Human capital accumulation and environmental preservation in a globalized economy

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Abstract

This paper analyzes the impact of trade openness on education and environmental preservation choices in a two country North-South model. Parents may invest in their children's education increasing their probability to become skilled and in maintenance investment in order to preserve present and future environmental quality. In autarky, unskilled individuals in the South economy are unable to invest in education due to endogenous borrowing constraints. Moreover, unskilled individuals in both countries choose not to invest in environmental preservation. Openness to trade modifies relative factor prices and increases pollution. This allows for human capital convergence between both economies and induce all individuals to contribute to environmental preservation in the post-trade equilibrium. In the latter, all individuals choose to invest in education while environmental quality can be both larger or smaller than in autarky. We also focus on the optimal allocation under free trade and conclude that a maintenance investment subsidy dependent on the skill level must be implemented together with appropriate lump-sum taxes.

1 Introduction

The intergenerational aspects of education and environmental maintenance decisions have given rise to a large set of theoretical contributions in economics. In this paper, we study the impact of trade openness on individuals' decisions concerning these particular intergenerational investments.

The literature on education and social mobility has highlighted the importance of indivisible investment and borrowing constraints. Under these assumptions the models of Galor and Zeira (1993) and Eckstein and Zilcha (1994) generate different skill classes and intergenerational income inequality. Individuals with lower wages might not be able to invest in their children's education thus generating persistence in intergenerational income inequality. On the empirical side, using cross country and panel data, Flug et al. (1998) show that the lack of financial markets seems to have a negative impact on human capital accumulation. Using a sample of 78 countries, Christou (2001) documents that the severity of borrowing constraints is inversely related to human capital accumulation. While these observations might be mostly important in developing countries, the intergenerational income correlation in countries like the U.S. also seem to point out to the importance of borrowing constraints.[1] However, by affecting relative factor prices, openness to trade might play an important role in relaxing borrowing constraints and fostering human capital accumulation. Owen (1999) provides empirical evidence on the positive relationship between trade openness and investment in human capital. Arbache et al. (2004) find that education levels rose in Brazil after trade liberalization while Edmonds et al. (2010) document that trade liberalization in India reduced the costs of schooling. The theoretical contributions of Cartiglia (1997) and Ranjan (2001) consider the possibility that trade might enhance human capital accumulation. In Cartiglia (1997), trade liberalization in a skilled-scarce country reduces the returns to education but also the cost of latter. The weakening of credit constraints results in higher investment in human capital. In Ranjan (2001) the effect operates through the changes in the distribution of income and wealth. However, these papers do not solve for the autarky equilibrium but instead analyze the impact of trade openness on a small open economy. Our contribution is closer to the work of Ranjan (2003) who considers a North-South model and show that trade might allow an economy stuck in a poverty trap to converge to the equilibrium of the high-income economy. We depart from the framework of Ranjan (2003) in two respects. First, while Ranjan (2003) considers warm-glow altruism, we assume dynastic altruism as in Barro (1974).[2] Second and most importantly, we consider that skill-intensive activities generate pollution externalities. We thus study jointly the implications of trade openness on both investment in education and environmental preservation choices.

The literature on environmental preservation has highlighted the limited capacity of short-lived individuals to take into account the impact of their decisions on future generations (see, e.g., John and Pecchenino, 1994; Bovenberg and Heijdra, 1998). In most overlapping generations (OLG) models that consider short-lived individuals, the latter are not altruistic towards their children and only take into account the impact of environmental quality on their own utility. A few contributions have however integrated altruistic behavior in models with environmental constraints. For example, Jouvet et al. (2000) introduce dynastic altruism following Barro (1974) into a standard OLG model while Karp (2017) studies a model with both pure and paternalistic altruism in a differential game setting. In both cases, the presence of altruism might not be sufficient to achieve the first-best outcome due to the public good nature of environmental quality, a point already highlighted earlier by Howarth and Norgaard (1995). In this paper, we follow the approach of Jouvet et al. (2000) and consider dynastic altruism. This implies that there exists an individual threshold value for the altruism factor, above which individuals are ready to contribute to environmental maintenance investment. This threshold value depends on individual income and on the level of environmental quality that would prevail in the absence of environmental investment. By affecting the income of individuals and the level of environmental quality in the absence of environmental investment, openness to trade modifies the threshold value and in turn the incentive of individuals to contribute to environmental preservation. There has been an increasing number of empirical studies investigating the link between openness to trade and environmental quality. Most of the evidence from these studies is mixed with some authors finding a positive impact of trade openness and others a negative one. For example, Frankel and Rose (2005) estimate the

[1]Concerning the U.S., Solon (1992) finds an approximate value around 0.4 while Charles and Hurst (2003) find the pre-bequest correlation in log wealth to be equal to 0.37. Keane and Wolpin (2001) estimate a structural model for the U.S. and estimate that borrowing constraints are indeed severe.

[2]Dynastic altruism implies that parents' utility depends on their children's utility, which in turn depends on expected wages.

effect of trade on environmental quality for a given level of income per capita and conclude that there is little evidence of environmental degradation. Baek et al. (2009) show that trade and income positively affected environmental quality in developed countries and China. Managi et al. (2009) find that most of the results depend on the pollutant and the country considered. Trade is found to benefit environmental quality in OECD countries and the authors highlight that the impact is large in the long-run, after the dynamic adjustment process has taken place. Finally, Le et al. (2016) use cross-country panel data and find a negative relationship between trade openness and environmental quality for their global sample of 98 countries. However, results seem to vary significantly with income differences. Empirical evidence thus seems to suggest that environmental quality might increase or decrease following trade openness. In our framework, trade openness will affect both pollution and the willingness to invest in environmental quality. Therefore, the full impact of trade on environmental quality will depend on the magnitude of those two effects.

An account of the results is as follows. In the autarky equilibrium, all individuals in the North are able to invest in their children's education while unskilled workers are constrained in the South. Concerning environmental maintenance, our autarky equilibrium is characterized by positive investment from skilled individuals in both countries and no contribution from unskilled ones. Openness to trade modifies relative factor prices and allows unskilled individuals in the South to invest in education while preserving the initial situation in the North. The additional number of skilled workers increases the pollution level and this combined with the change in wages modifies the willingness of individuals to invest in environmental preservation. However, the post-trade equilibrium level of environmental quality can both increase or decrease depending on the magnitude of both effects. We then focus on the optimal allocation under free trade and conclude that a maintenance investment subsidy must be implemented together with appropriate lump-sum transfers guaranteeing that all individuals can still invest in their children's education.

The paper is organized as follows. Section 2 presents the model with the production structure and household behavior. In Section 3 we characterize the equilibrium under autarky. In Section 4 we study the impact of trade openness on education and environmental preservation investment. Section 5 presents some numerical simulations in order to highlight the main results of the paper. Section 6 focuses on the optimal allocation under free trade while section 7 presents our concluding remarks. A final appendix contains all the proofs of the results.

2 Model

Consider a world consisting of two economies labeled North and South where variables in the South are indexed by an asterisk.

2.1 Production

The production side of the economy is similar to Ranjan (2003). Each economy produces a unique non-tradable final good Y using two tradable intermediate goods X^s and X^u. The production of the final good is given by

$$Y = A(X^s)^{1-\alpha}(X^u)^{\alpha} \tag{1}$$

with $\alpha \in [0,1)$. The final good is used for consumption and chosen as the numeraire. The prices of the two intermediate inputs are denoted by p^s and p^u. There is perfect competition in the three product markets. The optimal choice for intermediate inputs then implies

$$\begin{aligned} p^s &= (1-\alpha)A(X^s)^{-\alpha}(X^u)^{\alpha} && (2) \\ p^u &= \alpha A(X^s)^{1-\alpha}(X^u)^{\alpha-1}. && (3) \end{aligned}$$

Then, we can easily obtain the demand for the two inputs as

$$\begin{aligned} X^s &= \frac{(1-\alpha)Y}{p^s} && (4) \\ X^u &= \frac{\alpha Y}{p^u} && (5) \end{aligned}$$

implying that relative demand is given by

$$\frac{X^s}{X^u} = \frac{(1-\alpha)p^u}{\alpha p^s}.$$

There are two factors of production: skilled labor S and unskilled labor U, which are used to produce the intermediate goods. The total population in both economies is equal to $L > 1$ so that $S + U = L$. The wage of a skilled worker is denoted by w^s and the one of an unskilled worker by w^u.

As in Cartiglia (1997), there is an education sector which requires skilled workers as teachers. We suppose that a constant teacher-students ratio γ is needed, with $\gamma \in [0,1)$, so that

$$S^e = \gamma M,$$

where M is the number of students and S^e the number of teachers. In each period, the supply of skilled workers will be divided between the education sector S^e and the production of intermediate inputs S^s, so that $S = S^e + S^s$.

For algebraic simplicity and following Ventura (1997), we assume that X^s only uses skilled labor while X^u uses only unskilled labor. The production functions for the intermediate inputs are then given by $X^s = S^s$ and $X^u = U$. As we also assume perfect perfect competition in the two factors markets, optimality implies that $p^s = w^s$ and $p^u = w^u$.

From the full-employment condition for the two factors of production, the relative supply of intermediate inputs is

$$\frac{X^s}{X^u} = \frac{S^s}{U}.$$

The market clearing condition for intermediate goods implies therefore the following

$$\frac{p^u}{p^s} = \frac{w^u}{w^s} = \frac{\alpha(S - S^e)}{(1-\alpha)(L-S)}, \tag{6}$$

so that relative prices and wages are uniquely determined by the number of skilled workers and teachers in the economy.

2.2 Household behavior

We consider an OLG model where each individual lives for two periods, having a descendant at the beginning of the second period of life. We also assume that each individual only makes decisions, consumes and works during the second period of his life. Young individuals just go to school, and can become skilled following the investment of their parents in education. Individuals are supposed to be altruistic towards their children, which may lead them to invest in their descendant's education and in environmental maintenance. We assume the following utility function with dynastic altruism, for an adult individual at period t:

$$V_t = u(c_t) + v(N_t) + \beta E(V_{t+1}).$$

where c_t is his consumption level, N_t the aggregate level of environmental quality and V_{t+1} the utility of his direct descendant. Finally $\beta \in [0,1)$ is the altruism factor. We assume that both functions $u(.)$ and $v(.)$ are twice continuously differentiable with $u'(.) > 0$, $u''(.) < 0$, $v'(.) > 0$, $v''(.) < 0$ for all $c > 0$ and $N > 0$. In addition, the Inada conditions $\lim_{c\to 0} u'(c) = \infty$, $\lim_{c\to\infty} u'(c) = 0$, $\lim_{N\to 0} v'(N) = \infty$ and $\lim_{N\to\infty} v'(N) = 0$ hold. Finally, the coefficient of relative risk aversion in consumption is constant and equal to σ.

The evolution of environmental quality is given by

$$N_t = N_{t-1} + b(H - N_{t-1}) - \kappa(X_t^{s*} + X_t^s) + \eta Z_t,$$

where $H > 0$ is the natural level of environmental quality and $b \in (0,1)$ is the recovery speed of the environment. We assume that the world production of X^s is the only polluting activity and that pollution abatement occurs according to a linear technology. Denoting by $Z_t \geq 0$ the total amount of resources devoted to environmental maintenance investment, the improvement of environmental quality at time t amounts to ηZ_t.

We take the standard approach of a Cournot-Nash equilibrium where each individual takes the others' contributions to the public good as given. From the point of view of an individual, environmental quality evolves according to

$$N_t = N_{t-1} + b(H - N_{t-1}) - \kappa(X_t^{s*} + X_t^s) + \eta \overline{Z}_t + \eta z_t^h,$$

where $\overline{Z} \geq 0$ is the sum of other individuals' contributions and z the individual contribution of agent h. At each period, environmental maintenance contributions must be non-negative, i.e., $z_t^h \geq 0$ for all h and t.

Since education employs skilled workers, the individual cost of tuition is given by the wage of a teacher multiplied by the teacher-students ratio, that is γw_s. The budget constraint of an individual of type i, where $i = \{s, u\}$, is then given by $w_t^i = c_t^i + \gamma w_t^s + z_t^i$ if the individual invests in education and by $w_t^i = c_t^i + z_t^i$ if she does not invest in education. When an individual invests in education, her child will obtain a level of education $e = 1$ and become a skilled worker with probability π. When parents do not invest in their children's education, a young individual receives a lower education level $\overline{e}$, which is also its probability to become skilled. For education to remain valuable, we impose $\overline{e} < \pi$.

It should be noted that since $\gamma < 1$, a skilled individual can always choose to invest in the education of its child. On the contrary, it is possible that an unskilled worker is constrained. This happens when $w^u < \gamma w^s$. We should thus distinguish between the case where unskilled workers are constrained, $w^u < \gamma w^s$, and the case where no one is constrained, $\gamma w^s < w^u < w^s$.

Since the educational investment decision of parents is a discrete choice, the individual must compare its utility in both situations $j = \{\varepsilon, n\varepsilon\}$, where $j = \varepsilon$ if she invests in the education of her child and $j = n\varepsilon$ if she does not. The value function of an individual of type i, with $i = \{s, u\}$, such that $w^i > \gamma w^s$, is then given by

$$V_t^i = \max\left\{V_t^{i,\varepsilon}, V_t^{i,n\varepsilon}\right\} \tag{7}$$

where $V_t^{i,j}$, with $j = \{\varepsilon, n\varepsilon\}$, is the solution of the following problem

$$\begin{aligned} V_t^{i,j}(N_{t-1}) &= \max\left\{u(c_t^{i,j}) + v(N_t) + \beta\phi_j V_{t+1}^s(N_t) + \beta(1-\phi_j)V_{t+1}^u(N_t)\right\}, \\ s.t. \qquad z_t^{i,j} &\geq 0, & (8) \\ c_t^{i,j} &= w_t^i - \gamma d_j w_t^s - z_t^{i,j}, & (9) \\ N_t &= N_{t-1} + b(H - N_{t-1}) - \kappa(X_t^{s*} + X_t^s) + \eta\overline{Z}_t^{i,j} + \eta z_t^{i,j}. & (10) \end{aligned}$$

$z_t^{i,j}$ denotes the individual contribution to environmental maintenance at time t of an individual of type i with education decision j, while $\overline{Z}_t^{i,j}$ denotes the contributions of the other individuals so that $Z_t = \overline{Z}_t^{i,j} + z_t^{i,j}$. Moreover $\phi_j = \pi$ if $j = \varepsilon$ and takes the value $\overline{e}$ otherwise. Also d_j is a variable that takes the value 1 if $j = \varepsilon$ and takes the value zero otherwise.

Let $\rho_t^{i,j}$, $\lambda_t^{i,j}$, and μ_t be the multipliers associated to the constraints (8), (9) and (10) respectively. The first-order and envelope conditions are

$$\begin{aligned} u'(c_t^{i,j}) &= \lambda_t^{i,j}, & (11) \\ \eta\mu_t + \rho_t^{i,j} &= \lambda_t^{i,j}, & (12) \\ v'(N_t) + \beta V_{t+1}'(N_t) &= \mu_t, & (13) \\ \rho_t^{i,j} z_t^{i,j} &= 0 & (14) \\ V_t'(N_{t-1}) &= (1-b)\mu_t & (15) \end{aligned}$$

Combining (11), (12) and (13), we obtain

$$\rho_t^{i,j} = u'(c_t^{i,j}) - \eta[v'(N_t) + \beta V_{t+1}'(N_t)]. \tag{16}$$

This expression characterizes the intratemporal allocation between consumption and environmental quality. Since individuals are altruistic, the latter will take into account the impact of environmental preservation on their descendants' welfare. If $z_t^{i,j} > 0$, as $\rho_t^{i,j} = 0$, we obtain

$$u'(c_t^{i,j}) = \eta[v'(N_t) + \beta V_{t+1}'(N_t)], \tag{17}$$

implying that the marginal utility of consumption is equal to marginal benefit of investing in environmental preservation. If $z_t^{i,j} = 0$, as $\rho_t^{i,j} > 0$, we obtain

$$u'(c_t^{i,j}) > \eta[v'(N_t) + \beta V_{t+1}'(N_t)]. \tag{18}$$

implying that the marginal utility of consumption is larger than the marginal benefit of investing in environmental preservation.

Returning now to the educational investment of parents we have that

$$\begin{aligned} V_t^{i,\varepsilon} &= u(w_t^i - \gamma w_t^s - z_t^{i,\varepsilon}) + v(N_t) + \beta\pi V_{t+1}^s + \beta(1-\pi)V_{t+1}^u, & (19) \\ V_t^{i,n\varepsilon} &= u(w^i - z_t^{i,n\varepsilon}) + v(N_t) + \beta\overline{e}V_{t+1}^s + \beta(1-\overline{e})V_{t+1}^u & (20) \end{aligned}$$

where $z_t^{i,\varepsilon}$ and $z_t^{i,n\varepsilon}$ denote the optimal choices. Therefore, an individual of type i with $w^i > \gamma w^s$ will invest in education if and only if

$$\beta(\pi - \bar{e})(V_{t+1}^s - V_{t+1}^u) \geq u(w_t^i - z_t^{i,n\varepsilon}) - u(w_t^i - \gamma w_t^s - z_t^{i,\varepsilon}). \tag{21}$$

This expression states that the discounted benefit of investing in education must be larger or equal to the current utility loss due to this investment. We assume that in the case where the individual is indifferent, he will invest in education.

3 Autarky equilibrium

In the following, we restrict our attention to steady-state equilibria. This is the standard approach in models with dynastic altruism (see, for example Jouvet et al., 2000; Alonso-Carrera et al., 2007). In addition, as highlighted in the introduction, the impact of trade openness on environmental quality seems to be large in the long-run when the dynamic adjustment has already taken place (Managi et al., 2009).

We first focus on investment in environmental maintenance. At the steady-state, using expressions (13) and (15) we obtain

$$\mu = \frac{v'(N)}{1 - \beta(1-b)},$$

implying

$$\rho^{i,j} = u'(c^{i,j}) - \frac{\eta}{1-\beta(1-b)} v'(N), \tag{22}$$

$$\rho^{i,j} z^{i,j} = 0, \tag{23}$$

which are similar expressions to the ones obtained in Jouvet et al. (2000) where there is no intragenerational heterogeneity.

When an individual of type i with education choice j chooses to not contribute to environmental maintenance, $z^{i,j} = 0$, as $\rho^{i,j} > 0$, we have that

$$\frac{\eta}{1-\beta(1-b)} v'\left(H - \frac{1}{b}[\kappa(X^{s*} + X^s) - \eta \overline{Z}^{i,j}]\right) \leq u'(w^i - \gamma d_j w^s).$$

Therefore, provided that $b \neq 1$, the willingness to invest in environmental maintenance is a function of the altruism factor β. Indeed, there exists a threshold value

$$\overline{\beta}^{i,j} = \frac{1}{(1-b)}\left(1 - \frac{\eta v'(H - \frac{1}{b}[\kappa(X^{s*} + X^s) - \eta \overline{Z}^{i,j}])}{u'(w^i - \gamma d_j w^s)}\right), \tag{24}$$

above which individuals of type i with education decision j are ready to contribute to environmental maintenance. If $\beta < \overline{\beta}^{i,j}$, the marginal cost of maintenance investment is larger than the marginal utility of environmental quality and the optimal level of contribution is equal to zero. If $\beta > \overline{\beta}^{i,j}$, the marginal cost of maintenance investment is equal to the marginal utility of environmental quality and the optimal level of contributions is positive. In this last case $\rho^{i,j} = 0$ and the optimal level of contributions for an individual of type i with education choice j, $z^{i,j} > 0$, solves

$$u'(w^i - \gamma d_j w^s - z^{i,j}) = \frac{\eta}{1-\beta(1-b)} v'(N), \tag{25}$$

where

$$N = H - \frac{1}{b}[\kappa(X^{s*} + X^s) - \eta(\overline{Z}^{i,j} + z^{i,j})]. \tag{26}$$

Since $\overline{Z}^{i,j} + z^{i,j} = Z$ for all i and j, expressions (25) and (26) imply that when individuals are sufficiently altruistic to invest in environmental quality, we obtain that $c^{i,j}$ is constant across all i and j. The consumption levels of individuals that invest in environmental preservation are equal and the difference in income is allocated to additional environmental contributions. If the individual is sufficiently altruistic, the weight of its descendants' utility in terms of environmental quality and education level outweighs a possible increase in consumption. If we interpret the altruism factor as a more standard discount factor, the model highlights that if the discount factor is sufficiently large, individuals are ready to reduce consumption today in order to provide additional utility to their descendants tomorrow. In addition, we can derive the impact of the level of the altruism factor on consumption and environmental contributions. By differentiating the arbitrage condition (25), we obtain

$$\frac{dz^{i,j}}{d\beta} = -\frac{(1-b)u'(c^{i,j})}{[1-\beta(1-b)]u''(c^{i,j}) + \eta^2 v''(N)/b} \geq 0. \tag{27}$$

Since $z^{i,j} = w^i - \gamma d_j w^s - c^{i,j}$, this result also implies $dc^{i,j}/d\beta \leq 0$. If $b = 1$, the level of consumption does not depend on the level of altruism. In this case, the level of environmental quality is not transmitted from one generation to the next and the only incentive to invest in environmental maintenance is related to life-cycle utility. If $b < 1$, the level of consumption decreases with the level of altruism while the level of individual contribution increases. The more altruistic are individual agents, the more they are ready to reduce their consumption level in order to increase their offspring's welfare.

Given the environmental contribution decision rule, the following Lemma establishes a ranking of environmental contributions depending on net wages $w^i - \gamma d_j w^s$.

Lemma 1. *Consider two agents 1 and 2 that differ in terms of net wages. If $w^1 - \gamma d_j^1 w^s > w^2 - \gamma d_j^2 w^s$, then either $z^{1,j} > z^{2,j} \geq 0$ or $z^{1,j} = z^{2,j} = 0$.*

Proof. See Appendix A □

In order to establish a ranking of net wages, we need to analyze the education investment decision of individuals agents. We start by deriving a condition under which unskilled individuals are constrained which is equivalent to

$$\frac{w^u}{w^s} = \frac{\alpha(S - S^e)}{(1-\alpha)(L-S)} < \gamma.$$

When the latter inequality is satisfied, unskilled individuals do not invest in education so that, as $M = S$, we have $S^e = \gamma S$ and the previous condition can be written as

$$S < \underline{S} = \frac{\gamma(1-\alpha)L}{\alpha(1-\gamma) + \gamma(1-\alpha)}. \tag{28}$$

We are interested in an equilibrium where in the South economy, unskilled individuals are constrained so that $S^* < \underline{S}$ while in the North economy, all individuals are able to invest in education so that $S > \underline{S}$. However, in the latter economy we also need to ensure that

$w_s > w_u$ so that there is an incentive to invest in a child's education. This is always the case provided that

$$\frac{w^u}{w^s} = \frac{\alpha(S - S^e)}{(1-\alpha)(L - S)} < 1.$$

In the following we will focus on the case where all individuals invest in education in the North economy so that $S^e = \gamma L$, the previous condition being equivalent to

$$S < \overline{S} = [1 - \alpha(1-\gamma)]L, \tag{29}$$

so that in the North economy $\underline{S} < S < \overline{S}$. The next Proposition derives a condition under which all non-constrained individuals decide to invest in their children's education.

Proposition 1. *In the North and the South, all non-constrained agents invest and education if and only if*

$$\sigma \leq \min\{\sigma_s, \sigma_n\},$$

where for the South economy σ_s solves

$$\beta(\pi - \overline{e}) = \frac{u(w^s - z^{s,n\varepsilon}) - u[(1-\gamma)w^s - z^{s,\varepsilon}]}{u(w^s - z^{s,n\varepsilon}) - u(w^u - z^{u,n\varepsilon})},$$

while for the North economy σ_n solves

$$\beta(\pi - \overline{e}) = \frac{u(w^u - z^{u,n\varepsilon}) - u(w^u - \gamma w^s - z^{u,\varepsilon})}{u[(1-\gamma)w^s - z^{s,\varepsilon}] - u(w^u - \gamma w^s - z^{u,\varepsilon})}.$$

Proof. See Appendix B □

Proposition 1 shows that all non-constrained individuals will decide to invest in education if the steady state coefficient of relative risk aversion in consumption σ is sufficiently small. In this case, individuals are ready to reduce present consumption in order to increase their children's probability to become skilled from $\overline{e}$ to π. Individuals have a higher incentive to invest in education the larger the altruism factor β as well as the difference between π and $\overline{e}$.

In the following, we focus on a steady-state equilibrium where Proposition 1 is always satisfied. However, we still need to ensure that the steady-state values of skilled workers in both economies are compatible with restrictions (28) and (29). In the South, where only skilled agents invest in education, the dynamics governing the number of skilled workers is given by

$$S^*_{t+1} = \pi S^*_t + \overline{e}(L - S^*_t),$$

which at the steady-state equilibrium implies

$$S^* = \frac{\overline{e}L}{1 - \pi + \overline{e}}. \tag{30}$$

In the North, where all agents invest in education, the dynamics governing the number of skilled workers is given by

$$S_{t+1} = \pi L,$$

which directly implies

$$S = \pi L. \tag{31}$$

Using expressions (28), (29), (30) and (31), to ensure that $S^* < \underline{S} < S < \overline{S}$ we formulate the following assumption.

Assumption 1. *The following parameter restriction is satisfied:*

$$\frac{\overline{e}}{1-\pi+\overline{e}} < \frac{\gamma(1-\alpha)}{\alpha(1-\gamma)+\gamma(1-\alpha)} < \pi < 1-\alpha(1-\gamma).$$

The latter assumption ensures that unskilled individuals are constrained in the South while they are not in the North but the wage of skilled workers is still larger than the one of unskilled workers in the latter economy. With the number of skilled workers S and the number of teachers S^e in each country, we are able to derive a ranking of net wages for all types of individuals.

Lemma 2. *In the equilibrium without trade,* $w^{u*} < w^u - \gamma w^s < (1-\gamma)w^s < (1-\gamma)w^{s*}$ *if and only if*

$$(1-\gamma)\overline{e} < \pi - \gamma, \tag{32}$$

and

$$\frac{\gamma(1-\alpha)(1-\pi)}{\alpha(\pi-\gamma)} + \left[\frac{(1-\gamma)\overline{e}}{\pi-\gamma}\right]^{1-\alpha} < 1. \tag{33}$$

Proof. See Appendix C □

We now return to the decision to contribute to environmental maintenance. We denote by β^i the threshold value of the altruism factor of an individual of type i in the North and by β^{i*} the corresponding threshold value in the South. Since in the South only skilled individuals invest in education, while in the North both skilled and unskilled workers invest in the education of their offsprings, we have that $\beta^s = \overline{\beta}^{s,\varepsilon}$, $\beta^u = \overline{\beta}^{u,\varepsilon}$, $\beta^{s*} = \overline{\beta}^{s*,\varepsilon}$ and $\beta^{u*} = \overline{\beta}^{u*,n\varepsilon}$, with $\overline{\beta}^{i,j}$ given in (24). Recall that a given individual will only invest in environmental maintenance if the altruism factor is larger than its personal threshold value.

Proposition 2. *In the equilibrium without trade:*

1. *The threshold values of the altruism factor satisfy* $\beta^{s*} < \beta^s < \beta^u < \beta^{u*}$ *if*

$$\frac{1-\pi}{1-\pi+\overline{e}} z^{u*} \leq (1-\pi)z^u \leq \pi z^s \leq \frac{\overline{e}}{1-\pi+\overline{e}} z^{s*}. \tag{34}$$

2. *The level of private environmental contributions for an individual of type i with education decision j is positive if and only if* $b < 1$ *and*

$$\beta > \frac{1}{(1-b)}\left(1 - \frac{\eta v'[H - \frac{\kappa}{b}\left(\frac{(\pi-\gamma)(1-\pi+\overline{e})+\overline{e}(1-\gamma)}{1-\pi+\overline{e}}\right)N + \frac{\eta}{b}\overline{Z}^{i,j}]}{u'(w^i - \gamma d_j w^s)}\right). \tag{35}$$

Proof. See Appendix D □

If the ranking of the altruism factors presented in Proposition 2 is satisfied, environmental maintenance investment is more probable for wealthier individuals. This seems intuitive since environmental quality being a normal good, the demand for environmental preservation should increase with income. However, this ranking is only satisfied if the total contribution from wealthier individuals is larger than the one of poorer ones. For example, it is not always guaranteed that the willingness to contribute to the environmental good of skilled workers in the South is larger than the one in the North. In addition to income and the level of pollution, the threshold values depend on the potential contribution from other types of individuals. While the individual contribution of a skilled individual in the South z^{s*} is larger or equal to the one of a skilled individual in the North z^s due to Lemma 1, the number of skilled individuals in the North πL is larger then the number of skilled individuals in the South, $\overline{e}L/(1-\pi+\overline{e})$. It is then possible that the total contribution of skilled individuals in the North increases the threshold value β^{s*} above β^s despite the fact that $w^s < w^{s*}$.

We now focus on the equilibrium values when a particular set of agents invests in environmental quality. For a given individual, if $\overline{\beta}^{i,j} < \beta < 1$, the level of private voluntary contributions $z^{i,j}$ solves (25)-(26). Depending on the actual level of the altruism factor β, several cases could be considered. We then formulate the following assumption.

Assumption 2. *The common altruism factor satisfies*

$$\beta^{s*} < \beta^s < \beta < \beta^u < \beta^{u*}.$$

The latter assumption implies that in the autarky equilibrium, only skilled agents in both countries decide to invest in environmental maintenance. By using (26) we can compute the steady-state value of environmental quality which is given by

$$N = H - \frac{\kappa[(\pi-\gamma)(1-\pi+\overline{e})+\overline{e}(1-\gamma)]+\eta[\pi(1-\pi+\overline{e})z^s+\overline{e}z^{s*}]}{b(1-\pi+\overline{e})}L, \tag{36}$$

where $z^s$and z^{s*} solve the following system of equations:

$$u'[(1-\gamma)w^s - z^s] = \frac{\eta}{1-\beta(1-b)}v'(N), \tag{37}$$

$$u'[(1-\gamma)w^{s*} - z^{s*}] = \frac{\eta}{1-\beta(1-b)}v'(N). \tag{38}$$

As explained before, this implies that $c^{s*} = c^s$ and $z^{s*} > z^s$. In Appendix E, we derive implicit solutions for z^s and z^{s*} as functions of the parameters of the model.

This concludes our analysis of the steady-state equilibrium in autarky. We are now ready to focus on the implications of free trade in the current framework.

4 Trade

Suppose that in period t, both economies are in their respective steady-state equilibria so that $S = \pi L$ and $S^* = \overline{e}L/(1-\pi+\overline{e})$. This implies, using (6), that the relative price of X^u, that is $p^u/p^s = w^u/w^s$ is smaller in the South if and only if

$$(1-\gamma)\overline{e} < \pi - \gamma,$$

which is the same condition as the one guaranteeing that $w^s < w^{s*}$ in the equilibrium without trade. In this case, the South has a comparative advantage in the unskilled labor intermediate good while the North has a comparative advantage in the skilled labor intermediate good.

We now suppose that both economies open up to trade at a specific time T_W. Since agents are forward looking, trade liberalization must be unanticipated. After opening to trade, the world will behave like a closed economy with the following initial condition $S_t^W = (S_t + S_t^*)/2$. From (6) the relative prices and wages in period T_W for the world economy will be given by

$$\left(\frac{p_t^u}{p_t^s}\right)^W = \left(\frac{w_t^u}{w_t^s}\right)^W = \frac{\alpha(S_t^W - S_t^{eW})}{(1-\alpha)(L - S_t^W)}.$$

Both countries will face these relative prices from T_W onward. Depending on the value of S_t^W, there are two possible outcomes.

If $S_t^W < \underline{S}$, unskilled individuals of both countries become constrained. Factor prices do not allow for any intergenerational mobility and the amount of skilled workers in both countries converges to

$$S = S^* = \frac{\overline{e}L}{1 - \pi + \overline{e}}.$$

If $S_t^W > \underline{S}$, unskilled individuals in the South become unconstrained and invest in education. Using (28), (31) and (30) a necessary and sufficient condition for this outcome to occur is

$$\pi + \frac{\overline{e}}{1 - \pi + \overline{e}} > \frac{2\gamma(1-\alpha)}{\alpha(1-\gamma) + \gamma(1-\alpha)}.$$

If the latter restriction is satisfied, the world economy converges to the level of human capital of the North: $S^W = \pi L$. We know from Assumption 1 that

$$\frac{\overline{e}}{1 - \pi + \overline{e}} < \frac{\gamma(1-\alpha)}{\alpha(1-\gamma) + \gamma(1-\alpha)} < \pi,$$

implying that convergence to the level of human capital of the North is guaranteed either if π is sufficiently large or if the difference in the number of skilled workers across the two countries is not too large.

In this case, from (6), we have that in the post-trade steady-state, relative prices and wages are equal across countries and given by

$$\left(\frac{p^u}{p^s}\right)^W = \left(\frac{w^u}{w^s}\right)^W = \frac{\alpha(\pi - \gamma)}{(1-\alpha)(1-\pi)}. \tag{39}$$

In addition, the number of skilled workers is also the same across countries. It should be noted that the post-trade steady-state is reached in one period since once trade liberalization occurs, unskilled workers in the South immediately invest in education in period T_W. The accumulation of skilled workers become $S_{T_W+1} = \pi L$ and the steady-state is reached at period $T_W + 1$. Note that in our model one period is the time needed to educate one generation.

The decision concerning environmental maintenance investment is the same as in the closed economy case. However, the threshold values under which individuals are ready to provide environmental bequests are different. Since the South experiences an increase in skilled labor while the number of skilled in the North is unchanged, the level of pollution differs in the two steady-state equilibria. Using the steady-values for both equilibria of X^s and X^{s*}, we can show that the level of pollution in the post-trade equilibrium is larger than in autarky if and only if

$$\pi > \gamma + \overline{e}. \tag{40}$$

We conclude that pollution will be larger in the post-trade equilibrium if the teacher-students ratio γ is not too large. This is due to the fact that pollution is the result of the number of skilled individuals working in the dirty input sector. When the number of skilled individuals in the South, S^*, increases, a share of these individuals will need to work in the education sector. If the requirement in terms of teachers is sufficiently large, it is possible that the pollution level is smaller in the post-trade equilibrium. In the following we will focus on the case where expression (40) is always satisfied.

The impact of trade openness is thus driven by the increase in skilled labor in the South, S^*. The latter affects the wage levels in the South w^{s*} and w^{u*}. We denote by z_W^s and z_W^u the environmental contributions in the North in the post trade equilibrium and equivalently z_W^{s*} and z_W^{u*} in the South. The next Proposition identifies the impact of trade openness on the threshold values of the altruism factor.

Proposition 3. *In the post-trade equilibrium:*

1. *Individuals of type i in both countries are identical so that $z_W^s = z_W^{s*}$ and $z_W^u = z_W^{u*}$ while the threshold values of the altruism factor satisfy $\beta_W^s = \beta_W^{s*} < \beta_W^u = \beta_W^{u*}$ if*

$$(1-\pi)z_W^u \leq \pi z_W^s. \tag{41}$$

2. *$\beta_W^u < \beta^u < \beta^{u*}$ if and only if*

$$2\pi z_W^s < \frac{\kappa(\pi - \overline{e} - \gamma)(1-\pi)}{\eta(1-\pi+\overline{e})} + \frac{\overline{e}}{1-\pi+\overline{e}} z^{s*} + \pi z^s. \tag{42}$$

3. *$\beta_W^s < \beta^s$ if and only if*

$$2(1-\pi)z_W^u < \frac{\kappa(\pi - \overline{e} - \gamma)(1-\pi)}{\eta(1-\pi+\overline{e})} + \frac{\overline{e}}{1-\pi+\overline{e}} z^{s*}, \tag{43}$$

where z^s and z^{s*} are the solutions to the system of equations composed of expressions (37) and (38).

Proof. See Appendix F □

Similarly to the autarky equilibrium, the willingness to contribute to environmental preservation is larger for the skilled if the total contribution of the latter individuals is larger or equal to the one of the unskilled. Since $z_W^u \leq z_W^s$ due to Lemma 1, condition (41) is always satisfied provided that π is not too small. Conditions (42) and (43) define upper bounds for z_W^s and z_W^u such that the willingness to contribute to environmental preservation is larger in the free trade equilibrium. In this case, it is more likely that all individuals decide to invest in environmental maintenance. One of the implications of the last Proposition is that an increase in pollution makes more probable that individuals enjoying an increase in income after trade (or keeping their income constant) invest in environmental quality if the remaining contribution of other individuals is not too large. For example, we will only observe a decrease in the threshold of unskilled individuals, $\beta_W^u < \beta^u$, if the total contribution of skilled individuals $2\pi z_W^s$ is not too large. This argument is not necessarily valid for individuals experimenting a decrease in income (skilled workers in the South) since the impact of a decrease in environmental quality on the threshold value might be compensated by the decrease in income. However, in the present framework, as $\beta^s_W = \beta^{s*}_W$, if skilled individuals in the North decide to invest in maintenance investment, skilled individuals in the South will also decide to do so despite of the decrease in income.

Under Assumption 2, $\beta^{s*} < \beta^s < \beta < \beta^u < \beta^{u*}$. Assuming now that conditions (42) and (43) in Proposition 5 are verified, so that $\beta^u_W < \beta^u$ and $\beta^s_W < \beta^s$, two different free-trade equilibria are possible. In both $\beta^s_W < \beta$, so that skilled individuals always contribute to environmental maintenance. However, in the first one $\beta < \beta^u_W$ and unskilled individuals do not invest in environmental maintenance. In the second one, as $\beta^u_W < \beta$, these agents decide to do so.

Case I. In this case, we have $\beta^s_W < \beta < \beta^u_W$, implying that $z^u_W = 0$ and $z^s_W > 0$. Using the steady-state values of w^s, X^s and X^{s*}, the solution for z^s_W in this case is implicitly given by

$$u'\left[(1-\gamma)(1-\alpha)A\left(\frac{1-\pi}{\pi-\gamma}\right)^{\alpha} - z^s_W\right] = \frac{\eta}{1-\beta(1-b)}v'\left(H - \frac{2L}{b}[\kappa(\pi-\gamma) - \eta\pi z^s_W]\right), \tag{44}$$

Case II. In the second equilibrium, all individuals decide to invest in environmental maintenance so that $\beta^s_W < \beta^u_W < \beta$. In this case, the steady-state value of environmental quality is given by

$$N = H - \frac{2L}{b}\left\{\kappa(\pi-\gamma) - \eta[\pi z^s_W + (1-\pi)z^u_W]\right\}, \tag{45}$$

where z^s_W and z^u_W solve the following system of equations:

$$u'[w^s(1-\gamma) - z^s_W] = \frac{\eta}{1-\beta(1-b)}v'(N), \tag{46}$$

$$u'(w^u - \gamma w^s - z^u_W) = \frac{\eta}{1-\beta(1-b)}v'(N). \tag{47}$$

Similarly to the autarky case, we obtain $c^s_W = c^u_W$ and $z^s_W > z^u_W$. In Appendix G, we derive implicit solutions for z^s_W and z^u_W as functions of the parameters of the model.

The implementation of free trade has several implications for welfare in both countries. One of the main effects consists in increasing the level of human capital in the South which reduces income inequality in this country since $w^s - w^u < w^{s*} - w^{u*}$. However, in the North, the distribution of wages remains the same after opening the borders to free trade. Since skilled labor is used in the production of the dirty input, the reduction in income inequality in the South is associated with an increase in pollution at the world level. This increase in pollution modifies in turn the willingness of individuals to contribute to environmental preservation. As derived in Proposition 5, free trade can lead to a situation where unskilled individuals decide to contribute to the public good. In addition, the increase in pollution will also affect the level of contributions from skilled individuals in both countries.

We now compare the steady-state values of environmental quality in the autarky and free-trade equilibria. If we are in Case II where $\beta^u_W < \beta$, then $z^u_W > 0$ and unskilled individuals start to contribute to the public good. From expressions (36) and (45), we notice that the steady-state value of environmental quality is larger in the post-trade equilibrium if and only if

$$2[\pi z^s_W + (1-\pi)z^u_W] - \left(\frac{\bar{e}}{1-\pi+\bar{e}}z^{s*} + \pi z^s\right) > \frac{\kappa(\pi-\bar{e}-\gamma)(1-\pi)}{\eta(1-\pi+\bar{e})}. \tag{48}$$

i.e., if and only if the difference between the total environmental contribution under free trade and the total environmental contribution in autarky is larger than the increase in pollution due to free trade. In the following section, we focus on a numerical example in order to assess the impact of free trade on environmental quality and welfare for realistic parameter values.

5 Numerical simulations

We proceed with some numerical simulations of the autarky and free trade steady-states in order to compare the implication of trade openness on environmental quality and welfare. We first need to choose specific functional forms for our functions $u(.)$ and $v(.)$. For $u(.)$, we have our CRRA specification

$$u(c) = \frac{c^{1-\sigma}}{1-\sigma},$$

while for $v(.)$, we assume

$$v(N) = \delta \ln(N).$$

The parameters are calibrated in order to reproduce some of the empirical facts of developed economies. Table 1 provides the benchmark values of the parameters that we use in our numerical simulations.

We focus first on the parameters related to the preferences of individual agents. The coefficient of relative risk aversion in consumption σ is set at 2 which is in the range of empirical estimates provided by Ogaki and Reinhart (1998). This value also ensures that all non-constrained individuals decide to invest in education. The relative preference for environmental quality δ is set at 0.8 so that consumption is slightly more important than environmental quality in adulthood. Finally, the altruism factor β is set at 0.99 since it is a relatively standard approach to consider that the the altruism factor of individual agents is equal to planner's discount factor (see, for example Jouvet et al., 2000; Alonso-Carrera et al., 2008).

The parameters governing the dynamics of environmental quality take the following values: $\kappa = 0.1$, $\eta = 0.2$, $b = 0.5$ and $H = 10$. The value of κ implies that one tenth of the world production of the skilled intermediate input is transformed into pollution. We assume that η is larger since Z consists of specific resources allocated to environmental preservation while the skilled intermediate input generates pollution as an externality. The value of b is difficult to choose since it depends on the definition of environmental quality and the specificity of the pollution process (see, for example Jouvet et al., 2010). We then choose to follow Acemoglu et al. (2012) and set a value of 0.5 for b. The parameter H influences the level of N at the steady-state. We set $H = 10$ to ensure a positive level of environmental quality at the steady-state.

We now focus on the production and education parameters. We first normalize $A = 1$ since the size of our economies do not affect our results. We assume that the share of skilled intermediate goods in production α is equal to 1/2 while the probability to become skilled following parental investment π is equal to 0.45. Furthermore, the teacher-student ratio γ is set at 0.1. Since in the North

$$\frac{w^u}{w^s} = \frac{\alpha(\pi - \gamma)}{(1-\alpha)(1-\pi)},$$

this combination of parameters implies a ratio $w^u/w^s = 0.63$ or equivalently a skill premium of 1.58 which is in line with average skill premia in OECD countries. The value of γ implies that in the South it must be the case that $w^{u*}/w^{s*} < 0.1$ so that unskilled individuals are constrained and unable to invest in their children's education. Since in the South

$$\frac{w^{u*}}{w^{s*}} = \frac{\alpha(1-\gamma)\overline{e}}{(1-\alpha)(1-\pi)},$$

we choose $\overline{e} = 0.05$ implying $w^{u*}/w^{s*} = 0.08$ and unskilled individuals are unable to invest in education due to borrowing constraints. Finally, we choose to set $L = 2$ as a benchmark in order to ensure some free-riding behavior on the part of individual agents.

Table 1: Values for the parameters

Parameter	Notation	Value
Coefficient of relative risk aversion in consumption	σ	2
Relative preference for environmental quality	δ	0.8
Altruism factor	β	0.99
Impact of pollution	κ	0.1
Impact of maintenance investment	η	0.2
Regeneration rate of the environment	b	0.5
Natural level of environmental quality	H	10
TFP	A	1
Share of skilled intermediate good in production	α	0.5
Share of skilled workers in the population	π	0.45
Teacher-student ratio	γ	0.1
Probability to get skilled without investment in education	$\overline{e}$	0.05
Total population in each country	L	2

The benchmark values of the parameters ensure that trade openness implies convergence of the world economy to the level of human capital of the North since

$$\pi > \frac{2\gamma(1-\alpha)}{\alpha(1-\gamma)+\gamma(1-\alpha)}.$$

To be completed

6 Optimal allocation with trade

We now consider a social planner that maximizes the weighted sum of utilities of skilled and unskilled individuals. The weights associated to each type of individuals represents their share of the world population. We assume that the planner's intergenerational discount factor is equal to the altruism factor of individual agents β. The social welfare function is given by

$$W_t = \pi V_t^s + (1-\pi)V_t^u. \tag{49}$$

where π is the share of skilled individuals in the world population and $(1-\pi)$ the share of unskilled individuals under free trade. Since we are focusing on the optimal allocation with trade, the competitive equilibrium is characterized by investment in education from skilled and unskilled individuals in both countries. Free-trade is sufficient to relax borrowing constraints so that in terms of education choices, the planner allocation will correspond to the competitive equilibrium. We then choose to consider a planning problem where individuals already invest in education. The role of the social planner is to solve the potential suboptimality related to the levels of environmental quality and consumption.

6.1 Characterizing the optimal allocation

The welfare function being separable between the utilities of both type of agents, we solve for each type of individual separately. The value function for an individual of type i, with $i = \{s, u\}$ such that $w_t^i > \gamma w_t^s$, is then given by

$$\begin{aligned} V_t^i(N_{t-1}) &= \max\left\{u(c_t^i) + v(N_t) + \beta\pi V_{t+1}^s(N_t) + \beta(1-\pi)V_{t+1}^u(N_t)\right\}, \\ s.t. \quad z_t^i &\geq 0, && (50) \\ c_t^i &= w_t^i - \gamma w_t^s - z_t^i, && (51) \\ N_t &= N_{t-1} + b(H - N_{t-1}) - 2L\{\kappa(\pi-\gamma) + \eta[\pi z_t^s + (1-\pi)z_t^u]\}. && (52) \end{aligned}$$

Proceeding as before, at the steady-state, we obtain for skilled individuals

$$\rho^s = u'(c^s) - \frac{2\pi L\eta}{1-\beta(1-b)}v'(N), \quad (53)$$

$$\rho^s z^s = 0, \quad (54)$$

and for unskilled ones

$$\rho^u = u'(c^u) - \frac{2(1-\pi)L\eta}{1-\beta(1-b)}v'(N), \quad (55)$$

$$\rho^u z^u = 0. \quad (56)$$

Comparing expressions (53) and (55) with expression (22), we notice that the marginal utility of investing in environmental maintenance is larger in the planner's case. This is due to the standard free-riding behavior on the part of all agents in the competitive equilibrium. However, this does not imply that the planner will decide that all agents need to invest in environmental maintenance since the level of contribution of unskilled agents depends on the one of skilled agents and vice-versa.

We proceed as before and compare the threshold values of the social discount factor with the threshold values of the private altruism factor under free trade. The next Proposition derives necessary and sufficient conditions under which both type of agents must invest in environmental quality in the optimal case.

Proposition 4. *Concerning the optimal allocation:*

1. *The social discount factor thresholds satisfy $\beta_o^s < \beta_o^u$ if*

$$(1-\pi)z_o^u \leq \pi z_o^s. \quad (57)$$

2. $\beta_o^u < \beta_W^u$ *if and only if*

$$2(1-\pi)L > v'\left(\frac{H - \frac{\kappa}{b}2(\pi-\gamma)L + \frac{\eta}{b}2\pi L z_W^s}{H - \frac{\kappa}{b}2(\pi-\gamma)L + \frac{\eta}{b}2\pi L z_o^s}\right), \tag{58}$$

3. $\beta_o^s < \beta_W^s$ *if and only if*

$$2\pi L > v'\left(\frac{H - \frac{\kappa}{b}2(\pi-\gamma)L + \frac{\eta}{b}2(1-\pi)L z_W^u}{H - \frac{\kappa}{b}2(\pi-\gamma)L + \frac{\eta}{b}2(1-\pi)L z_o^u}\right), \tag{59}$$

where z_W^u and z_W^s are the solutions to the system of equations composed of expressions (46) and (47).

Proof. Appendix H □

From Proposition 4 we conclude that, similarly to the competitive equilibrium with trade, the planner will find optimal that skilled individuals contribute to the public good for a smaller social discount factor if their optimal total contribution is larger or equal to the one of the unskilled. Once again, conditions (58) and (59) define upper bounds for z_o^s and z_0^u so that the threshold value of the social discount factor is smaller than the threshold value of the private altruism factor. In this case, the social planner is more willing to invest in environmental maintenance then both types of individual agents.

Since in the competitive equilibrium with trade $\beta_W^s < \beta$, when the conditions stated in Proposition 4 are satisfied, $\beta_o^s < \beta$ and the planner will always find necessary for skilled individuals to invest in environmental maintenance. Concerning unskilled individuals, in case I of the competitive equilibrium with trade $\beta < \beta_W^u$ while in case II, $\beta_W^u < \beta$. We suppose that for the optimal allocation we always have $\beta_o^u < \beta$ so that the planner finds it optimal that unskilled individuals invest in environmental maintenance.

Notice from expressions (53) and (55) that when the planner implements environmental contributions from both types of individuals we obtain

$$\pi u'(w^u - \gamma w^s - z^u) = (1-\pi)u'[(1-\gamma)w^s - z^s],$$

implying that $c^u < c^s$ if and only if $\pi < 1/2$. The equality of consumption levels obtained in the competitive case is not valid for the optimal allocation. This is due to free-riding behavior concerning voluntary contributions to the public good. If $\pi < 1/2$, free-riding behavior is more important on the part of unskilled individuals since they represent a larger share of the population.

6.2 Decentralization of the optimal allocation

We now discuss how the optimal allocation can be decentralized. Since free-trade solves the issue related to borrowing constraints, only the externality related to the suboptimal level of environmental quality must be internalized. In order to solve the problem related to free-riding in the provision of the public good (environmental quality), the planner needs to subsidize the private contribution of individual agents. The budget constraint of the government is balanced by imposing lump-sum taxes on the same individuals. The budget constraint for an agent of type i is given by

$$w_t^i - \gamma w_t^s = c_t^s + (1-\tau_t^i)z_t^i + \theta_t^i,$$

where τ_t^i represents an environmental maintenance subsidy and θ_t^i a lump-sum tax. The government faces a budget constraint for each type of individual i which is given by $\tau_t^i z_t^i = \theta_t^i$. By solving the optimization problem of skilled and unskilled individuals with taxes, we obtain at the steady-state

$$(1-\tau^s)u'(c^s) = \frac{\eta}{1-\beta(1-b)}v'(N),$$

and

$$(1-\tau^u)u'(c^u) = \frac{\eta}{1-\beta(1-b)}v'(N).$$

Comparing the latter expressions with the ones characterizing the optimal allocation, i.e. expressions (53) and (55), we obtain

$$\tau^s = 1 - \frac{1}{2\pi L},$$

and

$$\tau^u = 1 - \frac{1}{2(1-\pi)L}.$$

Moreover, we conclude that $\tau^s < \tau^u$ if and only if $\pi < 1/2$. In the intuitive case where skilled individuals represent less than half of the population, the planner needs to subsidize more unskilled individuals.

7 Conclusion

To be completed

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Appendix

A Proof of Lemma 1

From expression (25), we know that when both agents 1 and 2 invest in environmental maintenance it must be the case that

$$u'(w^1 - \gamma d_j^1 w^s - z^{1,j}) = u'(w^2 - \gamma d_j^2 w^s - z^{2,j}),$$

so that $z^{1,j} > z^{2,j}$. Moreover, we need to prove that the case where agents of type 1 do not invest while agents of type 2 do so is ruled out. If $z^{2,j} > 0$ while $z^{1,j} = 0$ it must be the case that

$$u'(w^1 - \gamma d_j^1 w^s) > u'(w^2 - \gamma d_j^2 w^s - z^{2,j}),$$

which cannot be satisfied since $w^1 - \gamma d_j^1 w^s > w^2 - \gamma d_j^2 w^s$.

B Proof of Proposition 1

We start with the South economy where $w^u < \gamma w^s$. At a steady-state equilibrium, all successive generations of the same type take the same decision concerning educational investment. Unskilled individuals are constrained implying that all generations of unskilled individuals will not invest in education. Concerning skilled individuals, we look for conditions ensuring that inequality (21) is satisfied at the steady state, that is

$$\beta(\pi - \overline{e})(V^s - V^u) \geq u(w^s - z^{s,n\varepsilon}) - [(1-\gamma)w^s - z^{s,\varepsilon}].$$

where

$$\begin{aligned} V^s &= u[(1-\gamma)w^s - z^{s,\varepsilon}] + v(E) + \beta\pi V^s + \beta(1-\pi)V^u, && (60) \\ V^u &= u(w^u - z^{u,n\varepsilon}) + v(E) + \beta\overline{e}V^s + \beta(1-\overline{e})V^u, && (61) \end{aligned}$$

so that all generations of skilled individuals invest in education. If this is the case, we obtain the following:

$$\frac{\beta(\pi-\overline{e})}{1-\beta(\pi-\overline{e})} \geq \frac{u(w^s - z^{s,n\varepsilon}) - u[(1-\gamma)w^s - z^{s,\varepsilon}]}{u[(1-\gamma)w^s - z^{s,\varepsilon})] - u(w^u - z^{u,n\varepsilon})}.$$

or

$$\beta(\pi-\overline{e}) \geq \frac{u(w^s - z^{s,n\varepsilon}) - u[(1-\gamma)w^s - z^{s,\varepsilon}]}{u(w^s - z^{s,n\varepsilon}) - u(w^u - z^{u,n\varepsilon})}. \tag{62}$$

Consider two utility functions $u_1(.)$ and $u_2(.)$ that differ in terms of their coefficient of relative risk aversion such that $\sigma_1 < \sigma_2$. Due to the concavity of the utility function, we obtain

$$\frac{u_1(w^s - z^{s,n\varepsilon}) - u_1[(1-\gamma)w^s - z^{s,\varepsilon}]}{u_1(w^s - z^{s,n\varepsilon}) - u_1(w^u - z^{u,n\varepsilon})} < \frac{u_2(w^s - z^{s,n\varepsilon}) - u_2[(1-\gamma)w^s - z^{s,\varepsilon}]}{u_2(w^s - z^{s,n\varepsilon}) - u_2(w^u - z^{u,n\varepsilon})}, \tag{63}$$

implying that there exists a sufficiently small value for σ under which skilled individuals always invest in education. We then define σ_s as the value of σ that solves

$$\beta(\pi-\overline{e}) = \frac{u(w^s - z^{s,n\varepsilon}) - u[(1-\gamma)w^s - z^{s,\varepsilon}]}{u(w^s - z^{s,n\varepsilon}) - u(w^u - z^{u,n\varepsilon})}. \tag{64}$$

We next focus on the North economy where $w^u > \gamma w^s$. We proceed in the same way in this case and look for conditions guaranteeing that expression (21) is satisfied for both skilled and unskilled individuals. In this case, as both skilled and unskilled individuals invest in education at the steady state

$$V^s = u[(1-\gamma)w^s - z^{s,\varepsilon}] + v(E_t) + \beta\pi V^s + \beta(1-\pi)V^u, \tag{65}$$

$$V^u = u(w^u - \gamma w^s - z^{u,\varepsilon}) + v(E_t) + \beta\pi V^s + \beta(1-\pi)V^u, \tag{66}$$

we obtain:

$$V^s - V^u = u[(1-\gamma)w^s - z^{s,\varepsilon}] - u(w^u - \gamma w^s - z^{u,\varepsilon}).$$

For any skilled individual, the investment condition becomes

$$\beta(\pi-\overline{e}) \geq \frac{u(w^s - z^{s,n\varepsilon}) - u[(1-\gamma)w^s - z^{s,\varepsilon}]}{u[(1-\gamma)w^s - z^{s,\varepsilon}] - u(w^u - \gamma w^s - z^{u,\varepsilon})}, \tag{67}$$

For any unskilled individual, the investment condition becomes

$$\beta(\pi-\overline{e}) \geq \frac{u(w^u - z^{u,n\varepsilon}) - u(w^u - \gamma w^s - z^{u,\varepsilon})}{u[(1-\gamma)w^s - z^{s,\varepsilon}] - u(w^u - \gamma w^s - z^{u,\varepsilon})}. \tag{68}$$

Comparing the last two expressions, we notice that skilled individuals will decide to invest in education when unskilled individuals do so provided that

$$u(w^s - z^{s,n\varepsilon}) - u[(1-\gamma)w^s - z^{s,\varepsilon}] \leq u(w^u - z^{u,n\varepsilon}) - u(w^u - \gamma w^s - z^{u,\varepsilon}).$$

Using the results from Lemma 1, it can noticed that this condition is always satisfied. Consider once again the two utility functions $u_1(.)$ and $u_2(.)$ with $\sigma_1 < \sigma_2$. Due to the concavity of the utility function, we obtain

$$\frac{u_1(w^u - z^{u,n\varepsilon}) - u_1(w^u - \gamma w^s - z^{u,\varepsilon})}{u_1[(1-\gamma)w^s - z^{s,\varepsilon}] - u_1(w^u - \gamma w^s - z^{u,\varepsilon})} < \frac{u_2(w^u - z^{u,n\varepsilon}) - u_2(w^u - \gamma w^s - z^{u,\varepsilon})}{u_2[(1-\gamma)w^s - z^{s,\varepsilon}] - u_2(w^u - \gamma w^s - z^{u,\varepsilon})},$$

implying that there exists a sufficiently small value for σ under which unskilled individuals always invest in education. Let us define σ_n as the value that solves

$$\beta(\pi - \bar{e}) = \frac{u(w^u - z^{u,n\varepsilon}) - u(w^u - \gamma w^s - z^{u,\varepsilon})}{u[(1-\gamma)w^s - z^{s,\varepsilon}] - u(w^u - \gamma_u w^s - z^{u,\varepsilon})}.$$

It is straightforward to conclude that all non-constrained individuals will invest in education if σ is smaller than both σ_s and σ_n.

C Proof of Lemma 2

We start by deriving a condition under which $w^s < w^s_*$. In equilibrium, using expressions (2), (30) and (31), the wages of both types of skilled individuals are given by

$$w^s = (1-\alpha)A\left(\frac{1-\pi}{\pi-\gamma}\right)^{\alpha}, \tag{69}$$

and

$$w^{s*} = (1-\alpha)A\left[\frac{1-\pi}{(1-\gamma)\bar{e}}\right]^{\alpha}, \tag{70}$$

implying that $w^s < w^{s*}$ if and only if condition (32) is satisfied.

We know that $w^u < w^s$ from Assumption 1 and finally we need to derive a condition under which $w^{u*} < w^u - \gamma w^s$. Using expressions (3), (30) and (31), the equilibrium wages of unskilled individuals in both countries are given by

$$w^u = \alpha A\left(\frac{\pi-\gamma}{1-\pi}\right)^{1-\alpha}, \tag{71}$$

and

$$w^{u*} = \alpha A\left[\frac{(1-\gamma)\bar{e}}{1-\pi}\right]^{1-\alpha}. \tag{72}$$

implying that $w^{u*} < w^u - \gamma w^s$ if and only if condition (33) is satisfied.

D Proof of Proposition 2

We start with the first part of the Proposition. As $z^{s*} = z^{s*,\varepsilon}$, $z^s = z^{s,\varepsilon}$, $z^{u*} = z^{u*,n\varepsilon}$, $z^u = z^{u,\varepsilon}$, and as in equilibrium $X^s = S - S^e = (\pi - \gamma)L$, and $X^{s*} = S^* - S^{e*} = (1-\gamma)\bar{e}L/(1-\pi+\bar{e})$, we have that in the South

$$\beta^{u*} = \frac{1}{(1-b)}\left[1 - \frac{\eta v'(N^{u*})}{u'(w^{u*})}\right],$$

where

$$N^{u*} = H - \left\{\frac{\kappa}{b}\frac{[(\pi-\gamma)(1-\pi+\bar{e}) + \bar{e}(1-\gamma)]}{(1-\pi+\bar{e})} - \frac{\eta}{b}\left[\frac{\bar{e}}{(1-\pi+\bar{e})}z^{s*} + \pi z^s + (1-\pi)z^u\right]\right\}L,$$

and

$$\beta^{s*} = \frac{1}{(1-b)}\left\{1 - \frac{\eta v'(N^{s*})}{u'[(1-\gamma)w^{s*}]}\right\},$$

where

$$N^{s*} = H - \left\{ \frac{\kappa}{b} \frac{[(\pi-\gamma)(1-\pi+\overline{e}) + \overline{e}(1-\gamma)]}{(1-\pi+\overline{e})} - \frac{\eta}{b} \left[\frac{1-\pi}{1-\pi+\overline{e}} z^{u*} + \pi z^s + (1-\pi) z^u \right] \right\} L,$$

while in the North we have

$$\beta^u = \frac{1}{(1-b)} \left[1 - \frac{\eta v'(N^u)}{u'(w^u - \gamma w^s)} \right],$$

where

$$N^u = H - \left\{ \frac{\kappa}{b} \frac{[(\pi-\gamma)(1-\pi+\overline{e}) + \overline{e}(1-\gamma)]}{(1-\pi+\overline{e})} - \frac{\eta}{b} \left[\frac{(1-\pi) z_*^u + \overline{e} z_*^s}{1-\pi+\overline{e}} + \pi z^s \right] \right\} L,$$

and

$$\beta^s = \frac{1}{(1-b)} \left\{ 1 - \frac{\eta v'(N^s)}{u'[w^s(1-\gamma)]} \right\},$$

where

$$N^s = H - \left\{ \frac{\kappa}{b} \frac{[(\pi-\gamma)(1-\pi+\overline{e}) + \overline{e}(1-\gamma)]}{(1-\pi+\overline{e})} - \frac{\eta}{b} \left[\frac{(1-\pi) z_*^u + \overline{e} z_*^s}{1-\pi+\overline{e}} + (1-\pi) z^u \right] \right\} L.$$

We start by comparing β^s and β^u. Using the results from Lemma 1 and 2, either $z^s > z^u \geq 0$ or $z^s = z^u = 0$. Since $u(.)$ and $v(.)$ are both concave functions, it can then be noticed that $\beta^s < \beta^u$ if $(1-\pi) z^u \leq \pi z^s$. Applying the same reasoning to all altruism factor thresholds comparisons we obtain the ranking from expression (34).

Concerning the second part of the Proposition, the level of private environmental contributions is positive if and only if $\overline{\beta}^{i,j} < \beta$. Substituting the steady-state values of $S - S^e$ and $S^* - S^{e*}$ in expression (24) we obtain expression (35).

E Solutions for z^s and z^{s*}

We start with the wages of both types of skilled individuals in equilibrium which are given by

$$w^s = (1-\alpha) A \left(\frac{1-\pi}{\pi-\gamma} \right)^{\alpha}, \tag{73}$$

and

$$w^{s*} = (1-\alpha) A \left[\frac{1-\pi}{(1-\gamma)\overline{e}} \right]^{\alpha}. \tag{74}$$

Combining the wages, we obtain

$$\frac{w^s}{w^{s*}} = \left[\frac{(1-\gamma)\overline{e}}{\pi-\gamma} \right]^{\alpha}, \tag{75}$$

and by using the equality of marginal utilities of consumption

$$\left\{ 1 - \left[\frac{(1-\gamma)\overline{e}}{\pi-\gamma} \right]^{\alpha} \right\} (1-\gamma) w^{s*} = z^{s*} - z^s, \tag{76}$$

or

$$(1-\gamma)(1-\alpha)A\left\{\left[\frac{1-\pi}{(1-\gamma)\overline{e}}\right]^{\alpha}-\left[\frac{1-\pi}{\pi-\gamma}\right]^{\alpha}\right\}=(z^{s*}-z^{s})\,. \tag{77}$$

Finally, substituting this expression in expression (37), we obtain

$$u'[(1-\alpha)(1-\gamma)A\left(\frac{1-\pi}{\pi-\gamma}\right)^{\alpha}-z^{s}]=\frac{\eta}{1-\beta(1-b)}v'(N), \tag{78}$$

where

$$\begin{aligned} N &= H-\frac{\kappa[(\pi-\gamma)(1-\pi+\overline{e})+\overline{e}(1-\gamma)]-\eta[\pi(1-\pi)+\overline{e}(1+\pi)]z^{s}}{b(1-\pi+\overline{e})}L \\ &\quad +\frac{\eta\overline{e}(1-\gamma)(1-\alpha)A}{b(1-\pi+\overline{e})}\left\{\left[\frac{1-\pi}{(1-\gamma)\overline{e}}\right]^{\alpha}-\left[\frac{1-\pi}{\pi-\gamma}\right]^{\alpha}\right\}L. \end{aligned} \tag{79}$$

This expression provides an implicit solution for z^s as a function of the parameters of the model. We proceed in the same way to compute a similar expression for z^{s*} and we obtain

$$u'\{(1-\alpha)(1-\gamma)A\left[\frac{1-\pi}{(1-\gamma)\overline{e}}\right]^{\alpha}-z^{s*}\}=\frac{\eta}{1-\beta(1-b)}v'(N), \tag{80}$$

where

$$\begin{aligned} N &= H-\frac{\kappa[(\pi-\gamma)(1-\pi+\overline{e})+\overline{e}(1-\gamma)]-\eta[\pi(1-\pi)+\overline{e}(1+\pi)]z^{s*}}{b(1-\pi+\overline{e})}L \\ &\quad -\frac{\eta\pi(1-\gamma)(1-\alpha)A}{b}\left\{\left[\frac{1-\pi}{(1-\gamma)\overline{e}}\right]^{\alpha}-\left[\frac{1-\pi}{\pi-\gamma}\right]^{\alpha}\right\}L. \end{aligned} \tag{81}$$

F Proof of Proposition 3

We start with the first part of the Proposition. In the post-trade equilibrium $S=S^*=\pi L$ implying $w^s=w^{s*}$ and $w^u=w^{u*}$. This implies that individuals of type i are identical across countries. Therefore using (24) we obtain:

$$\beta_W^u=\beta_W^{u*}=\frac{1}{(1-b)}\left(1-\frac{\eta v'[H-\frac{\kappa}{b}2(\pi-\gamma)L+\frac{\eta}{b}2\pi L z_W^s)]}{u'(w^u-\gamma w^s)}\right), \tag{82}$$

and

$$\beta_W^s=\beta_W^{s*}=\frac{1}{(1-b)}\left(1-\frac{\eta v'[H-\frac{\kappa}{b}2(\pi-\gamma)L+\frac{\eta}{b}2(1-\pi)L z_W^u]}{u'[w^s(1-\gamma)]}\right). \tag{83}$$

Using the results from Lemma 1 and the fact that $u(.)$ and $v(.)$ are concave functions, we obtain that $\beta_W^s<\beta_W^u$ if $(1-\pi)z_W^u\leq\pi z_W^s$.

Concerning the second part of the Proposition, we start by deriving the conditions under which $\beta_W^u<\beta^u$ in the post-trade equilibrium. Since the net wage $w^u-\gamma w^s$ is the same in both equilibria and from Assumption 2, $z^u=z^{u*}=0$, a necessary and sufficient condition for $\beta_W^u<\beta^u$ is

$$\frac{\kappa[(\pi-\gamma)(1-\pi+\overline{e})+\overline{e}(1-\gamma)]}{b(1-\pi+\overline{e})}-\frac{\eta}{b}\left(\pi z^s+\frac{\overline{e}}{1-\pi-\overline{e}}z^{s*}\right)<\frac{2}{b}[\kappa(\pi-\gamma)-\eta\pi z_W^s],$$

which is equivalent to condition (42). From Lemma 2, we know that in the autarky equilibrium $\beta^u < \beta^{u*}$ while in the post-trade equilibrium $\beta^u_W = \beta^{u*}_W$ implying that if $\beta^u_W < \beta^u$, it is also the case that $\beta^u_W < \beta^{u*}$.

We proceed in the same way concerning β_s. Since the net wage $(1-\gamma)w^s$ is the same in both equilibria and $z^u = z^{u*} = 0$, a necessary and sufficient condition for $\beta^s_W < \beta^s$ is

$$\frac{\kappa[(\pi-\gamma)(1-\pi+\bar{e})+\bar{e}(1-\gamma)]}{b(1-\pi+\bar{e})} - \frac{\eta\bar{e}}{b(1-\pi+\bar{e})}z^{s*} < \frac{2}{b}[\kappa(\pi-\gamma)-\eta(1-\pi)z^u_W],$$

which is equivalent to condition (43).

G Solutions for z^s_W and z^u_W

We start with the wages of both types of individuals in equilibrium which are given by

$$w^s = (1-\alpha)A\left(\frac{1-\pi}{\pi-\gamma}\right)^{\alpha}, \tag{84}$$

and

$$w^u = \alpha A\left(\frac{\pi-\gamma}{1-\pi}\right)^{1-\alpha}. \tag{85}$$

Combining the wages, we obtain

$$\frac{w^u}{w^s} = \frac{\alpha(\pi-\gamma)}{(1-\alpha)(1-\pi)},$$

and by using the equality of marginal utilities of consumption

$$\left[\frac{(1-\alpha)(1-\pi)-\alpha(\pi-\gamma)}{(1-\alpha)(1-\pi)}\right]w^s = z^s_W - z^u_W,$$

or

$$A\left[\frac{(1-\alpha)(1-\pi)-\alpha(\pi-\gamma)}{(1-\pi)}\right]\left(\frac{1-\pi}{\pi-\gamma}\right)^{\alpha} = z^s_W - z^u_W.$$

Finally, substituting this last expression in (47) we obtain

$$u'\left\{(1-\alpha)A\left[1-\frac{\gamma(1-\alpha)(1-\pi)}{\alpha(\pi-\gamma)}\right]\left(\frac{\pi-\gamma}{1-\pi}\right)^{1-\alpha} - z^u_W\right\} = \frac{\eta}{1-\beta(1-b)}v'(N),$$

where

$$N = H - \frac{2L}{b}\left\{\kappa(\pi-\gamma) - \eta z^u_W - \eta\pi A\left[\frac{(1-\alpha)(1-\pi)-\alpha(\pi-\gamma)}{(1-\pi)}\right]\left(\frac{1-\pi}{\pi-\gamma}\right)^{\alpha}\right\}.$$

We proceed in the same way to compute a similar expression for z^s_W and we obtain

$$u'\left[(1-\gamma)(1-\alpha)A\left(\frac{1-\pi}{\pi-\gamma}\right)^{\alpha} - z^s_W\right] = \frac{\eta}{1-\beta(1-b)}v'(N),$$

where

$$N = H - \frac{2L}{b}\left\{\kappa(\pi-\gamma) - \eta z^s_W + \eta(1-\pi)A\left[\frac{(1-\alpha)(1-\pi)-\alpha(\pi-\gamma)}{(1-\pi)}\right]\left(\frac{1-\pi}{\pi-\gamma}\right)^{\alpha}\right\}.$$

H Proof of Proposition 4

We start with the first part of the Proposition. The threshold values of the social discount factor are given by

$$\beta_o^u = \frac{1}{(1-b)}\left(1 - \frac{2(1-\pi)L\eta v'[H - \frac{\kappa}{b}2(\pi-\gamma)L + \frac{\eta}{b}2\pi L z_o^s]}{u'(w^u - \gamma w^s)}\right), \tag{86}$$

and

$$\beta_o^s = \frac{1}{(1-b)}\left(1 - \frac{2\pi L\eta v'[H - \frac{\kappa}{b}2(\pi-\gamma)L + \frac{\eta}{b}2(1-\pi)L z_o^u]}{u'[w^s(1-\gamma)]}\right). \tag{87}$$

Using the results from Lemma 1 and the fact $u(.)$ and $v(.)$ are concave functions, we obtain that $\beta_o^s < \beta_o^u$ if $(1-\pi)z_o^u \leq \pi z_o^s$.

Concerning the second part of the Proposition, since the net wage $w^u - \gamma w^s$ is the same in the post-trade equilibria and the optimal allocation, a necessary and sufficient condition for $\beta_o^u < \beta_W^u$ is that condition (58) is satisfied.

A similar argument can be used for the third part of the Proposition, where a necessary and sufficient condition for $\beta_o^s < \beta_W^s$ is that condition (59) is satisfied.