The energy transition and fossil energy use

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Abstract

Achieving the energy transition is among global priorities of the 21^{st} century, one key element to success being the development of affordable renewable technology to compete with fossil energy. While technological progress seems already biased in favor of the renewable sector since the 70's, we had to wait until 2005 to observe a sharp increase of its share in the US energy mix. In this paper I develop a theoretical model of energy transition, with exogenously biased technological change, able to explain this delay through a lasting capital effect in favor of fossil energy. The presence of embodied technical change in the energy sector creates a lasting capital effect in favor of the fossil sector, which slows down the capacity to close polluting units. This mechanism postpones the effect of directed technical change, therefore technological progress by itself is not enough to achieve a quick energy transition unless its rate is really high. I also show that in this framework, the optimal carbon tax is less sensitive to the discount rate.

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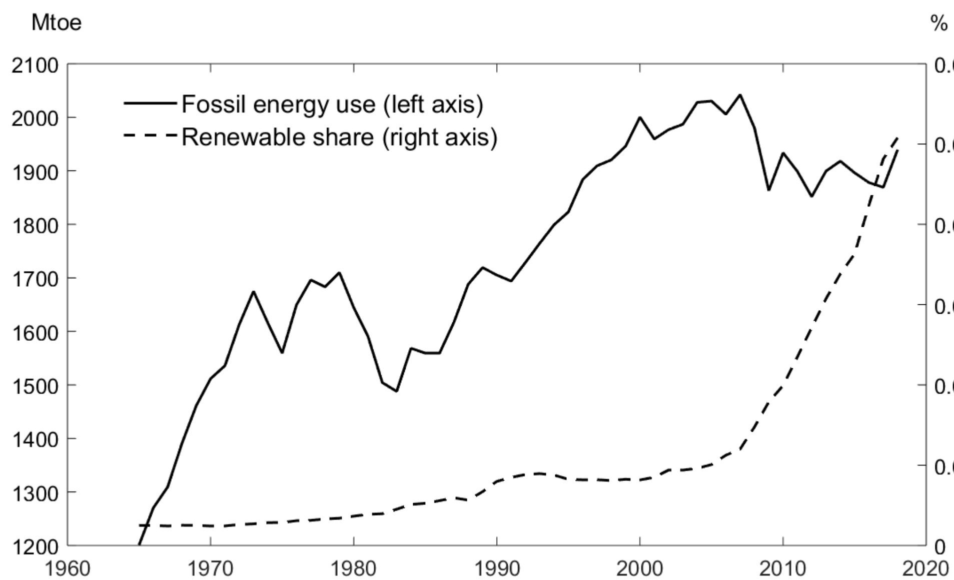
1 Introduction

Technological and Institutional co-evolution in our use of fossil fuel during the 20^{th} century has led to today's policy inertia towards mitigation of climate change (Unruh (2000)). Escaping this "carbon lock-in" situation and achieving the energy transition is among global priorities of the 21^{st} century and requires to decrease our use of carbonized energy. Figure 1 highlights that although the US is increasing its share of renewable energy, the country is also increasing its use of fossil energy. The absence of a sharp drop in our use of carbonized energy might become problematic in a context of climate urgency,[2] pollution emissions depending on the level of fossil energy use and not on its share. In this context, 2 interesting questions might be raised: i) is the increase of the renewable energy share sufficient to limit risks of climate change ? ii) Why does the US exhibit an increase of both its share of renewable energy and its use of fossil energy? I examine how these questions can be answered in a structural change model with embodied technological progress.



source: BP statistical review of world energy

Figure 1: Fossil energy use and renewable energy share in the US

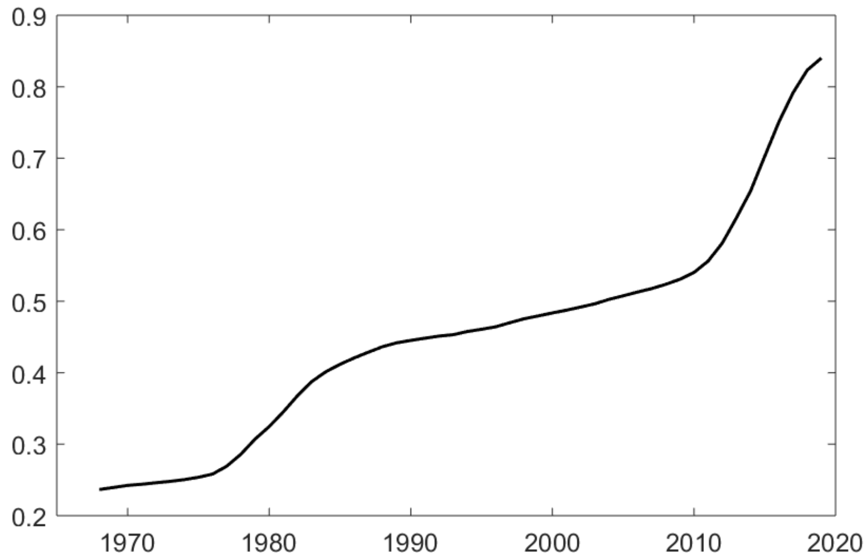
Directed technical change literature (Acemoglu et al. (2012), Lennox and Witajewski-Baltvilks (2017), Hassler et al. (2019) or Hötte (2020)) states that the use of both a

[2] 6^{th} IPCC report, Tsur and Zemel (2008), van den Bijgaart et al. (2016), Rezai and van der Ploeg (2017), Tol (2018) or Botzen et al. (2019) show how the actual path of energy use can lead us to a disastrous situation

carbon tax and a research subsidy helps to redirect R&D towards carbon free energy, increasing investment in the renewable sector. In figure 2 I show that research in the US has already been biased in favor of the clean sector since the end of the 70's using patents data as a proxy for technological progress. I have collected data from the USPTO (United States Patent and Trademark Office) that I have classified into 2 categories: clean and dirty patents. Then I have plot the stock of clean patents over the stock of dirty patents to obtain figure 2, more details on these data are available in the Appendix. Research is redirected toward carbon-free energy since the end of the 70's but, as observed in figure 1, directed technical change mechanism seems to have had delayed impact on energy investment. The share of renewable energy has only grown more rapidly after 2005, pointing out a delay of approximately 25 years between technology differential and its direct impact on the energy mix. In this context I argue there exists an underlying mechanism able to explain why we observe such a delay and why it is compatible with an increase of both the renewable energy share and the level of fossil energy use. When technological change is embodied, and power plants long living, the trade-off between efficient fossil energy[3] and less-efficient carbon-free alternative creates a lasting capital effect in favor of carbonized sources of energy. At the beginning of the period, the economy is investing massively in long-living fossil energy plants to sustain growth, before carbon-free alternatives are able to catch-up. Power plants being meant to operate for at least 40 years (see appendix A), the closing of previously built units will be delayed through time and a quick transition become difficult. In my paper I argue that this mechanism is at the origin of the directed technical change delay observe in the energy sector.

My paper studies the existence of a lasting capital effect able to explain the delayed impact of biased technical change on the energy mix. For this purpose I use a multi-sector exogenous growth model with climate economics *à la* Nordhaus and Boyer (2000) and embodied technical change *à la* Krusell (1998). The two energy sectors differ in the inputs needed to produce one unit of capital ; productivity difference between these sectors being exogenous. Technological progress is embedded in new capital units, each investment is then meant to stay into the economy before it fully depreciates after 40 years (Appendix A). In my analysis I follow the idea of Lennox and Witajewski-Baltvilks (2017) by adding the embodied technical change structure. Whereas their paper looks at the policies required to incentivise firms to redirect innovation toward carbon-free technology, and at the effect of embodied technical change compared to disembodied in the framework of Acemoglu et al. (2012), I depart from directed technical change literature by using a model of structural change close to Acemoglu and Guerrieri (2008) and **?**. I consider technology is already redirected toward the clean sector and I then add a Nordhaus' damage function, following the one used by Golosov et al. (2014). Utilization of fossil sources of energy increases the stock of pollution, which has a

[3]In the 80's the levelized cost of energy was smaller for fossil fuels. Coal, gas and oil create more energy for a cheaper price than renewable



Author computations based on USPTO data

Figure 2: Technological delay of the renewable sector

negative impact on the GDP. The presence of both an exogenous technology differential and a negative externality incentivize the economy to rely more intensively on the carbon free alternative but I will show it is also compatible with an increasing use of fossil energy.

This paper has 3 main contributions. Firstly, I show there exists a lasting capital effect able to explain the persistence of fossil energy accumulation in the US. In the early 80's the use of carbon-free alternatives was delayed compare to fossil energy and were noncompetitive, creating a trade-off between efficient polluting energy and less efficient carbon-free alternative. The technology differential takes time to be corrected and the economy continues to invest in fossil energy in order to sustain growth. This trade-off, paired with an extended lifetime of power plants, tends to slow down closing of fossil power-plants in the economy.

Secondly, I show that technological progress is not enough to provide a quick energy transition. It is therefore a key to help the economy to switch from carbonized to carbon free energy production but the increase of the renewable energy share do not offset our reliance on fossil energy sources. In this paper I show that the evolution of the stock of fossil energy depends on growth, rate of technological progress and the pollution flow. Because "clean" and "dirty" capital are imperfect substitute, when renewable energy is not enough efficient compare to fossil energy relying too much on it would be costly in term of growth. I will show what are the conditions for the economy to decrease its use of fossil energy.

Lastly, I provide a simple, adaptive and somehow original theoretical framework of growth to study energy transition. The final good being produced through a Cobb-Douglas-CES production function, properties differ from a more standard case. I focus on the transitional process but I also characterize the long-run equilibrium of the model, providing the set of Non-balanced growth rate of my dynamic model. It is a flexible framework which can be enhanced with new assumptions like damage uncertainty of pollution, endogenous scrapping of capital or any other relevant hypothesis in the context of the energy transition.

For the theoretical analysis I use an exogenous growth model in a formulation close to Acemoglu and Guerrieri (2008), with two capital sectors. The final good, which can be either consumed or invested in the intermediary sectors, is produced using labor and capital. In my paper, capital can be viewed as total energy production and it is an aggregate of 2 intermediate inputs, "clean" (carbon-free energy) and "dirty" (fossil energy) capital. These two are produced using an investment specific accumulation equation *à la* Krusell (1998), with a technology differential variable set on "clean" investment. Technology differential is characterized by the relative performance of "clean" investments compare to "dirty", investing one unit of the final good in the "dirty" sector creates one unit of capital, while it depends on the state of the technology in the "clean" sector. By construction I insure "clean" capital to be less efficient at the starting period. Technological progress is exogenous and is embodied in new units of capital as in Greenwood et al. (1997). Use of "dirty" capital emits pollutants in the atmosphere, added to the pollution stock of the economy. Finally, pollution stock has a direct impact on GDP though a damage function equivalent to Golosov et al. (2014), which is an exponential version of Nordhaus' mapping from pollution to damages. Solving the social planner allows me to simulate both the "dirty" capital stock and the share of "clean" energy in the energy mix. By doing so I am able to reproduce dynamics of the US, explaining the delayed impact of directed technical change.

Energy questions with embodied technical change have been somewhat treated by vintage capital literature, with papers by Hritonenko and Yatsenko (2012) or Díaz and Puch (2019). These papers look at the impact of energy price shocks on macroeconomic aggregates when energy efficiency is dependent of the capital vintage. Lennox and Witajewski-Baltvilks (2017) also use embodied technical change, they study optimal policy to redirect innovation toward carbon-free technology instead of polluting one. They show that with embodied technical change there is a difference for optimal tax, subsidies. Pollution damages are also greater in their model compare to a disembodied framework. My approach uses embodied technical change to study its impact when technological progress is already biased. My methodology is then closer to Greenwood et al. (1997) but with an emphasis on the energy transition while the latter is developed to account for post-war growth differential in the US. Additionally, my paper shows embodied technical change may have a negative impact on transition speed, while in the existing literature it is use to account for growth processes.

A second branch of the literature tries to compute the optimal tax rate of the economy and estimate the social cost of carbon. Golosov et al. (2014) uses a DSGE model to show that the optimal tax rate is proportional to GDP when some plausible assumptions hold, making taxation dynamic. Li et al. (2016) enhances this framework by adding an uncertainty measure on future damages from pollution to GDP, using robust control theory. The robust path slows down significantly the use of coal in the economy but the carbon tax is still dynamic. Acemoglu et al. (2016) builds a tractable microeconomic model of endogenous growth, estimated with microdata, to study optimal environmental policy required to accompany the energy transition, they find that relying only on a carbon tax or delaying the intervention has significant welfare costs on the economy. Acemoglu and Rafey (2018) assess the risks implied by geoengineering alternatives to reduce climate change damages as a way to only postpone the problem instead of solving it. In my paper I am also able to develop a dynamic taxation scheme but I differ in the theoretical framework use, my theory is based on a structural change model instead of directed technical change, the social cost of carbon is then implicit and characterized by the damage function of the economy.

The paper is organized as follow, section 2 details the economic model of structural change while section 3 characterizes optimal growth path and theoretical results. Section 4 presents calibration and numerical results of the paper.

2 The model

To study the drivers of the energy transition I develop a multi-sector growth model of structural change with both exogenous growth and climate economics. The baseline model is a mix of Lennox and Witajewski-Baltvilks (2017) and Acemoglu and Guerrieri (2008). Time is continuous and intermediate energy capital is accumulated through an investment specific accumulation equation, which is a continuous version of Krusell (1998). Structural change comes from the supply side and is then induced by a price effect from a productivity differential. This mechanism is therefore close to Ngai and Pissarides (2007) but is also shaped by the presence of a negative externality from pollution. As in the directed technical change literature, the presence of a negative externality incites the economy to limit its investment toward the polluting sector. The intermediary "clean" and "dirty" sectors are aggregated through a CES function to produce the capital good of the model, associated to labor within a Cobb-Douglas function to produce the final good. Intermediary sectors are imperfect substitutes and here I am focusing on the case of substitutable inputs, based on the work by Papageorgiou et al. (2017).

2.1 Household

The representative household maximizes an instantaneous separable logarithmic utility function by choosing her consumption and labor participation, discounted through time.

$$U = \max \quad \int_0^{\infty} (ln(c_t) - \chi ln(L_t))e^{-\rho t} dt \tag{1}$$

Where ρ is the discount factor, c_t the instantaneous consumption, L_t labor participation and χ is a scale parameter for disutility of work. Preferences are homogeneous and compatible with both exogenous and endogenous growth. I exclude the possibility to add environment quality in the utility function, it is let for future research.

The representative household also owns firms and decide on the amount to invest on new machines, therefore she maximizes her lifetime utility subject to the following budget constraint,

$$c(t) = Y(t) - I(t) \tag{2}$$

Where $Y(t)$ is the production of the economy, and $I(t)$ is the total amount invested in intermediate goods. Therefore, the revenue can be either consumed or invested by the representative household. $I(t)$ will be used to invest in two intermediate goods, either "clean" or "dirty" capital and also serve as savings.

2.2 Production sector

The final good is produced through a standard Cobb-Douglas function, without aggregate technological progress. Labor, $L(t)$, and capital, $K(t)$, are used with constant return to scale. In the model the final good is also used as the numéraire.

$$\tilde{Y}(t) = L(t)^{1-\alpha} K(t)^{\alpha}$$

Where $0 < \alpha < 1$ is the capital intensity of final good production, $L(t)$ is labor and $K(t)$ is an aggregate of "dirty" and "clean" capital. These two intermediary goods represent energy capital units and are aggregated through a CES function to produce the final capital good.

$$K(t) = (K_c(t)^{\sigma} + K_d(t)^{\sigma})^{\frac{1}{\sigma}} \tag{3}$$

$K_c(t)$ and $K_d(t)$ are respectively clean and dirty capital, and are considered as imperfect substitutes. They are produced using an investment specific accumulation equation *à la* Krusell (1998). $-1 < \sigma < 1$ is a transformation of the elasticity of substitution between clean and dirty inputs, such as $\sigma = \frac{\varepsilon - 1}{\varepsilon}$ where ε is the elasticity of substitution.

Assumption 1 *"Clean" and "dirty" capital are substitutable inputs,* $0 < \sigma < 1$

Assumption 1 is based on the empirical paper by Papageorgiou et al. (2017) in which they show that in the framework of Acemoglu et al. (2012) "clean" and "dirty" capital units are substitutable. Based on this finding I will only consider the substitutable case between the two intermediate inputs, which make them imperfect substitutes. This framework will create a trade-off mechanism on capital use that will be derive from investments decisions.

Accumulation of both type of capital is done using an investment specific accumulation equation, which is a continuous time version of Greenwood et al. (1997) or Krusell (1998), as mentioned in Greenwood and Jovanovic (2001). Technological progress is embodied, new technologies are incorporated in new capital units and are unable to spread over already existing capital. I then have the following accumulation equations:

$$\dot{K}_d = i_{dt} - \delta K_{dt} \tag{4}$$
$$\dot{K}_c = q_t i_{ct} - \delta K_{ct} \tag{5}$$

Where δ is the depreciation rate of capital and is the same for both type and i_j is the amount invested in new machines for sector $j = c, d$. As observed in 2, the representative household decides how much she wants to invest in the acquisition of new capital units, this amount is then allocated to "clean" and "dirty" capital accumulation through the following equality,

$$I(t) = i_c(t) + i_d(t)$$

The amount invested is optimally allocated between the two different type of capital, such that the budget constraint can be rewritten as:

$$c(t) = Y(t) - i_c(t) - i_d(t)$$

The variable q in equation 5 is the relative efficiency of clean sector, it determines the amount of "clean" capital produced with one unit of the final good. Here I do not assume there is no technological progress within the "dirty" sector, this $q(t)$ is a variable of relative performance of the "clean" sector compare to the "dirty" one. The first paper using this double accumulation equation is the quasi-accountability paper by Greenwood et al. (1997) in which they use embodied technical change to account for post-war growth in the US. In their paper they consider 2 types of capital, structure and equipment, the latter being the one concerned by embodied technical change. As they argue, the relative performance variable, $q(t)$, might be interpreted in two different ways: i) $1/q$ could be interpreted as the relative cost of producing one unit of "clean" capital in terms of final output. ii) q represents the relative productivity of a new unit of "clean" capital, and, because I consider technological progress is biased in favor of the clean sector, it is increasing over time. However, the following assumption ensures a productivity gap in favor of the "dirty" sector at the starting point.

Assumption 2 *The initial condition of relative efficiency in the clean sector is such that:* $q(0) < 1$

Assumption 2 creates inertia in the dirty sector, it ensures "dirty" capital to be more efficient in a first time creating the actual trade-off in energy investment: fossil sources are cheaper but more polluting on the long term while carbon-free alternative are more expensive. Renewable technology catch-up through an exogenous process, the relative efficiency evolves at a constant rate γ such that,

$$\dot{q}_t = \gamma q_t \tag{6}$$

Imposing a restriction on q_0 allows existence of both capital at the same time but only the "dirty" type will have an impact on the pollution level of the economy, creating a negative externality. As it will be detailed in the next section, the production suffers from the level of pollution in the economy, this mechanism is introduced through a Nordhaus' damage function.

I can then incorporate equation 3 in the final good production function to obtain a Cobb-Douglas-CES form in the final good sector.

$$\tilde{Y}_t = L_t^{1-\alpha} \left(K_{ct}^{\sigma} + K_{dt}^{\sigma}\right)^{\alpha/\sigma} \tag{7}$$

The final good is produced using both labor and a capital good aggregated from both type of intermediate inputs. It ends up with a "CES-Cobb-Douglas" formulation which is not standard in macroeconomic literature. Capital dynamics implied by this formulation will be at the core of the structural change mechanism. $1/q$ being the price of investment in the "clean" sector and q being increasing it will results into a structural change mechanism in favor of the "clean" sector due to assumption 1, but it will be shaped by the imperfect substitution imposed by the CES part of the function, additionally to the capital share of the model. As mentioned, the use of "dirty" capital emits pollutant in the atmosphere that are added to the pollution stock. The latter being a negative externality it will reduced the level of GDP through a damage function introduced in the following section.

2.3 Pollution stock and damage function

Following the literature on energy transition and climate change, see Acemoglu et al. (2012), Golosov et al. (2014), Li et al. (2016), Nordhaus (2014b) or Lennox and Witajewski-Baltvilks (2017) among other, a pollution stock equation is introduced. Carbon accumulation, through use of "dirty" capital has a negative impact on the economy. Justification of this effect can be found in Graff Zivin and Neidell (2012), Chang et al. (2019), Pindyck (2019) or Nordhaus and Boyer (2000). The baseline model assumes "dirty" capital is the only source of new pollution, accumulated in the global carbon stock. The environment is regenerating itself at a constant rate through photosynthesis and other carbon absorption mechanism, $S(t)$ represents the carbon stock of the economy and is described by,

$$\dot{S}(t) = -\varphi_1 S(t) + \varphi_2 K_d(t) \tag{8}$$

Where φ_1 is the natural rate of absorption and φ_2 the linear transformation rate of dirty capital into carbon. If "dirty" capital stock falls under a sufficiently low level, the pollution stock starts to decrease as $\varphi_1 S(t) > \varphi_2 K_d(t)$. This assumption about pollution stock might seems too simple but allows to reproduce short term behavior of the economy, which is the objective here compare to asymptotic properties. In section 5.1 I am studying a multi-level pollution equation with a permanent and a transitory part to avoid this simplification equation about pollution accumulation.

In this model, pollution stock has a negative impact on GDP, the damage function is an exponential version of Nordhaus mapping and is the same than Golosov et al. (2014), such that GDP is given by $Y(t) = (1 - d(S(t)))\tilde{Y}(t)$ where $d(S(t))$ is the fraction of GDP lost because of pollution. The damage function is then characterized by $1 - d(S(t)) = exp(\theta(S(t) - \bar{S}))$ with $\bar{S}$ the pre-industrial level of pollution and θ the scale parameter for the mapping from pollution to damages to GDP. Everything being considered, I can rewrite the final expression for GDP as:

$$Y_t = L^{1-\alpha} \left(K_c^\sigma + K_d^\sigma\right)^{\alpha/\sigma} \exp(-\theta(S_t - \bar{S})) \tag{9}$$

This model aims at computing the timing of the energy transition according to 3 phenomena. First, the decreasing price of investment in the "clean" sector favors the use of "clean" capital, which will be observed as an increasing share for this type of input. Second, the dirty capital sector exhibits an higher relative efficiency in a first time, characterizing the advancement of fossil technologies, at the starting point the economy is still relying massively on fossil energy, which have a lasting effect due to capital lifetime. Third, the accumulation of dirty capital increases the damages to GDP, there will be a trade-off between growth and environment preservation. These three effects are characterizing the optimal growth of the economy and evolution of the energy transition, the dynamic implied by this model is consistent with the data about fossil energy use and renewable energy share. The embodied technical change added to the CES-Cobb-Douglas production function creates a lasting effect in favor of the "dirty" capital and postpone the energy transition implied by biased technical change. Each effect will be then described in the simulation section of the paper.

3 Optimal energy transition

3.1 The planner problem

I analyze the first best solution, the social planner maximizes utility of the representative household.

$$\begin{aligned} \max_{L_t, i_{ct}, i_{dt}} & \int_0^{+\infty} (ln(c_t) - \chi ln(L_t))\, e^{-\rho t} dt \\ s.t. \quad & (3) - (7) \text{ and} \\ & c_t = y_t - i_{ct} - i_{dt} \end{aligned} \tag{10}$$

The central planner solves the Hamiltonian in current value,

$$\begin{aligned} \mathcal{H} &= ln(Y - i_c - i_d) - \chi ln(L) + P[L^{1-\alpha} (K_c^\sigma + K_d^\sigma)^{\alpha/\sigma} e^{-\theta(S_t - \bar{S})} - Y] \\ &+ P_d[i_d - \delta K_d] + P_c[q i_c - \delta K_c] + Q_t[-\varphi_1 S_t + \varphi_2 K_{dt}] \end{aligned}$$

Giving first order conditions,

$$Y = \frac{\chi}{(1-\alpha)P} \tag{11}$$

$$P = P_d = qP_c = \frac{1}{Y - i_c - i_d} \tag{12}$$

Equation (12) shows shadow prices of production and dirty capital are the same and they are equal to the product $P_c q$. At t_0, $q(0) < 1$ means $P_c(0) > P_d(0)$ validating actual empirical facts of cheapest fossil energy compared to renewable. Equation (11) show the direct relationship between the shadow price of the final good and production itself in a straightforward equation.
Dynamic equations of state variables are the following,

$$\begin{aligned} \frac{\dot{P}_c}{P_c} &= \rho + \delta - \alpha q K_c^{\sigma-1} \frac{Y}{K_c^\sigma + K_d^\sigma} \\ \frac{\dot{P}_d}{P_d} &= \rho + \delta - \alpha K_d^{\sigma-1} \frac{Y}{K_c^\sigma + K_d^\sigma} - \varphi_2 \frac{Q}{P_d} \\ \frac{\dot{Q}}{Q} &= \rho + \varphi_1 + \frac{\theta Y P_d}{Q} \end{aligned}$$

As expected evolution of both shadow prices are similar but differ in the presence of q for the clean sector and the term $-\varphi_2 \frac{Q}{P_d}$, the decentralized equilibrium will show that the latter is equivalent to the carbon tax. Further in the paper we will see $Q < 0$, ensuring the dynamic equation for the shadow price of “dirty" capital to be bigger with carbon emissions than without.

In order to go further in the analysis the ratio κ is introduced, such that

$$\kappa \equiv \frac{K_c^\sigma}{K_c^\sigma + K_d^\sigma}$$

This ratio will be the proxy for energy transition, the closer from 1 κ is, the higher the share of "clean" energy. Calibration of the model will match $\kappa(0)$ with renewable share in energy mix in 2010.

This proxy allows to rewrite dynamic equation of both P_d and P_c,

$$\frac{\dot{P}_c}{P_c} = \rho + \delta - \alpha q \kappa \frac{Y}{K_c} \tag{13}$$

$$\frac{\dot{P}_d}{P_d} = \rho + \delta - \alpha(1-\kappa)\frac{Y}{K_d} - \varphi_2 \frac{Q}{P_d} \tag{14}$$

Shadow price of both type of capital depends on clean energy ratio. As one might expect, the closer from 1 κ is, the higher the growth difference will be. Next section will characterize the steady growth path and transitional patterns of the model, κ will be the central element of the analysis as it drives energy transition and all other variables.

3.2 Steady growth path

This section aims at computing the asymptotic behavior of the model, to derive steady growth rate is the first step before characterizing the transitional path of the economy. Model behavior is in line with some papers of structural changes like Acemoglu and Guerrieri (2008) or **?**. Economic transition and structural change occur along the growth path of other variables. In this paper, the energy transition takes place along with constant growth of prices, labor and GDP. As mentioned above, κ is the proxy for energy transition, therefore the final goal of this section is to derive the asymptotic and transitional growth rate of the "clean" capital ratio. By differentiating its definition one obtains,

$$\frac{\dot{\kappa}}{\kappa} = \sigma(1-\kappa)(g_{K_c} - g_{K_d})$$

In the following the term g_x refers to growth rate of variable x. κ growth rate depends on its own value and on the difference between "clean" and "dirty" capital growth. If "clean" capital grows faster (slower) than "dirty" one, κ is increasing (decreasing), and there are no inconsistent behavior because if κ is equal to one, the growth rate is equal to 0. To derive the complete characterization of κ's growth rate one uses the following statement: growth rate of shadow prices are assumed to be constant, such that $\dot{g_{P_c}} = \dot{g_{P_d}} = 0$. Using this property on equations (13) and (14) gives,

$$g_{K_c} = \gamma + \frac{\dot{\kappa}}{\kappa} + g_Y \tag{15}$$

$$g_{K_d} = g_Y - \frac{\dot{\kappa}}{\kappa}\frac{\kappa}{1-\kappa} + \frac{\varphi_2(1-\alpha)QK_d}{\alpha(1-\kappa)\chi}(g_Q - g_{P_d}) \tag{16}$$

g_{K_d} can be simplified by differentiating (11) and using the following proposition,

Proposition 1 *The shadow price of pollution, Q, is always at its steady-state value and is negative.*

Proof. Using (11) and (12), the shadow price of pollution growth can be rewritten as $\frac{\dot{Q}}{Q} = \rho + \varphi_1 + \frac{\theta\chi}{(1-\alpha)Q}$, it depends on the value of Q and on model's parameter. As for every variable of the model, asymptotically this growth rate should be constant, $\dot{g}_Q = 0 \Leftrightarrow g_Q = 0$. One is able to derive $Q^* = -\frac{\theta\chi}{(1-\alpha)(\rho+\varphi_1)}$, the steady state value of the shadow price of pollution, and this value is negative. In the long-run, Q must converge to its steady-state level, however it appears that if Q deviates from this value, its trend is explosive and cannot converge. The conclusion is there exist only one value for the shadow price of pollution leading to a stable steady-state, Q is a jump variable and is always at its steady-state level. □

At first, proposition 1 seems counter-intuitive, one should expects the constraint on pollution stock to variate with the level of pollution, to capture the constraint induced, by definition, by a shadow price. However, each new unit of pollutant emitted in the atmosphere has the same impact on this economy because of (11) and (12), the marginal impact of pollution is expected to co-move with the level of GDP($\frac{\partial Y(t)}{\partial S(t)} = -\theta Y(t)$), but the equivalence between $Y(t)$ and $P(t)$ is cutting this co-movement. The increasing impact of one unit of pollution is compensate by a drop in prices, such that the constraint is always the same, $Q(t)$ is then constant.

Equation (16) can be rewritten using proposition 1 and differentiating (11)

$$g_{K_d} = g_Y \left(1 - \frac{\varphi_2\theta K_d}{\alpha(1-\kappa)(\rho+\varphi_1)}\right) - \frac{\dot{\kappa}}{\kappa}\frac{\kappa}{1-\kappa} \quad \text{(16*)}$$

Output growth, dirty capital level and evolution of the clean share are the three variables defining "dirty" capital growth rate. Because of equation (11) and the assumption made on g_{P_d} the output growth rate is constant, then the only variables at play are K_d and κ. Combining this result with equation (15) in κ's growth rate and rearranging it characterizes the rhythm of energy transition,

$$\frac{\dot{\kappa}}{\kappa} = \frac{\sigma}{1-\sigma}\gamma(1-\kappa) + \frac{\sigma\varphi_2\theta K_d}{(1-\sigma)\alpha(\rho+\varphi_1)}g_Y \quad (17)$$

This expression can be divided in 2 parts, $\frac{\sigma}{1-\sigma}\gamma(1-\kappa)$ represents the transition implied by technology, it relies on γ, the efficiency differential between the two intermediates, and on the imperfect substitution reflected by the parameter σ. Without any environmental damages, growth rate of κ is only defined by this first part. $\frac{\sigma\varphi_2\theta K_d}{(1-\sigma)\alpha(\rho+\varphi_1)}g_Y$ represents the second part in which transition is implied by damages from pollution. The term $\theta\varphi_2 K_d$ represents the damages induced by "dirty" capital on GDP, the higher it is the faster transition is, Pollution has an acceleration effect on the energy transition. There are also 2 straightforward remarks, i) it appears that the bigger κ is, the slower the transition, which is due to scale effect ; ii) $\frac{\sigma}{1-\sigma}$ is present in each part, it represents the substituability effect implied by the CES function for capital aggregation.

κ growth rate depends on $(1-\kappa)$ and on K_d, it is straightforward that asymptotically κ will tend to 1. Growth rate of the clean ratio proxy is always larger than 0 and κ cannot be higher than 1 by construction. We have $\kappa^* = 1$ as asymptotic condition. When $t \to \infty$ the clean technology will be the dominant on the energy market, it does not mean that dirty technologies will totally disappear but its level will be non significant in the energy mix. Their existence will be discussed below and is of first importance when it comes to the energy sector. Structural change occurs, production becomes relatively more green but the economy will continue to buy new dirty inputs if $g_{K_d}(t)$ is positive. Using 17 I am able to rewrite 16,

$$g_{K_d} = g_Y - \frac{\sigma}{1-\sigma}\gamma\kappa(t) - \frac{\theta\varphi_2 K_d(t)}{\alpha(1-\kappa(t))(1-\alpha)(\rho+\varphi_1)}\left(1 + \frac{\sigma}{1-\sigma}\kappa(t)\right)$$

This equation will be represented in the simulation section but we can see that the accumulation of "dirty" capital depends on three main variables: i) growth, to keep a constant growth rate the economy needs to build new capital units and therefore some of them are "dirty". ii) technological progress, especially through parameter γ, the highest the rate of biased technological progress the lowest the growth rate of dirty capital accumulation. ii) damages, $\theta\varphi_2 K_d(t)$ represents the damages from the flow of pollution emissions and it has a direct impact on "dirty" capital accumulation. It is also shaped by the degree of substitution, σ, and by the share of renewable energy in the economy. The latter being implicitly defined by q_t, the technology delay of the "clean" sector.

Using the clean capital ratio result, I am able to derive the other growth rates of the economy. When $t \to \infty$ economy tends toward the Non Balanced Growth Path (NBGP) detailed in following theorem.

Theorem 1 *Under assumptions 1 and 2, the set of Non Balanced Growth Rates (NBGR) of this model are as follow:*

$$\begin{aligned} g_Y &= \frac{\alpha\gamma}{1-\alpha} \quad ; \quad g_{K_c} = \gamma + g_Y \quad ; \quad g_{K_d} = g_Y - \frac{\sigma\gamma}{1-\sigma} \\ g_{P_c} &= -g_Y - \gamma \quad ; \quad g_{P_d} = -g_Y \quad ; \quad g_{i_c} = g_Y \\ g_{i_d} &= g_{K_d} \end{aligned}$$

Theorem 1 shows the set of non-balanced growth rates, when energy transition has been completed $\kappa \to 1$, some features can be derived from the asymptotic behavior of the model. In the long run, "dirty" capital might still be increasing even if its share becomes non significant, if $\alpha > \sigma$ one obtains $g_{K_d} > 0$ which has no consequences for the energy transition but has some disastrous effect on environment quality, through the carbon stock. In the illustrative calibration in section **4.4**, $\alpha = 0.4$ and $\sigma = 0.44$, it coincides with an asymptotically decreasing growth of "dirty" capital. Nevertheless, it seems difficult to imagine an infinitely increasing fossil energy due to resources limitations, however this paper omits intentionally to include a resource stock because it is not the problematic here. Jaffe et al. (2011) survey about world oil reserves lets one think resource constraint will not be the major problematic of tomorrow. The idea of a negligible Hotelling effect in the short run is also present in Hart and Spiro (2011), in which they argue that scarcity rents do not dominate prices of fossil resources . Therefore, the main limitation of the non-balanced growth path is the lack of an Hotelling rule, but in the short and middle-run this absence seems less problematic.

3.3 Stability analysis

In the previous section I have characterized the asymptotic properties of the model, in this section I will show this Non-balanced growth path (NBGP) is stable and unique. For this puprpose I will reformulate the dynamical system of my model by using the normalization of variables introduced by Caballé and Santos (1993). I obtain the stationarized NBGP by deflating my variables by their long run growth rate, I then obtain: $k_c(t) = K_c(t)e^{-g_{K_c}}, k_d(t) = K_d(t)e^{-g_{K_d}}$ and $p_c(t) = P_c(t)e^{-g_{p_c}}$, for all $t > 0$, with $k_c(t), k_d(t)$ and $p_c(t)$ the stationarized values of $K_c(t), K_d(t)$ and $P_c(t)$.

Substituting these values into (5), (6) and (13) I obtain a stationarized system of differential equations able to characterize the equilibrium path. The expression of this 3 variables is sufficient to describe the full dynamic of the model, and in this stationarized system I am still assuming an elasticity of substitution such as: $0 < \sigma < 1$.

Lemma 1 *Let assumptions 1 and 2 hold. Along a stationarized equilibrium path and for any given q_0, "clean" capital k_c, "dirty" capital k_d and price of the "clean" capital p_c are solutions of the following dynamical system*

$$\begin{array}{rcl} \frac{\dot{k}_c}{k_c} & = & \frac{q_0 i_c(k_c,k_c,p_c)}{k_c} - \delta - g_{k_c} \\ \frac{\dot{k}_d}{k_d} & = & \frac{i_d(k_c,k_d,p_c)}{k_d} - \delta - g_{k_d} \\ \frac{\dot{p}_c}{p_c} & = & \rho + \delta - \alpha\kappa(k_c,k_d)\frac{\chi}{(1-\alpha)p_c k_c} - g_{p_c} \end{array} \tag{18}$$

Proof: See appendix □

I now have a stationarized dynamical system characterized in Lemma 1, I can prove the existence of a unique steady-state that will corresponds to the set of non balanced growth rate given by theorem 1.

Theorem 2 *Let q_0 be given and suppose assumptions 1 and 2 hold. There exists a unique steady-state (k_c^*, k_d^*, p_c^*) solution of the dynamical system (18).*

Proof: See appendix □

Theorem 2 proves there exists a unique and stable steady state for the "clean" capital stock k_c, the "dirty" capital stock k_d and the price of "clean" capital p_c.

3.4 Decentralized equilibrium

Previous section aimed at solving the social planner problem, correcting for pollution damages from use of a "dirty" technology. To solve the decentralized equilibrium will help at characterizing the optimal tax rate required for this economy to reach the optimal growth path. In this model there is only one externality, pollution from use of dirty capital, which needs one instrument to be corrected, the tax rate. The final good is used as a numéraire, its price is normalized to 1.

3.4.1 Household

The representative household owns the intermediate firms and lend his work to the final good producer, he exhibits the same utility function than for the social planner but his budget constraint is: $c(t) = L(t)w(t) + r_c(t)K_c(t) + r_d(t)K_d(t)$. The representative household maximizes its utility with respect to consumption, labor and capital investment,

$$
\begin{array}{lll}
U = & \max\limits_{L(t),c(t),i_c(t),i_d(t)} & \displaystyle\int_0^{+\infty} \left(ln(c_t) - \chi ln(L_t)\right) e^{-\rho t} dt \\
 & s.t. & Y(t) = L(t)w(t) + r_c(t)K_c(t) + r_d(t)K_d(t) \\
 & & c(t) = Y(t) - i_c(t) - i_d(t) \\
 & & \dot{K}_c = q(t)i_c(t) - \delta K_c(t) \\
 & & \dot{K}_d = i_d(t) - \delta K_d(t)
\end{array}
\tag{19}
$$

First order conditions state:

$$
\frac{\chi}{L(t)} = w(t)P(t) \qquad ; \qquad P_c(t)q(t) = P_d(t) = P(t)
$$

Where $P(t), P_d, P_c$ are the multiplier of respectively the budget constraint, the "dirty" capital accumulation and the "clean" capital accumulation. FOC conditions in the decentralized equilibrium are similar to social planner.

Dynamic equations are also similar to what one can found in the centralized equilibrium,

$$
\begin{aligned}
\frac{\dot{P}_c}{P_c} &= \rho + \delta - q r_c \\
\frac{\dot{P}_d}{P_d} &= \rho + \delta - r_d
\end{aligned}
$$

Household do not take into account damages from pollution to the global production, the externality will need to be corrected by a tax rate as it will be shown below. The main difference here is for dynamic equation for the shadow price of the "dirty" capital because in the suboptimal equilibrium adverse pollution effects are not taken into account.

3.4.2 Final good

The final good firm maximizes its profit, it sells its production, buy work of the household and rent capital units,

$$\max_{L(t),K_c(t),K_d(t)} \pi = L^{1-\alpha}\left(K_c^{\sigma}+K_d^{\sigma}\right)^{\alpha/\sigma} - w(t)L(t) - r_c(t)K_c(t) - r_dK_d(t) \tag{20}$$

Deriving first order conditions leads to,

$$w(t) = (1-\alpha)\frac{Y(t)}{L(t)} \tag{21}$$

$$r_c(t) = \alpha\kappa(t)\frac{Y(t)}{K_c(t)} \tag{22}$$

$$r_d(t) = \alpha(1-\kappa(t))\frac{Y(t)}{K_d(t)} \tag{23}$$

$w(t)$, $r_c(t)$ and $r_d(t)$ represent, respectively, wages, rental price of "clean" capital and rental price of "dirty" capital. The next section will look at the optimal tax rate needed to coincide with the socail planner equilibrium and correct for the externality.

3.4.3 Optimal tax rate

Pollution accumulation due to use of "dirty" capital destroys a share, $d(t)$, of the production such that $D(t) = 1 - e^{-\theta(S(t)-\bar{S})}$ is the damage function. In order to correct for this externality a government needs to introduce a tax on "dirty" capital for decentralized equilibrium to coincide with central planner scheme.

The tax will apply on the rental price of dirty capital, a slower rate of return for each "dirty" unit slow-down the investment in this kind of capital. The government modifies the household maximization (17) such that the budget constraint becomes

$$Y(t) = L(t)w(t) + r_c(t)K_c(t) + (r_d(t) - \tau(t))K_d(t)$$

Using (23) and solving the new maximization for dynamic equation it appears,

$$\frac{\dot{P}_d}{P_d} = \rho + \delta - \alpha(1-\kappa(t))\frac{Y(t)}{K_d(t)} + \tau(t)$$

Comparing to the central planner results for the dynamic equation of the shadow price of "dirty" capital it appears clearly that the tax rate is such that,

$$\tau(t) = \frac{-\varphi_2 Q(t)}{P_d(t)}$$

Proposition 1 states $Q(t)$ is constant and negative, and it still hold in the decentralized equilibrium making the tax rate dependent of the shadow price of "dirty" capital. The lower $P_d(t)$, the higher the tax rate. $\tau(t)$ is inversely proportional to $P_d(t)$, and so is proportional to $Y(t)$ because of (11) and (12). Using this and proposition 1 the expression for the tax rate can be rewritten,

$$\tau(t) = \frac{\varphi_2 \theta}{\rho + \varphi_1} Y(t) \tag{24}$$

To reach the optimal growth rate asks for an increasing tax rate on "dirty" capital, proportional to production in the economy, such a result is in line with recent literature on the topic of taxation of fossil energies, like Golosov et al. (2014) or Li et al. (2016). It might seems surprising that the taxation only depends on production output, but the intuition behind this result is quite simple as damages are proportional to output and marginal utility is inversely proportional to output. If GDP rises, damages are greater but marginal utility is lower, then the second effect offset the first and taxation only depends on the current level of output.

4 Numerical illustration

4.1 Calibration

Compared to a Ramsey model, this paper differs in its double capital market with investment-specific accumulation equations and the presence of a damage function, linked to the emissions of pollutants. The model is then characterized by the parameters summarized in table 1 and by 3 initial conditions, $Y(0)$, $K_c(0)$ and $K_d(0)$. Technological progress is computed using the patent stock measure detailed in Appendix B, figure 2 in the introduction shows the evolution of the technology differential between "clean" and "dirty" technologies. Looking at figure 2 we observe two distinct periods: 1970-2008, with a slow evolution of the technology differential, and 2009-2019 with a sharp increase of this variable. I observe an annual growth rate of 2.3% during the first period and a growth rate of 5.5% during the second period. The growth differential is calibrated such as $\gamma_1 = 0.023$ for 1970-2008, and $\gamma_2 = 0.055$ for 2009-2019. The value for α, the labor share is given directly by the bureau of labor and statistics, its average on the considered period is such as $\alpha = 0.4$. Depreciation rate of capital and discount factor, respectively δ and ρ are calibrated following Barro and Sala-i Martin

(2004), the values $\delta = 0.05$ and $\rho = 0.02$ are widely use in macroeconomic literature and calibration of Ramsey models, this paper does not innovate in this regard. The value for the elasticity of substitution is chosen following Papageorgiou et al. (2017), they estimate its value in AABH framework which is close to the one in this paper, regarding to their computations the elasticity of substitution is calibrated as $\sigma = 0.44$. And lastly, the 3 parameters associated to environment are calibrated using the last IAM model used by Nordhaus, such that $\varphi_1 = 0.1$, $\varphi_2 = 0.0228$ and $\theta = 0.02$.

Parameter	Value	Data
γ_1	0.023	USPTO patents data (1970-2008)
γ_2	0.055	USPTO patents data (2009-2019)
α	0.4	Bureau of Labor statistics
σ	0.33	Papageorgiou et al. (2017)
δ	0.05	Barro and Sala-i Martin (2004)
ρ	0.02	Barro and Sala-i Martin (2004)
φ_1	0.1	*Nordhaus (2014a)*
φ_2	0.0228	*Nordhaus (2014a)*
θ	0.02	*Nordhaus (2014a)*

Table 1: Parameters value

Then I will calibrate the starting values for clean and dirty capital in the US. I will use the data provided by BP statistical review of world energy 2019, expressed in Mtoe (Million tonne of oil equivalent). I aggregate coal, oil and gas consumption data for the "dirty" capital input, I obtain a value of 1562.11 Mtoe in 1970 in the US. For renewable energy I take the same data source and I obtain a consumption of renewable energy of 3.67 Mtoe in 1970. Using year 1970 as the starting point of my simulations and reducing the unit to hundreds of Mtoe I calibrate the model such as $K_c(0) = 0.0367$ and $K_d(0) = 15.1162$. It implies a share of proxy for the share of renewable energy such as $\kappa(0) = 0.0642$ and a true share of renewable energy of 0.00242. By doing so I will be able to both the evolution of the share of renewable energy and the level of fossil energy in the US from 1970 to 2019. Then by applying some assumptions on the future of technological progress I will also try to provide middle run forecast for the future of the energy transition.

4.2 Simulations

This sections aims at providing some useful insight on the transitional pattern of this model. Without any doubt the economy will switch from a fossil energy dominance to a renewable world, but in the presence of climate change the major problem is how long this switching will take. This section will first deal with the timing of the energy transition providing simulations of the clean energy share, then it will study the evolution of the stock of dirty capital and lastly I will provide few insights about taxation and the possible evolution in the near future.

4.2.1 Renewable energy share

Next simulation (figure 3) displays the real share ($\frac{K_c}{K_c+K_d}$) of clean capital into the energy mix, both theoretically and in the data. The two trends are really close and the model captures quite well the movements observed in the data.

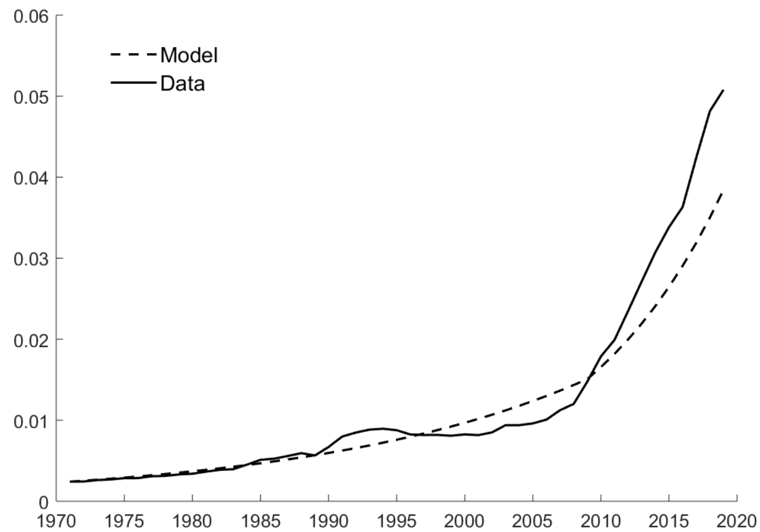


Figure 3: Share of clean technology across time

The change in the rate of technological progress coincide with the highest growth observed in the data. The model do not fit perfectly with the data after 2008 but it is mainly due to the 2008 crisis that have reduced the activity and the energy used in the US, unfortunately this model is not able to take this effect into account. Even with an exogenously biased technological progress in favor of the clean sector, the transition take time to occur due to the lasting effect of capital accumulation in the energy sector.

4.2.2 Level of fossil energy use

The next plot (figure 4) is about absolute level of “dirty" capital and its evolution through time. Here also I provide evolution of both the model and the data.

For the level of “dirty” capital the trend are quite similar and the fit seems good. However, as for the previous plot the model is unable to take into account different crisis, especially the two oil crisis in 1973 and 1979, and the financial crisis of 2008.

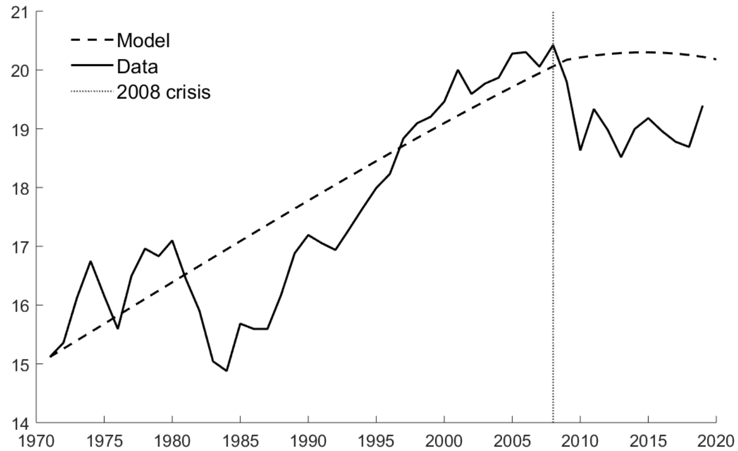


Figure 4: Use of “dirty” capital

After 1980 the model overestimate the US consumption and is not able to account for the drop after 2008. However, on the 2008-20019 period the model reproduces quite well the apparent stability of the fossil energy consumption in the US. The overall trend is also fitting quite well with the data and highlight the important role of the rate of technological progress. Even with a biased technological progress of 5.5% the use of fossil energy do not drop in 10 years, it only stays constant during this period. The next plot provides a simple forecast of the model according to the future value of technological progress, 3 scenarios are taken into account: i) it stays high at 5.5%, ii) it comes back to its former value of 2.3% and iii) it stops between at 4%.

The future level of fossil energy use is highly dependent on the rate of technological progress, with a difference of 3500 Mtoe of fossil energy used between the 2 extreme scenario in 2040. The more interesting result lies in the ii) scenario, when technological progress come back to its 1970-2008 level the use of dirty capital starts to increase again, which seems a little bit counter intuitive. In this model, the two type of capital are gross substitutes and technology is exogenous, investment decisions are an arbitrage across time. With a lower rate of technological progress it mean that future renewable technologies will be less efficient than expected, the representative household then decide to invest on dirty capital units tu sustain growth.

4.2.3 Growth rate of fossil energy accumulation

As mentioned earlier, the growth rate of “dirty” capital can be positive, even if the share of “clean” capital is growing, as n figure 1 presented in the introduction. The following graph represents the growth of fossil capital accumulation according to the level of “clean” capital share.

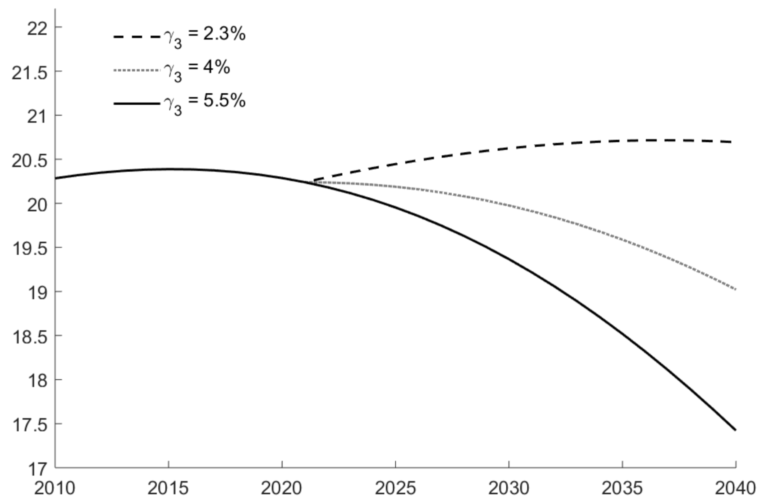


Figure 5: Forecast level of fossil energy use

We observe that this growth rate is decreasing and can be negative depending on the level of he renewable energy share and on the rate of technological progress. I have applied the 3 same scenarios than previously to the computation of this growth rate and we observe slightly different results. As mentioned previously, a low level of technolgical progress means that the economy needs to sustain GDP growth by accumulating "dirty" capital again, until the "clean" sector being competitive and independent enough.

4.2.4 Taxation scheme

Using equation (24), I am able to simulate the value of the tax rate. In this section I am trying to confront the results of my model to Golosov et al. (2014) and Li et al. (2016). As these two papers, I consider a constant level of GDP at its 2010 level and then I compute the tax in $ per ton of CO2 emitted. I apply the same 3 scenarios for technological progress after 2020.

In the literature about optimal carbon taxation the discount factor is of first importance, for this plot this discount factor is set at a level such that $\rho = 0.02$, the left panel of figure 8 shows how the tax per ton of carbon evolves according to the discount rate. In this model I find a carbon tax of around 74$ per ton of carbon, which is close to the results observed in Golosov et al. (2014) and Li et al. (2016). As I compute the tax rate at constant GDP I cut the growth effect of taxation and it evolves only according to the flow of pollution emitted, explaining why it is decreasing in a first place. Then, the taxation level highly depends on the level of biased technological progress as it influ-

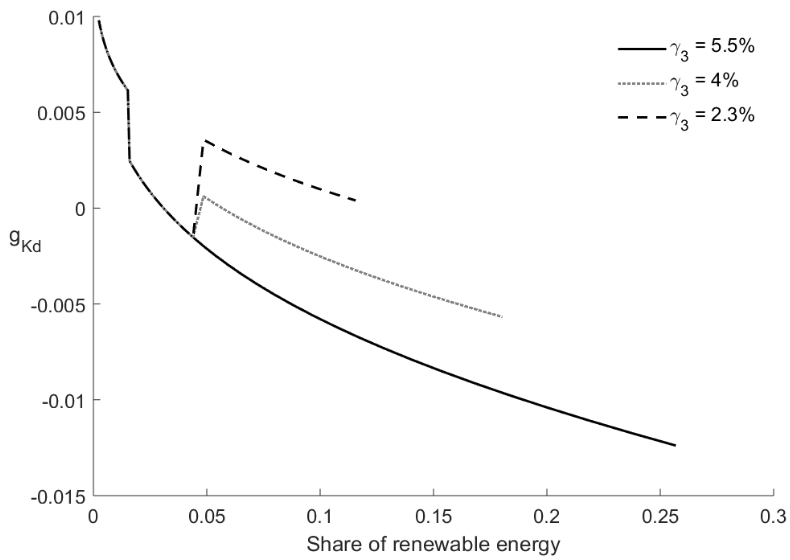


Figure 6: Growth rate of "dirty" capital accumulation

ences the stock of "dirty" capital and the pollution flow. The next graph provides some sensitivity analysis of the tax per ton of CO2 according to the level of the discount rate, ρ (left panel) and to the natural absorption rate of carbon, φ_1 (right panel).

Contrarily to papers in optimal carbon taxation, my results are less sensitive to the discount rate. In Golosov et al. (2014) for a discount rate $\rho = 0.001$ the carbon tax is comprised between \$221/ton and \$4,263/ton while in my case it reaches \$87.75/ton. In my paper, the taxation is more sensitive to the natural absorption rate of carbon, φ_1. As observed in figure 8, right panel, for a low level of natural absorption rate the tax is going above \$200/ton while it can fall below \$50/ton for higher values. These results are explained by the tax equation (24), both the discount rate and the natural absorption rate are at the denominator of the equation. The tax is more sensitive to he sum of these parameters more than to only one of them. Such framework leads to less extreme results in the characterization of the tax per ton of carbon, which is a critic from Pindyck (2017) in which the author consider the discount factor as being arbitrary. One strength of my paper is therefore to provide a framework in which arbitrary parameters have less impacts on the overall result.

The quantitative result that can be kept from these simulation is the very slow transitional process. Even in this simple framework with completely exogenous growth and expected outcome, there is an inertia effect from investment specific capital accumulation and initial conditions, calibrated on actual and historical data. Going back to the carbon lock-in argument of Unruh (2000), the pessimistic view would say that in practice we cannot escape carbon lock-in easily due to capital inertia. Looking at the

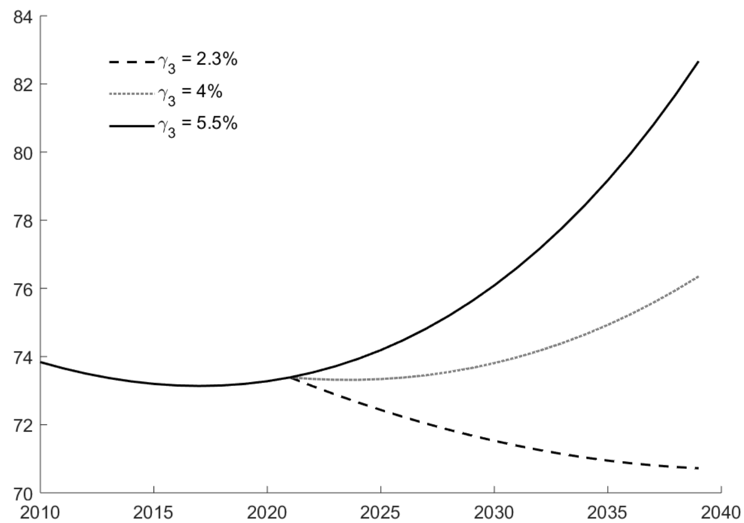
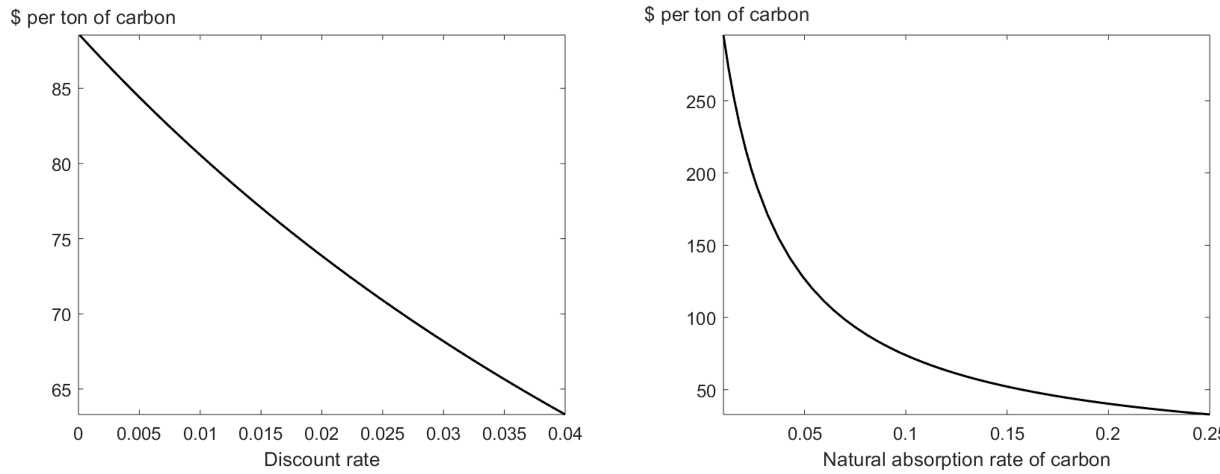


Figure 7: Tax in $ per ton of CO2 emitted

actual discussion around carbon taxation and the work by Clements et al. (2013) and Coady et al. (2015), the economy is not so close for such a taxation scheme. However, the optimistic view would say this paper do not capture the complete inertia of the energy sector, nor political decisions, nor consumers behavior with respect to climate change, nor uncertainties linked to the energy sector. Also, there are a couple of parameters of first importance in this model, like technology differential and damages to GDP, a government would be able to affect and change their value. Increasing public research, carbon capture storage or geoengineering technology may have significant consequences on the short and middle-run (on this topic, Acemoglu and Rafey (2018) shows that relying on geoengineering technology is suboptimal).



(a) Sensitivity to discount rate

(b) Sensitivity to CO2 absorption rate

Figure 8: 2010 tax sensitivity

5 Model extensions

5.1 Multi-level pollution equation

A way to enhance the model and be closer to relative literature, especially Li et al. (2016) and Adao et al. (2017), is to transform the pollution accumulation equation into a multi-level function. In this framework pollution has two components: T, which is the transitory part that can be absorbed by the atmosphere, and H, a permanent stock that can never disappear. Therefore I have the $S = T + H$, and I will detail how these component are accumulated in such framework.

Pollution is still coming from the use of dirty capital and the pollution flow is still $\varphi_2 K_d$. This flow will split in two part, a share ϕ of new emissions will stay forever in the atmosphere and be part of the permanent component, and a share $(1 - \phi)$ will be added to the transitory pollution part. I then obtain:

$$\dot{H} = \phi\varphi_2 K_d \tag{25}$$
$$\dot{T} = -\varphi_1 T + (1-\phi)\varphi_2 K_d \tag{26}$$
$$\dot{S} = -\varphi_1 T + \varphi_2 K_d \tag{27}$$

The global pollution equation is not very different from the previous one, the main difference lies in the fact that φ_1 applies only to the transitory part. This global pollution accumulation equation can be rewritten as follow

$$\dot{S} = -\varphi_1 (S - P) + \varphi_2 K_d$$

Once I have these new equations for pollution accumulation, I rewrite the Hamiltonian in current value with one more constraint on the permanent pollution stock:

$$\begin{aligned}\mathcal{H} &= ln(Y - i_c - i_d) - \chi ln(L) + P[L^{1-\alpha}\left(K_c^\sigma + K_d^\sigma\right)^{\alpha/\sigma} e^{-\theta(S_t - \bar{S})} - Y] \\ &+ P_d[i_d - \delta K_d] + P_c[q i_c - \delta K_c] + Q_S[-\varphi_1(S - H) + \varphi_2 K_{dt}] + Q_H[\phi\varphi_2 K_d]\end{aligned}$$

The first order conditions with respect to the control variables are not different from the previous specification, however there are few changes on the F.O.C on the state variables, here are the updated version of the dynamic equations

$$\frac{\dot{P}_d}{P_d} = \rho + \delta - \alpha(1-\kappa)\frac{Y}{K_d} - \varphi_2\frac{Q_S + \phi Q_H}{P_d} \tag{28}$$

$$\frac{\dot{Q}_H}{Q_H} = \rho - \varphi_1\frac{Q_S}{Q_H} \tag{29}$$

The dynamic equation associated to S multiplier did not change with this new version of the pollution accumulation equation, meaning that Q_S is constant as proved in section 3. I have $Q_S^* = -\frac{\theta\chi}{(1-\alpha)(\rho+\varphi_1)}$ that can be inserted into 29 to obtain that Q_H growth rate only depends on its own value. I then apply the same reasoning than for Q_S^* and I can easily show that $Q_H^* = \frac{\varphi_1}{\rho}Q_S^* = -\frac{\theta\chi\varphi_1}{(1-\alpha)(\rho+\varphi_1)\rho}$ and prove that this value is unique as Q_H cannot converge to its equilibrium value if it deviates from it.

Introduction of a multi-level pollution equation does not impact the qualitative behavior of the model, but it has some quantitative implications that I will explain below. For this purpose I am using exactly the same methodology than in section 3. The multi-level pollution equation does not impact the characterization of the "clean" sector but it implies some changes for the "dirty" one. I obtain the following dynamic equation for $K_c and K_d$:

$$g_{K_c} = \gamma + \frac{\dot{\kappa}}{\kappa} + g_Y \tag{30}$$

$$g_{K_d} = g_Y\left(1 - \frac{\varphi_2\theta K_d}{\alpha(\rho+\varphi_1)(1-\kappa)}(1 + \frac{\phi\varphi_1}{\rho})\right) - \frac{\dot{\kappa}}{\kappa}\frac{\kappa}{1-\kappa} \tag{31}$$

The only difference lies in the characterization of g_{K_d} with the appearance of $\frac{\phi\varphi_1}{\rho}$ which reduces the accumulation of dirty capital. By splitting pollution accumulation equation in two, with a share of carbon that stays forever in the atmosphere, I reduce the natural absorption capacity and I add infinite damages due to their permanent nature. Therefore it increases the transition speed of the model, as one can see in the new version of κ's growth rate:

$$\frac{\dot{\kappa}}{\kappa} = \frac{\sigma}{1-\sigma}\gamma(1-\kappa) + \frac{\sigma\varphi_2\theta K_d}{(1-\sigma)\alpha(\rho+\varphi_1)}\left(1+\frac{\phi\varphi_1}{\rho}\right)g_Y \tag{32}$$

The value of ϕ, the share of pollution flow that will stay forever in the atmosphere, plays an important role in the transition speed, the greater this parameter is, the faster the transition takes place due to greater infinite damages along time.

To invest the implication of a multi level pollution accumulation equation I will simulate the share of renewable energy in the model for two different value of ϕ and I will compare them to the baseline scenario, without the multi-level pollution accumulation equation.

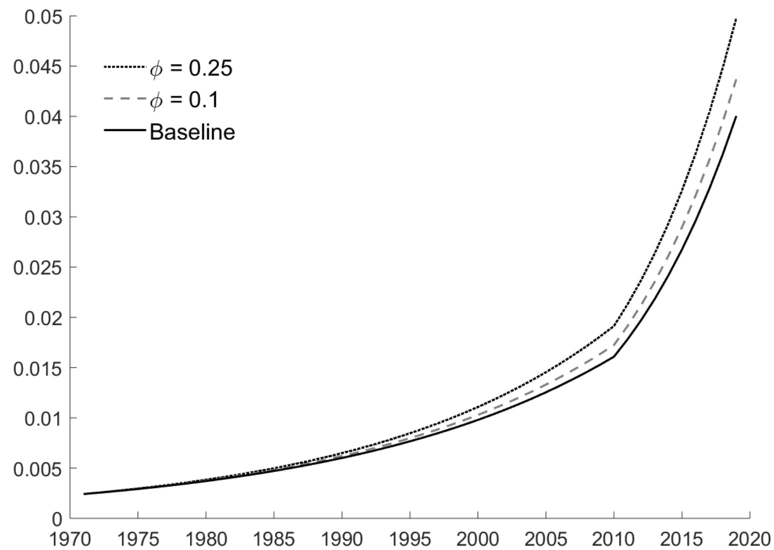


Figure 9: Share of renewable energy with permanent pollution component

The incorporation of a multi-level pollution equation does not affect massively the share of renewable energy in the economy, even when 25% of the emissions are turned into permanent pollution stock, the share of renewable energy is only 0.5% greater after 40 years (between 1970 and 2010).

This extension do not change qualitatively the results of the model, it just have a small quantitative impact on the transitional rate of the economy. Renewable energy is accumulated more rapidly but the effect is quite low compared to the baseline model. It means that the persistence effect implied by embodied technical change cannot be corrected only by considering a permanent pollution part in the model, and the biased technological change effect remains postponed due to embodied technical change.

6 Conclusion

In this paper, I use an exogenous growth model with climate economics and embodied technical change to show that the existence of a lasting capital effect can explain why, even if technological progress is biased in favor of the renewable sector, we observe an increase of both the share of renewable energy and the level of fossil energy use. The key mechanism here is a trade-off between efficient "dirty" capital and less efficient carbon-free alternative, which last due to the extended lifetime of capital units. I was also able to characterize a level of tax per ton of carbon in line with the actual literature on this topic, but less dependent to my parameter values. My carbon tax is around $74/ton and slightly vary with the level of the discount factor, it is more sensitive to the parameter associated to the natural rate of carbon but it never reach extreme values. In this simple framework I've shown that in the context of the energy transition, relying only on technological progress is not enough to engage a quick transition unless its rate is really high. A promising way of research would be t analyze the different tools available to speed-up the energy transition and decarbonize the economy.

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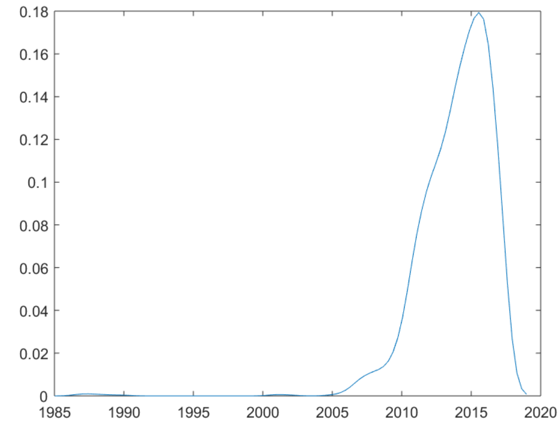
Appendix

A Capital Inertia and lifetime of energy plants

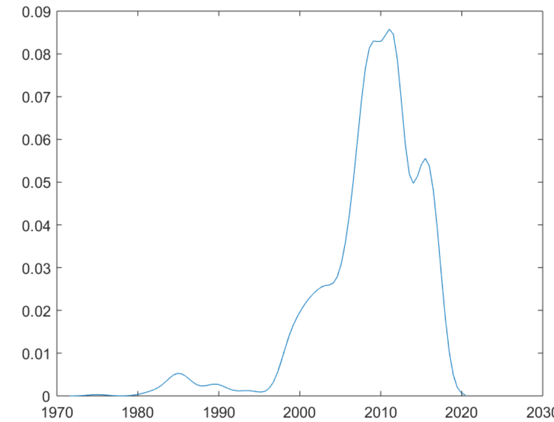
This section aims to document more consistently capital inertia from energy power plants. The motivation of this paper is to look if the lifetime of energy power plants may be an issue for our transition from fossil fuel to renewable energies. Ii is argue that when capital units are long living, with embodied technology, there are frictions in the transition process.

Power plants are using different type of energy sources with their own characteristics in term of production capacity, pollutant emission, geographical preferences,... Each energy source depends on one or several technologies, like solar energy, it can be transformed using photovoltaic (PV) or concentrating solar power (CSP) units. Technological progress is then embedded in each power plant, PV panels are formed of numerous photovoltaic modules which convert sun light into electricity using the photovoltaic effect. Performance of PV panels can be enhanced using more recent modules or coupling the system with an heat pump for example, but it is not possible to apply better modules or provide heat pump association on already engaged PV farms: technology is embedded in each generation (vintage) of panels, and is incompatible with other kind of solar energy like concentrating solar power (CSP). This special feature advocates for models with embodied technical change.

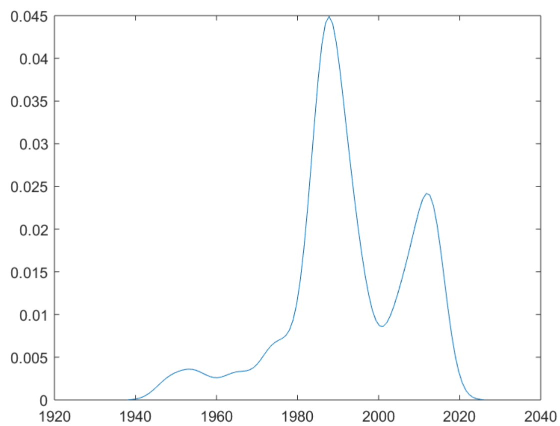
Using NEEDS data one can study, at least for US, lifetime of energy power plants. This dataset contains information about the commissioning year of actual power plants according to energy they use. Figures 2 and 3 show kernel density of On Line Year for renewable and fossil energy plants.



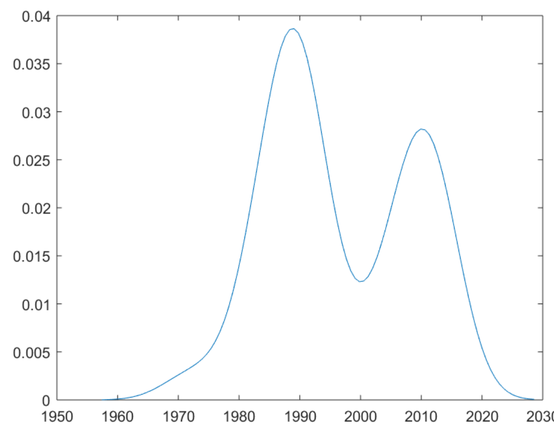
(a) Solar



(b) Wind



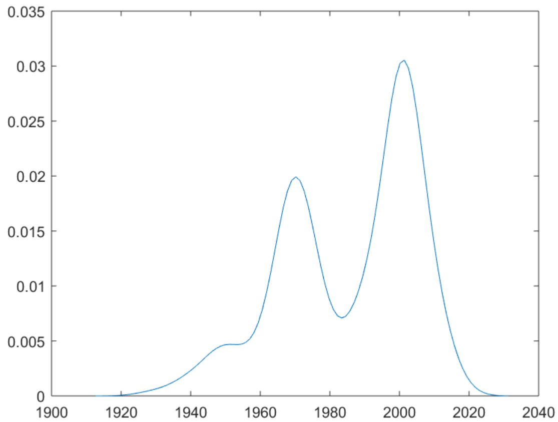
(c) Biomass



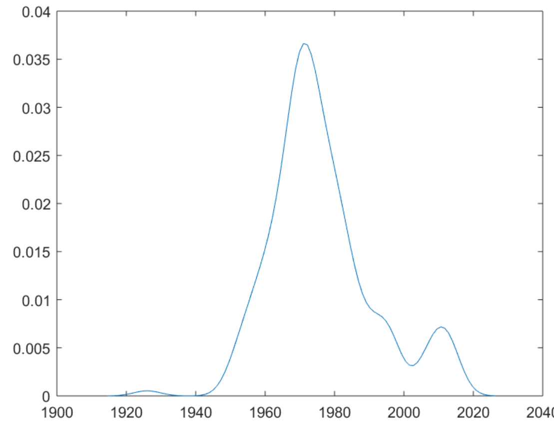
(d) Geothermal

Figure 10: Renewable energy sources - On line year kernel density

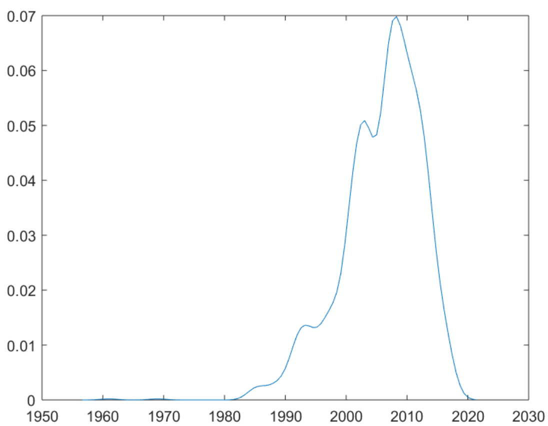
We observe that fossil energy plants are older than renewable ones, but the later are still long living. especially for geothermal and biomass, there is a non-negligible share of them which between 40 and 60 years old. For gas power plants a majority of them were built around 2000 but some are a little bit older, for coal and oil power plants a big proportions of them are aged between 40 and 60 years. In conclusion, lifetime of power plants in US can be very long, almost 100 years for some specific units, slowing down the capacity to scrap old plants to build new ones.



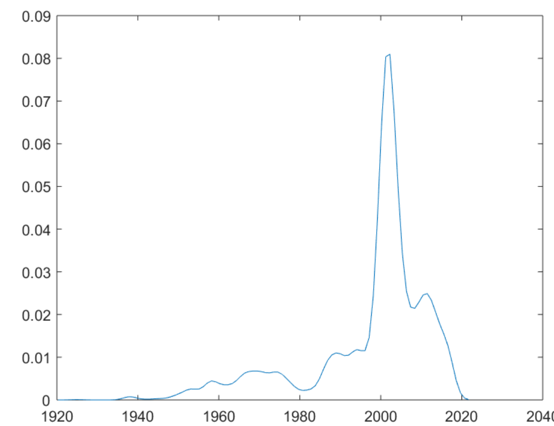
(a) Oil



(b) Coal



(c) Landfill gas



(d) Natural gas

Figure 11: Fossil energy sources - On line year kernel density

B Proxy for technological progress

Usually technological progress is derived from TFP growth, but there are no TFP index for renewable or fossil energy sectors. Based on Acemoglu et al. (2016), this paper uses patent data to proxy the level of technological progress in the US economy. This approach aims at capturing research output instead of input (research and development budget). I use USPTO data to create two technology variables: "clean" and "dirty" research. I construct this index using the CPC (Cooperative Patent Classification) system for US patents. Clean technology variable is build using 5 classes of patent: Y02E technologies or applications for mitigation or adaptation against climate change ; H02S generation of electric power by conversion of infra-red radiation, visible light or ultraviolet light, e.g. using photovoltaic modules ; F03D wind motors ; F24S solar heat collectors, solar heat systems ; F24T geothermal collector, geothermal systems. The number of patents granted each year in all these categories are used as a proxy of clean technological progress. For the dirty technology variable the methodology is the same, categories used are: E21B earth drilling e.g. deep drilling, obtaining oil, gas, water, soluble or meltable materials or a slurry of minerals from wells ; F02C gas-turbine plants, air intake for jet-propulsion plants, controlling fuel supply in air breathing jet-propulsion plants ; F17C vessels for containing or storing compressed, liquefied or solidified gases, fixed-capacity gas-holders, filling vessels with, or discharging from vessels, compressed, liquefied or solidified gases ; F23D Burners ; F23C methods or apparatus for combustion using fluid fuel or solid fuels suspended in (a carrier gas or) air. Once I have collected data I aggregate them into two stock variables, “clean” and “dirty” research, and my measure for technology differential is such that,

$$q_t = \frac{\text{Stock of clean patents}}{\text{Stock of dirty patents}}$$

This measure represent my proxy for the technology differential between the two sources of energy as the higher this measure is, the more competitive renewable energy is.

C Embodied technical change in the data

Using The National Electric Energy Data System (NEEDS) v6 I am able to justify the functional forms used in the theoretical part of the paper. I am documenting here the validity of embodied technical change, that older power plants are less efficient and develop less power than new ones.

The NEEDS dataset contains information about all power plants operating in US and producing electricity. It details the capacity developed, the energy source used, an efficiency proxy (The net heat input required to generate 1 kilowatt hour of electricity), in which state the power plant is operating, when it started to operate and if the power plant is subject to pollution control (NOx and particulate matter). I then regress the efficiency measure and the power capacity on the other variables, using state and plant type fixed effect to show the negative relationship between lifetime of power plants and their capacity and efficiency. For this purpose I am using a simple OLS model, results can be found in tables 9, for efficiency, and 10 for capacity.

Variables used are the following,

• Efficiency: the neat heat input required to generate 1 kilowatt hour of electricity. This value is a proxy for efficiency of electricity generators, the higher this value is the lower the efficiency?. I have then inverse values, to have a positive trend for this variable. However this measure is not applicable for every power plants, then Photovoltaic panels and wind turbines are not concerned by this measure.

• Capacity (MW): the power developed by power plants, in megawatts

• renewable: dummy variable, it is equal to 1 if the plant is using renewable energy, 0 if it using fossil source. It is used to control for the capacity and efficiency delay of a power plant.

Then I use pollution control variable, for both NOx and particulate matter as other control variables. I am also controlling for a state fixed effect, because energy policies are different according to the state considered. California is using more intensively renewable energy and may develop learning by doing effect for renewable energies, while Texas have the same advantage with oil. And then I have a plant type fixed effect to control for each energy specificity.

Figure 12: Efficiency of Electricity power plants

	(1)	(2)	(3)	(4)
lifetime	-52.88***	-50.03***	-48.21***	-49.60***
	(-21.53)	(-19.97)	(-18.63)	(-18.98)
Capacity (MW)		2.825***	2.609***	2.756***
		(15.52)	(14.67)	(14.20)
renewable		1589.7**	1557.0**	1432.6*
		(3.01)	(3.02)	(1.96)
Constant	22237.7***	20284.0***	20305.6***	20787.4***
	(130.36)	(41.17)	(42.12)	(28.86)
Other control	No	No	Yes	Yes
Plant Type fixed effect	Yes	Yes	Yes	Yes
State fixed effect	No	No	No	Yes
Observations	10771	10771	10771	10771
R^2	0.41	0.42	0.42	0.44

t statistics in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

More recent US power plants are more efficient than older one, this result is robust at the 0.1% threshold. it holds under many specifications and under state and plant type fixed effect. The assumption of embodied technical change fits quite well with energy power plants: older power plants are less efficient, it highlights the fact that technology do not spread over already built structures. The choice of an investment specific accumulation equation for both “clean” and “dirty” capital is confirmed by those results.

Additionally it seems 1) there is a scale effect. Capacity has a positive impact on efficiency, large power plants are then more efficient than smaller ones. 2) Renewable power plants are more efficient than fossil ones. It is logical as efficiency is constructed through a net heat input, we can think renewable power plants need less heat to produce electricity.

Figure 13: Capacity regression

	(1)	(2)	(3)	(4)
lifetime	-1.064***	-1.064***	-0.888***	-0.854***
	(-20.91)	(-20.91)	(-17.07)	(-15.53)
renewable		-48.84	-50.59	-28.43
		(-1.69)	(-1.72)	(-0.81)
Constant	129.9***	178.8***	177.1***	204.2***
	(8.01)	(7.44)	(7.26)	(6.38)
Other control	No	No	Yes	Yes
Plant Type fixed effect	Yes	Yes	Yes	Yes
State fixed effect	No	No	No	Yes
Observations	14434	14434	14434	14434
R^2	0.51	0.51	0.52	0.54

t statistics in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

I find the same impact from lifetime on capacity generation than for efficiency measure. Older power plants are delivering less power than more recent ones, and this result holds under several specifications. It also validates the embodied technical change assumption for the theoretical model. We can also conclude that once we control for both power plant type and state fixed effect, there is no evidence that renewable plants are delivering a lowest amount of power. It justify the trade-off made in the final good production function, taking both “clean” and “dirty” capital as equal and imperfectly substitutable inputs.

D Stability analysis

D.1 Proof of lemma 1

In this section I aim to prove that my system can be reduced to 3 dimensions with only the variables K_c, K_d and P_c. Because, by definition $\kappa = \kappa(K_c, K_d)$, I will only show that i_c and i_d can be rewritten according to the 3 variables of my model.

First, by rearranging 11 and 12 I derive the following equation:

$$i_d = \frac{1}{qP_c}\left(\frac{\chi}{1-\alpha} - 1\right) - i_c \tag{33}$$

Then I rewrite 4 and 5 such as

$$g_{K_c} = \frac{qi_c}{K_c} - \delta \quad ; \quad g_{K_d} = \frac{i_d}{K_d} - \delta$$

That I replace into the differentiated version of κ such that

$$\frac{\dot{\kappa}}{\kappa} = \sigma(1-\kappa)\left(\frac{qi_c}{K_c} - \frac{i_d}{K_d}\right)$$

Using this result with 17 and 33, and applying tedious but straightforward computations finally allow me to write i_c as a function of K_c, K_d, P_c that is characterized by:

$$i_c = \frac{K_c}{qK_d + K_c}\left(\frac{\sigma}{1-\sigma}\left(\gamma(1-\kappa) + \frac{\varphi_2\theta K_d}{\alpha(\rho+\varphi_1)}g_Y\right)\frac{K_d}{\sigma(1-\kappa)} + \frac{1}{qP_c}\left(\frac{\chi}{1-\alpha} - 1\right)\right) \quad (34)$$

With 33 and 34 I have proved that $i_c = i_c(K_c, K_d, P_c)$ and $i_d = i_d(K_c, K_d, P_c)$

D.2 Proof of Theorem 2

I will start by deriving the steady state value of K_c, K_d, P_c that will be defined by k_c^*, k_d^*, p_c^* respectively, using the fact that at steady-state $\kappa = 1$. We rewrite the stationarized form of 13 such as:

$$k_c^* = \frac{\alpha q_0 \chi}{(\rho + \delta - g_{P_c})(1-\alpha)p_c^*} \equiv \frac{\psi_1}{p_c^*}$$

Once I have this result I can use it in combination with 4, 5 and 33 to express k_d^* using p_c^* as for k_c^*.

$$i_c + i_d = \frac{1}{q_0 p_c^*}\left(\frac{\chi}{1-\alpha} - 1\right) \Leftrightarrow (g_{K_c}+\delta)\frac{\alpha\chi}{(\rho+\delta-g_{P_c})(1-\alpha)p_c^*} + (g_{K_d}+\delta)k_d^* = \frac{1}{q_0 p_c^*}\left(\frac{\chi}{1-\alpha} - 1\right)$$

Which allow me to obtain the following definition of k_d^*

$$k_d^* = \frac{1}{p_c^*}\left(\frac{1}{q_0}\left(\frac{\chi}{1-\alpha} - 1\right) - (g_{K_c}+\delta)\frac{\alpha\chi}{(\rho+\delta-g_{P_c})(1-\alpha)}\right) \equiv \frac{\psi_2}{p_c^*}$$

Lastly I am using 14 to isolate p_c^* and characterize the full equilibrium by proceeding as follow:

$$\begin{aligned}
& g_{P_d} = \rho + \delta + \frac{\chi}{(1-\alpha)q_0 p_c^*}\left(\frac{\varphi_2\theta}{\rho+\varphi_1} - \frac{k_d^{*\sigma-1}}{k_c^{*\sigma}+k_d^{*\sigma}}\right) \\
\Leftrightarrow\ & g_{P_d} = \rho + \delta + \frac{\chi}{(1-\alpha)q_0 p_c^*}\left(\frac{\varphi_2\theta}{\rho+\varphi_1} - \frac{p_c^*\psi_1^{\sigma-1}}{\psi_1^{\sigma}+\psi_2^{\sigma}}\right) \\
\Leftrightarrow\ & p_c^*\left(\frac{\psi_1^{\sigma-1}}{\psi_1^{\sigma}+\psi_2^{\sigma}} - \frac{(\rho+\delta-g_{P_d})(1-\alpha)q_0}{\chi}\right) = \frac{\varphi_2\theta}{\rho+\varphi_1} \\
\Leftrightarrow\ & p_c^* = \frac{\varphi_2\theta}{\rho+\varphi_1}\left(\frac{\psi_1^{\sigma-1}}{\psi_1^{\sigma}+\psi_2^{\sigma}} - \frac{(\rho+\delta-g_{P_d})(1-\alpha)q_0}{\chi}\right)^{-1} \equiv \psi_3
\end{aligned}$$

I have now the full characterization of my steady state values such as

$$p_c^* = \psi_3 \quad ; \quad k_c^* = \frac{\psi_1}{\psi_3} \quad ; \quad k_d^* = \frac{\psi_2}{\psi_3}$$

The steady-state of this model is unique and has been derived from steady-state properties of the dynamic system. In the next part I will show that the dynamic system is characterized by a Saddle-point and the economy always converges to this unique equilibrium. For this purpose let me consider the dynamic system, in its stationarized formulation:

$$\begin{aligned}
\dot{k_c} &= k_c\left\{\frac{q_0 i_c(k_c,k_c,p_c)}{k_c} - \delta - g_{k_c}\right\} \equiv \mathcal{F}(k_c,k_d,p_c) \\
\dot{k_d} &= k_d\left\{\frac{i_d(k_c,k_d,p_c)}{k_d} - \delta - g_{k_d}\right\} \equiv \mathcal{G}(k_c,k_d,p_c) \\
\dot{p_c} &= p_c\left\{\rho + \delta - \alpha\kappa(k_c,k_d)\frac{\chi}{(1-\alpha)p_c k_c} - g_{p_c}\right\} \equiv \mathcal{H}(k_c,k_d,p_c)
\end{aligned}$$

The Jacobian matrix is then defined by linearazition around the steady-state

$$\mathcal{J} = \begin{pmatrix}
\mathcal{F}_1(k_c^*,k_d^*,p_c^*) & \mathcal{F}_2(k_c^*,k_d^*,p_c^*) & \mathcal{F}_3(k_c^*,k_d^*,p_c^*) \\
\mathcal{G}_1(k_c^*,k_d^*,p_c^*) & \mathcal{G}_2(k_c^*,k_d^*,p_c^*) & \mathcal{G}_3(k_c^*,k_d^*,p_c^*) \\
\mathcal{H}_1(k_c^*,k_d^*,p_c^*) & \mathcal{H}_2(k_c^*,k_d^*,p_c^*) & \mathcal{H}_3(k_c^*,k_d^*,p_c^*)
\end{pmatrix}$$

As the sign analysis is not straightforward, I am using the parameter's values defined in the calibration section, in order to infer the sign of each derivative and then characterize the sign of the trace and the determinant:

$$\begin{aligned}
\mathcal{F}_1 &= \frac{q_0^2 k_d^*}{(qk_d^* + k_c^*)^2}\left(\frac{1}{q_0 p_c^*}\left(\frac{\chi}{1-\alpha} - 1\right) + \frac{k_d^*}{1-\sigma}\left(\gamma + \frac{\varphi_2 \theta {k_d^*}^{1-\sigma}(k_c^{*\sigma} + k_d^{*\sigma})}{\alpha(\rho + \varphi_2)} g_Y\right)\right) - \delta - g_{K_c} > 0 \\
\mathcal{F}_2 &= \frac{\psi_1}{(1-\alpha)(q_0\psi_2 + \psi_1)^2}\left(\chi\left(\frac{\gamma\alpha q_0}{\rho + \delta - g_{P_c}} - 1\right) + (1-\alpha)\right) < 0 \\
\mathcal{F}_3 &= -\frac{k_c^*}{p_c^{*2}(k_c^* + q_0 k_d^*)}\left(\frac{\chi}{1-\alpha} - 1\right) < 0 \\
\mathcal{G}_1 &= -\frac{q_0^2 k_d^*}{(qk_d^* + k_c^*)^2}\left(\frac{1}{q_0 p_c^*}\left(\frac{\chi}{1-\alpha} - 1\right) + \frac{k_d^*}{1-\sigma}\left(\gamma + \frac{\varphi_2 \theta {k_d^*}^{1-\sigma}(k_c^{*\sigma} + k_d^{*\sigma})}{\alpha(\rho + \varphi_2)} g_Y\right)\right) < 0 \\
\mathcal{G}_2 &= -\frac{\psi_1}{(1-\alpha)(q_0\psi_2 + \psi_1)^2}\left(\chi\left(\frac{\gamma\alpha q_0}{\rho + \delta - g_{P_c}} - 1\right) + (1-\alpha)\right) - \delta - g_{K_d} < 0 \\
\mathcal{G}_3 &= \frac{1}{q_0 p_c^{*2}}\left(\frac{k_c^*}{k_c^* + q_0 k_d^*} - 1\right)\left(\frac{\chi}{1-\alpha} - 1\right) < 0 \\
\mathcal{H}_1 &= \frac{\alpha\chi}{(1-\alpha)k_c^{*2}} > 0 \\
\mathcal{H}_2 &= 0 \\
\mathcal{H}_3 &= \rho + \delta - g_{P_c} > 0
\end{aligned}$$

Once these signs are defined I can easily show that the trace is positive and the determinant is negative.

$$\mathcal{T} = \mathcal{F}_1 + \mathcal{G}_2 + \mathcal{H}_3$$

As $g_{P_c} = -g_{K_c} = -g_{K_d} - \gamma$ I have that $\mathcal{G}_2 + \mathcal{H}_3$ is always positive, then by adding $\mathcal{F}_1$, which is positive, I finally get $\mathcal{T} > 0$. Then I compute the determinant of the Jacobian matrix:

$$\mathcal{D} = -\mathcal{F}_2[\mathcal{G}_1\mathcal{H}_3 - \mathcal{H}_1\mathcal{G}_3] + \mathcal{G}_2[\mathcal{F}_1\mathcal{H}_3 - \mathcal{H}_1\mathcal{F}_3]$$

I am able to derive the sign of this equation only by looking at the sign of each component. It clearly appears that $\mathcal{D} < 0$. I can then conclude there exist a unique negative eigenvalue, any given steady-state located on the manifold is a saddle-point. The model is stable around the steady-state.

E Embodied vs Disembodied technological change

The model uses embodied technical change for the accumulation of both “clean” and “dirty” capital, this feature is also present in Lennox and Witajewski-Baltvilks (2017). here I will make some adjustment to obtain a disembodied technical change model and study differences with the embodied version.

To do so I am reducing the dimension of the model by removing one state equation, there is one type of capital which can be divided between "clean" or "dirty" intermediate such that

$$K(t) = K_c(t) + K_d(t)$$

This unique capital stock stock is accumulating as follow

$$\dot{K} = Y(t) - \delta K(t) - c(t)$$

And lastly, technological progress is set on all type of capital through the production function

$$Y(t) = L(t)^{1-\alpha} \left((q(t)K_c(t))^{\sigma} + K_d(t)^{\sigma}\right)^{\frac{\alpha}{\sigma}} exp(-\theta(S(t) - \bar{S}))$$

I then solve this modified version of the model using an hamiltonian in current value, to obtain the dynamic system. I simulate the dynamic equations of the model to compare disembodied and embodied technical change. Using the US calibration I obtain the following graphs:

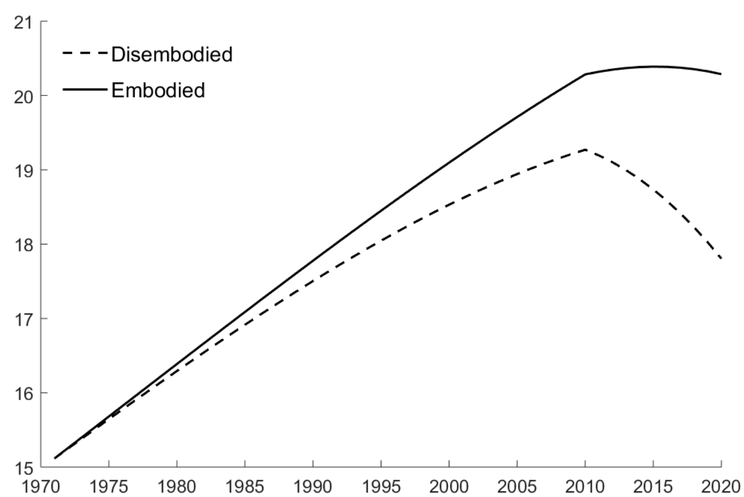


Figure 14: Level of fossil energy use

Since for the disembodied version technological progress is assigned to the stock of "clean" capital, I cannot look at the real share of renewable energy because efficiency increases each period. I then use a proxy to compare the 2 patterns. It appears that the proxy with disembodied technical change is growing faster than the embodied version. It seems reasonable since in this version technological progress spread on each unit of "clean" capital, while in the embodied technical change it affects only newly installed units.

Share of renewable energy is increasing more rapidly but it might also be the case for the stock of "dirty" capital. It would coincides with an higher level of pollution.

When technology is disembodied, the level of dirty capital starts to decrease earlier than for embodied technology. It also reach a lower maximum with the same parameters with a 1970 level almost reached in 2030 according to these simulations.

These differences between embodied and disembodied technical change are highlighting the presence of the lasting capital effect described previously. By considering 2 distinct capital accumulation equation I impeach capital to move freely between the 2 sectors and each decision is lasting over approximately 40 years. This mechanism is absent from the disembodied version because the only capital good can be freely dedicated to “clean” or “dirty” sector.