higher spending per child (and vice-versa). This means that measures of health (survival chances) and skills move in the same direction across individuals without any causal link between them. Thus, estimates that do not control for the internalization of child mortality may be overestimated or be even spurious.

The model also predicts a discontinuity in the optimal household allocations induced by poverty. Its empirical relevance lies in the fact that some of the gradients disappear or change its direction. The composition of the sample thus gains critical importance. If it consists mainly of poor families or undeveloped economies, the results would be the opposite than if it consists of families or economies out of poverty. For instance, for poor economies, the model suggest that an exogenous positive shock to health (survival chances) would have no meaningful impact on human capital, and hence, on economic growth (e.g. Acemoglu and Johnson (2014)). However, if the sample is mainly composed by rich economies, the same shock has a significant positive impact on human capital and economic growth (e.g. Bloom et al. (2014)).

The main empirical implication derived from this paper regards the importance of testing the hypothesis of whether families internalize their children's survival chances. If the answer is no, then this kind of unified growth model and its insights can be straightly rejected. But if the answer is yes, besides the array of considerations just mentioned, measuring the elasticity of the survival function becomes a crucial parameter for public policy. The reason is that it determines the magnitude with which families react to exogenous changes in earnings, taxation, transfers, provision of public goods...

Appendices

Appendix 1: Characterization of the optimal household allocations

Substituting out equations (9) and (10) into the utility function, and taking into account the time constraint, we obtain that the Lagrangian to the maximization problem described in Definition 1 reads as

$$L[\phi] \equiv \alpha_c \ln(c_t)+\alpha_z \ln(z_t)+\alpha_q \left(lnb_t + lnM[z_t] + ln(\theta+\epsilon_t)\right)+\Psi+\lambda_t \left(w_t h_t (T-(\epsilon_t+\bar{\epsilon})b_t)-c_t-z_t b_t\right),$$

where λ_t is the typical shadow value and Ψ is a composition of parameters that does not affect the optimal choices. The first order conditions with respect to each control are then

$$\frac{\alpha_c}{c_t} = \lambda_t, \tag{24}$$

$$\frac{\alpha_q}{b_t(w_t h_t(\epsilon_t+\bar{\epsilon})+z_t)} = \lambda_t, \tag{25}$$

$$\frac{\alpha_z+\alpha_q \varepsilon_{M,z}}{b_t z_t} = \lambda_t. \tag{26}$$

$$\frac{\alpha_q}{(\theta+\epsilon_t)w_t h_t b_t} = \lambda_t. \tag{27}$$

Our first step is to obtain the optimal adult consumption level c_t^* and the marginal utility of income at the optimum λ_t^*. To that end, we begin by combining equations (24) and (25) to derive the expression

$$c_t = \frac{\alpha_c}{\alpha_q} b_t(w_t h_t(\epsilon_t+\bar{\epsilon})+z_t). \tag{28}$$

Plugging equation (28) into the budget constraint yields that

$$b_t^*(w_t h_t(\epsilon_t^*+\bar{\epsilon})+z_t^*) = \frac{\alpha_q}{\alpha_c+\alpha_q} w_t h_t T. \tag{29}$$

Now, if expression (29) is substituted out back into (28), it follows that the optimal adult consumption level reads as

$$c_t^* = \frac{\alpha_c}{\alpha_c+\alpha_q} w_t h_t T, \tag{30}$$

while the marginal utility of income at the optimum is

$$\lambda_t^* = \frac{\alpha_c+\alpha_q}{w_t h_t T}. \tag{31}$$

Using the optimal adult consumption level and the marginal utility of income at the optimum, we can obtain the equation relating child rearing time and child consumption given in the main text. If we take into account the marginal utility of income at the optimum, equation (26) implies that the optimal fraction of income spend on children must satisfy

$$\frac{b_t^* z_t^*}{w_t h_t} = \frac{(\alpha_z+\alpha_q \varepsilon_{M,z})T}{\alpha_c+\alpha_q}. \tag{32}$$

Re-arranging the FOC of fertility (equation (25)), we obtain expression

$$b_t(\epsilon_t + \bar{\epsilon}) = \frac{\alpha_q w_t h_t}{\lambda_t} - \frac{b_t z_t}{w_t h_t},$$

which plugging in equations (31) and (32) becomes

$$b_t^*(\epsilon_t^* + \bar{\epsilon}) = \frac{\alpha_q T}{\alpha_c + \alpha_q} - \frac{(\alpha_z + \alpha_q \varepsilon_{M,z})T}{\alpha_c + \alpha_q}. \tag{33}$$

Equation (33) establishes then that $b_t^*(\epsilon_t^* + \bar{\epsilon})$ is a strictly increasing function of z_t^* through $\varepsilon_{M,z}$ (properties i and ii).

The next step is to establish the complementary relationship between z_t^* and ϵ_t^*, which follows directly from combining equations (25) and (26):

$$\frac{z_t^*}{\alpha_z + \alpha_q \varepsilon_{M,z}} - z_t^* = \frac{w_t h_t(\epsilon_t^* + \bar{\epsilon})}{\alpha_q}. \tag{34}$$

Due to properties *i)* and *ii)*, the left hand-side of the previous equation is a strictly convex-increasing and differentiable function of z_t^*. So, the inverse function theorem applies, and z_t^* must be then a strictly concave-increasing function of $w_t h_t(\epsilon_t^* + \bar{\epsilon})$, which we denote as $z[w_t h_t(\epsilon_t^* + \bar{\epsilon})]$. Consequently, the optimal child consumption level z_t^* will be an increasing function of the full wage as long as $\partial \epsilon_t^* / \partial w_t h_t > -1$, which we will show to be true.

Combining equations (25) and (27), we obtain that the optimal investment in child quality must be a root of

$$\frac{z[w_t h_t(\epsilon_t^* + \bar{\epsilon})]}{w_t h_t} = \theta - \bar{\epsilon}.$$

Notice that the left hand-side is a strictly increasing function of ϵ_t^*, whereas the right hand-side is a flat line. If an interior root exists, it must be unique by virtue of the mean value theorem. Recall that $z[w_t h_t(\epsilon_t^* + \bar{\epsilon})]$ is concave in $w_t h_t$, so the left hand-side must be decreasing in $w_t h_t$. Consequently, if an interior root exists, the implicit function theorem establishes that $\partial \epsilon_t^* / \partial w_t h_t \geq 0$. Thus, the optimal spending level per child z_t^* must be a strictly increasing function of the full wage $w_t h_t$. In turn, the latter implies that the optimal number of hours devoted to childrearing $b_t^*(\epsilon_t^* + \bar{\epsilon})$ must be also a strictly increasing function the full wage $w_t h_t$. This completes the proofs of Lemmata 1 and 2.

Interior Solution. Suppose that an interior solution exists, i.e. $\epsilon_t^* > 0$. Then, equations (25) and (27) imply that

$$z_t^* = w_t h_t(\theta - \bar{\epsilon}).$$

Using this spending rule, equations (25) and (26) yield the optimal time investment in child quality

$$\epsilon_t^* = (\theta - \bar{\epsilon}) \left(\frac{1}{\alpha_z + \alpha_q G[w_t h_t]} - \frac{1}{\alpha_q} \right)$$

where $G[w_t h_t] = \varepsilon_{M,z}$. From the last line, it follows that an interior solution exists only if $w_t h_t \geq k > 0$, being k an implicit parameter determined by mapping $M[\bullet]$. With this information, we can exploit equations (24), (25) and the budget constraint to obtain the optimal fertility rule

$$b_t^* = \frac{\alpha_q}{\alpha_c + \alpha_q} \frac{T}{\theta + \epsilon_t^* + \bar{\epsilon}}.$$

Corner Solution. As noted above, if $w_t h_t \leq k$, then the optimal investment in child quality is zero:

$$\epsilon_t^* = 0.$$

Taking into account the last line, equations (25) and (26) imply that the optimal child consumption level must be the root of

$$\frac{\alpha_q z_t^*}{\alpha_z + \alpha_q \varepsilon_{M,z}} - z_t^* = w_t h_t \bar{\epsilon}.$$

Due to the above asserted hypothesis on $\varepsilon_{M,z}$ (properties i) and ii)), the left hand-side of the previous equation is a strictly continuous, convex-increasing function of z_t^*. Thus, its inverse function exists, establishing that z_t^* is a unit-value, continuous and concave-increasing function of $w_t h_t \bar{\epsilon}$. This spending rule jointly with equations (24), (25) and the budget constraint imply now an optimal fertility level equal to

$$b_t^* = \frac{\alpha_q}{\alpha_c + \alpha_q} \frac{T}{\bar{\epsilon} + \frac{z_t^*}{w_t h_t}}.$$

Given the concavity of z_t^* with respect to $w_t h_t$, it holds that $\partial b_t^* / \partial w_t h_t > 0$.

Appendix 2: Proof of Proposition 2

In the first place, we will show that the wage rate grows secularly during the Malthusian Regime, bringing earnings above the poverty line. Recall that the equilibrium wage rate under Assumption 1 is

$$w_t^* = \left(\frac{I_t^*}{h_t^*} \right)^\beta,$$

whose rate of growth reads as

$$\gamma_t^w = \left(\frac{I_{t+1}^*}{I_t^*} \right)^\beta \left(\frac{h_t^*}{h_{t+1}^*} \right)^\beta - 1.$$

Since there is no parental investment in child quality during the Malthusian Regime (Proposition 1), human capital remains constant over generations at its initial level $h_t^* = h_0$, whereas the law governing innovation becomes

$$\gamma_t^I = \left(\frac{h_0}{I_t^*} \right)^\varphi.$$

Plunging this information in the law of wage growth yields that

$$\gamma_t^w = \left(\left(\frac{h_0}{I_t^*} \right)^\varphi + 1 \right)^\beta - 1 > 0, \quad \forall h_0 / I_t^* > 0,$$

which proves that the equilibrium wage rate grows sustainedly over generations, and hence, there is a period $\bar{t} < \infty$ at which earnings surpass the poverty threshold, ending the Malthusian regime.

Now we shall establish that the rate of innovation is slower than exponential during the Malthusian Regime. Recall that the law governing innovation during this stage reads as

$$\gamma_t^I = \left(\frac{h_0}{I_t^*}\right)^{\varphi}.$$

A sustained exponential rate of innovation means that

$$\gamma_t^I = \gamma_{t+1}^I = ... = \gamma_n^I,$$

which requires that the ratio human capital to ideas remains constant over generations,

$$\frac{h_0}{I_t^*} = \frac{h_0}{I_{t+1}^*} = ... = \frac{h_0}{I_n^*}.$$

However, notice that the process of innovation always yields new ideas despite stagnant human capital

$$\gamma_t^I = \left(\frac{h_0}{I_t^*}\right)^{\varphi} > 0, \quad \forall h_0/I_t^*,$$

so the stock of ideas grows sustainedly over generations

$$I_t^* < I_{t+1}^* < ... < I_n^*.$$

The combination of stagnant human capital and innovation implies that the ratio of human capital to ideas declines over generations, and thus, the rate of innovation falls over generations,

$$\gamma_t^I > \gamma_{t+1}^I > ... > \gamma_n^I,$$

and the rate of innovation is below exponential during the Malthusian Regime.

Finally, we will prove that innovation is partially offset by fertility during the Malthusian Regime, so income per capita varies less than the stock of ideas. Income per capita along the equilibrium is given by

$$\bar{y}_t^* = \frac{Y_t^*}{P_t^*} = (I_t^*)^{\beta} (h_t^*)^{1-\beta} \frac{n_t^*}{(1+b_t^*)},$$

where the last line takes into account equations (1), (3) and (4). The premise that we want to prove false is

$$\frac{\bar{y}_{t+1}^*}{\bar{y}_t^*} \geq \frac{I_{t+1}^*}{I_t^*}.$$

Under Assumption 1, the premise implies that

$$\frac{1+b_t^*}{1+b_{t+1}^*} \geq \left(\frac{I_{t+1}^*}{I_t^*}\right)^{1-\beta}.$$

From the above paragraphs, we know that

$$\left(\frac{I_{t+1}^*}{I_t^*}\right)^{1-\beta} > 1,$$

whereas Corollary 3 implies that the fertility rate grows over generations driven by wage increases, so it holds that

$$\frac{1+b_t^*}{1+b_{t+1}^*} < 1, \quad \forall t \leq \bar{t}.$$

We thus reach a contradiction and the premise must be false. So, it must be true that income per capita grows over generations less than the stock of ideas:

$$\frac{\bar{y}^*_{t+1}}{\bar{y}^*_t} < \frac{I^*_{t+1}}{I^*_t}.$$

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