Bank Recovery and Resolution, Liquidity Management and Fragility

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Abstract

We propose a novel theory for the interaction of banks' liquidity management, financial fragility, and regulation. Banks invest in illiquid productive assets and hold liquidity to repay depositors with idiosyncratic needs to withdraw funds during an interim period. Moreover, strategic complementarities in the depositors' withdrawal decisions make banks subject to self-fulfilling runs, modeled as "global games". Under different regulatory regimes, we study liquidity holdings and the pecking order in which banks use assets to repay depositors during a run. In an unregulated economy, in which banks choose the pecking order as a run unfolds, they first deploy liquidity and then liquidate productive assets, and hold excess liquidity in anticipation of runs. In an economy with recovery and resolution planning, banks commit to the asset pecking order that maximizes expected welfare, and a resolution authority resolves banks if a run occurs by suspending deposit convertibility and creating a good bank at a cost. Under this regime, banks commit to liquidating productive assets first and then deploying liquidity, and hold no excess liquidity in anticipation of runs. The difference between the two regimes only depends on the coexistence of recovery and resolution planning.

Keywords: banks, liquidity, financial fragility, self-fulfilling runs, resolution

JEL codes: G01, G21, G28

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1 Introduction

It is well known that banks' liquidity management plays a crucial role in anticipation of and at times of financial distress. Several arguments have been proposed to explain this, all based on the assumption that banks face fundamental uncertainty against which they might want to hold liquidity (Ashcraft et al., 2011; Acharya and Merrouche, 2013). However, extensive evidence shows that banks are prone to financial fragility induced by depositors' self-fulfilling expectations. Indeed, the very essence of banking, i.e., liquidity and maturity transformation, creates financial fragility through a mismatch in banks' balance sheets, leading to the possibility of depositors' self-fulfilling runs. Financial fragility and self-fulfilling runs are not a phenomenon of the past: for example, Argentina in 2001 and Greece in 2015 faced such systemic events. Furthermore, there is a broad consensus that both the 2007-2009 global financial crisis and the 2010-2012 joint bank and sovereign debt crisis in the EU had a significant self-fulfilling component (Gorton and Metrick, 2012; Baldwin et al., 2015), thus justifying an extensive government intervention to calm investors' expectations and guarantee financial stability. These considerations call for a theory of the interaction between banks' liquidity management, self-fulfilling financial fragility, and financial regulation. This is the aim of the present paper.

Our argument's distinctive feature is that the interaction between financial fragility and banks' liquidity management is bidirectional and crucially depends on the regulatory regime that banks face. On the one hand, financial fragility might have nontrivial effects on liquidity management, as banks anticipate being able to cope with excessive withdrawals during periods of distress either by rolling over liquidity or by liquidating the more productive assets on their balance sheets. On the other hand, banks' liquidity management affects investors' perception of how resilient banks are to aggregate shocks, which feeds back into financial fragility. Financial regulation affects this two-way interaction in several ways. In particular, the Financial Stability Board (2011) has been coordinating international efforts to introduce bank recovery and resolution planning. This intervention aims to provide banks with a framework to address severe financial shocks and set rules for orderly resolution of distressed banks while maintaining such banks' continuity. The present paper uncovers the channels through which recovery and resolution planning impact banks' liquidity management, financial fragility, and their interaction.

To this end, we propose a theory of banking, featuring two types of fundamental uncertainty, namely aggregate productivity shocks that hit banks' investments and idiosyncratic liquidity shocks that force depositors to withdraw funds during an interim period, i.e., before the maturity of the investment. Banks also face self-fulfilling financial fragility. Due to incomplete contractibility related to idiosyncratic liquidity shocks and imperfect information about the aggregate productivity shocks, the economy features multiple equilibria with the possibility of runs by banks' depositors in the interim period. These runs are self-fulfilling because depositors withdraw funds in the interim period since they expect that all the other depositors also withdraw funds and fear that if they do not do the same, they might end up with zero consumption.[1] We resolve the multiplicity of equilibria following the "global game" approach by Carlsson and van Damme (1993) and Morris and Shin (1998). We assume that the depositors observe a noisy signal about the realization of the economy's aggregate state, based on which they create beliefs about the behavior of all the other depositors and decide whether to withdraw funds during the interim period.

Competitive banks collect deposits and offer standard deposit contracts, financed by investing deposits in liquidity and a partially illiquid but productive asset. Characterizing both sides of banks' balance sheets complicates the analysis, which represents a methodological contribution of the paper. More importantly, it enables us to offer a complete analysis of banks' liquidity management from an ex-ante perspective, i.e., in anticipation of fundamental and self-fulfilling uncertainty, as well as from an ex-post perspective, when the latter eventually is realized.

Apart from the deposit contract and the asset portfolio, we further allow banks to choose the asset pecking order that they can follow to satisfy depositors' withdrawals in the interim period. In particular, banks can either first deploy liquidity and then liquidate productive assets or liquidate productive assets and then deploy liquidity. Both strategies entail some costs, as deploying liquidity reduces more productive investments, while liquidating the productive asset is costly in terms of resources forgone at liquidation and forgone future consumption. Studying the pecking order as part of banks' liquidity management is the main contribution of this paper and is crucial for the paper's results because such order influences depositors' withdrawal decisions.

[1]For this argument to hold, we need to assume that there exists no deposit insurance. This assumption can be justified by the growing role of uninsured bank deposits and the shadow banking system, which offers bank services – and in particular liquidity and maturity transformation – without any regulation or government assistance (Pozsar et al., 2010).

We characterize the game's unique symmetric equilibrium between banks and depositors, starting from an unregulated equilibrium in which banks choose the asset pecking order without commitment, i.e., as depositors withdraw funds during the interim period and a self-fulfilling run eventually unfolds. If only the depositors hit by the idiosyncratic liquidity shocks withdraw funds, no run is realized at date 1: banks are solvent and use only liquidity to cover depositors' withdrawals without liquidating productive assets. If all depositors withdraw funds, a run is realized: banks are insolvent and are forced to liquidate all of the portfolios' productive assets and close down. If instead a partial depositors' run occurs, and banks are illiquid but solvent, we show that they will always prefer to first deploy liquidity and then liquidate productive assets to cover depositors' withdrawals. The reason is that in this scenario, depositors' incentives to run are lower than under the opposite strategy of first liquidating productive assets and then deploying liquidity for any possible fraction of depositors engaging in a bank run. In the unregulated equilibrium, banks anticipate this mechanism, and recognize that if they increase liquidity, depositors' incentives to run are reduced. Thus, banks choose to hold excess liquidity, i.e., a higher liquidity than they would need to cover depositors' idiosyncratic liquidity needs if no runs were to occur.

We then introduce recovery and resolution planning. In particular, we assume that a regulatory intervention forces banks to commit to a recovery plan. Accordingly, to maximize the depositors' expected welfare the banks fix on date 0 an asset pecking order to serve depositors on date 1 if a run occurs. This assumption can be justified by the recommendations of the Financial Stability Board that recovery plans should "provide a covered bank with a framework to effectively and efficiently address the financial effects of severe stress events and avoid failure or resolution" (Office of the Comptroller of the Currency, 2019) through the identification of "credible options that a covered bank could undertake in response to the stress to restore its financial strength and viability" (Office of the Comptroller of the Currency, 2018).

The regulatory intervention further establishes a resolution authority (hereafter, RA). After observing banks' recovery plans, the RA commits on date 0 to a resolution plan that also maximizes depositors' expected welfare. According to the plan, the RA has the power to suspend deposit convertibility when a run is underway, and the fraction of depositors withdrawing funds on date 1 reaches a certain threshold.[2] After suspension, the RA pays a verification cost to establish which

[2]This assumption is consistent with common resolution practice around the world. For example, the European

depositors need liquidity on date 1 among those who have not already withdrawn funds and reallocates the available resources between them and the remaining depositors. In particular, depositors with early liquidity needs still receive what banks have promised them according to the deposit contract. Accordingly, such a resolution procedure resembles the creation of a "good bank" that shields insured deposits. This is consistent with the recommendations by the Basel Committee on Banking Supervision (2010) and is a crucial part of several resolution plans worldwide. The verification cost instead represents mechanisms that might delay bank resolution, such as costly resolution tools, the RA's lack of commitment leading to resolution forbearance, or imperfect coordination between different regulatory layers. In the model, the role of costly verification is also crucial because a credible commitment to a costless suspension of deposit convertibility could rule out depositors' runs altogether (Diamond and Dybvig, 1983).

Our main result shows that in an economy with recovery and resolution planning, banks in equilibrium choose a pecking order different from that in the unregulated economy: they commit first to liquidate productive assets and then deploy liquidity when facing a depositors' run. Intuitively, this occurs because banks recognize that the choice of pecking order affects not only depositors' incentives to withdraw funds but also the type and amount of resources left to the good bank after resolution. These two channels influence banks' choice of asset portfolios on date 0 and their expected return when a run does not occur, thus further affecting the expected welfare. With recovery and resolution planning, banks do not hold excess liquidity differently from the case of the unregulated equilibrium. The reason is that by committing first to liquidate productive assets and then deploy liquidity in case of a run, banks realize that higher liquidity is detrimental to financial fragility because it increases depositors' incentives to withdraw early. Thus, banks are willing to keep the minimum level of liquidity. Significantly, all of these results depend on the complementarities between recovery and resolution planning: neither resolution planning alone nor recovery planning alone can induce banks to change asset pecking order and reduce excess liquidity compared to the case of the unregulated equilibrium.

To summarize, the introduction of recovery and resolution planning significantly changes banks'

Systemic Risk Board states that "resolution occurs at the point where the authorities determine that a bank is failing or likely to fail, that there is no other supervisory or private sector intervention that can restore the institution to viability (for example by applying measures set out in a so-called recovery plan, which all banks are required to draft) within a short time frame and that normal insolvency proceedings would cause financial instability while having an impact on the public interest". Source: https://srb.europa.eu/en/content/what-bank-resolution.

liquidity management in anticipation of self-fulfilling uncertainty and its eventual realization, thus curbing financial fragility and guaranteeing a more valuable allocation of resources in the economy.

The present paper contributes to several strands of the literature. Our study is the first to examine banks' asset pecking orders during self-fulfilling depositors' runs in the literature on banks' liquidity and financial fragility. In recent models of bank runs, this issue is completely overlooked. For example, in Goldstein and Pauzner (2005) banks hold no liquidity because it is dominated by the productive asset, which has zero liquidation costs. In contrast, in Rochet and Vives (2004) and Vives (2014) banks indeed hold liquidity and productive assets, but the structure of their balance sheets and the pecking order during a run are exogenous. Kashyap et al. (2017) develop a model of both sides of banks' balance sheets as we do. However, differently from our study, banks do not hold excess liquidity against self-fulfilling uncertainty and do not choose asset pecking orders. Ahnert and Elamin (2020) also study banks' portfolio choice but assume that banks can liquidate the productive asset at zero cost in the interim period and do not have any liquidity available in the initial period. Moreover, banks have access to liquidity only after depositors' decisions to withdraw funds and eventually run and after receiving a perfectly informative signal about the realization of the economy's aggregate state.

More generally, our study contributes to the literature on liquidity management of financial institutions prone to self-fulfilling uncertainty. Chen et al. (2010) develop a theory of investors' strategic complementarities to rationalize the empirical evidence on the connection between mutual funds' liquidity and performance. Zeng (2017) studies the optimal rebuilding of cash buffers by open-end mutual funds in a dynamic framework. Liu and Mello (2011) analyze the liquidity management of hedge funds facing coordination risk. Distinctly from all of these papers, our focus is on banks and their maturity transformation and fragility. In this sense, we characterize the equilibrium deposit contract and the optimal asset pecking order, which the cited studies instead all leave to be exogenous.

Finally, our paper contributes to the literature on the economics of bank resolution regimes. Keister and Mitkov (2019) show how the RA's lack of commitment in resolving banks leads to banks' delay in bail-ins. Colliard and Gromb (2018) instead explore the effect of the government's involvement in the private restructuring of distressed banks. Schilling (2018) examines the optimal delay of bank resolution and its effects on financial fragility. These studies do not analyze the consequences of recovery and resolution planning for banks' choices of asset pecking orders and liquidity management as we do.

The rest of the paper is organized as follows. Section 2 states the basic features of the environment. Section 3 analyzes banks' liquidity management in the unregulated economy. Section 4 introduces recovery and resolution planning. Section 5 describes the equilibrium choice of liquidity and asset pecking order. Finally, section 6 concludes the paper.

2 Environment

The economy lasts for three periods, labeled $t = 0, 1, 2$, and is populated by a unitary continuum of ex-ante identical agents and a large number of banks. Each agent is endowed with 1 unit of a consumption good on date 0, and 0 afterwards. On date 1, all agents are hit by a privately observed idiosyncratic liquidity shock θ of value 0 with probability λ and value 1 with probability $1-\lambda$. The law of large numbers holds. Hence, the probability distribution of idiosyncratic liquidity shocks is equivalent to the cross-sectional distribution of their realization: on date 1, there is a fraction λ of agents in the whole economy who experience the realized shock $\theta = 0$, and a fraction $1-\lambda$ of agents who experience the realized shock $\theta = 1$. Agents are risk-neutral, and each agent's individual utility function is $U(c_1, c_2, \theta) = (1-\theta)c_1 + \theta c_2$. Accordingly, the idiosyncratic liquidity shock θ affects the point in time when agents want to consume. Agents experiencing the shock $\theta = 0$ are only willing to consume on date 1, and those experiencing the shock $\theta = 1$ are only willing to consume on date 2. Consistently with the literature, we refer to these two categories as early consumers and late consumers, respectively.

There are two technologies available in the economy. The first is a storage technology, which we call "liquidity"; it yields 1 unit of consumption on date $t+1$ for each unit invested on date t. The second is a productive asset that, for each unit invested on date 0, yields a stochastic return Z on date 2. The stochastic return takes values $R > 1$ with probability p, and 0 with probability $1-p$, and its realization is publicly revealed in the beginning of date 2. The probability of success, p, represents the aggregate state of the economy, and is uniformly distributed over the interval $[0, 1]$, with $\mathbb{E}[p]R > 1$. The productive asset can be liquidated on date 1 via a liquidation technology that allows one to recover $r < 1$ units of consumption for each unit of liquidation. In other words, the economy features a liquid asset with low but safe yields, and an illiquid asset that yields a low return in the short run, and a possibly high return in the long run, subject to the realization of an aggregate productivity shock.

Banks operate in a perfectly competitive market with free entry. They collect the agents' endowments by accepting deposits and invest the proceeds to maximize the expected profits. Perfect competition and free entry ensure that banks solve the equivalent dual problem of maximizing the expected welfare of the agents/depositors, subject to their budget constraint. To this end, we assume that banks offer a standard deposit contract, stating the amount of early consumption c that depositors can withdraw on date 1, and the amount $c_2(Z)$ that they can withdraw on date 2. The banks' offer of a noncontingent amount of early consumption depends on the true realization of the aggregate state being revealed only in the beginning of date 2.[3] Moreover, we assume that banks' choice of early consumption is subject to $c \geq 1$, to reflect the observed reluctance of banks in the real world to pass on negative rates to their depositors (Heider et al., 2019).[4]

To repay the depositors according to the deposit contract, the banks on date 0 invest the deposits – that are the only liability on their balance sheets – in a portfolio of L units of liquidity and $1 - L$ units of the productive asset. Then, given the deposit contract and the asset portfolio, the banks on date 1 pay c to all the depositors who withdraw funds early until the banks' resources are exhausted. Finally, on date 2 the depositors who have not withdrawn funds on date 1 become residual claimants on equal shares of the remaining resources. If resources are exhausted on date 1, and the banks cannot fulfill their contractual obligations to the depositors anymore, the banks become insolvent. In this case, they are forced to liquidate all the productive assets in their portfolios, and serve the depositors according to an "equal service constraint" such that all of them receive equal shares of the available resources.

Banks also choose a pecking order to use the available assets to cover withdrawals on date 1. Under the pecking order {Liquidation, Liquidity}, banks first liquidate the productive asset and then deploy the available liquidity. Under the pecking order {Liquidity, Liquidation}, they instead first deploy liquidity and then liquidate the productive asset. In what follows, we denote the pecking

[3] More generally, the rationale for the optimality of this assumption is the presence of asymmetric information. In a Diamond-Dybvig environment, Farhi et al. (2009) show that an uncontingent amount of early consumption endogenously emerges in equilibrium with non-exclusive contracts.

[4] With a linear utility, banks in equilibrium would choose the lowest possible value of early consumption. Then, the assumption of $c \geq 1$ allows us to maintain a rationale for liquidity management. Carletti et al. (2018), who also study a banking model with a linear utility, encounter the same issue and resolve it by assuming an exogenous amount of early consumption equal to 1 instead.

orders {Liquidation, Liquidity} and {Liquidity, Liquidation} by subscripts (1) and (2), respectively.

We assume that the depositors cannot observe the true value of the realization of the aggregate state of the economy p, but receive on date 1 a signal $\sigma = p+e$ about it. The term e is an idiosyncratic noise, uniformly distributed over the interval $[-\epsilon, +\epsilon]$, where ϵ is positive but arbitrarily close to zero. Given the received signal, late consumers decide whether to wait and withdraw funds from the bank on date 2 or "run on the bank" and withdraw funds on date 1. In particular, we assume that late consumers follow a threshold strategy: they run if the signal they receive is lower than a threshold signal σ^*. This strategy is based on the expected advantage of waiting over running, which explicitly depends on depositors' beliefs, the deposit contract, and bank asset portfolio.

We solve the model by backward induction and characterize a pure-strategy symmetric perfect Bayesian equilibrium. Hence, we focus our attention on the behavior of a representative bank. The definition of equilibrium is as follows:

Definition 1. *Given the distributions of idiosyncratic liquidity shocks θ, the aggregate productivity shock Z and the individual signals σ, a banking equilibrium comprises an amount of early consumption c, an asset portfolio $\{L, 1-L\}$, a pecking order and a set of depositors' withdrawal decisions such that*

- *The depositors' withdrawal decisions maximize their expected welfare;*
- *The pecking order, the deposit contract, and the asset portfolio maximize the depositors' expected welfare, subject to the bank's budget constraints, and*
- *The bank's and depositors' beliefs are updated according to the strategies used and the Bayes' rule.*

2.1 Equilibrium with perfect information

As a benchmark, in this section we characterize a banking equilibrium in which the representative bank is perfectly informed about depositors' types, i.e., it can observe and verify the realizations of the idiosyncratic liquidity shocks hitting the depositors (but not the realization of the aggregate state), and it maximizes their expected welfare subject to budget constraints. This is a reasonable benchmark, as its equilibrium outcome is equivalent to that of an economy with imperfect information in which banks cannot observe depositors' types and must induce truth-telling, but depositors do not observe noisy signals about the aggregate state.[5] Formally, the bank solves the following:

$$\max_{c,c_2(Z),L,D} \lambda c + (1-\lambda)\mathbb{E}[c_2(Z)], \tag{1}$$

subject to $c \geq 1$ and to

$$L + rD \geq \lambda c, \tag{2}$$

$$0 \leq D \leq 1 - L, \tag{3}$$

$$L \geq 0, \tag{4}$$

$$(1-\lambda)c_2(Z) = Z(1-L-D) + L + rD - \lambda c, \tag{5}$$

where the last constraint has to hold for any $Z \in \{0, R\}$. On date 0, the bank collects all endowments and invests them in an amount L of liquidity and $1-L$ of productive assets. On date 1, the liquidity constraint (2) states that the amount of liquid assets, given by the sum of liquidity and the resources generated by liquidating an amount D of productive assets at rate r, must be sufficient to pay early consumption c to λ early consumers. Any resource $L + rD - \lambda c$ left constitutes excess liquidity and is rolled over to date 2. The excess liquidity, together with the return from the remaining productive assets, pays for late consumption according to (5) for any realization of the aggregate productivity shock $Z \in \{0, R\}$. The bank cannot liquidate a negative amount of assets and cannot liquidate more assets than are available on its balance sheet.

Substituting the budget constraints into the objective function results in the following banking problem:

$$\max_{c,L,D} \lambda c + (1-\lambda)\int_0^1 \left[p\frac{R(1-L-D)+L+rD-\lambda c}{1-\lambda} + (1-p)\frac{L+rD-\lambda c}{1-\lambda}\right] dp, \tag{6}$$

subject to $L + rD \geq \lambda c$, $L \geq 0$, $c \geq 1$ and $0 \leq D \leq 1 - L$. The following result holds:

Lemma 1. *In the banking equilibrium with perfect information, there is no liquidation of the productive asset and no excess liquidity, i.e., $D = 0$ and $L = \lambda c$. Moreover, in equilibrium $c = 1$.*

Proof. In Appendix B. ■

[5] In Appendix A, we compare the banking equilibrium to autarky and study the viability of the banking system.

The above lemma states that liquidating productive assets to create liquidity on date 1 is never part of an equilibrium in the case of perfect information because the recovery rate $r < 1$ implies that liquidation is too costly. Consequently, with perfect information, banks only use liquidity to serve early consumers, which is a result that will be important for the characterization of the endogenous pecking order in the subsequent sections. Moreover, the advantage of holding excess liquidity, in terms of consumption in the bad state of the world when the productive asset yields zero, is dominated by its cost in terms of forgone return in the good state of the world. Hence, the bank in equilibrium holds just enough liquidity to cover the total early consumption λc. Finally, as depositors are risk-neutral, the bank only cares about its investment's expected total return. This means that it chooses to offer the minimum amount of early consumption possible, i.e., $c = 1$.

3 Unregulated economy

We now proceed to the analysis of the competitive banking equilibrium in the unregulated economy. First, for a given asset pecking order, we study the withdrawal decisions of late consumers (as early consumers will certainly want to withdraw funds on date 1) who choose whether to withdraw funds on date 1 (i.e., "run") or wait until date 2. Then, we characterize the equilibrium asset pecking order, and finally, the choice of deposit contract and bank asset portfolio.

Assume that the depositors observe their private types and signals in the beginning of date 1, and decide whether to withdraw funds early. If so, they arrive at the bank in a random order and are served sequentially. They do not know how many of them are in line. Hence, they do not accept a contract contingent on either their position in the line or the number of early withdrawals. At the same time, the bank does not know that a run is underway until the fraction of depositors withdrawing funds exceeds λ. Hence, it serves the first λ early withdrawers with liquidity as in the equilibrium with perfect information and has to choose the asset pecking order to cover further withdrawals.

For clarity, we summarize here the timing of actions. On date 0, the bank chooses the asset portfolio $\{L, 1 - L\}$ and the deposit contract $\{c, c_2(Z)\}$. On date 1, all depositors become aware of their private types and signals and decide whether to withdraw funds. Then, they take positions in line and start withdrawing funds until resources are exhausted, while the bank chooses the asset pecking order as a run unfolds. Finally, on date 2, those late consumers who have not withdrawn funds on date 1 withdraw equal shares of the available resources left.

3.1 Asset pecking orders

The presence of noisy signals allows us to apply global-game techniques. Late consumers act based on their private signals σ on date 1, and take as given the deposit contract and the bank asset portfolio fixed on date 0. Based on this information, they create posterior beliefs about the probability of the realization of the aggregate productivity shock Z and about the number n of depositors withdrawing funds at time 1, and decide whether to join them.

We assume the existence of two extreme regions such that, for signal realizations within them, the withdrawal decisions of late consumers are independent of such consumers' posterior beliefs. Following Goldstein and Pauzner (2005), we assume an "upper dominance region" above a threshold $\bar{\sigma}$ where the productive asset is safe, i.e., $p = 1$, and yields the same return R on dates 1 and 2. In this way, late consumers who wait are sure to receive $(R(1-L) + RL - nc)/(1-n) > c$ on date 2. Hence, they will never run for any fraction n of depositors withdrawing early. In the "lower dominance region" instead, the signal is so low that late consumers always withdraw funds on date 1 irrespective of the behavior of the other depositors. This occurs below the threshold signal $\underline{\sigma}$ that makes late consumers indifferent between withdrawing or not:

$$c = \underline{\sigma}\frac{R(1-L) + L - \lambda c}{1-\lambda} + (1-\underline{\sigma})\frac{L - \lambda c}{1-\lambda}. \tag{7}$$

Hence:

$$\underline{\sigma}(c, L) = \frac{c - L}{R(1-L)}. \tag{8}$$

The existence of the lower and upper dominance regions ensures the existence of an equilibrium in the intermediate region $[\underline{\sigma}, \bar{\sigma}]$, where late consumers decide whether to run or not based on their beliefs about the true realization of the aggregate state and other depositors' actions.

To characterize depositors' withdrawal behavior, we first study the utility advantage of waiting over running under both pecking orders, for a given fraction n of depositors who withdraw funds on date 1. Table 1 reports payoffs under both pecking orders. In Table 1a, $n_1^* = \lambda + (r(1-L))/c$ and $n^{**} = (r(1-L) + L)/c$ are the maximum fractions of depositors that a bank can serve with early consumption c on date 1, and either liquidating the whole amount of productive assets in the

Table 1: Depositors' ex-post payoffs in the unregulated economy.

(a) Pecking order {Liquidation, Liquidity}

Date	$n \in [\lambda, n_1^*)$	$n \in [n_1^*, n^{**})$	$n \in [n^{**}, 1]$
$t = 1$	c	c	$\frac{r(1-L)+L}{n}$
$t = 2$	$\frac{Z\left(1-L-\frac{(n-\lambda)c}{r}\right)+L-\lambda c}{1-n}$ $\forall Z \in \{0, R\}$	$\frac{r(1-L)+L-nc}{1-n}$	0

(b) Pecking order {Liquidity, Liquidation}

Date	$n \in [\lambda, n_2^*)$	$n \in [n_2^*, n^{**})$	$n \in [n^{**}, 1]$
$t = 1$	c	c	$\frac{r(1-L)+L}{n}$
$t = 2$	$\frac{Z(1-L)+L-nc}{1-n}$ $\forall Z \in \{0, R\}$	$\frac{Z\left(1-L-\frac{nc-L}{r}\right)}{1-n}$ $\forall Z \in \{0, R\}$	0

portfolio (up to n_1^*) or also using liquidity (up to n^{**}). If the fraction of depositors who withdraw funds on date 1 is in the interval $[\lambda, n_1^*)$, the bank fulfills its contractual obligation on date 1 by liquidating productive assets first. It needs to pay an amount of early consumption c to $n - \lambda$ depositors using resources of rD from liquidation. Hence the amount of the productive asset to be liquidated is $D = (n - \lambda)c/r$. Then, if n depositors withdraw funds on date 1, consumption of a late consumer who waits until date 2 is

$$c_L(Z, n) = \frac{Z\left(1 - L - \frac{(n-\lambda)c}{r}\right) + L - \lambda c}{1 - n}, \tag{9}$$

depending on the realization of the aggregate productivity shock $Z \in \{0, R\}$, as the remaining liquidity $L - \lambda c$ is rolled over from date 1 to date 2. If the fraction of depositors who withdraw funds on date 1 is in the interval $[n_1^*, n^{**})$, the bank instead fulfills its contractual obligation by liquidating all productive assets in its portfolio (thus, generating resources equal to $r(1 - L)$) and by deploying liquidity. Thus, if n depositors withdraw funds on date 1, the consumption of a late consumer who waits until date 2 is independent of the realization of the aggregate productivity shock Z (as the productive assets have all been liquidated) and is equal to $c_L^L(n) = (r(1 - L) + L - nc)/(1 - n)$. Finally, if the fraction of depositors who withdraw funds on date 1 is in the interval $[n^{**}, 1]$, the bank becomes insolvent, as it does not hold sufficient resources to pay c to all depositors. In this case, the bank is forced to liquidate all productive assets and close down. Therefore, a late consumer who waits until date 2 receives zero. The total liquidation value of the bank's assets (equal to $r(1-L)+L$) is split pro-rata among the n depositors withdrawing funds on date 1. Accordingly, consumption at insolvency is $c^B(n) = (r(1-L)+L)/n$.

In Table 1b instead, $n_2^* = L/c$ and $n^{**} = (r(1-L)+L)/c$ are the maximum fractions of depositors that a bank can serve on date 1 with early consumption c and using liquidity (up to n_2^*), and also liquidating the whole amount of productive assets in its portfolio (up to n^{**}). If the fraction of depositors who withdraw funds on date 1 is in the interval $[\lambda, n_2^*)$, the bank fulfills its contractual obligation by using liquidity, while retaining the productive asset. Hence, in this case the consumption of a late consumer who waits until date 2 is either $c_L(R,n) = (R(1-L)+L-nc)/(1-n)$ or $c_L(0,n) = (L-nc)/(1-n)$, depending on the realization of the aggregate productivity shock Z. If the fraction of depositors who withdraw funds on date 1 is instead in the interval $[n_2^*, n^{**})$, the bank is forced to fulfill its contractual obligation by also liquidating the productive assets in its portfolio. Hence, the total resources available to provide early consumption c are $L + rD$, and the amount that the bank liquidates is equal to $D = \frac{nc-L}{r}$. Moreover, as the liquidity has been exhausted, the consumption of a late consumer who waits until date 2 is

$$c_L^D(Z,n) = \frac{Z\left(1-L-\frac{nc-L}{r}\right)}{1-n}, \tag{10}$$

for any $Z \in \{0, R\}$. Finally, if the fraction of depositors who withdraw funds on date 1 is in the interval $[n^{**}, 1]$, the bank is insolvent. As before, all n depositors who withdraw funds on date 1 receive $c^B(n) = (r(1-L)+L)/n$, and those $1-n$ who do not withdraw funds receive zero.

Given the described payoff structure, under the pecking order {Liquidation, Liquidity} the utility advantage of waiting over running is

$$v_1(\sigma, n) = \begin{cases} \sigma \frac{R\left(1-L-\frac{(n-\lambda)c}{r}\right)+L-\lambda c}{1-n} + (1-\sigma)\frac{L-\lambda c}{1-n} - c & \text{if } \lambda \le n < n_1^*, \\ \sigma \frac{r(1-L)+L-nc}{1-n} + (1-\sigma)\frac{r(1-L)+L-nc}{1-n} - c & \text{if } n_1^* \le n < n^{**}, \\ -\frac{r(1-L)+L}{n} & \text{if } n^{**} \le n \le 1, \end{cases} \tag{11}$$

while under the pecking order {Liquidity, Liquidation} it is

$$v_2(\sigma,n)=\begin{cases}\sigma\frac{R(1-L)+L-nc}{1-n}+(1-\sigma)\frac{L-nc}{1-n}-c & \text{if } \lambda\le n<n_2^*,\\ \sigma\frac{R(1-L-D)}{1-n}-u(c)=\sigma\frac{R\left(1-L-\frac{nc-L}{r}\right)}{1-n}-c & \text{if } n_2^*\le n<n^{**},\\ -\frac{r(1-L)+L}{n} & \text{if } n^{**}\le n\le 1.\end{cases} \tag{12}$$

The strategic complementarities affecting a late consumer's decision to run depend on how the advantage of waiting over running varies with the fraction of depositors n withdrawing funds on date 1. Under both pecking orders, in the interval $[n^{**},1]$ the advantage of waiting over running is increasing in n, as after insolvency, equal service prescribes that the total resources be shared pro-rata with all depositors. The following lemma characterizes the strategic complementarities in the other intervals under the two pecking orders.

Lemma 2. *The function $v_1(\sigma,n)$ is decreasing in $n\in[\lambda,n^{**})$. The function $v_2(\sigma,n)$ is increasing in $n\in[\lambda,n_2^*)$ and decreasing in $[n_2^*,n^{**})$.*

Proof. In Appendix B. ■

Figure 1 shows that under the pecking order {Liquidation, Liquidity} the economy exhibits one-sided strategic complementarities. The advantage of waiting over running is decreasing in the fraction of depositors running before insolvency and increasing after insolvency. In contrast, under the pecking order {Liquidity, Liquidation} the advantage of waiting over running is increasing in the fraction of depositors running as long as the bank holds sufficient liquidity to serve them, i.e., in the interval $[\lambda,n_1^*)$. This occurs because the positive effect on the incentives to wait given by the lower number of depositors who wait dominates the negative effect given by the lower amount of liquidity rolled over from date 1. In any case, both functions $v_1(\sigma,n)$ and $v_2(\sigma,n)$ cross zero only once, and are increasing in σ. Altogether, these properties guarantee the existence and uniqueness of the equilibrium under both pecking orders (Goldstein and Pauzner, 2005).

3.2 Optimal pecking order

By backward induction, we start the characterization of the unregulated equilibrium with the bank's decision as to the optimal pecking order as the run unfolds, i.e., for any possible fraction n

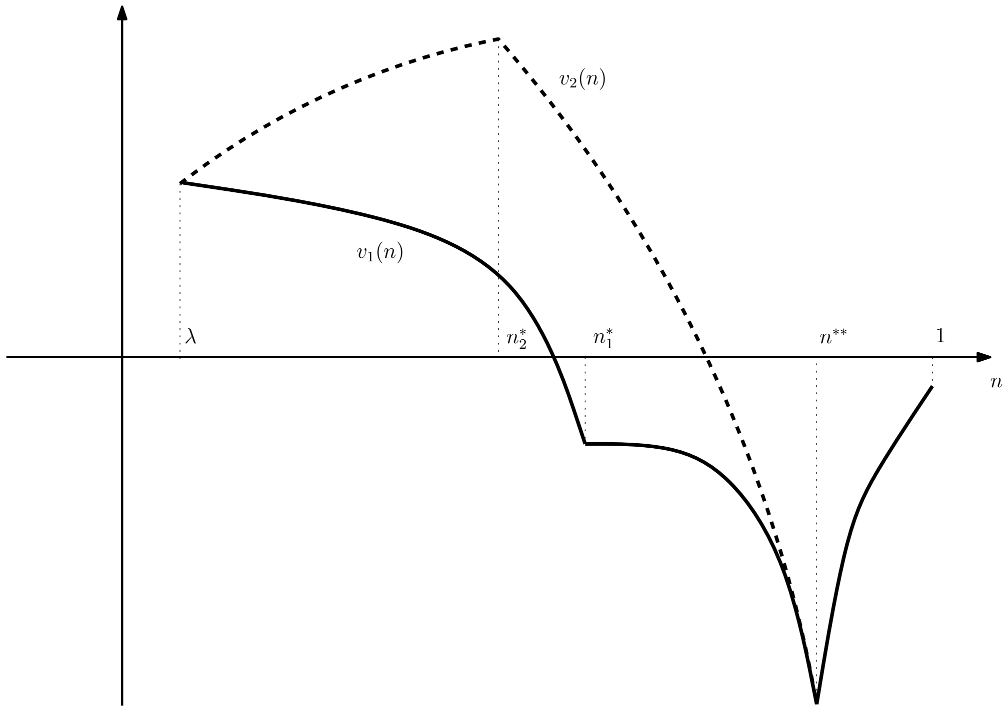

Figure 1: Advantage of waiting over running, as a function of the fraction of depositors running, under the pecking orders {Liquidation, Liquidity} (solid line) and {Liquidity, Liquidation} (dashed line).

of depositors running. Note that as we solve for a perfect Bayesian equilibrium, it is necessary to consider the bank's strategy for any possible subgame. This is crucial because the pecking-order strategy defines the equilibrium threshold strategy that the depositors follow on date 1, and in turn, the deposit contract and the asset portfolio that the bank chooses on date 0.

Suppose that either no depositor runs (i.e., $n = \lambda$) and the bank is solvent or all depositors run (i.e., $n = 1$) and the bank is insolvent. In the first case, the outcome is equivalent to that with perfect information, where the bank never liquidates the productive asset, as shown in Lemma 1. In the second case, the bank is instead forced to liquidate all of the portfolio's productive assets. If the fraction n of depositors running is in the interval $[n^{**}, 1]$, the bank is also insolvent and again must liquidate all productive assets. The following proposition characterizes the preferred asset pecking order in the remaining interval $[\lambda, n^{**})$.

Proposition 1. *If $n \in (\lambda, n^{**})$, the pecking order {Liquidity, Liquidation} is always preferred to {Liquidation, Liquidity}.*

Proof. In Appendix B. ■

The intuition for this result is straightforward. For a given deposit contract and asset portfolio, the bank decides on the optimal pecking order as its best response to the depositors' withdrawal decisions. These depend on the threshold strategy that depositors follow, as characterized by threshold signals $\sigma_j^*(c, L)$ under asset pecking order $j \in \{1, 2\}$. Such threshold signals are those that make depositors indifferent between withdrawing or not, given their beliefs under asset pecking order j, i.e., those that solve $\mathbb{E}[v_j(\sigma, n)|\sigma_j^*] = 0$. As we focus on the limit case of vanishing signal noises, the probability of a run under asset pecking order j is equal to the probability that the signal σ falls below σ_j^*. Since $\sigma \sim U[0, 1]$, then $Pr(\sigma \leq \sigma_j^*) = \sigma_j^*$ for any $j \in \{1, 2\}$. Due to perfect competition, banks' objective function is depositors' expected welfare, which is decreasing in the probability of a run for a given deposit contract and asset portfolio. Hence, depositors' expected welfare is maximized at $\sigma^* = \min\{\sigma_1^*(c, L), \sigma_2^*(c, L)\}$. Put differently, the bank wants to induce in the depositors the greatest possible incentives to wait as a run unfolds.

The proposition shows that under {Liquidity, Liquidation} the depositors always have stronger incentives to wait than under {Liquidation, Liquidity}, i.e., $v_2(\sigma, n) \geq v_1(\sigma, n)$ for any $n \in (\lambda, n^{**})$. Since for $n = \lambda$ and $n \in [n^{**}, 1]$ the bank behaves identically under the two pecking orders, then $\sigma^*(c, L) = \min\{\sigma_1^*(c, L), \sigma_2^*(c, L)\} = \sigma_2^*(c, L)$, which solves $\mathbb{E}[v_2(\sigma, n)|\sigma_2^*] = 0$, i.e.,

$$\sigma^*(c, L) = \frac{(n^{**} - \lambda)c - \displaystyle\int_{\lambda}^{n_2^*} \tfrac{L - nc}{1-n} dn + \int_{n^{**}}^{1} \tfrac{r(1-L)+L}{n} dn}{\displaystyle\int_{\lambda}^{n_2^*} \tfrac{R(1-L)}{1-n} dn + \int_{n_2^*}^{n^{**}} \tfrac{R\left(1-L-\frac{nc-L}{r}\right)}{1-n} dn}. \tag{13}$$

A comparative-statics exercise clarifies the different channels through which the value of early consumption c associated with the deposit contract and bank liquidity L affect this threshold, and as a consequence the depositors' incentives to run:

$$\frac{\partial \sigma^*}{\partial c} = \frac{1}{DEN_{\sigma^*}} \left[\int_{\lambda}^{n_2^*} \frac{n}{1-n} dn + \sigma^*(c, L) \int_{n_2^*}^{n^{**}} \frac{R}{r} \frac{n}{1-n} dn \right], \tag{14}$$

where DEN_{σ^*} is the denominator of $\sigma^*(c, L)$ in (13). This expression is clearly positive. A higher early consumption c lowers late consumption by lowering the amount of liquidity $L - nc$ rolled over from date 1 to date 2, and by forcing the bank to liquidate more productive assets after it has depleted all liquidity. Both channels have the effect of increasing the depositors' incentives to run.

In contrast,

$$\frac{\partial \sigma^*}{\partial L} = \frac{1}{DEN_{\sigma^*}} \left[(\sigma^* R - 1) \int_{\lambda}^{n_2^*} \frac{1}{1-n} dn - \sigma^* R \left(\frac{1}{r} - 1 \right) \int_{n_2^*}^{n^{**}} \frac{1}{1-n} dn \right]. \tag{15}$$

Two forces are at play here. On the one hand, higher liquidity reduces the investment in the productive asset, and therefore the depositors' incentives to wait. On the other, higher liquidity reduces the need to liquidate productive assets when the bank is illiquid but solvent (i.e., if the fraction of depositors running is in the interval $[n_2^*, n^{**}]$), and this increases the depositors' incentives to wait.

3.3 Bank liquidity

To complete the characterization of the unregulated equilibrium, by backward induction we solve for the equilibrium deposit contract and bank asset portfolio. The bank solves the following:

$$\begin{aligned} \max_{c,L} & \int_0^{\sigma^*(c,L)} \big[L + r(1-L) \big] dp + \\ & + \int_{\sigma^*(c,L)}^{1} \left[\lambda c + (1-\lambda) \left[p \frac{R(1-L) + L - \lambda c}{1-\lambda} + (1-p) \frac{L - \lambda c}{1-\lambda} \right] \right] dp, \end{aligned} \tag{16}$$

subject to $L \geq \lambda c$, $c \geq 1$ and $L \leq 1$. If $p \leq \sigma^*(c, L)$, a run occurs:[6] all depositors receive equal shares of the liquidation value of the whole asset portfolio, equal to $L + r(1 - L)$. If instead the signal is above the threshold $\sigma^*(c, L)$, a run does not occur: a fraction λ of depositors are early consumers who receive c, while a fraction $1 - \lambda$ of depositors are late consumers who consume either $c_2(R) = (R(1 - L) + L - \lambda c)/(1 - \lambda)$ if the productive assets yields a positive return, or $c_L(0) = (L - \lambda c)/(1 - \lambda)$ if it yields zero.

If $c \geq L + r(1 - L)$ and $L < 1$, the objective function is decreasing in $\sigma^*(c, L)$ for any possible values of c and L. Hence, it is easy to argue that in equilibrium $c = 1$, as the threshold signal $\sigma^*(c, L)$ is increasing in c. To consider liquidity, define the difference between the utility in the no-run case and the utility in the case of a run as

$$\Delta U = L + \sigma^*(c, L) R(1 - L) - L - r(1 - L) = (\sigma^*(c, L) R - r)(1 - L). \tag{17}$$

[6]This holds since we focus on the case of vanishing signal noises.

Then, the first-order condition of the above optimization problem with respect to L is

$$\sigma^*(c,L)(1-r)+\int_{\sigma^*}^{1}\big[-pR+1\big]dp-\frac{\partial\sigma^*}{\partial L}\Delta U+\xi-\chi=0, \tag{18}$$

where ξ and χ are the respective Lagrange multipliers on the two constraints. Compared to the equilibrium with perfect information, in this case the bank takes into account that liquidity affects what the depositors consume in the event of a run as well as their incentives to run, as represented by the marginal effect of L on $\sigma^*(c,L)$. While the first effect is unquestionably positive, the sign of the second is ambiguous, as argued above. This means that it is not straightforward to determine whether in the unregulated equilibrium the bank holds excess liquidity.

4 Bank recovery and resolution planning

In this section, we proceed to the analysis of a regulated equilibrium in which banks are subject to recovery and resolution planning. To this end, we extend the previous environment as follows. Assume that a regulatory intervention forces the bank to commit on date 0 to a "recovery plan". Accordingly, the bank has to fix an asset pecking order to serve depositors on date 1 that maximizes their expected welfare.[7] The regulatory intervention further establishes a resolution authority (RA). After observing the bank recovery plan, the RA commits on date 0 to a "resolution plan" that maximizes depositors' expected welfare. According to the plan, the RA has the power to suspend deposit convertibility when the fraction of depositors withdrawing funds on date 1 reaches a certain threshold ϕ. After suspension, the RA creates a "good bank" that observes the types of depositors that have not participated in the run after paying a verification cost $\psi(1-\phi)$, increasing in the fraction $1-\phi$ of depositors to be verified. Then, the good bank pays c to early consumers and equally shares the resources left among the remaining late consumers who have not withdrawn early.

As before, we solve the model by backward induction. Hence, we start from the characterization of threshold strategies σ_j^* under pecking order $j\in\{1,2\}$ for the given resolution plan, bank asset portfolio, and recovery plan. Then, we solve for the equilibrium resolution plan for the given bank asset portfolio and recovery plan, and finally characterize the bank's decisions.

[7]The depositors are aware of the mechanism of government intervention. Therefore, they can infer the bank's strategy in reaction to it, including the choice of the asset pecking order, without directly observing it.

We guess that resolution arrives too late to stop a run, but early enough to avoid bank insolvency (we will verify that the guess is correct in equilibrium). Under pecking order j, this occurs if $n_j^0 < \phi_j < n^{**}$, where n_j^0 is the fraction of depositors such that $v_j(n_j^0) = 0$. Under the two pecking orders, these values are[8]

$$n_1^0 = \frac{\sigma R \left(1 - L_1 + \frac{\lambda c_1}{r}\right) + L_1 - \lambda c_1 - c_1}{\left(\frac{\sigma R}{r} - 1\right) c_1}, \tag{19}$$

$$n_2^0 = \frac{\sigma R \left(1 - L_2 + \frac{L_2}{r}\right) - c_2}{\left(\frac{\sigma R}{r} - 1\right) c_2}. \tag{20}$$

The creation of the good bank by the RA changes depositors' ex-post payoffs, according to Table 2. Under the pecking order {Liquidation, Liquidity} (Table 2a), after ϕ_1 withdrawals the RA suspends deposit convertibility. At resolution, the good bank pays early consumption c_1 to the early consumers, and equally shares among $(1-\lambda)(1-\phi_1)$ remaining late consumers the available resources. These are equal to the total liquidation value of the bank's assets, less $\phi_1 c_1$ withdrawals before resolution and $(1-\phi_1)\lambda c_1$ withdrawals after resolution. Under the pecking order {Liquidity, Liquidation} instead (Table 2b), a suspension is implemented after ϕ_2 withdrawals. At resolution, the good bank pays early consumption c_2 to the early consumers, and equally shares among $(1-\lambda)(1-\phi_2)$ remaining late consumers the available resources. These are only productive assets since the bank has depleted liquidity before resolution. Hence, the late consumption offered by the good bank is equal to

$$c_L^R(Z,n) = \frac{Z\left(1 - L_2 - \frac{\phi_2 c_2 - L_2}{r} - \frac{(1-\phi_2)\lambda c_2}{r}\right)}{(1-\lambda)(1-\phi_2)}, \tag{21}$$

for every $Z \in \{0, R\}$. Intuitively, of $1 - L_2 - (\phi_2 c_2 - L_2)/r$ units of productive asset received, the good bank has to further liquidate an amount $(1-\phi_2)\lambda c_2$ to pay early consumers.

[8]In the regulated economy, the bank commits to an asset pecking order $j \in \{1,2\}$ on date 0 and then, contingent on that, chooses early consumption c and liquidity L. For this reason, in this section c and L have a subscript $j \in \{1,2\}$.

Table 2: Depositors' ex-post payoffs in the economy with recovery and resolution planning.

(a) Pecking order {Liquidation, Liquidity}

Date	$n \in [\lambda, n_1^*)$		$n \in [n_1^*, \phi_1)$	$n \in [\phi_1, 1]$
$t=1$	c_1		c_1	c_1
$t=2$	$\frac{Z\left(1-L_1-\frac{(n-\lambda)c_1}{r}\right)+L_1-\lambda c_1}{1-n}$	$\forall Z \in \{0, R\}$	$\frac{r(1-L_1)+L_1-nc_1}{1-n}$	$\frac{L_1+r(1-L_1)-\phi_1 c_1-(1-\phi_1)\lambda c_1}{(1-\lambda)(1-\phi_1)}$

(b) Pecking order {Liquidity, Liquidation}

Date	$n \in [\lambda, n_2^*)$	$n \in [n_2^*, \phi_2)$	$n \in [\phi_2, 1]$	
$t=1$	c_2	c_2	c_2	
$t=2$	$\frac{Z(1-L_2)+L_2-nc_2}{1-n}$	$\frac{Z\left(1-L_2-\frac{nc_2-L_2}{r}\right)}{1-n}$	$\frac{Z\left(1-L_2-\frac{\phi_2 c_2-L_2}{r}-\frac{(1-\phi_2)\lambda c_2}{r}\right)}{(1-\lambda)(1-\phi_2)}$	$\forall Z \in \{0, R\}$

4.1 Asset pecking orders

Under the pecking order {Liquidation, Liquidity}, the advantage of waiting over running becomes

$$v_1(\sigma, n) = \begin{cases} \sigma \frac{R\left(1-L_1-\frac{(n-\lambda)c_1}{r}\right)+L_1-\lambda c_1}{1-n} + (1-\sigma)\frac{L_1-\lambda c_1}{1-n} - c_1 & \text{if } \lambda \leq n < n_1^*, \\ \sigma \frac{r(1-L_1)+L_1-nc_1}{1-n} + (1-\sigma)\frac{r(1-L_1)+L_1-nc_1}{1-n} - c_1 & \text{if } n_1^* \leq n < \phi_1, \end{cases} \tag{22}$$

where (as before) $n_1^* = \lambda + (r(1-L_1))/c_1$. Under the pecking order {Liquidity, Liquidation}, we instead obtain

$$v_2(\sigma, n) = \begin{cases} \sigma \frac{R(1-L_2)+L_2-nc_2}{1-n} + (1-\sigma)\frac{L_2-nc_2}{1-n} - c_2 & \text{if } \lambda \leq n < n_2^*, \\ \sigma \frac{R(1-L_2-D_2)}{1-n} - c_2 = \sigma \frac{R\left(1-L_2-\frac{nc_2-L_2}{r}\right)}{1-n} - c & \text{if } n_2^* \leq n < \phi_2, \end{cases} \tag{23}$$

where $n_2^* = L_2/c_2$. The only difference between (11)-(12) and (22)-(23) is in the fact that resolution stops a run before the bank becomes insolvent (i.e., $\phi_j < n^{**}$), but not early enough to avoid a run (i.e., $\phi_j > n_j^0$).

We solve for the threshold strategies under the two pecking orders by imposing $\mathbb{E}[v_j(\sigma, n)|\sigma_j^*] =$

0, and obtain

$$\sigma_1^*(c_1, L_1, \phi_1) = \frac{(\phi_1 - \lambda)c_1 - \int_{\lambda}^{n_1^*} \frac{L_1 - \lambda c_1}{1-n} dn - \int_{n_1^*}^{\phi_1} \frac{r(1-L_1)+L_1 - nc_1}{1-n} dn}{\int_{\lambda}^{n_1^*} \frac{R\left(1-L_1-\frac{(n-\lambda)c_1}{r}\right)}{1-n} dn}, \tag{24}$$

$$\sigma_2^*(c_2, L_2, \phi_2) = \frac{(\phi_2 - \lambda)c_2 - \int_{\lambda}^{n_2^*} \frac{L_2 - nc_2}{1-n} dn}{\int_{\lambda}^{\phi_2} \frac{R(1-L_2)}{1-n} dn - \int_{n_2^*}^{\phi_2} \frac{R}{r} \frac{nc_2 - L_2}{1-n} dn}. \tag{25}$$

These two expressions allow us to perform several preliminary comparative-statics exercises. First, it is easy to observe that both threshold signals are increasing in early consumption c: a higher c increases necessary early repayments, lowers the excess liquidity, and increases the amount of productive assets that the bank eventually needs to liquidate. Second, we study how the threshold signals depend on bank liquidity:

$$\frac{\partial \sigma_1^*}{\partial L_1} = \frac{1}{DEN_{\sigma_1^*}} \left[-\int_{\lambda}^{n_1^*} \frac{1}{1-n} dn - (1-r) \int_{n_1^*}^{\phi_1} \frac{1}{1-n} dn + \sigma_1^* R \int_{\lambda}^{n_1^*} \frac{1}{1-n} dn \right], \tag{26}$$

$$\frac{\partial \sigma_2^*}{\partial L_2} = \frac{1}{DEN_{\sigma_2^*}} \left[-\int_{\lambda}^{n_2^*} \frac{1}{1-n} dn - \sigma_2^* R \left(-\int_{\lambda}^{\phi_2} \frac{1}{1-n} dn + \frac{1}{r} \int_{n_2^*}^{\phi_2} \frac{1}{1-n} dn \right) \right], \tag{27}$$

where $DEN_{\sigma_j^*}$ is the denominator of σ_j^* for any $j \in \{1, 2\}$. The signs of these expressions are again undetermined. In fact, under the pecking order {Liquidation, Liquidity} a higher liquidity not only reduces the investment in the productive asset (thus, increasing the incentives to run) but also increases the total liquidation value of bank assets $L_1 + r(1 - L_1)$ and excess liquidity $L_1 - \lambda c_1$ (thus, reducing the incentives to run). Similarly, under the pecking order {Liquidity, Liquidation} a higher liquidity reduces the investment in the productive asset, but allows a higher excess liquidity and leads to less liquidation when the bank is illiquid but solvent.

The analysis of the effect of resolution on the depositors' incentives to run yields clearer results.

Lemma 3. *The threshold signals σ_1^* and σ_2^* are increasing in the suspension points ϕ_1 and ϕ_2, respectively.*

Proof. In Appendix B. ■

Intuitively, as the suspension is too late to stop a run (i.e., $n_j^0 < \phi_j$ for every $j \in \{1, 2\}$), the later it arrives, the smaller the resources available for late consumption, and therefore the lower the depositors' incentive to run. This implies that the RA has incentives to suspend convertibility as soon as possible: in principle, it could announce the suspension at n_j^0 under asset pecking order j, and rule out runs altogether.[9] However, by suspending convertibility the RA incurs verification costs to create the good bank, and this pushes the RA to postpone the intervention optimally.

4.2 Resolution planning

Given the depositors' threshold strategies, we now use backward induction to characterize the RA's choice of a resolution plan on date 0. Under the pecking order {Liquidation, Liquidity}, resolution occurs after the bank has liquidated all of its assets but before it has depleted all liquidity and become insolvent. Therefore, depositors' utility at resolution is

$$\begin{aligned}\phi_1 c_1 + (1-\phi_1)\left[\lambda c_1 + (1-\lambda)\frac{L_1 + r(1-L_1) - \phi_1 c_1 - (1-\phi_1)\lambda c_1}{(1-\lambda)(1-\phi_1)}\right] - \psi(1-\phi_1) = \\ = L_1 + r(1-L_1) - \psi(1-\phi_1). \qquad (28)\end{aligned}$$

The RA suspends convertibility after ϕ_1 depositors have withdrawn c. Then, it pays the verification cost $\psi(1-\phi_1)$ to create the good bank. Among $1-\phi_1$ remaining depositors under the good bank, λ are early consumers who receive c_1 on date 1, while on date 2 the remaining late consumers receive equal shares of the available liquidity left, i.e., $L_1 + r(1-L_1) - \phi_1 c_1 - (1-\phi_1)\lambda c$.

Under the pecking order {Liquidity, Liquidation} instead, resolution occurs after the bank has deployed all of its liquidity, so the RA is left only with productive assets to liquidate in order to serve depositors on date 1. Then, depositors' expected utility at resolution is

$$\begin{aligned}\int_0^{\sigma^*(c_2,L_2,\phi_2)} \left[\phi_2 c_2 + (1-\phi_2)\left[\lambda c_2 + (1-\lambda)p\frac{R\left(1 - L_2 - \frac{\phi_2 c_2 - L_2}{r} - \frac{(1-\phi_2)\lambda c_2}{r}\right)}{(1-\lambda)(1-\phi_2)}\right] - \psi(1-\phi_2)\right] dp = \\ = \int_0^{\sigma^*(c_2,L_2,\phi_2)} \left[\left[\phi_2 + (1-\phi_2)\lambda\right] c_2 \left(1 - \frac{pR}{r}\right) + pR\left(1 - L_2 + \frac{L_2}{r}\right) - \psi(1-\phi_2)\right] dp. \qquad (29)\end{aligned}$$

The RA is left with an amount $(1 - L_2 - (\phi_2 c_2 - L_2)/r)$ of productive assets, from which it further

[9] Note that $n_j^0 > \lambda$ for every $j \in \{1, 2\}$. The reason is that to stop a run, it suffices to suspend convertibility at a fraction of early withdrawers implying a non-negative expected advantage of waiting over running.

liquidates an amount $((1-\phi_2)\lambda c_2)/r$ to serve the remaining early consumers. As before, remaining late consumers receive equal shares of the residual resources.

Under the pecking order {Liquidation, Liquidity}, the RA chooses the optimal suspension point ϕ_1 that solves

$$\begin{aligned}\max_{\phi_1} & \int_0^{\sigma_1^*(c_1,L_1,\phi_1)} \big[L_1 + r(1-L_1) - \psi(1-\phi_1)\big] dp + \\ & + \int_{\sigma_1^*(c_1,L_1,\phi_1)}^1 \left[\lambda c_1 + (1-\lambda)\left[p\frac{R(1-L_1)+L_1-\lambda c_1}{1-\lambda} + (1-p)\frac{L_1-\lambda c_1}{1-\lambda}\right]\right] dp. \end{aligned} \tag{30}$$

The first-order condition of the program yields the optimal suspension point ϕ_1 as the value that equalizes the expected marginal verification costs and its marginal benefits, in terms of lower threshold signal $\sigma_1^*(c_1, L_1, \phi_1)$:

$$\sigma_1^*(c_1, L_1, \phi_1)\psi'(1-\phi_1) = \frac{\partial \sigma_1^*}{\partial \phi_1}\Delta U_1, \tag{31}$$

where

$$\begin{aligned}\Delta U_1 &= \lambda c_1 + \sigma_1^* R(1-L_1) + L_1 - \lambda c_1 - L_1 - r(1-L_1) + \psi(1-\phi_1) = \\ &= (\sigma_1^* R - r)(1-L_1) + \psi(1-\phi_1)\end{aligned} \tag{32}$$

is the difference between the utility in the case of no run and the utility in the case of a run. From this equilibrium condition, we derive the following result:

Lemma 4. *Under the asset pecking order {Liquidation, Liquidity}, the equilibrium suspension point ϕ_1 is independent of the return on the productive asset R.*

Proof. In Appendix B. ■

The intuition for this is straightforward. On the one hand, R affects the expected marginal cost of the suspension (the left side of (31)) through the threshold signal σ_1^*. On the other hand, R also affects the marginal benefit of the suspension (the right side of (31)) through the marginal effect of ϕ_1 on σ_1^*. The two effects perfectly offset each other. Hence, the expected marginal costs of the suspension are independent of R.

Under the pecking order {Liquidity, Liquidation} instead, the RA chooses the optimal suspension point ϕ_2 that solves

$$\max_{\phi_2} \int_0^{\sigma_2^*(c_2,L_2,\phi_2)} \left[\big[\phi_2 + (1-\phi_2)\lambda\big] c_2 \left(1 - \frac{pR}{r}\right) + pR\left(1 - L_2 + \frac{L_2}{r}\right) - \psi(1-\phi_2) \right] dp + \\ + \int_{\sigma_2^*(c_2,L_2,\phi_2)}^1 \left[\lambda c_2 + (1-\lambda)\left[p\frac{R(1-L_2)+L_2-\lambda c_2}{1-\lambda} + (1-p)\frac{L_2 - \lambda c_2}{1-\lambda} \right] \right] dp. \quad (33)$$

As before, by considering the first-order condition, we implicitly derive the optimal suspension point:

$$\sigma_2^*(c_2, L_2, \phi_2)\psi'(1-\phi_2) = (1-\lambda)\left[\frac{\sigma_2^{*2}(c_2, L_2, \phi_2)R}{r} - \sigma_2^*(c_2, L_2, \phi_2) \right] c_2 + \frac{\partial \sigma_2^*}{\partial \phi_2}\Delta U_2, \quad (34)$$

where

$$\Delta U_2 = \sigma_2^* R(1-L_2) + L_2 + \big[\phi_2 + (1-\phi_2)\lambda\big] c_2 \left(\frac{\sigma_2^* R}{r} - 1 \right) - \sigma_2^* R\left(1 - L_2 + \frac{L_2}{r}\right) + \psi(1-\phi_2) = \\ = \Big[\big[\phi_2 + (1-\phi_2)\lambda\big] c_2 - L_2 \Big] \left(\frac{\sigma_2^* R}{r} - 1 \right) + \psi(1-\phi_2). \quad (35)$$

Again, the RA chooses the suspension point ϕ_2 so that the expected marginal verification costs are equal to its marginal benefits, now in terms of a lower threshold signal $\sigma_2^*(c_2, L_2, \phi_2)$ as well as the smaller amount of resources lost as a result of the liquidation of the productive asset.

4.3 Bank liquidity and recovery planning

Finally, as the last step of the backward induction, we conclude the analysis with the characterization of the bank's choices on date 0.

Under the pecking order {Liquidation, Liquidity}, the bank chooses the amount of early consumption c_1 and liquidity L_1 to maximize depositors' expected welfare in (30) subject to $L_1 \geq \lambda c_1$, $c_1 \geq 1$ and $L_1 \leq 1$. As before, in equilibrium the bank chooses $c_1 = 1$, as the objective function is decreasing in c_1. The first-order condition of the problem with respect to L_1 instead is

$$\sigma_1^*(1-r) + \int_{\sigma_1^*}^1 \big[-pR + 1\big] dp - \frac{\partial \sigma_1^*}{\partial L_1}\Delta U_1 + \xi_1 - \chi_1 = 0, \quad (36)$$

where ξ_1 and χ_1 are the respective Lagrange multipliers on the two constraints.

Under the pecking order {Liquidity, Liquidation} instead, the bank chooses the amount of early consumption c and liquidity L to maximize depositors' expected welfare in (33), again subject to $L_2 \geq \lambda c_2$, $c_2 \geq 1$ and $L_2 \leq 1$. As under the other pecking order, the bank in equilibrium chooses $c_2 = 1$. The first-order condition of the problem with respect to L instead is

$$\frac{\sigma_2^{*2}}{2} R\left(\frac{1}{r} - 1\right) + \int_{\sigma_2^*}^{1} \big[-pR + 1\big] dp - \frac{\partial \sigma_2^*}{\partial L_2} \Delta U_2 + \xi_2 - \chi_2 = 0, \tag{37}$$

where ξ_2 and χ_2 are again Lagrange multipliers on the constraints.

Compared to the equilibrium with perfect information, where the bank has no incentives to hold excess liquidity, with imperfect information and recovery and resolution planning, there are two reasons why the bank might want to hold excess liquidity. First, the bank considers that liquidity can directly influence depositors' self-fulfilling expectations of a run. Second, the bank internalizes the idea that by increasing liquidity, it can provide more resources to the good bank at resolution, which indirectly affects depositors' incentives to run.

Finally, we characterize the choice on date 0 of the bank recovery plan, i.e., the choice of the pecking order that maximizes depositors' expected welfare, according to

$$W = \max \left\{W_1(1, L_1, \phi_1), W_2(1, L_2, \phi_2)\right\}. \tag{38}$$

It is important to note the differences between the choice of the pecking order here and in the unregulated equilibrium. In the latter case, the asset pecking order is not observed as an equilibrium outcome, as in the only two possible outcomes (the bank being either solvent or insolvent) the bank either never liquidates productive assets or is forced to do so. With bank recovery and resolution planning instead, the asset pecking order is observable as an equilibrium outcome because it affects the RA's choice of the suspension point and the type and amount of resources that the good bank obtains at resolution, which ultimately influences what late consumers not participating in the run can obtain.

5 Banks' liquidity management with recovery and resolution planning

The aim of this section is twofold. First, we want to study how banks' liquidity management – both in anticipation of and during a run – changes under the two different regulatory regimes of the preceding sections. To this end, we compare the equilibrium in the unregulated economy to that with recovery and resolution planning. Second, we want to perform comparative statics exercises. As it is impossible to derive closed-form solutions for the expected welfare under the two pecking orders and obtain the solution of (38), we rely on a numerical analysis.

Assume quadratic verification costs for the RA of the form $\psi(1-\phi_j) = (1-\phi_j)^2/2$ for both pecking orders $j \in \{1, 2\}$. We fix the probability of being an early consumer λ to 0.02, as in Mattana and Panetti (2020), and early consumption c_j to 1 for both pecking orders $j \in \{1, 2\}$, as proven in the previous section. We solve the model for values of the return on the productive asset R in the interval $[2.02, 2.08]$. In this way, the expected net return is between 1 and 4 percent, which is roughly the range of the cost of bank credit for nonfinancial corporations observed during 2003-2019 in the Euro Area.[10] Finally, we show different results for values of recovery rate r in the interval $[.70, .95]$ to include the average recovery rate for bank loans observed by Acharya et al. (2007).

We start from the analysis of a bank's behavior with recovery and resolution planning, performed separately under the two pecking orders. We report results in Tables 3 and 4. Tables 3a and 4a show that financial fragility, as represented by the threshold signals σ_1^* and σ_2^*, is decreasing in R for a given recovery rate r under both pecking orders because a higher return on the productive asset lowers depositors' incentives to run. In contrast, σ_1^* is first increasing and then decreasing in the recovery rate r for a given R, while σ_2^* is first decreasing and then increasing. The reason is that three forces are in play. A higher r raises the bank's value at bankruptcy and delays resolution (i.e., it increases the suspension points ϕ_1 and ϕ_2), thus increasing depositors' incentives to run. On the other hand, a higher r also raises the amount of productive assets that are not liquidated, thus reducing depositors' incentives to run.

Suspension points ϕ_1 and ϕ_2 in Tables 3b and 4b are increasing in recovery rate r because

[10] Using return on assets yields qualitatively similar results. Source: MFI Interest Rate Statistics, European Central Bank.

Table 3: Banking equilibrium with pecking order {Liquidation, Liquidity}

.

(a) Financial fragility (σ_1^*)

	$r = 0.70$	$r = 0.75$	$r = 0.80$	$r = 0.85$	$r = 0.90$	$r = 0.95$
$R = 2.02$	0.5280	0.5569	0.5703	0.5715	0.5620	0.5410
$R = 2.04$	0.5228	0.5514	0.5647	0.5659	0.5565	0.5357
$R = 2.06$	0.5178	0.5461	0.5592	0.5604	0.5511	0.5305
$R = 2.08$	0.5128	0.5408	0.5538	0.5550	0.5458	0.5254

(b) Suspension point (ϕ_1)

	$r = 0.70$	$r = 0.75$	$r = 0.80$	$r = 0.85$	$r = 0.90$	$r = 0.95$
$R = 2.02$	0.4575	0.5626	0.6576	0.7459	0.8297	0.9116
$R = 2.04$	0.4575	0.5626	0.6576	0.7459	0.8297	0.9116
$R = 2.06$	0.4575	0.5626	0.6576	0.7459	0.8297	0.9116
$R = 2.08$	0.4575	0.5626	0.6576	0.7459	0.8297	0.9116

(c) Liquidity (L_1)

	$r = 0.70$	$r = 0.75$	$r = 0.80$	$r = 0.85$	$r = 0.90$	$r = 0.95$
$R = 2.02$	0.0200	0.0200	0.0200	0.0200	0.0200	0.0200
$R = 2.04$	0.0200	0.0200	0.0200	0.0200	0.0200	0.0200
$R = 2.06$	0.0200	0.0200	0.0200	0.0200	0.0200	0.0200
$R = 2.08$	0.0200	0.0200	0.0200	0.0200	0.0200	0.0200

(d) Expected welfare (W_1)

	$r = 0.70$	$r = 0.75$	$r = 0.80$	$r = 0.85$	$r = 0.90$	$r = 0.95$
$R = 2.02$	0.9489	1.0165	1.0764	1.1303	1.1784	1.2199
$R = 2.04$	0.9579	1.0253	1.0850	1.1386	1.1864	1.2276
$R = 2.06$	0.9669	1.0341	1.0936	1.1469	1.1944	1.2353
$R = 2.08$	0.9758	1.0429	1.1022	1.1553	1.2025	1.2431

a higher r lowers the marginal benefits of an early suspension, and raises its expected marginal costs by increasing σ_1^* and σ_2^*. The suspension point ϕ_1 is independent of R, as proven in Lemma 4. An increase in R has instead the effect of anticipating the suspension point under {Liquidity, Liquidation} (i.e., ϕ_2 declines) except for low values of r. This non-monotonicity depends on the expected return on the productive asset at resolution relative to the recovery rate, i.e., $\sigma_2^* R/r$, that affects the first term on the right side of (34).[11]

[11] Under both pecking orders, the guess that $n_j^0 < \phi_j < n^{**}$ is verified.

Table 4: Banking equilibrium with pecking order {Liquidity, Liquidation}

.

(a) Financial fragility (σ_2^*)

	$r = 0.70$	$r = 0.75$	$r = 0.80$	$r = 0.85$	$r = 0.90$	$r = 0.95$
$R = 2.02$	0.5101	0.4951	0.5003	0.5095	0.5183	0.5230
$R = 2.04$	0.4989	0.4905	0.4967	0.5059	0.5146	0.5188
$R = 2.06$	0.4904	0.4862	0.4931	0.5024	0.5108	0.5148
$R = 2.08$	0.4834	0.4822	0.4896	0.4990	0.5072	0.5108

(b) Suspension point (ϕ_2)

	$r = 0.70$	$r = 0.75$	$r = 0.80$	$r = 0.85$	$r = 0.90$	$r = 0.95$
$R = 2.02$	0.5645	0.6724	0.7287	0.7725	0.8145	0.8647
$R = 2.04$	0.5769	0.6693	0.7234	0.7671	0.8098	0.8614
$R = 2.06$	0.5831	0.6654	0.7178	0.7616	0.8051	0.8581
$R = 2.08$	0.5858	0.6608	0.7120	0.7560	0.8004	0.8547

(c) Liquidity (L_2)

	$r = 0.70$	$r = 0.75$	$r = 0.80$	$r = 0.85$	$r = 0.90$	$r = 0.95$
$R = 2.02$	0.6794	0.6670	0.6183	0.5368	0.4032	0.1496
$R = 2.04$	0.6691	0.6488	0.5972	0.5138	0.3788	0.1247
$R = 2.06$	0.6561	0.6300	0.5759	0.4909	0.3546	0.1002
$R = 2.08$	0.6411	0.6107	0.5545	0.4681	0.3308	0.0764

(d) Expected welfare (W_2)

	$r = 0.70$	$r = 0.75$	$r = 0.80$	$r = 0.85$	$r = 0.90$	$r = 0.95$
$R = 2.02$	0.8941	0.9516	0.9884	1.0271	1.0787	1.1660
$R = 2.04$	0.9057	0.9567	0.9938	1.0339	1.0876	1.1778
$R = 2.06$	0.9146	0.9619	0.9995	1.0410	1.0968	1.1899
$R = 2.08$	0.9222	0.9672	1.0054	1.0484	1.1062	1.2023

5.1 Equilibrium liquidity and asset pecking order

If there were no runs, as in the equilibrium with perfect information, the bank would want to hold the minimum amount of liquidity. However, the possibility of runs affects this decision. Under pecking order {Liquidation, Liquidity} in Table 3c, two further channels affect the choice of liquidity: the bank's willingness to leave resources to the good bank at resolution, and that to affect depositors' incentives to run via the threshold signal σ_1^*. While the former raises liquidity, the latter turns out to lower it. The threshold signal σ_1^* rises with liquidity because the increasing effect of a higher

Table 5: Components of expected welfare with pecking order {Liquidation, Liquidity}

.

(a) Expected welfare at resolution

	$r = 0.70$	$r = 0.75$	$r = 0.80$	$r = 0.85$	$r = 0.90$	$r = 0.95$
$R = 2.02$	0.2256	0.3248	0.3999	0.4552	0.4924	0.5106
$R = 2.04$	0.2220	0.3207	0.3954	0.4504	0.4874	0.5055
$R = 2.06$	0.2184	0.3166	0.391	0.4457	0.4826	0.5006
$R = 2.08$	0.2149	0.3127	0.3867	0.4411	0.4778	0.4957

(b) Expected welfare from liquidity in the case of "no run"

	$r = 0.70$	$r = 0.75$	$r = 0.80$	$r = 0.85$	$r = 0.90$	$r = 0.95$
$R = 2.02$	0.0094	0.0088	0.0086	0.0086	0.0088	0.0092
$R = 2.04$	0.0095	0.0090	0.0087	0.0087	0.0089	0.0093
$R = 2.06$	0.0096	0.0091	0.0088	0.0088	0.0090	0.0094
$R = 2.08$	0.0097	0.0092	0.0089	0.0089	0.0091	0.0095

(c) Expected welfare from productive assets in the case of "no run"

	$r = 0.70$	$r = 0.75$	$r = 0.80$	$r = 0.85$	$r = 0.90$	$r = 0.95$
$R = 2.02$	0.7139	0.6828	0.6679	0.6666	0.6772	0.7001
$R = 2.04$	0.7264	0.6956	0.6809	0.6795	0.6901	0.7128
$R = 2.06$	0.7388	0.7084	0.6938	0.6924	0.7029	0.7253
$R = 2.08$	0.7512	0.7211	0.7066	0.7053	0.7156	0.7379

liquidity on depositors' incentives to run, due to a lower investment in the productive asset, is stronger than the decreasing effect due to a higher total liquidation value and a higher excess liquidity (see Equation (26)). The three channels' overall effect is to keep bank liquidity at its minimum equilibrium level $L_1 = \lambda c_1 = \lambda$ irrespective of the return on the productive asset and the recovery rate. Under pecking order {Liquidity, Liquidation} in Table 4c instead, the threshold signal σ_2^* decreases with liquidity, as the increasing effect of a higher liquidity on depositors' incentives to run, due to a lower investment in the productive asset, is weaker than the decreasing effect due to a higher excess liquidity and a reduced asset liquidation when the bank is illiquid but solvent (see Equation (27)). The effect of liquidity on σ_2^* is so strong that it causes the bank to hold excess liquidity $L > \lambda c$, i.e., more than it would need to repay early consumers if a run does not occur. Intuitively, the excess liquidity is decreasing both in R and in r because these variables lower the relative value of liquidity with respect to the productive asset.

Finally, we study the choice of pecking order under recovery and resolution planning by comparing the expected welfare under the two pecking orders in Tables 3d and 4d. Unsurprisingly, both measures are increasing in R and r, as they both raise the per-unit return on bank investment. The pecking order {Liquidation, Liquidity} is always preferred to {Liquidity, Liquidation} for any possible combination of parameters. This is an unexpected result, given that the economy exhibits a higher financial fragility and (for most parameter values) a more delayed resolution under {Liquidation, Liquidity} than under {Liquidity, Liquidation}. To understand the rationale for the choice of {Liquidation, Liquidity}, in Tables 5 and 6 we decompose the expected welfare into three components, namely the values (i) at resolution, (ii) from the investment in liquidity in the case of "no run", and (iii) from the investment in productive assets in the case of "no run". The difference between the two pecking orders does not seem to arise from the first component, as the expected welfare becomes higher under {Liquidation, Liquidity} than under {Liquidity, Liquidation} for high values of r, but otherwise is quite similar in the two cases. In contrast, under {Liquidity, Liquidation} the bank holds excess liquidity, and this substantially distorts expected welfare in the case of no run because it forces the bank to lower its investment in productive assets, thus losing a significant amount of welfare.

Table 7 reports the main features of the equilibrium in the unregulated economy to compare them to those of the economy with recovery and resolution. Recall that by Proposition 1, in the unregulated economy the bank chooses the pecking order {Liquidity, Liquidation} as a run unfolds. Accordingly, the bank recognizes that financial fragility is decreasing in liquidity. For $r \leq 0.85$, this is sufficient to push the bank to hold excess liquidity, i.e., $L > \lambda c$. For higher values of r instead, a higher liquidity induces a relatively more significant loss in expected bank revenues, which dominates the benefit of a lower financial fragility. Hence, the bank chooses to hold the minimum amount of liquidity necessary to repay early consumers, i.e., $L = \lambda c = 0.02$.

To summarize, introducing a regulatory regime with recovery and resolution planning has two effects on banks' liquidity management. On the one hand, providing a framework for orderly resolution allows the bank to address financial fragility better and hold no excess liquidity. On the other hand, recovery and resolution planning push the bank to change asset pecking order compared to the case of the unregulated economy because of the distortion of the asset portfolio that following the pecking order {Liquidity, Liquidation} would entail when a run does not occur.

These results leave open the question as to whether the different liquidity management depends

Table 6: Components of expected welfare with pecking order {Liquidity, Liquidation}.

(a) Expected welfare at resolution

	$r = 0.70$	$r = 0.75$	$r = 0.80$	$r = 0.85$	$r = 0.90$	$r = 0.95$
$R = 2.02$	0.3217	0.3610	0.3904	0.4174	0.4437	0.4707
$R = 2.04$	0.3169	0.3541	0.3837	0.4110	0.4379	0.4654
$R = 2.06$	0.3112	0.3472	0.3769	0.4047	0.4320	0.4601
$R = 2.08$	0.3050	0.3402	0.3702	0.3984	0.4263	0.4550

(b) Expected welfare from liquidity in the case of "no run"

	$r = 0.70$	$r = 0.75$	$r = 0.80$	$r = 0.85$	$r = 0.90$	$r = 0.95$
$R = 2.02$	0.3329	0.3368	0.3089	0.2633	0.1942	0.0714
$R = 2.04$	0.3353	0.3306	0.3006	0.2539	0.1839	0.0560
$R = 2.06$	0.3343	0.3237	0.2920	0.2443	0.1735	0.0486
$R = 2.08$	0.3312	0.3162	0.2830	0.2345	0.1630	0.0374

(c) Expected welfare from productive assets in the case of "no run"

	$r = 0.70$	$r = 0.75$	$r = 0.80$	$r = 0.85$	$r = 0.90$	$r = 0.95$
$R = 2.02$	0.2395	0.2539	0.2890	0.3464	0.4408	0.6240
$R = 2.04$	0.2535	0.2720	0.3095	0.3690	0.4659	0.6525
$R = 2.06$	0.2691	0.2910	0.3306	0.3920	0.4913	0.6812
$R = 2.08$	0.2860	0.3108	0.3522	0.4155	0.5169	0.7100

on recovery or resolution planning or both. To answer it, we characterize liquidity management under two intermediate regulatory regimes, namely either resolution planning alone or recovery planning alone.

Under resolution planning alone, the RA chooses on date 0 the optimal suspension point that maximizes depositors' expected welfare but allows the bank to choose the asset pecking order while a run unfolds. Solving for the equilibrium by backward induction, it is easy to argue that the choice of the asset pecking order is unaffected by resolution planning alone. In fact, when the bank is solvent ($n = \lambda$), the choice of pecking order is not observed in equilibrium, as in the unregulated economy. When instead the bank is illiquid but solvent ($n \in [\lambda, \phi]$), it always prefers the pecking order {Liquidity, Liquidation} to {Liquidation, Liquidity}. The reason is that the suspension, regardless of being chosen on date 0, does not change the result of Proposition 1: the depositors' utility advantage of waiting over running is always higher under {Liquidity, Liquidation} than under {Liquidation, Liquidity}. This also means that under resolution planning alone, the equilibrium is

Table 7: Banking equilibrium in the unregulated economy.

(a) Financial fragility (σ^*)

	$r = 0.70$	$r = 0.75$	$r = 0.80$	$r = 0.85$	$r = 0.90$	$r = 0.95$
$R = 2.02$	0.6376	0.6580	0.6743	0.6858	0.6619	0.5870
$R = 2.04$	0.6384	0.6576	0.6729	0.6835	0.6554	0.5812
$R = 2.06$	0.6389	0.6570	0.6714	0.6812	0.6490	0.5756
$R = 2.08$	0.6393	0.6563	0.6698	0.6789	0.6428	0.5701

(b) Liquidity (L)

	$r = 0.70$	$r = 0.75$	$r = 0.80$	$r = 0.85$	$r = 0.90$	$r = 0.95$
$R = 2.02$	0.7231	0.5701	0.3927	0.1802	0.0200	0.0200
$R = 2.04$	0.6978	0.5474	0.3732	0.1638	0.0200	0.0200
$R = 2.06$	0.6737	0.5259	0.3545	0.1480	0.0200	0.0200
$R = 2.08$	0.6507	0.5054	0.3368	0.1329	0.0200	0.0200

equivalent to that in the case of {Liquidity, Liquidation} in Table 4, in which the bank holds excess liquidity.

In contrast, in an economy with recovery planning alone, the bank chooses on date 0 the pecking order that maximizes the expected welfare, taking into account that the allocation at insolvency is independent of this choice: if the bank becomes insolvent, it is forced to liquidate all assets and equally share the proceeds among all depositors, irrespective of the asset pecking order. The expected welfare under {Liquidation, Liquidity} is the value function of a problem similar to (16), in which the threshold signal σ_1^* is derived from the indifference condition $\mathbb{E}[v_1(\sigma, n)|\sigma] = 0$. In contrast, the expected welfare under {Liquidity, Liquidation} is equal to that in the unregulated economy due to Proposition 1. A comparison of the two welfare measures in Tables 8b and 8d shows that {Liquidity, Liquidation} is still preferred to {Liquidation, Liquidity} particularly because the former is associated with lower financial fragility. This also implies that under recovery planning only, the bank keeps holding excess liquidity, as the comparison between Tables 8a and 8c makes clear. Interestingly, as recovery rate r increases, the welfare difference between the two pecking orders tends to vanish, as the underlying investment technologies tend to be more and more similar.

Overall, neither recovery planning alone nor resolution planning alone seems to affect banks' liquidity management as to liquidity holdings and asset pecking order. What explains the different

Table 8: Expected welfare with recovery planning alone.

(a) Liquidity with {Liquidation, Liquidity}

	$r = 0.70$	$r = 0.75$	$r = 0.80$	$r = 0.85$	$r = 0.90$	$r = 0.95$
$R = 2.02$	0.0200	0.0200	0.0200	0.0200	0.0200	0.0200
$R = 2.04$	0.0200	0.0200	0.0200	0.0200	0.0200	0.0200
$R = 2.06$	0.0200	0.0200	0.0200	0.0200	0.0200	0.0200
$R = 2.08$	0.0200	0.0200	0.0200	0.0200	0.0200	0.0200

(b) Expected welfare with {Liquidation, Liquidity}

	$r = 0.70$	$r = 0.75$	$r = 0.80$	$r = 0.85$	$r = 0.90$	$r = 0.95$
$R = 2.02$	0.7408	0.8804	0.9947	1.0870	1.1600	1.2152
$R = 2.04$	0.7533	0.8915	1.0046	1.0960	1.1683	1.2230
$R = 2.06$	0.7657	0.9025	1.0146	1.1051	1.1766	1.2309
$R = 2.08$	0.7780	0.9136	1.0245	1.1142	1.1850	1.2387

(c) Liquidity with {Liquidity, Liquidation}

	$r = 0.70$	$r = 0.75$	$r = 0.80$	$r = 0.85$	$r = 0.90$	$r = 0.95$
$R = 2.02$	0.7231	0.5701	0.3927	0.1802	0.0200	0.0200
$R = 2.04$	0.6978	0.5474	0.3732	0.1638	0.0200	0.0200
$R = 2.06$	0.6737	0.5259	0.3545	0.1480	0.0200	0.0200
$R = 2.08$	0.6507	0.5054	0.3368	0.1329	0.0200	0.0200

(d) Expected welfare with {Liquidity, Liquidation}

	$r = 0.70$	$r = 0.75$	$r = 0.80$	$r = 0.85$	$r = 0.90$	$r = 0.95$
$R = 2.02$	1.0127	1.0285	1.0548	1.0966	1.1600	1.2152
$R = 2.04$	1.0155	1.0327	1.0605	1.1041	1.1683	1.2230
$R = 2.06$	1.0185	1.0371	1.0664	1.1117	1.1766	1.2309
$R = 2.08$	1.0218	1.0417	1.0725	1.1194	1.1850	1.2387

bank's behavior in the case of recovery and resolution planning compared to that of the unregulated economy is the complementarity between the two regulatory interventions, i.e., that banks take into account the effects of the pecking order on the expected welfare as well as on the availability of resources for the good bank at resolution.

6 Concluding remarks

We have proposed a theory to explain how banks manage liquidity during periods of financial fragility and in anticipation of them under different regulatory regimes. The novelty of our contribution is in the analysis of the asset pecking order that banks follow to cover withdrawals during depositors' runs, and its interaction with excess liquidity under recovery and resolution planning. Our results show that recovery and resolution planning lead to a more valuable resource allocation by allowing a more effective liquidity management by banks, thereby reducing the need for excess liquidity. The phenomenon that in the real world, banks, although subject to recovery and resolution planning, hold significant amounts of liquidity is not at odds with this, as banks are also subject to tight liquidity requirements, from which we abstract in the model.

In principle, a part of the asset pecking order mechanism studied here could be extended to other financial intermediaries subject to investors' strategic withdrawals. In this sense, our argument has the potential to explain the contrasting empirical evidence on liquidity cushioning (Chernenko and Sunderam, 2016) and liquidity hoarding (Morris et al., 2017) in mutual funds.

Notably, a comparison of our results to the case of an unregulated economy cannot allow us to derive normative implications on the efficiency of excess liquidity and of recovery and resolution planning. Nevertheless, a simple policy implication can definitely be drawn. Changing the architecture of recovery and resolution planning by limiting banks' independence and giving all powers to the RA should not affect liquidity management as long as banks are competitive and share with the RA the objective of maximizing depositors' welfare. The effect of introducing market power instead remains an open question, and we leave it to future research.

Note that in our framework, there is no deposit insurance. As already mentioned, this can be justified by the increasing role of uninsured deposits on banks' balance sheets.[12] Moreover, Allen et al. (2018) show that deposit insurance functioning as it does in the real world (i.e., guaranteeing a fixed repayment in any possible state of the economy) would not completely rule out panic runs. A similar argument holds regarding liquidity requirements. Only forcing banks to be "narrow" would make them immune from financial fragility. However, that would come at the cost of losing maturity transformation and possibly making banks redundant (Wallace, 1996). This is why in the real world, liquidity requirements are milder and leave some financial fragility unresolved. This means that the introduction of plausible deposit insurance schemes and liquidity requirements would alter neither the mechanism of our argument nor its conclusions.

[12]The total uninsured chackable, time, and savings deposits held by the U.S. chartered commercial banks in 2020 represented almost 40 percent of total U.S. bank liabilities, after reaching their lowest value of approximately 10 percent in mid-2009. Source: Financial Accounts of the United States.

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References

Acharya, V. V., S. T. Bharath, and A. Snirivasan (2007, September). Does industry-wide distress affect defaulted firms? Evidence from creditor recoveries. Journal of Financial Economics 85(3), 787–821.

Acharya, V. V. and O. Merrouche (2013). Precautionary Hoarding of Liquidity and Interbank Markets: Evidence from the Subprime Crisis. Review of Finance 17(1), 107–160.

Ahnert, T. and M. Elamin (2020, October). Bank Runs, Portfolio Choice and Liquidity Provision. Journal of Financial Stability 50.

Allen, F., E. Carletti, I. Goldstein, and A. Leonello (2018, September). Government Guarantees and Financial Stability. Journal of Economic Theory 177, 518–557.

Ashcraft, A., J. McAndrews, and D. Skeie (2011, October). Precautionary Reserves and the Interbank Market. Journal of Money, Credit and Banking 43(Supplement s2), 311–348.

Baldwin, R. E., T. Beck, A. Benassy-Quere, O. Blanchard, G. Corsetti, P. De Grauwe, W. Den Haan, F. Giavazzi, D. Gros, S. Kalemli-Ozcan, S. Micossi, E. Papaioannou, P. Pesenti, C. A. Pissarides, G. Tabellini, and B. Weder di Mauro (2015, November). Rebooting the Eurozone: Step I – agreeing a crisis narrative. CEPR Policy Insight No. 85.

Basel Committee on Banking Supervision (2010, March). Report and Recommendations of the Cross-border Bank Resolution Group. Technical report, Bank for International Settlements.

Carletti, E., I. Goldstein, and A. Leonello (2018, June). The interdependence of bank capital and liquidity. mimeo.

Carlsson, H. and E. van Damme (1993, September). Global Games and Equilibrium Selection. Econometrica 61(5), 989–1018.

Chen, Q., I. Goldstein, and W. Jiang (2010, August). Payoff complementarities and financial fragility: Evidence from mutual fund outflows. Journal of Financial Economics 97(2), 239–262.

Chernenko, S. and A. Sunderam (2016, June). Liquidity Transformation in Asset Management: Evidence from the Cash Holdings of Mutual Funds. mimeo.

Colliard, J.-E. and D. Gromb (2018, May). Financial Restructuring and Resolution of Banks. mimeo.

Diamond, D. W. and P. H. Dybvig (1983, June). Bank Runs, Deposit Insurance, and Liquidity. Journal of Political Economy 91(3), 401–419.

Farhi, E., M. Golosov, and A. Tsyvinski (2009). A Theory of Liquidity and Regulation of Financial Intermediation. Review of Economic Studies 76, 973–992.

Financial Stability Board (2011). Key Attributes of Effective Resolution Regimes for Financial Institutions. Technical report, Financial Stability Board.

Goldstein, I. and A. Pauzner (2005, June). Demand-Deposit Contracts and the Probability of Bank Runs. Journal of Finance 60(3), 1293–1327.

Gorton, G. B. and A. Metrick (2012, March). Getting up to Speed on the Financial Crisis: A One-Weekend-Reader's Guide. Journal of Economic Literature 50(1), 128–150.

Heider, F., F. Saidi, and G. Schepens (2019, October). Life Below Zero: Bank Lending Under Negative Policy Rates. Review of Financial Studies 32(10), 3728–3761.

Kashyap, A. K., D. P. Tsomocos, and A. P. Vardoulakis (2017, August). Optimal Bank Regulation in the Presence of Credit and Run Risk. Federal Reserve Board Finance and Economics Discussion Series No. 2017-097.

Keister, T. and Y. Mitkov (2019, October). Bailouts, Bail-ins and Banking Crises. mimeo.

Liu, X. and A. S. Mello (2011, December). The Fragile Capital Structure of Hedge Funds and the Limits to Arbitrage. Journal of Financial Economics 102(3), 491–506.

Mattana, E. and E. Panetti (2020). The Welfare Costs of Self-Fulfilling Bank Runs. Journal of Money, Credit and Banking forthcoming.

Morris, S., I. Shim, and H. S. Shin (2017, August). Redemption risk and cash hoarding by asset managers. Journal of Monetary Economics 89(1), 71–87.

Morris, S. and H. S. Shin (1998, June). Unique Equilibrium in a Model of Self-Fulfilling Currency Attacks. American Economic Review 88(3), 587–597.

Office of the Comptroller of the Currency (2018, December). OCC Guidelines Establishing Standards for Recovery Planning by Certain Large Insured National Banks, Insured Federal Savings Associations, and Insured Federal Branches; Technical Amendments.

Office of the Comptroller of the Currency (2019, March). Recovery Planning. Technical report, Office of the Comptroller of the Currency.

Pozsar, Z., T. Adrian, A. Ashcraft, and H. Boesky (2010, July). Shadow Banking. Staff Report 458, Federal Reserve Bank of New York.

Rochet, J.-C. and X. Vives (2004, December). Coordination Failures and the Lender of Last: Was Bagehot Right After All? Journal of the European Economic Association 2(6), 1116–1147.

Schilling, L. M. (2018, December). Optimal Forbearance of Bank Resolution. Becker Friedman Institute for Research in Economics Working Paper No. 2018-15.

Vives, X. (2014). Strategic Complementarity, Fragility, and Regulation. Review of Financial Studies 27(12), 3547–3592.

Wallace, N. (1996, Winter). Narrow Banking Meets the Diamond-Dybvig Model. Federal Reserve Bank of Minneapolis Quarterly Review 20(1), 3–13.

Zeng, Y. (2017, January). A Dynamic Theory of Mutual Fund Runs and Liquidity Management. mimeo.

Appendices

A Viability of the banking system

As a benchmark for studying the viability of the banking equilibrium, in this appendix we analyze an autarkic equilibrium and compare it to the competitive equilibria of the main text. Assume that the agents cannot access the banking system on date 0, but can invest in a portfolio of liquidity L and productive assets $1-L$. Then, if agents turn out to be early consumers, they will consume the liquidation value of their assets, $c^A = L + r(1-L)$, which is clearly lower than or equal to 1, given that $r < 1$. If instead agents turn out to be late consumers, they will consume an amount that depends on the realization of the productivity shock Z and on the amount of liquidity that is rolled over to date 2, i.e., $c_2^A(R) = R(1-L) + L$ or $c_2^A(0) = L$. Then, on date 0, the portfolio problem is reduced to maximizing

$$\begin{aligned} W^A &= \lambda\big[L + r(1-L)\big] + (1-\lambda)\int_0^1 \big[p(R(1-L)+L) + (1-p)L\big]dp = \\ &= L + (1-L)\big[\lambda r + (1-\lambda)\mathbb{E}[p]R\big]. \end{aligned} \tag{39}$$

Clearly, if

$$1 > \big[\lambda r + (1-\lambda)\mathbb{E}[p]R\big] \tag{40}$$

then the objective function is increasing in L, and in equilibrium $L^A = 1$ and $W^A = 1$. Otherwise, it is decreasing, and $L^A = 0$ and $W^A = \lambda r + (1-\lambda)\mathbb{E}[p]R$. Intuitively, an agent in autarky chooses a corner solution depending on the expected returns per unit of investment: either 1 (from liquidity) or the sum of r with probability λ and $\mathbb{E}[p]R$ with probability $1-\lambda$ (from the productive asset).

How does the banking equilibrium compare to autarky? On the one hand, the agents in autarky choose either a fully liquid or a fully illiquid asset portfolio depending on the expected returns per unit of investment, taking into account that they must liquidate any productive asset they have if they turn out to be early consumers. On the other hand, banks can exploit the law of

Table 9: Viability of a fragile banking system: Expected welfare.

(a) Autarky

	$r = 0.70$	$r = 0.75$	$r = 0.80$	$r = 0.85$	$r = 0.90$	$r = 0.95$
$R = 2.02$	1.0038	1.0048	1.0058	1.0068	1.0078	1.0088
$R = 2.04$	1.0136	1.0146	1.0156	1.0166	1.0176	1.0186
$R = 2.06$	1.0234	1.0244	1.0254	1.0264	1.0274	1.0284
$R = 2.08$	1.0332	1.0342	1.0352	1.0362	1.0372	1.0382

(b) Unregulated equilibrium

	$r = 0.70$	$r = 0.75$	$r = 0.80$	$r = 0.85$	$r = 0.90$	$r = 0.95$
$R = 2.02$	1.0127	1.0285	1.0548	1.0966	1.1600	1.2152
$R = 2.04$	1.0155	1.0327	1.0605	1.1041	1.1683	1.2230
$R = 2.06$	1.0185	1.0371	1.0664	1.1117	1.1766	1.2309
$R = 2.08$	1.0218	1.0417	1.0725	1.1194	1.1850	1.2387

(c) Recovery and resolution planning

	$r = 0.70$	$r = 0.75$	$r = 0.80$	$r = 0.85$	$r = 0.90$	$r = 0.95$
$R = 2.02$	0.9489	1.0165	1.0764	1.1303	1.1784	1.2199
$R = 2.04$	0.9579	1.0253	1.0850	1.1386	1.1864	1.2276
$R = 2.06$	0.9669	1.0341	1.0836	1.1469	1.1944	1.2353
$R = 2.08$	0.9758	1.0429	1.1022	1553	1.2025	1.2431

large numbers and avoid liquidation altogether unless a run occurs. In the banking equilibrium with perfect information, the bank can always choose the same investment strategy as agents do in autarky. Put differently, the autarkic allocation is feasible for the bank, but is not chosen. Then, as perfectly competitive banks maximize the expected welfare of the depositors, this must mean that the banking equilibrium with perfect information Pareto-dominates autarky.

How do the equilibria with financial fragility instead compare to autarky? After some simple algebra, the value functions of the unregulated equilibrium and the equilibrium with recovery and resolution planning can be expressed as

$$W^{UNREG} = L + (1-L)\big[\sigma^* r + (1-\sigma^*)\mathbb{E}[p]R(1+\sigma^*)\big], \tag{41}$$

$$W^{RECRES} = L + (1-L)\big[\sigma^* r + (1-\sigma^*)\mathbb{E}[p]R(1+\sigma^*)\big] - \psi(1-\phi_1), \tag{42}$$

where we already took into account that with recovery and resolution planning the bank chooses the pecking order {Liquidation, Liquidity}. As $L \geq \lambda c$, then $L > \lambda r$; i.e., the bank is better at managing liquidity than are autarkic agents. However, (41) and (42) also show that the expected return on the productive asset in the equilibria with financial fragility depends on whether a run occurs. In other words, the cost for an agent of obtaining the better liquidity management offered by the bank is the fact that runs can happen. Table 9 shows which of the three equilibria dominates for different combinations of parameters. If R is high and r is small, autarky dominates and fragile banks are not viable independently of the presence of recovery and resolution planning. In all other cases, the equilibria with financial fragility dominate autarky.

B Proofs

Proof of Lemma 1. Simplify the objective function as follows:

$$\max_{L,D} L + rD + \int_0^1 pR(1 - L - D), \tag{43}$$

subject to $L + rD \geq \lambda c$, $L \geq 0$, $D \leq 1 - L$ and $D \geq 0$. Assign Lagrange multipliers ξ, μ, η and ζ, respectively. The first-order conditions are

$$L: \qquad 1 - \mathbb{E}[p]R + \xi + \mu - \eta = 0, \tag{44}$$

$$D: \qquad r - \mathbb{E}[p]R + r\xi - \eta + \zeta = 0. \tag{45}$$

Substituting the first into the second, we obtain

$$-\mathbb{E}[p](1 - r)R - r\mu - (1 - r)\eta + \zeta = 0, \tag{46}$$

hence it must be the case that $\zeta > 0$ and $D = 0$ by complementary slackness. According to the liquidity constraint, this means that $L \geq \lambda c > 0$. Thus, $\mu = 0$. Then, from the first of the first-order conditions, it follows that

$$\xi = \mathbb{E}[p]R - 1 + \eta, \tag{47}$$

which is strictly positive, as $\mathbb{E}[p]R > 1$. Therefore, $L = \lambda c < 1$ and $\eta = 0$. This completes the proof. ■

Proof of Lemma 2. In the interval $[\lambda, n_1^*)$,

$$\frac{\partial v_1(\sigma, n)}{\partial n} = \sigma \frac{-\frac{R}{r}c(1-n) + R\left(1 - L - \frac{(n-\lambda)c}{r}\right) + R(L - \lambda c) - R(L - \lambda c)}{(1-n)^2} + \frac{L - \lambda c}{(1-n)^2}. \tag{48}$$

This is negative if

$$(L - \lambda c)(1 - \sigma R) < \sigma R \left(\frac{\lambda r + (1-\lambda)}{r} c - 1\right). \tag{49}$$

As $\sigma > \underline{\sigma} = (c - L)/(R(1 - L))$, then $\sigma R > 1$, as $c \geq 1$. Hence, the left side is negative, and the right side is positive. In the interval $[n_1^*, n^{**})$, $v_1(\sigma, n)$ is decreasing, as $c > L + r(1 - L)$.

In the interval $[\lambda, n_2^*)$,

$$\frac{\partial v_2(\sigma, n)}{\partial n} = \sigma R \frac{1 - L}{(1-n)^2} + \frac{L - c}{(1-n)^2}. \tag{50}$$

This is negative if

$$c - L > \sigma R(1 - L), \tag{51}$$

which is impossible because $\sigma > \underline{\sigma}$. In the interval $[n_2^*, n^{**})$ instead,

$$\frac{\partial v_2(\sigma, n)}{\partial n} = \sigma \frac{-\frac{R}{r}c(1-n) + R\left(1 - L - \frac{nc-L}{r}\right)}{(1-n)^2} = \sigma R \frac{1 - L}{(1-n)^2}\left(1 - \frac{1}{r}\right) < 0. \tag{52}$$

■

Proof of Proposition 1. For a given deposit contract and the asset portfolio, the bank decides on the optimal pecking order as a best response to the withdrawal decisions of depositors. Recalling the timing of actions, note that a bank solving the dual problem under pecking order j maximizes

$$\int_0^{\sigma_j^*} L + r(1-L)dp + \int_{\sigma_j^*}^1 \left[\lambda c + (1-\lambda)\Big[pc_2(R) + (1-p)c_L(0)\Big]\right]dp. \tag{53}$$

This is the expected utility of depositors when the bank offers an amount c of early consumption, holds an amount L of liquidity, and chooses the pecking order j inducing the threshold signal σ_j^*. If $c \geq L + r(1 - L)$ and $L < 1$, the above expression is decreasing in σ_j^* for any possible values of c and L. Hence, the expected utility is maximized at $\sigma^* = \text{argmin}\ \{\sigma_1^*, \sigma_2^*\}$. Note that $v_1(\sigma, n)$ and $v_2(\sigma, n)$ are equal for $n \in [n^{**}, 1]$ and for $n = \lambda$. However, $v_1(\sigma, n)$ is decreasing everywhere in $n \in (\lambda, n^{**})$, and $v_2(\sigma, n)$ is first increasing (until n_2^*) and then decreasing (between n_2^* and n^{**}).

Then, we just need to compare the second arm of $v_2(\sigma, n)$ with $v_1(\sigma, n)$ for given c and L. In $[\lambda, n_1^*)$, $v_2(\sigma, n) \geq v_1(\sigma, n)$, as

$$\sigma \frac{R\left(1 - L - \frac{nc-L}{r}\right)}{1-n} \geq \sigma \frac{R\left(1 - L - \frac{(n-\lambda)c}{r}\right) + L - \lambda c}{1-n} + (1-\sigma)\frac{L - \lambda c}{1-n}. \tag{54}$$

Simplifying, we obtain

$$\left(\frac{\sigma R}{r} - 1\right) L \geq \left(\frac{\sigma R}{r} - 1\right) \lambda c, \tag{55}$$

which is always true, as $L \geq \lambda c$. In the interval $[n_1^*, n^{**}]$, we have $v_2(\sigma, n) > v_1(\sigma, n)$, as

$$\sigma \frac{R\left(1 - L - \frac{nc-L}{r}\right)}{1-n} > \frac{r(1-L) + L - nc}{1-n} \tag{56}$$

implies that

$$(\sigma R - r)(1-L) > (L - nc)\left(1 - \frac{\sigma R}{r}\right). \tag{57}$$

This holds, as $\sigma > \underline{\sigma}$ implies that $\sigma R > 1$. ■

Proof of Lemma 3. The derivatives of (24) and (25) with respect to ϕ_1 and ϕ_2 are

$$\frac{\partial \sigma_1^*}{\partial \phi_1} = \frac{1}{DEN_{\sigma_1^*}}\left[c_1 - \frac{r(1-L_1) + L_1 - \phi_1 c_1}{1-\phi_1}\right] = \frac{1}{DEN_{\sigma_1^*}} \frac{c_1 - r(1-L_1) - L_1}{1-\phi_1}, \tag{58}$$

$$\frac{\partial \sigma_2^*}{\partial \phi_2} = \frac{1}{DEN_{\sigma_2^*}}\left[c_2 - \sigma_2^* \frac{R\left(1 - L_2 - \frac{\phi_2 c_2 - L_2}{r}\right)}{1-\phi_2}\right], \tag{59}$$

respectively. The expression in (58) is positive, as $c_1 > L_1 + r(1-L_1)$. The expression in (59) is also positive, as $v_2(\sigma_2^*, \phi_2) < 0$ since $n_2^0 < \phi_2$. ■

Proof of Lemma 4. Substituting (24) and (58) into (31), we obtain

$$\left[(\phi_1 - \lambda)c_1 - \int_{\lambda}^{n_1^*} \frac{L_1 - \lambda c_1}{1-n} dn - \int_{n_1^*}^{\phi_1} \frac{r(1-L_1) + L_1 - nc_1}{1-n} dn\right] \psi'(1-\phi_1) = \frac{c_1 - r(1-L_1) - L_1}{1-\phi_1} \Delta U_1. \tag{60}$$

As the right side of this expression is independent of R, then also the left side must be so. Hence, the lemma follows. ■