Bequests, relative consumption, and social security*

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Abstract

We consider a simple overlapping generations model with bequests where agents care about their consumption relative to that of others. It is shown that, under some parameter values, the distribution of bequests diverges and only a fraction of agents leaves bequests in the long run. Then, the introduction of an unfunded social security system promotes capital accumulation, as opposed to the standard result in the literature. We also show that an expansion of the social security system causes the bequest distribution to diverge more slowly, suggesting its mitigating effect on wealth concentration.

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1 Introduction

In this paper, we challenge the common wisdom that unfunded social security has a negative effect on aggregate savings. Using a simple overlapping

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generations (OLG) framework, we show that if only a fraction of individuals chooses to leave bequests to their children, then social security increases aggregate savings in the long run.

It is well known that when agents do not have a bequest motive, the introduction of an unfunded social security system discourages capital accumulation. On the other hand, when agents are linked with their descendants through positive bequests, unfunded social security is known to be completely neutral with regard to capital accumulation. These are the implications of Diamond's (1965) and Barro's (1974) models, respectively, two canonical OLG models that provide a basis for the macroeconomic analysis of social security. It is worth noting, though, that both models have been criticized (see, e.g., Mankiw (2000)) for ignoring consumer heterogeneity, as is apparent in the data. Accounting for this fact, Laitner (2001) and Michel and Pestieau (1998) have considered a model with two types of agents – altruistic and non-altruistic – and showed that in the long run, the neutrality of unfunded social security still holds at the aggregate level.[1] The result holds even in more general models where agents are heterogeneous in both the degree of altruism and productivity level (Michel and Pestieau, 2005) or the degree of altruism and the preference for wealth (Pestieau and Thibault, 2012).

Similarly to Laitner (2001) and Michel and Pestieau (1998), we are interested in the analysis of unfunded social security when bequest behavior differs across agents. In contrast to their approach, however, ours does not require agents' preferences to be different. The heterogeneity in our model comes from the initial wealth of dynasties to which agents belong, so that newborn agents differ only in the bequest received from their parents.[2] At the same time, following the recent literature on income and wealth distribution, we assume that agents compare their consumption with some reference

[1]One can reach the same conclusion by alternatively assuming that all agents have a bequest motive, but one group of agents is more patient than the other (see Smetters (1999)).

[2]In a related study by Caballe and Fuster (2003), newborn agents are also identical in all respects except for their inheritance. However, they face uncertainty about whether they will be altruistic toward children or not. When old, agents learn their type. Using this idea, the authors show that the introduction of an unfunded social security system crowds out capital.

level. Several papers (see Bogliacino and Ortoleva (2015), Borissov (2016), Genicot and Ray (2017), Borissov and Kalk (2020)) have argued that the presence of a reference point in utility can lead to a polarized society, with the poor trapped in a low-income group. For example, Borissov (2016) has found that when individual consumption yields utility relative to the average consumption, poorer dynasties can become even poorer by keeping up with this reference. Eventually, they will save nothing, while the richest dynasties will accumulate the entire wealth. In the context of social security, the importance of relative consumption has been highlighted by Knell (2010) and Bilancinia and D'Antoni (2012).

We show that depending on the degree of relative concerns among agents, two equilibrium regimes are possible: one where all agents leave bequests and one where some agents do not leave bequests. In the first regime, the economy converges to full equality regardless of the initial wealth distribution and unfunded social security does not affect capital accumulation. As in the model of Barro (1974), an increase in the social security tax is exactly offset by an increase in bequests. On the contrary, in the second regime, from some time on the economy is characterized by the existence of two homogeneous groups of agents such that only those who have belonged to the wealthiest dynasties since the very beginning leave positive bequests. Then, in line with the results obtained by Laitner (2001) and Michel and Pestieau (1998), unfunded social security redistributes wealth from dynasties leaving no bequests to those leaving positive bequests. However, in our framework, this also has an impact on aggregate wealth accumulation. In particular, as dynasties with an operative bequest motive get richer, they save a larger fraction of their income, which increases the aggregate saving rate. Thus, a larger size of the social security system leads to a higher long-run capital stock.

The fact that saving rates rise with income has been well documented by Dynan et al. (2004). Furthermore, existing empirical evidence suggests that many individuals do not receive an inheritance, while the inheritance received by an individual is negatively related to her own income and positively related to the income of her parents. Our model is consistent with these observations. When modeling a bequest motive, we follow the approach of Becker and Tomes (1979) and assume that agents derive utility from the disposable income of their offspring. As a consequence, bequests in our economy serve merely to compensate children for their low levels of (net) earnings and can be zero if parents are poor. Compared to pure altruism of Barro (1974), this approach seems to be simpler, yet it remains consistent with Barro's neutrality result applied to unfunded social security.[3]

The rest of the paper is organized as follows. We present the model in Section 2 and characterize its equilibrium properties in Section 3. Section 4 examines how social security affects capital accumulation and the dynamics of the bequest distribution. In the final section, we conclude the paper.

2 The model

2.1 Demographics

Consider an OLG economy in discrete time indexed by $t \in \{0, 1, \ldots\}$. At any date t, a new generation of individuals is born. They live for two periods: young and old. At the end of "youth", each individual gives birth to one offspring belonging to the same dynasty. There exist L such dynasties and hence L individuals in each generation, which we index by $j \in \{1, \ldots, L\}$. Dynasties differ only in their initial endowment of capital.

2.2 Production

The economy produces a single good that is either consumed or invested. The production function takes the Cobb-Douglas form with a constant capital share $\alpha \in (0, 1)$:

$$Y_t = K_t^{\alpha} L_t^{1-\alpha},$$

where Y_t is total output, K_t is the capital stock, and L_t is labor input, all at date t. For simplicity, capital fully depreciates after one period.

[3]The use of the offspring's disposable income in the utility function for analyzing social security policies can be found in Lambrecht et al. (2005) and Kunze (2012).

All markets are perfectly competitive. Therefore, the rate of return to capital, R_t, and the wage rate, w_t, satisfy

$$R_t = \alpha k_t^{\alpha-1}, \; w_t = (1-\alpha)k_t^{\alpha}, \tag{1}$$

where $k_t = K_t/L_t$ is the capital stock per worker at date t.

2.3 Consumer problem

Each individual inelastically supplies one unit of labor when young and retires when old, as in Diamond (1965). Since there is no population growth, this implies $L_t = L$ for every t. During the working age, individual j born at date t receives a bequest b_t^j from her parent and earns the wage w_t on which she pays a payroll tax at rate $\tau \in [0, 1)$. The resulting income is allocated between young-age consumption c_t^j and savings s_t^j:

$$c_t^j + s_t^j = (1-\tau)w_t + b_t^j. \tag{2}$$

After retirement the individual uses the proceeds of her savings to leave a bequest b_{t+1}^j to her offspring. Furthermore, she is entitled to social security benefits θ_{t+1}:

$$b_{t+1}^j = R_{t+1}s_t^j + \theta_{t+1}, \tag{3}$$

where R_{t+1} corresponds to the gross interest rate at date $t+1$. Bequests cannot be negative, i.e.,

$$b_t^j \geq 0. \tag{4}$$

Similarly to Bogliacino and Ortoleva (2015), we assume that the desire to leave a bequest is the only motive for saving in our OLG economy. A natural interpretation of this would be that consumption when old is incorporated in the offspring's consumption. One can consider a more general setting where the provision of old-age consumption is an additional saving motive. In this case, however, the main result of the paper remains qualitatively unchanged, while the analysis becomes significantly more complex.

Individuals are altruistic toward their offspring and derive utility from her disposable income when she is young. They also care about their own young-age consumption, which we assume is subject to positional concerns. More precisely, following Alvarez-Cuadrado and Long (2011; 2012), we assume that individual j of generation t compares her c_t^j with that of the average individual in the same generation. Let us denote this benchmark value by $\bar{c}_t$. Preferences are represented by the following utility function:

$$U_t^j = \ln(c_t^j - \gamma \bar{c}_t) + \beta \ln((1-\tau)w_{t+1} + b_{t+1}^j),$$

where $\gamma \in [0,1)$ measures the importance of social comparisons and $\beta > 0$ is the degree of altruism.

Thus, the problem of individual j born at date t is to choose $(c_t^j, s_t^j, b_{t+1}^j)$ that maximizes the lifetime utility U_t^j subject to the budget constraints (2) and (3), and the non-negativity constraint on bequests (4). Assuming that the individual takes the reference level $\bar{c}_t$ as given, the first-order condition of this problem is

$$(1-\tau)w_{t+1} + b_{t+1}^j \geq \beta R_{t+1}(c_t^j - \gamma \bar{c}_t) \quad (= \text{if } b_{t+1}^j > 0). \tag{5}$$

2.4 Government

The government runs a balanced-budget unfunded social security system, collecting taxes from young individuals at date t and transferring those funds to the current old. With constant population, the government's budget constraint is simply written as

$$\theta_t = \tau w_t. \tag{6}$$

Our primary objective is to examine the aggregate and distributional effects of changes in τ, which can be interpreted as changes in the size of the social security system.

3 Temporary, intertemporal and steady-state equilibria

In this section, we give the definitions of equilibria in our model and describe two possible regimes of the economy. The definitions are fairly standard. First, we define a temporary equilibrium where all consumers maximize their utility, firms maximize their profits, the government balances its budget each period, and all markets clear. An intertemporal equilibrium is defined as a sequence of temporary equilibria.

Formally, let s_{t-1}^j be given for all j at some date t. Let further $k_t = \sum_{j=1}^L s_{t-1}^j/L > 0$. A tuple $\{(c_t^j, b_t^j, s_t^j)_{j=1}^L, \bar{c}_t, \bar{b}_t, k_{t+1}\}$ constitutes a *time-t temporary equilibrium* if

(i) for every $j = 1, \ldots, L$, (c_t^j, b_t^j, s_t^j) is determined by conditions (2)-(5) with θ_t and θ_{t+1} given by (6), (R_t, w_t) and (R_{t+1}, w_{t+1}) given by (1), and $\bar{c}_t = \sum_{j=1}^L c_t^j/L$;

(ii) $k_{t+1} = \sum_{j=1}^L s_t^j/L > 0$;

(iii) $\bar{b}_t = \sum_{j=1}^L b_t^j/L$.

Note that when the value of γ is close to 1, the temporary equilibrium may not exist. To ensure its existence, we need to have $c_t^j - \gamma\bar{c}_t > 0$ for any j. It is not difficult to show that the sufficient condition for this to occur is

$$\gamma < \tilde{\gamma}(\tau) \equiv \frac{(1-\alpha)[1-\tau+\beta(\alpha+\tau(1-\alpha))]}{1+\beta(1-\alpha)(\alpha+\tau(1-\alpha))}.$$

If it is satisfied, then the temporary equilibrium not only exists but is also unique.

Now suppose that we are given $\{(s_{-1}^j)_{j=1}^L, k_0\}$ such that $k_0 = \sum_{j=1}^L s_{-1}^j/L$ is positive. A recursively constructed sequence $\{(c_t^j, b_t^j, s_t^j)_{j=1}^L, \bar{c}_t, \bar{b}_t, k_{t+1}\}_{t=0}^\infty$ of temporary equilibria is called an *intertemporal equilibrium* starting from $(s_{-1}^j)_{j=1}^L$. Its *existence* and *uniqueness* follows from the existence and uniqueness of temporary equilibria.

We next turn to steady-state equilibria, where all variables are constant. We call a tuple $\{(c^j, b^j, s^j)_{j=1}^L, \bar{c}, \bar{b}, k\}$ a *steady-state equilibrium* if the sequence $\{(c_t^j, b_t^j, s_t^j)_{j=1}^L, \bar{c}_t, \bar{b}_t, k_{t+1}\}_{t=0}^\infty$ given for all $t \geq 0$ by

$$c_t^j = c^j,\ b_t^j = b^j,\ s_t^j = s^j,\ j = 1, \ldots, L,$$
$$\bar{c}_t = \bar{c},\ \bar{b}_t = \bar{b},\ k_{t+1} = k,$$

is an intertemporal equilibrium starting from $(s^j)_{j=1}^L$.

To describe the structure of steady-state equilibria, we define the following threshold:

$$\gamma^* \equiv \frac{\beta(1-\alpha)}{1+\beta(1-\alpha)}.$$

It is easy to see that the right-hand side increases with the share of labor income $(1-\alpha)$ and the degree of altruism (β). Also, one can easily verify that

$$\gamma^* < \tilde{\gamma}(\tau),\ \tau \in [0, 1).$$

It turns out that the long-run behavior of the economy crucially depends on whether γ is smaller or greater than γ^*. If $\gamma > \gamma^*$, then the steady-state equilibrium is characterized by the division of the set of dynasties into two groups. One group leaves positive bequests, whereas all other dynasties leave no bequests and so the members of those dynasties consume their entire lifetime income. We interpret the first group of dynasties as the rich, the second group as the poor, and the corresponding steady-state equilibrium as a *polarized* (or two-class) equilibrium. What is important is that the polarized equilibrium with any proportion between the rich and the poor is possible, except for the degenerate equilibrium where all dynasties are poor.[4] For given γ and τ, the aggregate saving rate associated with a steady-state equilibrium is fully determined by the share of the rich in the population, which we denote by m. It is given by

$$s(\gamma, \tau, m) = \frac{\beta\alpha[m(1-\gamma) + (1-m)(\alpha + \tau(1-\alpha))]}{\beta\alpha(1-m\gamma) + m + (1-m)(\alpha + \tau(1-\alpha)(1+\beta))}. \qquad (7)$$

[4] The polarized equilibrium where all dynasties are rich may exist, however. It occurs in a special case when initial endowments of capital are identical across dynasties.

Conversely, if $\gamma < \gamma^*$, there is a unique steady-state equilibrium, in which all agents leave positive bequests and, moreover, these bequests are the same. We therefore treat this equilibrium as *egalitarian.* Since there are no poor agents, the population share of the rich is equal to unity, so that the economy's saving rate is given by $s(\gamma, \tau, 1)$.

The following proposition formalizes the above description.

Proposition 1. *If $\gamma < \gamma^*$, then there exists a unique steady-state equilibrium satisfying*

$$k = s(\gamma, \tau, 1)k^\alpha; \; b^j = \bar{b}, \; s^j = k, \; j = 1, \ldots, L.$$

If $\gamma > \gamma^*$, then for any non-empty subset $J \subseteq \{1, \ldots, L\}$ with cardinality $|J|$, there exists a unique steady-state equilibrium satisfying

$$k = s\left(\gamma, \tau, \frac{L}{|J|}\right) k^\alpha; \; b^j = \frac{L}{|J|}\bar{b}, \; s^j = \frac{L}{|J|}k + \frac{L - |J|}{|J|}\tau\frac{1-\alpha}{\alpha}k, \; j \in J;$$

$$b^j = 0, \; s^j = -\tau\frac{1-\alpha}{\alpha}k, \quad j \notin J.$$

In the rest of the section, we characterize the dynamic and asymptotic properties of the equilibrium distribution of bequests across dynasties. These properties depend on the relationship between γ and γ^* and, in fact, determine the structure of steady-state equilibria. In particular, we show that if $\gamma < \gamma^*$, then an intertemporal equilibrium converges to the egalitarian steady-state equilibrium. In other words, the bequest distribution tends asymptotically towards a uniform distribution. Conversely, if $\gamma > \gamma^*$, then an intertemporal equilibrium converges to the polarized steady-state equilibrium. From some time on, all dynasties stop leaving bequests and become poor except for dynasties that had the largest capital endowments in the beginning.

Consider an intertemporal equilibrium $\{(c_t^j, b_t^j, s_t^j)_{j=1}^L, \bar{c}_t, \bar{b}_t, k_{t+1}\}_{t=0}^\infty$ starting from $(s_{-1}^j)_{j=1}^L$. Without loss of generality, we assume

$$s_{-1}^1 = \cdots = s_{-1}^{L'} > s_{-1}^{L'+1} \geq \cdots \geq s_{-1}^L,$$

where $L' \in \{1, \ldots, L\}$ denotes the number of dynasties having the highest initial capital endowment. Then, for any pair (j, h), we have that $s_{-1}^j = s_{-1}^h$ implies $b_0^j = b_0^h$, while $s_{-1}^j > s_{-1}^h$ implies $b_0^j \geq b_0^h$. Similarly, at any date t, we have that $b_t^j = b_t^h$ implies $b_{t+1}^j = b_{t+1}^h$, while $b_t^j > b_t^h$ implies $b_{t+1}^j \geq b_{t+1}^h$. Hence, the following relationship holds for all $t \geq 0$:

$$b_t^1 = \cdots = b_t^{L'} \geq b_t^{L'+1} \geq \cdots \geq b_t^L.$$

We use $M_t \in \{0, \ldots, L\}$ to denote the number of agents leaving positive bequests at date t, i.e.,

$$b_t^j > 0, \; j \leq M_t; \; b_t^j = 0, \; j > M_t,$$

and $m_t \equiv M_t/L$ to denote their share in the population. Following Michel and Pestieau (1998), we will call the agents leaving positive bequests *altruists*.

Now we can describe the dynamics of bequest distribution. The next proposition should be read as follows. In the case where $\gamma < \gamma^*$, irrespective of the initial distribution of capital endowments, all agents are altruists starting from date $t = 1$, and eventually the economy converges to the egalitarian steady-state equilibrium. On the contrary, in the case where $\gamma > \gamma^*$, the number of altruists is non-increasing over time, and eventually only agents belonging to the dynasties with the highest initial capital endowments leave positive bequests.

Proposition 2. *If $\gamma < \gamma^*$, then*

$$M_{t+1} = L \; (\textit{or } m_{t+1} = 1), \; k_{t+1} = s(\gamma, \tau, 1) k_t^\alpha, \; t \geq 0;$$

$$\lim_{t \to \infty} b_t^j / b_t = 1, \; j = 1, \ldots, L.$$

If $\gamma > \hat{\gamma}$, then $M_{t+1} \leq M_t$ for all $t \geq 0$ and there exists a date T such that for $t \geq T$,

$$M_t = L' \ (or\ m_t = L'/L),\ k_{t+1} = s(\gamma, \tau, L/L')k_t^{\alpha},$$
$$b_t^j/\bar{b}_t = L/L',\ j \leq L';\ b_t^j = 0,\ j > L'.$$

It is noteworthy that the number of altruists can increase only in the case where $\gamma < \gamma^*$ and only between dates 0 and 1. This happens if at date $t = 0$ some agents receive no bequests. In all other cases, the number of altruists cannot increase. As one can show, if the number of altruists does not decrease at some date $t + 1$, i.e., $M_{t+1} \geq M_t$ ($m_{t+1} \geq m_t$), then

$$k_{t+1} = s(\gamma, \tau, m_{t+1})k_t^{\alpha},$$

and for all $j \leq M_{t+1}$,

$$\frac{b_{t+1}^j}{\bar{b}_{t+1}} = A(\gamma, \tau, m_{t+1}) + B(\gamma, \tau, m_{t+1})\frac{b_t^j}{\bar{b}_t},$$

where the functions $A : \Omega \to \mathbb{R}$ and $B : \Omega \to \mathbb{R}_{++}$ are defined on

$$\Omega \equiv \{(\gamma, \tau) :\ 0 < \gamma < \tilde{\gamma}(\tau),\ 0 < \tau < 1\} \times [0, 1]$$

and have the following properties:

(i) for any $m \in (0, 1]$,

$$\gamma \lesseqgtr \gamma^* \ \Leftrightarrow\ A(\gamma, \tau, m) \gtreqless 0 \ \Leftrightarrow\ B(\gamma, \tau, m) \lesseqgtr 1;$$

(ii) for any $\tau \in [0, 1)$ and $\gamma \in [0, \tilde{\gamma}(\tau))$, the solution of the equation

$$x = A(\gamma, \tau, m) + B(\gamma, \tau, m)x$$

is given by $x = 1/m$.

In particular, if $\gamma < \gamma^*$, then the aggregate saving rate does not depend on the initial distribution of capital endowments; from date $t = 0$ on, it is equal to $s(\gamma, \tau, 1)$:

$$k_{t+1} = s(\gamma, \tau, 1) k_t^\alpha, \;\; t \geq 0.$$

As for the dynamics of bequest distribution in this case, for all $t \geq 0$, we have

$$\frac{b_{t+1}^j}{\bar{b}_{t+1}} = A(\gamma, \tau, 1) + B(\gamma, \tau, 1) \frac{b_t^j}{\bar{b}_t}, \;\; j = 1, \ldots, L,$$

and hence the speed of convergence to the egalitarian steady-state equilibrium is determined by $B(\gamma, \tau, 1)$. A higher value of $B(\gamma, \tau, 1)$ implies a lower speed of convergence.

The case where $\gamma > \gamma^*$ is completely different. Until some date, altruists belong to the same dynasties as altruists at $t = T_0 \equiv 0$. More precisely, there is a date T_1 such that $M_t = M_0$ for all $t = 0, \ldots, T_1 - 1$. Some dynasties drop out from this group at T_1, and there is a date T_2 such that $M_t = M_{T_1}$ for all $t = T_1, \ldots, T_2 - 1$, and so on. Eventually, there is a date T_S such that, from T_S on, only agents belonging to the dynasties with the highest initial capital endowments continue to leave positive bequests: $M_t = L'$ for all $t \geq T_S$. For each time span from T_s to $T_{s+1} - 1$ with $s < S$, the number of altruists remains constant and the bequest share of dynasty j leaving a positive bequest at date $T_s + 1$ evolves according to

$$\frac{b_{t+1}^j}{\bar{b}_{t+1}} = A(\gamma, \tau, m_{T_s+1}) + B(\gamma, \tau, m_{T_s+1}) \frac{b_t^j}{\bar{b}_t}.$$

If the bequest of dynasty j is higher than the average bequest among altruists, that is,

$$\frac{b_t^j}{\bar{b}_t} > \frac{1}{m_{T_s+1}},$$

dynasty j will increase its next period's share in aggregate bequests. In contrast, if the bequest of dynasty j is lower, that is,

$$0 < \frac{b_t^j}{\bar{b}_t} < \frac{1}{m_{T_s+1}},$$

this share will fall. Thus, the economy is characterized by the divergence in bequests. The speed of divergence over the time span from T_s to $T_{s+1} - 1$ is determined by $B(\gamma, \tau, m_{T_s+1})$. A higher value of $B(\gamma, \tau, m_{T_s+1})$ implies a higher speed of divergence. Note that starting from date T_S, the economy's saving rate is given by $s(\gamma, \tau, L'/L)$:

$$k_{t+1} = s\left(\gamma, \tau, \frac{L'}{L}\right) k_t^{\alpha}, \ t \geq T_S.$$

4 Changes in the size of the social security system τ

The aim of this section is to examine how the size of the social security system affects the aggregate saving rate and the dynamics of bequest distribution. To do this, it is useful to look first at the effects of changes in other parameters of the model.

The saving rate of the economy in the long run, $s(\gamma, \tau, m)$, depends on the importance of social comparisons, γ, the size of the social security system, τ, and the population share of the rich, m. The dependence of the saving rate on γ is negative:

$$\frac{\partial s(\gamma, \tau, m)}{\partial \gamma} < 0,$$

i.e., less intensive social comparisons lead to a higher saving rate. As for the dependence of the steady-state saving rate on the bequest distribution, note that the former is inversely related to m:

$$\frac{\partial s(\gamma, \tau, m)}{\partial m} < 0 \text{ if } \gamma > \gamma^*.$$

In other words, the saving rate is higher in the polarized equilibrium with a low m than with a high m. The intuition is straightforward. A lower population share of the rich implies that a "representative" rich agent receives a larger bequest and therefore consumes a smaller fraction of her lifetime income. Then, her saving rate increases, which, in turn, causes the aggregate saving rate to rise.

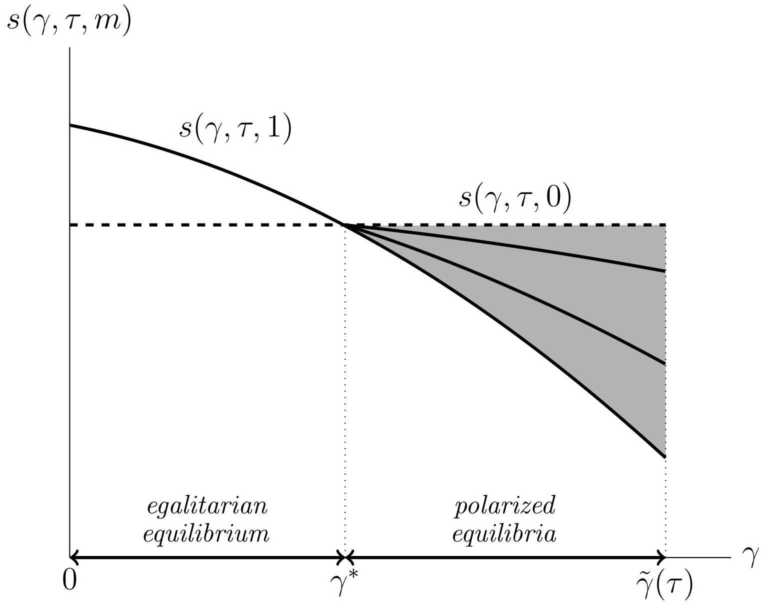

Figure 1: Effect of γ on the saving rate in two types of steady-state equilibrium

All this does not, however, mean that a higher saving rate is associated with a higher inequality. Indeed, it is readily verified that

$$\gamma \lesseqgtr \gamma^* \;\Leftrightarrow\; s(\gamma, \tau, 1) \gtreqless s(\gamma, \tau, 0).$$

As was noted above, $s(\gamma, \tau, 1)$ is decreasing in γ for all $\tau \in [0, 1)$, so that the aggregate saving rate in the egalitarian steady-state equilibrium (obtained under $\gamma < \gamma^*$) must be higher than that in any polarized equilibrium. The relationship among the importance of social comparisons, inequality, and growth is illustrated in Fig. 1, where the shaded area corresponds to various polarized equilibria.

At this point, we can present the main results of the paper and answer the question of what impact social security has on capital accumulation and the distribution of bequests. Note that $\tilde{\gamma}(\tau)$ is decreasing in τ. Thus, an expansion of the social security system leads to a narrower range of γ over which the bequest distribution necessarily diverges (see Fig. 2). Also, we

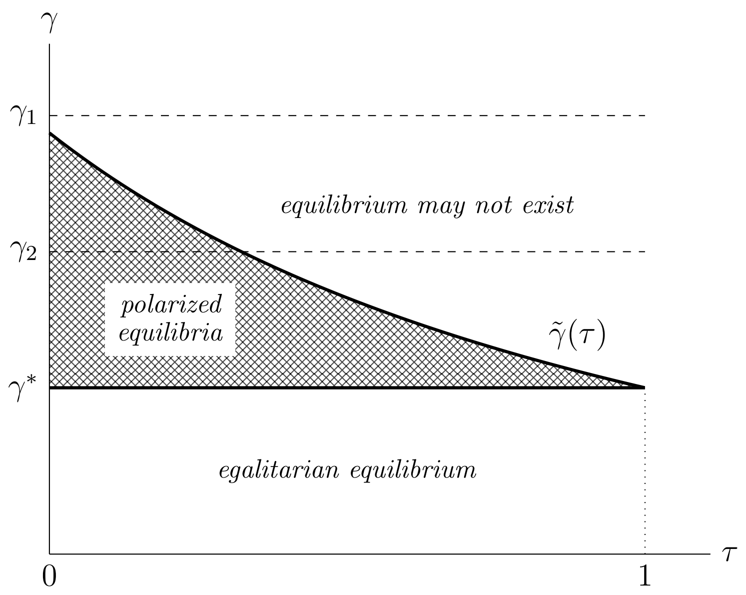

Figure 2: Effects of γ and τ on the type of steady-state equilibrium

have, for any $m \in (0, 1)$,

$$\frac{\partial B(\gamma, \tau, m)}{\partial \tau} < 0 \text{ if } \gamma > \gamma^*.$$

while $\partial B(\gamma, \tau, 1)/\partial \tau = 0$, regardless of the value of γ. The implications of these findings are as follows:

(1) For $\gamma > \bar{\gamma}(0)$ (e.g., $\gamma = \gamma_1$ in Fig. 2), a temporary equilibrium may not exist.

(2) For $\gamma^* < \gamma < \tilde{\gamma}(0)$ (e.g., $\gamma = \gamma_2$ in Fig. 2), there is a threshold $\tilde{\tau}(\gamma) \equiv \tilde{\gamma}^{-1}(\gamma)$. If the size of the social security system is below this level, then the economy eventually converges to the polarized steady-state equilibrium. If, in addition, initial capital endowments are unevenly distributed across dynasties, we know that as τ becomes closer to the threshold, the distribution of bequests diverges more slowly. All values of τ above $\tilde{\tau}(\gamma)$ may result in the non-existence of a temporary equilibrium.

(3) For $0 < \gamma < \gamma^*$, the economy converges to the egalitarian steady-state equilibrium with the speed of convergence being independent of τ.

To answer the question about the impact of social security on capital accumulation, suppose first that $\gamma < \gamma^*$. Then, all dynasties leave positive bequests starting from date $t = 1$, and it is straightforward to see from (7) that $\partial s(\gamma, \tau, 1)/\partial\tau = 0$. That is, social security does not affect capital accumulation. Let us now suppose that $\gamma^* < \gamma < \tilde{\gamma}(0)$. In this case, the dependence of the aggregate saving rate on τ is positive for any population share of the rich between 0 and 1, $m \in (0, 1)$:

$$\frac{\partial s(\gamma, \tau, m)}{\partial \tau} > 0 \text{ if } \gamma > \gamma^*.$$

Hence, we conclude that the introduction of social security has a positive effect on capital accumulation as long as only a fraction of dynasties leaves bequests. The latter occurs in the long run if the intensity of social comparisons does not take extreme values (i.e., $\gamma^* < \gamma < \tilde{\gamma}(0)$) and initial capital endowments are not identical across dynasties.

Summing up, we have:

Proposition 3. *If $\gamma < \gamma^*$, then all dynasties leave bequests starting from date $t = 1$ and the introduction of an unfunded social security system does not affect capital accumulation. If, on the contrary, $\gamma > \gamma^*$, then only a fraction of dynasties leaves bequests in the steady state and the introduction of an unfunded social security system increases the steady-state capital stock. Moreover, the larger is the size of the social security system, the lower is the speed with which the bequest distribution diverges.*

5 Conclusion

In this paper, we study how unfunded social security affects capital accumulation and the distribution of bequests across dynasties in the presence of the reference consumption in utility. Dynasties are identical in every respect except for their initial wealth. We show that if the importance of relative consumption is sufficiently low, the economy converges an egalitarian equilibrium where all dynasties leave bequests. Then, the introduction of an unfunded social security system does not affect capital accumulation. In contrast, if the importance of relative consumption is above some threshold, the economy converges to a polarized equilibrium, with poor dynasties leaving no bequests in the long run and rich dynasties leaving positive bequests and having the highest initial capital endowments. Since these endowments can be arbitrarily distributed, any proportion between the rich and the poor is possible in the polarized regime. The introduction of an unfunded social security system in the polarized regime promotes capital accumulation and reduces the divergence rate of the bequest distribution.

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