Essays in the Economics of Networks

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Thesis defence
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Two features of today's economies:

- long, interconnected **supply chains**;

- in many sectors **superstar firms** (market power?)

Two features of today's economies:

- long, interconnected **supply chains**;
  (Berlingieri (2013), Alfaro et al, 2019, ...)
- in many sectors **superstar firms** (market power?)
  (De Loecker and Eeckhout (2020), Autor et al. (2020), ...)

Big picture:
How do input-output connections matter for competition policy?

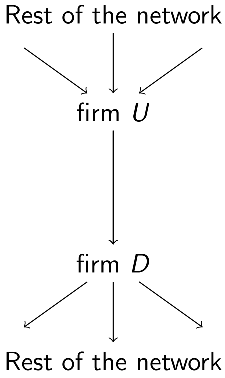
This paper: A tractable model of competition such that:

- network and technology determine **both** size and split of the surplus;
- firms strategically exploit their position in the supply chain.

Goal: provide a tool to assess market power: e.g. evaluate mergers.

- market power+network interesting for business cycles, monetary policy,...

Main contributions



How to model market power in firm-to-firm trade?

- e.g. firms set the price -

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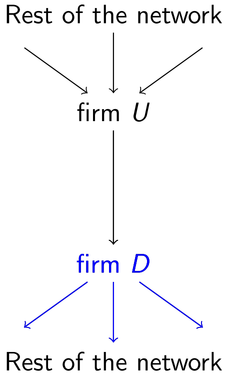

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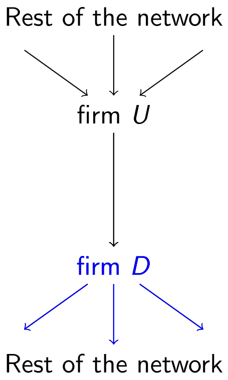

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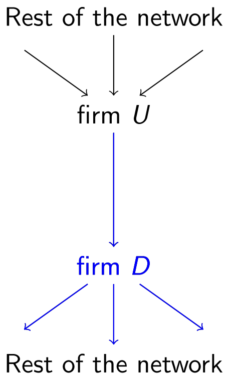

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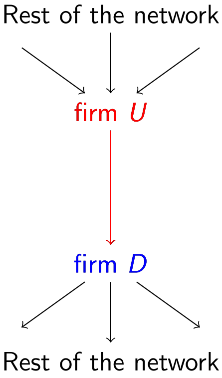

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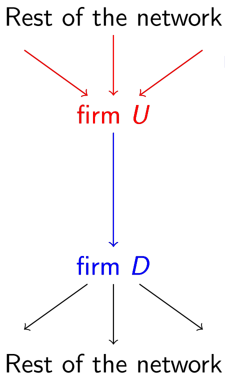
- e.g. firms set the price - output?
- **network** $\rightarrow$ firms are both **buyers** and **sellers**
- **but** also firms in U "want" to set the price;

Main contributions



Common approach:

- market power on one side: **inputs** or output;

  Carvalho et al. (WP), Hart and Tirole (1990), Grassi (WP)

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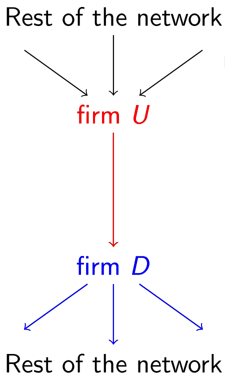

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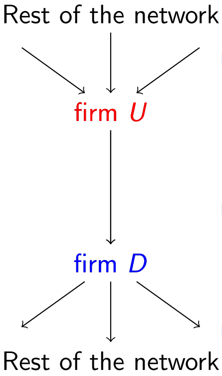

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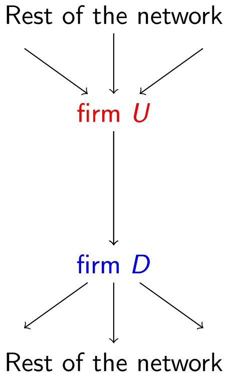
This paper:

- market power on inputs and outputs, endogenously;

Important to assess **relative** market power:

- choice of market where firms have power changes predictions

Main contributions



Classical (macro) approach:

- Firms neglect their network position.

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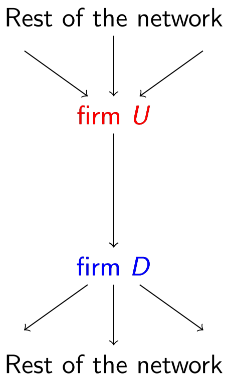

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Important for **aggregate** welfare impact of oligopolies:

- market power is **stronger** than if they don't (final price larger)

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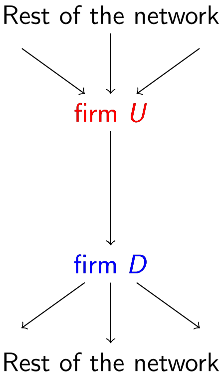

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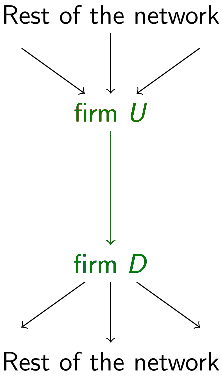
How? competition in **supply and demand functions**

Main contributions - S&D equilibrium



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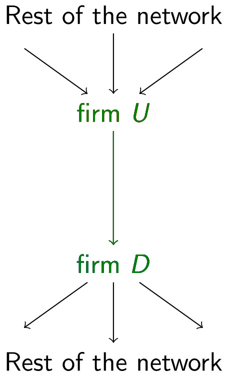

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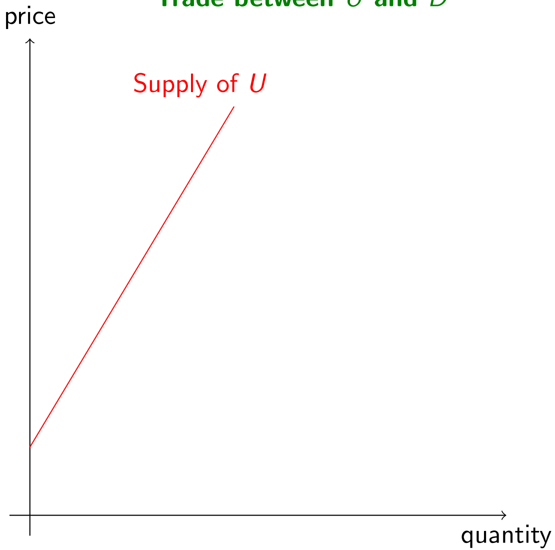

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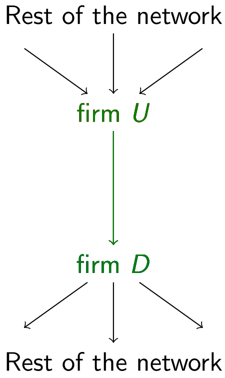

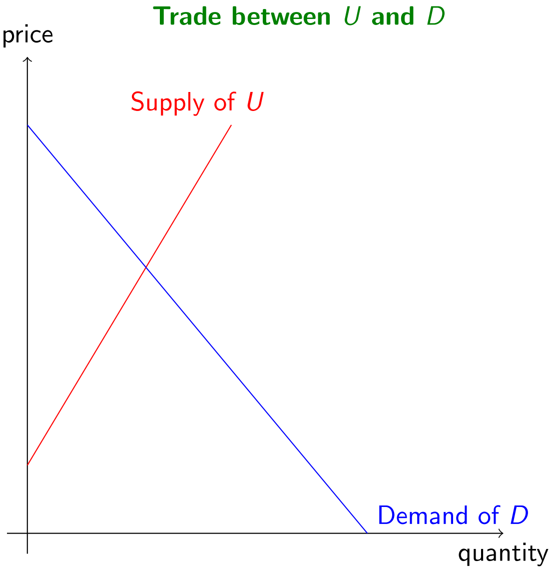

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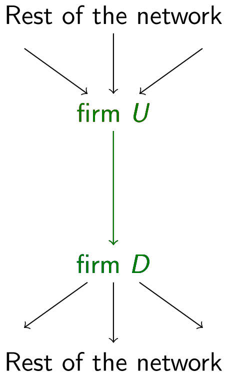

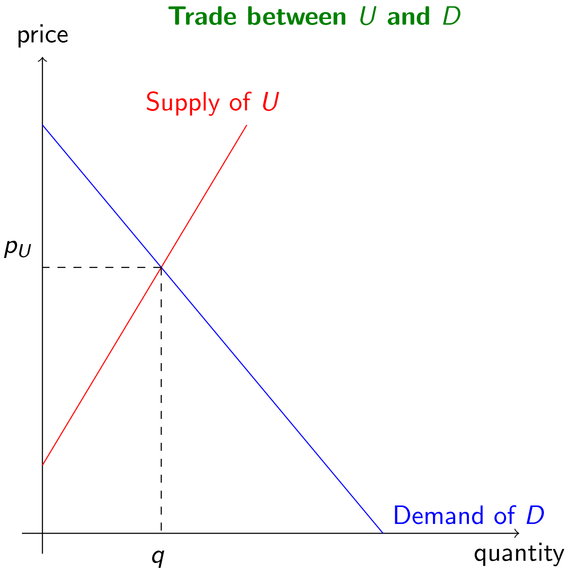

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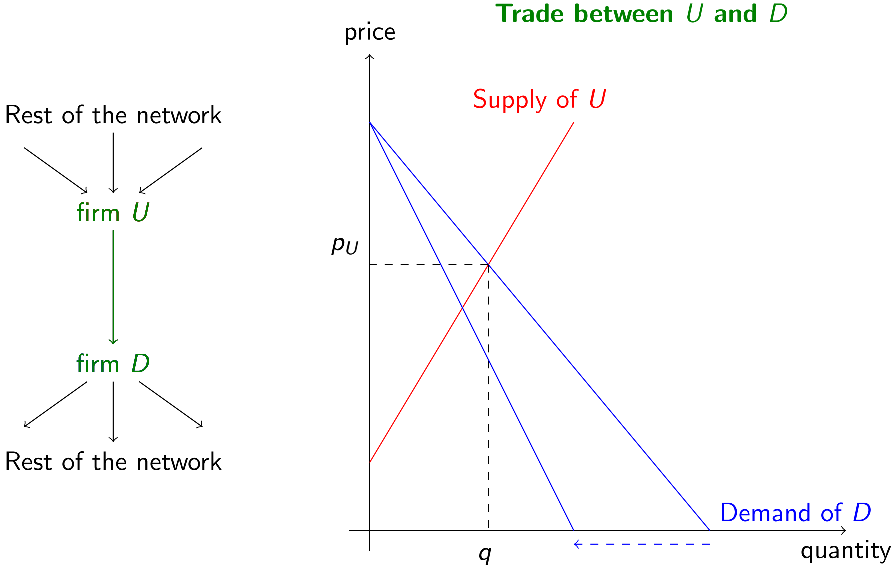

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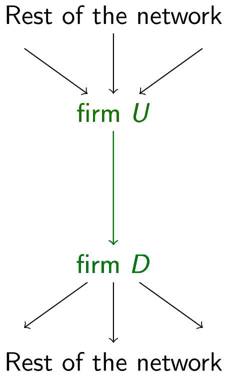

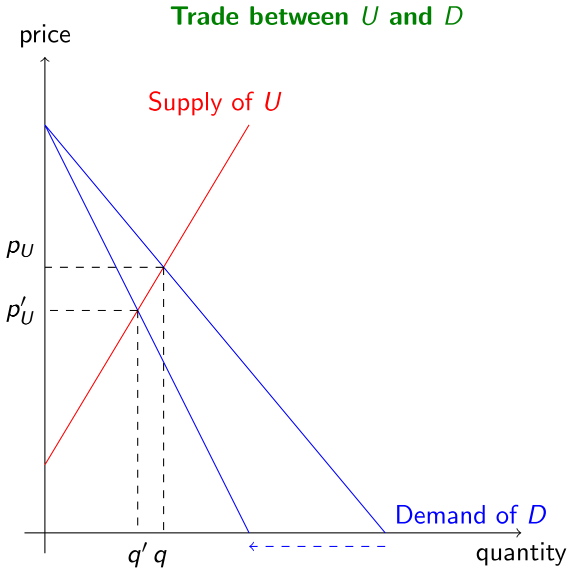

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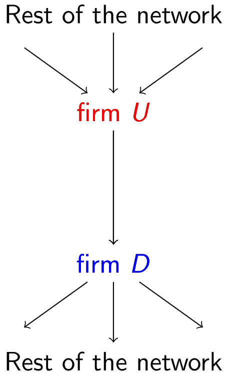

Supply and demand functions:

- physically used in e.g. finance, electricity auctions;
- here: any arrangements (contractual, managerial) that specify how firm reacts to different conditions in the market.

Results

- Existence for **any network** (e.g. sequential models need acyclic);
- **Horizontal mergers** always increase the final price;
  - even if **countervailing power** (Stigler ('52), Loertscher and Marx (WP));
- Final price **smaller** if firms neglect network ("macro" approach);
- (In a line) if market power only on outputs/inputs:
  - $\Longrightarrow$ **opposite** predictions on **markups**;
- Feasible algorithm to solve model numerically;
- **in progress**: proof of concept on US IO network.

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Related literature

Market power and efficiency (macro):

production networks **Acemoglu and Tahbaz-Salehi (WP)**, Grassi (WP), Kikkawa et al. (2020), Baqaee (2019), Baqaee and Farhi (2019, 2020), Pasten et al. (2018), Carvalho et. al (WP);

no input-output Pellegrino (WP), Azar and Vives (2020);

Market power and efficiency in networks:

mergers Loertscher and Marx (WP), Hinnosaar ('19); Bimpikis et al. ('20), Hart and Tirole ('90), Salinger ('90);

bargaining Condorelli et al. ('17, WP), Kotowski and Leister ('19), Manea ('18);

matching Hatfield et al. ('12), Fleiner et al. ('20), Fleiner et al. ('19);

Supply function competition/double auctions:

Supply function competition Klemperer and Meyer (1989), Green and Newbery (1992), Vives (2011);

Finance microstructure Kyle (1989), **Malamud and Rostek (2017)**;

Auctions Ausubel et al. (2014), Woodward (WP).

Plan

Model

Results

- Existence
- Horizontal mergers
- Relative market power
- Aggregate impact of market power

Setting I

Firms

- $i = 1, \ldots, N$ sectors, $\alpha = 1, \ldots, n_i$ **homogeneous** firms per sector;
- firms need specific goods as inputs - this defines the *input-output network* (exogenous);
- every link is a market - **customized** prices (Dhyne et al. (WP));
- **For ease of exposition**: 1 customer per sector (network is a **tree**).

Consumers

- continuum - **price taker representative consumer**
- consumers provide labor (L) and own the firms;
- competitive labor market: wage **taken as given** (normalized to 1).

Two assumptions allow tractability:

- **stochastic productivity** $\rightarrow$ unique best reply;
- **specific functional form** $\rightarrow$ linear schedules;

Analogous to most models with supply/demand functions (e.g. Klemperer and Meyer (89)).

Setting II - Parametric assumptions

The profit of firm α in sector i is:

$$\pi_{i\alpha} = p_i\Big(\sum_j \omega_{ij} q_{i\alpha,j}\Big) - \sum_j p_j q_{i\alpha,j} - \underbrace{\left(\varepsilon_i \sum_j q_{i\alpha,j} + \frac{1}{2}\sum_j q_{i\alpha,j}^2\right)}_{\text{Amount of labor hired}}$$

where ε_i is a **labor productivity shock**.

- Can be rationalized through a production function: Technology
- inputs **nor substitutes nor complements** generalization

Consumers: $\frac{A_c+\varepsilon_c}{B_c}c - \frac{1}{2}\frac{1}{B_c}c^2 - L$

$\boldsymbol{\varepsilon} = (\varepsilon_1, \dots, \varepsilon_N, \varepsilon_c) \sim F, \quad suppF$ is a neighborhood of 0

The Game I - Players and Actions

The firms play a **simultaneous** game $\mathcal{G}$. ε is realized at the end.

Players the **firms**;

Actions firm α in sector i chooses a profile of:

- a supply function $S_{i\alpha}$;
- demands for intermediate inputs $D_{i\alpha} = (D_{i\alpha,j})_j$;

such that:

1. defined on $(p_i, p_i^{in}, \varepsilon_i)$;
2. subject to the **technology constraint**:

$$S_{i\alpha}(p_i, p_i^{in}, \varepsilon_i) = \Phi_i\left(D_{i\alpha},\right) \text{ for any } (p_i, p_i^{in}, \varepsilon_i)$$

3. differentiable, p.d. Jacobian bounded away from 0; details

The Game II - Payoffs

Payoffs $u_{i\alpha} = \mathbb{E}\pi_{i\alpha}(p_i, p_i^{in}, \varepsilon_i, S_{i\alpha}(p_i, p_i^{in}, \varepsilon_i), D_{i\alpha}(p_i, p_i^{in}, \varepsilon_i)$

$p = ?$

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$p =?$ The **prices** are determined solving the **market clearing equations**:

$$\sum_\beta D_{j\beta,k}(p_j, p_j^{in}, \varepsilon_j) = \sum_\alpha S_{k\alpha}(p_k, p_k^{in}, \varepsilon_k) \quad \text{if } k \text{ sells to } j$$

$$A_c + \varepsilon_c - B_c p_c = \sum_\beta S_{0\beta}(p_c, p_0^{in}, \varepsilon_0) \text{ for final output producer}$$

Global implicit function theorem yields a function $p^*(\varepsilon)$.

Supply and Demand Function Equilibrium

I will use shocks $\varepsilon = ((\varepsilon_i)_i, \varepsilon_c)$ as **selection device**. Formally:

Consider a sequence of rv $\varepsilon^n \sim F^n$ such that $\cap_n suppF^n = \{0\}$, and consider the relative game $\mathcal{G}^n$, as above.

a Supply and Demand Function Equilibrium is a profile of prices $(p_i)_i$ and quantities $(q_{i\alpha})_{i\alpha}$ that arise in the Nash equilibrium of $\mathcal{G}^n$ for $n \to \infty$.

Reminiscent of **Trembling-hand equilibrium**.

Symmetric Linear Equilibrium

A **symmetric linear equilibrium** is a profile of functions $(D_{i\alpha})_{\alpha,i}$ defined in open sets $\mathcal{O}_i$ such that:

- is a Nash equilibrium of $\mathcal{G}^n$ for n large enough;
- firms in same sector behave identically: $\forall\alpha\, D_{i\alpha,j} = D_{ij}$;
- they are **linear**, that is for all links where there is trade:

$$D_i = B_i \begin{pmatrix} \vdots \\ \omega_{ij} p_i - p_j \\ \vdots \end{pmatrix} + \varepsilon_i B_{i,\varepsilon} \quad B_i \text{ \textbf{symmetric positive definite}}$$

details

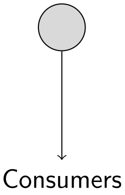
- $(p^*(0), 0) \in \cap_i \mathcal{O}_i$.

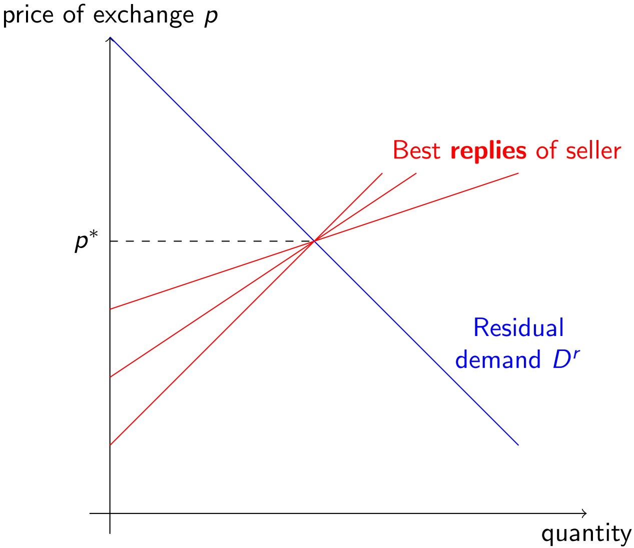
How to solve it?

Two steps:

- best reply problem equivalent to **ex-post price setting** against residual demand (and supply) (Klemperer and Meyer '89);
- $\rightarrow$ back up residual demand and supply from market clearing.
  - where **network** comes into play.

Solution I - Best reply





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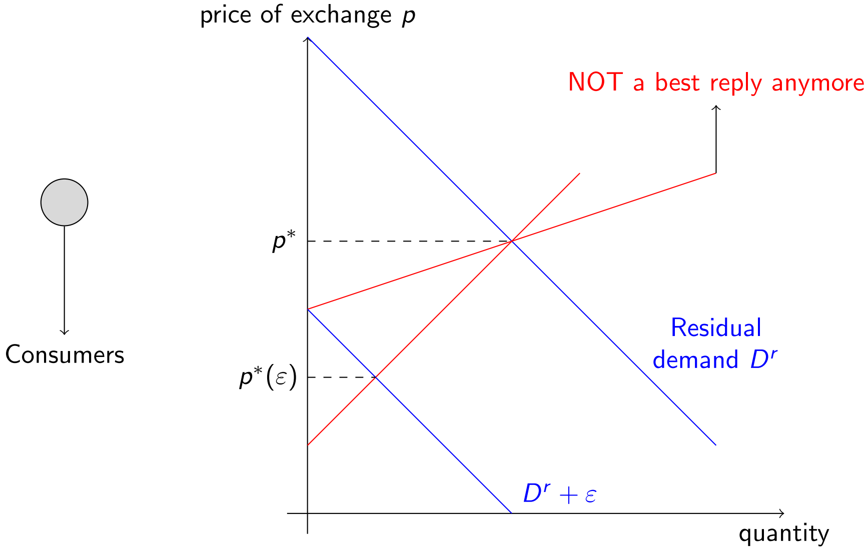

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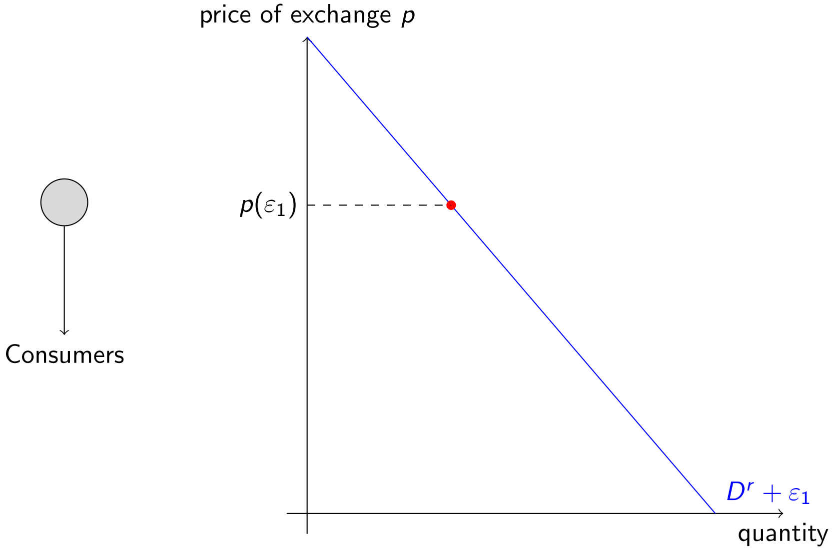

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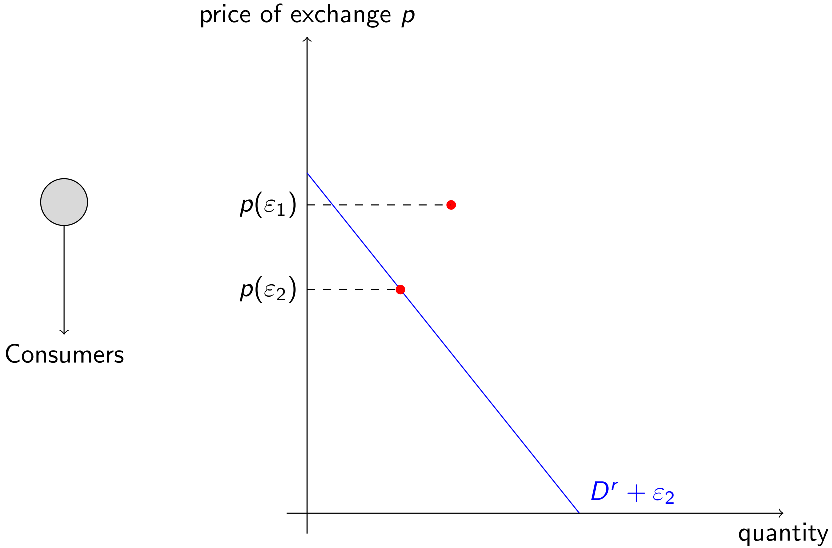

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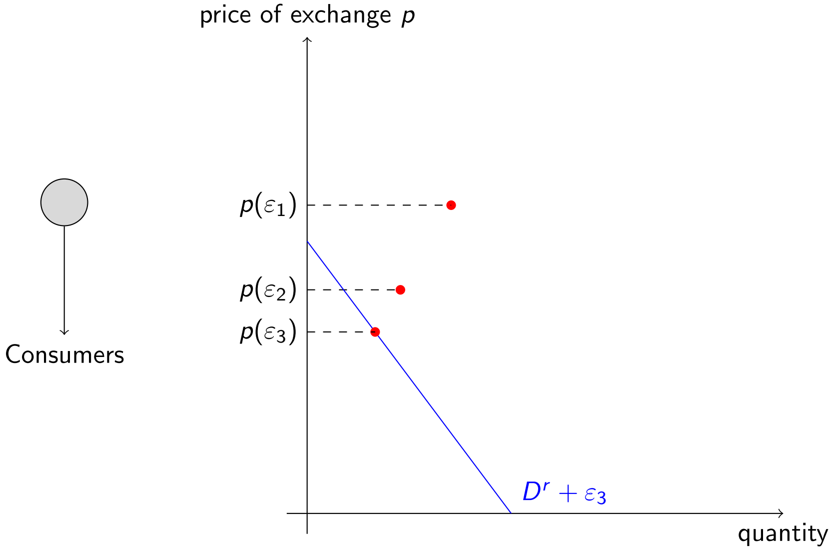

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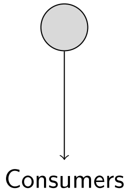
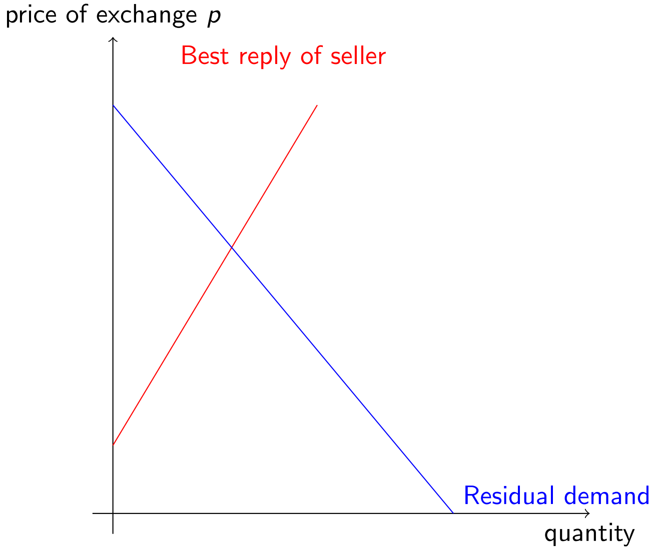
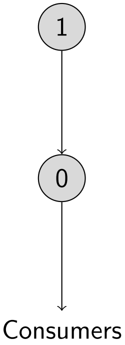




Solution II - Residual demand



Focus on firm α in 0.

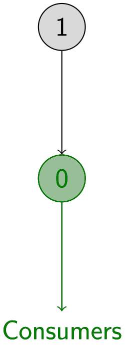
- Assume all other firms play **linear** schedules:

$$S_1 = B_1(p_1 - \varepsilon_1)$$
$$S_0 = B_0(p_0 - p_1 - \varepsilon_0)$$

To compute its best reply, firms in 1 compute:

$$\max_{p_1} \pi_{1\alpha}(p_1, D^r(p_1, \varepsilon_0), \varepsilon_1)$$

Solution II - Residual demand



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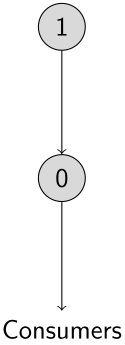
where:

$$D^r(p_1) = \underbrace{n_0 B_0(p_0^* - p_1 - \varepsilon_0)}_{\text{Demand from 0}} - \underbrace{(n_1 - 1)B_1(p_1 - \varepsilon_1)}_{\text{Supply of competitors}}$$

$p_0^*(p_1)$ got from **market clearing in final market**

Solution II - Residual demand



Market clearing conditions are a **linear system**:

$$Mp = \boldsymbol{A} \quad \text{where } M = f((B_i)_i)$$

Partially solving we get the **residual demand and supply**:

$$\begin{pmatrix} D_i^r \\ S_i^r \end{pmatrix} = A_i(\varepsilon) - \Lambda_i^{-1} \begin{pmatrix} p_i^{out} \\ -p_i^{in} \end{pmatrix} - (n_i - 1)B_i \begin{pmatrix} p_i^{out} \\ -p_i^{in} \end{pmatrix}$$

Λ_i is (endogenous) **price impact** of sector i:

- it represents amount of monopoly power in sector i.

Taking the first order conditions and solving:

- we get fixed point equation in coefficients.

Equilibrium

In equilibrium the coefficients $(B_i)_{i \in I}$ satisfie:

- $$B_i = \left(C_i^{-1} + \underbrace{\left((n_i - 1) B_i + \overline{\Lambda}_i^{-1} \right)^{-1}}_{\text{Deviation from perfect competition}} \right)^{-1}$$
- $\overline{\Lambda}_i$ **constrained price impact** (adjusted for technology constraint):

  $$\overline{\Lambda}_i = \Lambda_i - \frac{1}{\tilde{u}' \Lambda_i \tilde{u}} \Lambda_i \tilde{u} \tilde{u}' \Lambda_i$$

  where $\tilde{u}' = (1, -\omega_i)$.
- C_i are the coefficients of a firm that **takes prices as given**!
- if $n_i \to \infty$, it converges to perfect competition limit.

Back

Existence

Theorem

A non-trivial linear symmetric Supply and Demand Function equilibrium exists in any network for generic values of ωs if there are at least 2 firms per sector (sufficient condition);

Key elements of the proof:

- strategic complementarities in **slopes**:
  best reply coefficient matrices increasing in psd ordering;
- this also yields an algorithm to solve it (iterating the best reply).

Proof

Horizontal mergers are harmful for welfare

- an horizontal merger in sector i is a **decrease** in number of firms n_i
- **simplification:** mergers don't change the set of active links.

Proposition

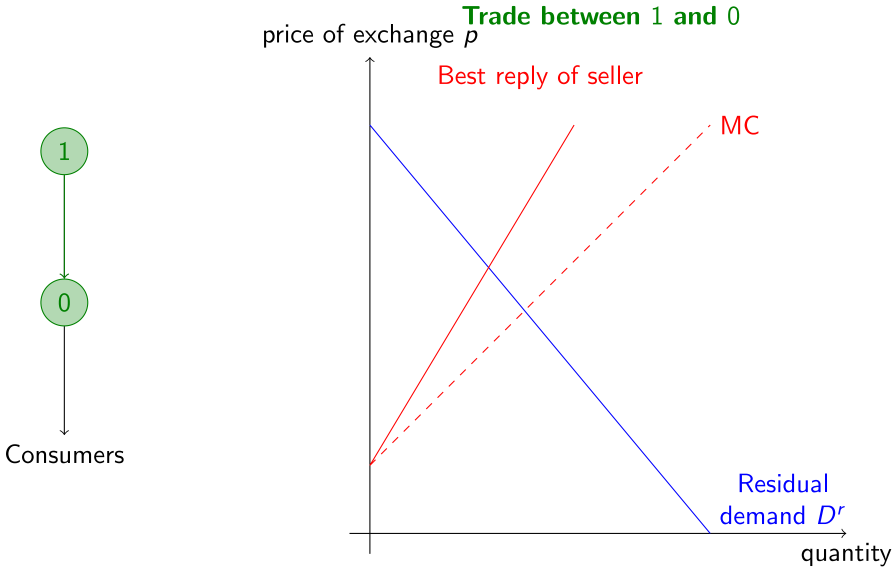
In the maximal equilibrium, if there is just **one final good** *any horizontal merger increases the price of the final good.*

Proof: strategic complementarities.

If network is a **line or regular tree**:

- any horizontal merger decreases total welfare.

Mergers - intuition



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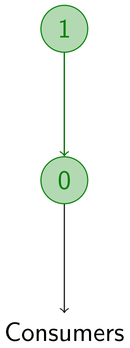

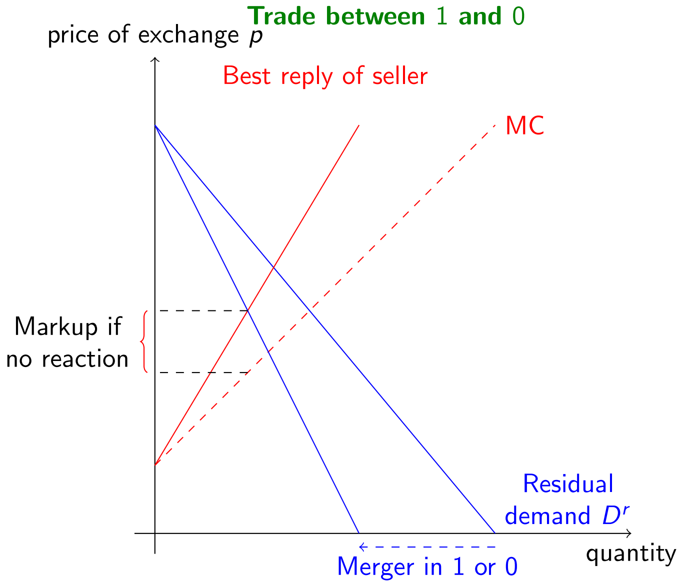

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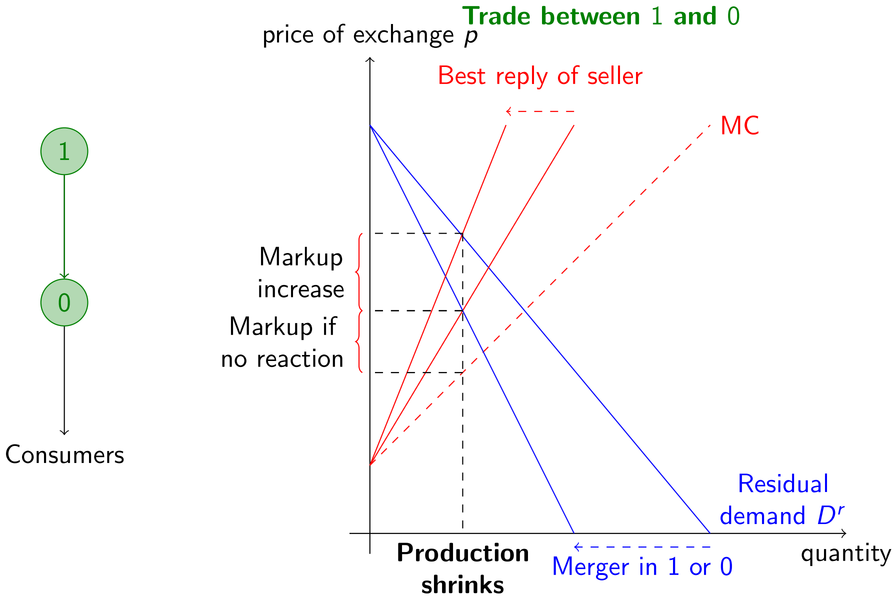
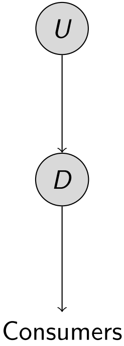


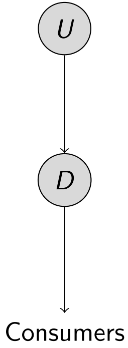
Relative market power



Assume $n_U = n_D$, $\omega_U = \omega_D = 1$.

In which sector firms have more market power?

Relative market power



Assume $n_U = n_D$, $\omega_U = \omega_D = 1$.

In which sector firms have more market power?

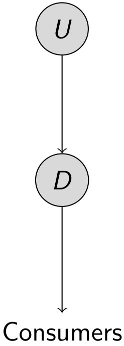
"Classical" approach (Belleflamme and Peitz (2010), Salinger (1990)..)

- **Solve sequentially**:

1. firms in 1 and 2 choose output quantity (à la Cournot);
2. firms in sector 0 do the same, **taking input price as given**.

- **Prediction**: firms in U have larger markup and profit;

Relative market power



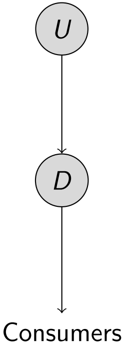
Mechanism:

- only U takes **pass-through** into account.

In general production networks who "moves first?" / "takes price as given?"

If firms set **input** quantities, in reverse order → **reverse predictions**!

Relative market power



Mechanism:

- only U takes **pass-through** into account.

In general production networks who "moves first?" / "takes price as given?"

If firms set **input** quantities, in reverse order → **reverse predictions**!

- in S&D equilibrium firms are symmetric, but for **network position** and **technology**.
- this example: U has larger markup, D larger markdown Details
- same profits and welfare impact of mergers.

Similar examples with welfare impact of mergers.

Are network effects important?

A widespread assumption:

- firms don't take network position into account;

(Grassi (2019), Baqaee (2018), . . .)

To see the implications assume:

- firms best respond **taking as given prices of other markets**.

Formal definition

Key change: neglect **pass-through** effect:

- **residual demand** (and supply) **less steep**. Intuition

Global vs Local

Theorem

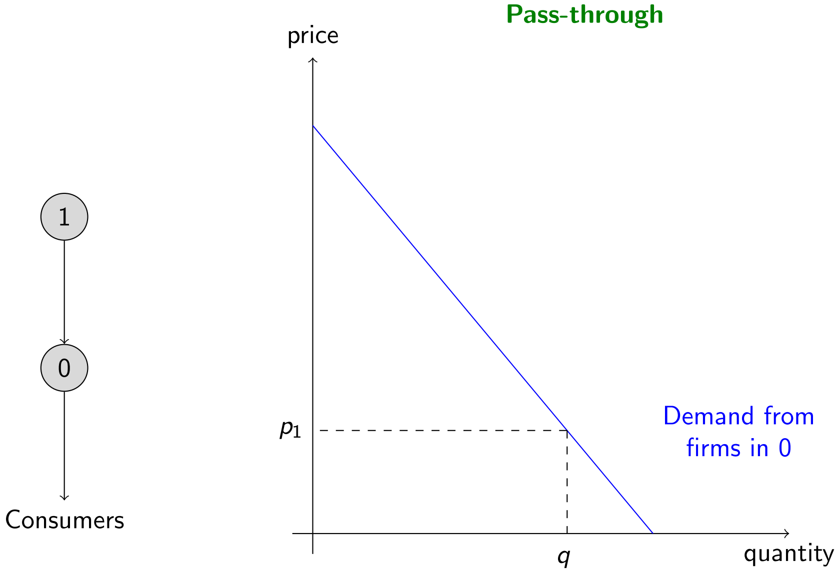
Assume the set of active links is the same in the standard and in the local S&D equilibrium. If there is a **unique final good***, in the maximal equilibrium under* **local** *competition, the price of the final good is* **smaller** *than under global competition.*

Proof: for given (B_j): $BR_i^{global}(B_{-i}) > BR_i^{local}(B_{-i})$ in p.s.d. sense. Details

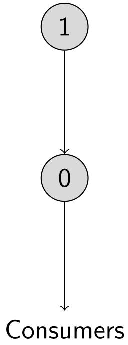
If network is a **line** or a **regular tree**:,

- the welfare is **larger** in the **local** S&D equilibrium.

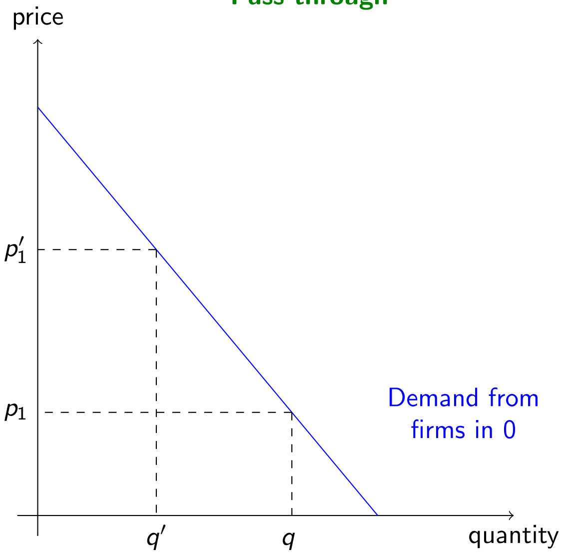
Pass-through - Intuition



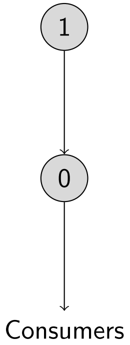
Pass-through - Intuition

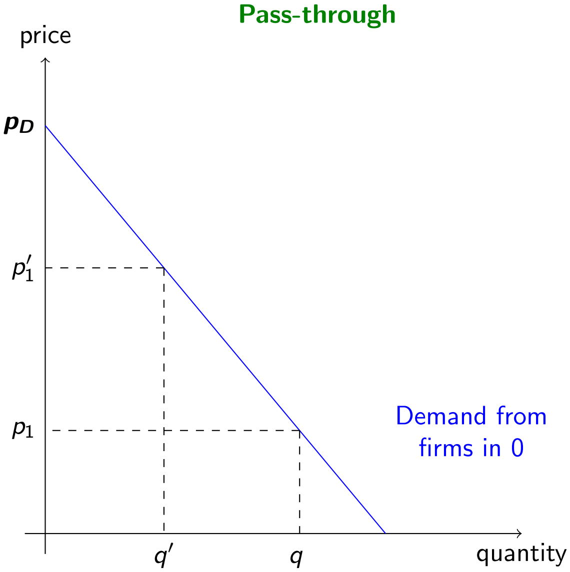


Pass-through

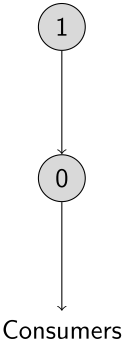


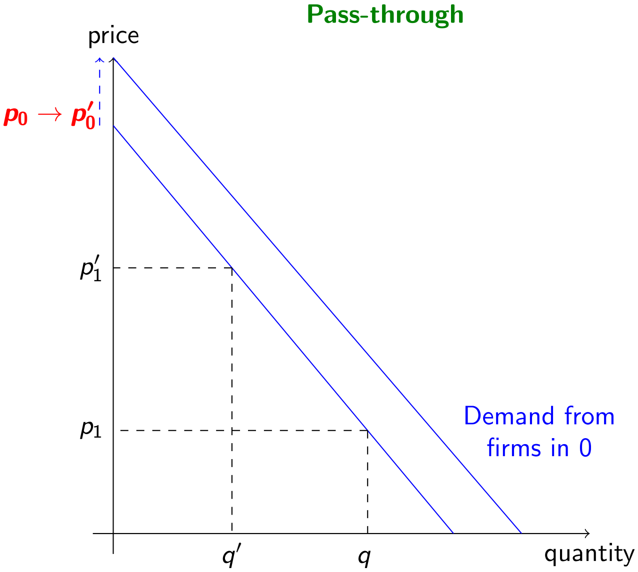
Pass-through - Intuition



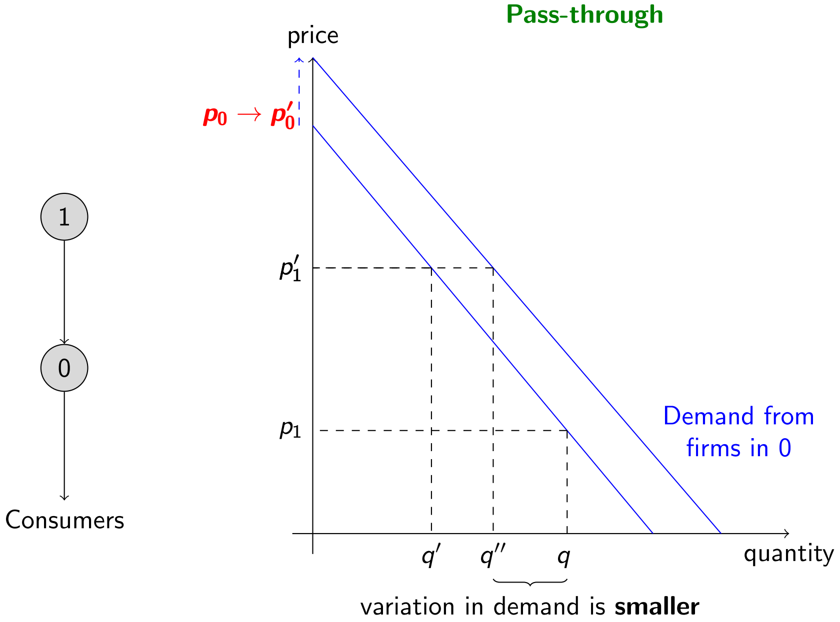


Pass-through - Intuition





Pass-through - Intuition



How different are they?

The relative difference can be **arbitrarily large**:

In a line network of length N, with 2 firms per sector, we can prove that:

$$\lim_{N \to \infty} \frac{\text{Welfare}^{global}}{\text{Welfare}^{local}} = 0$$

Numerical solution

Needed:

- data on network;
- specify number of firms, technology and demand parameters.

Algorithm:

1. Initialize all coefficients B_i to either:
   - "perfect competition" matrices C_i ($\rightarrow$ maximal equilibrium)
   - $\underline{c}I$ for $\underline{c}$ suff. small ($\rightarrow$ minimal equilibrium)
2. iterate best reply equations Equations
3. at convergence, check that all trades positive:
   - if not, cancel link and start back from 1.

Feasible: **2-3 mins** on 6-digit US IO network ($\sim$ 400 sectors);

Important: use Matrix inversion lemma to avoid inverting big matrices.

Conclusion

Key messages:

- competition in S&D schedules useful to model economywide strategic interaction;
- **inter**-sector strategic interaction important for **aggregate** market power;
- firms having power on **all** markets important for **relative** market power;

For the future:

- proof of concept on real IO network (in progress);
- corner solutions $\rightarrow$ a theory of **endogenous** production networks?

Structure of proof

a) The best reply to a linear symmetric profile is unique and still **linear**: defines fixed point equation in matrices $BR(B) = B$.

b) BR increasing in **positive semidefinite ordering** $\rightarrow$ iteration of best replies converges (though not a lattice!); Equation

c) If corner solutions, cancel links and find new solution:

- Computing best replies can only decrease set of **active links**.
- with empty network $\exists$ non-trivial equilibrium (KM (1989));
- $\Rightarrow$ procedure converges.

d) profile of matrices such that $B = BR(B) \rightarrow$ profile of functions D, that generically are linear in neighborhood of $\varepsilon = 0$, $p = p^*(0)$.

back

Technology

The production function of firms in sector i is:

$$\Phi_i((q_{i\alpha,j},\ell_{i\alpha,j})_j) = \sum_j \omega_{ij} \min\{\bar{\ell}_{i\alpha,j}(\varepsilon_i), q_{\alpha,j}\}$$

$$= \lim_{\substack{\sigma\to\infty,\\ \rho\to 0}} \left(\sum_j \omega_{ij} \left(\left(\bar{\ell}_{i,\alpha j}(\varepsilon_i)^{\frac{\rho}{(\rho-1)}} + q_{\alpha,j}^{\frac{\rho}{\rho-1}} \right)^{\frac{\rho-1}{\rho}} \right)^{\frac{\sigma}{\sigma-1}} \right)^{\frac{\sigma-1}{\sigma}}$$

- $\bar{\ell}_{i\alpha,j}(\varepsilon_i)$ **effective labor** hours allocated to input j;
- $\bar{\ell}_{i\alpha,j}(\varepsilon_i) = -\varepsilon_i + \sqrt{\varepsilon_i^2 + 2\ell_{i\alpha,j}}$, where ε_i "labor productivity shock";

back

Technology with non-specific labor

We can include the possibility of non-input-specific labor:

$$\Phi_i((q_{i\alpha,j}, \ell_{i\alpha,j})_j) = \sum_j \omega_{ij} \min\{\bar{\ell}_{i\alpha,j}(\varepsilon_i), q_{\alpha,j}\} + \omega_i^\ell \sqrt{\ell_i}$$

- important in case of corner solutions (existence theorem);
- expressions slightly different, but all results go through; in the presentation I set $\omega_i^\ell = 0$.

back

Technology with general substitution pattern

$$\Phi_i((q_{i\alpha,j},\ell_{i\alpha,j})_j) = \sum_j \omega_{ij} \min\{\bar{\ell}_{i\alpha,j}(\varepsilon_i), q_{\alpha,j}\} + s \sum_{j\neq k} q_{i\alpha,j} \sum q_{i\alpha,k}$$

- main case $s = 0$;
- $s > 0 \rightarrow$ substitutes;
- $s < 0 \rightarrow$ complements.

back

General networks - technology

- each **link** is a market: customized output price (Dhyne et al. (WP))

The **production possibility set** of all $(q_{ki})_k, (q_{ij})_j, (\ell_{i,kj})_{k,j}$ such that:

$$q_{ki} = \sum_j \omega_{ij} \min\{-\varepsilon_i + \sqrt{\varepsilon_i^2 + 2\ell_{i,kj}}, z_{i,kj}\} \quad k = 1, \ldots, d_i^{out}$$

for some subdivision $(z_{i,kj})$ such that: $q_{ij} = \sum_k z_{i,kj}$.

Intuition

You produce laptops and tablets using $q_1 =$ hardware and $q_2 =$ software. Set $\varepsilon_i = 0$.

You need some number of workers l_1 to deal with hardware, and some others l_2 to deal with software.

You need to allocate both inputs to both production lines, as long as a suitable number of labor hours:

$$q_1 = z_{11} + z_{21} \quad q_2 = z_{12} + z_{22}$$

$$\text{laptops: } z_{11} + z_{12}, \text{ if at least } \frac{1}{2}z_{11}^2 \text{ and } \frac{1}{2}z_{12}^2 \text{ units of specific labor}$$

$$\text{tablets: } z_{21} + z_{22}, \text{ if at least } \frac{1}{2}z_{21}^2 \text{ and } \frac{1}{2}z_{22}^2 \text{ units of specific labor}$$

back

Formally, the derivative matrix (Jacobian):

$$J_{i\alpha} = D_{i,p_i^{out},-p_i^{in}}(S_{i\alpha}D_{i,\alpha})_{i\alpha}$$

must be:

- positive semidefinite of rank at least $d_i - 1$;
- $\|J_{i\alpha}\| \geq k > 0$

for any i, α.

back

Best reply equations with Cobb Douglas technology

One output, for simplicity. Shock still additive.

Suppose residual demand and supply are:

$$D_i^r = (p_i^{out})^{\eta_{D,out}} \prod p_{ij}^{-\eta_{D,h}}$$
$$S_{ij}^r = (p_i^{out})^{-\eta_{S,out}} \prod p_{ij}^{\eta_{S,j}}$$

then the best response of i solves:

$$D_{ih} = \frac{D_i^r}{p_{ih}} \left(\left(1 - \frac{\lambda_i}{p_i^{out}}\right) \eta_{D,h} - \sum_j \left(\frac{p_{ij} S_{ij}^r}{p_i^{out} D_i^r} - \omega_{ij} \frac{\lambda_i S}{p_i^{out} D^r} \right) \eta_{S,j} \right)$$
$$S_i = - \frac{D_i^r}{p_i^{out}} \left(\left(1 - \frac{\lambda_i}{p_i^{out}}\right) \eta_{D,out} - \sum_j \left(\frac{p_{ij} S_{ij}^r}{p_i^{out} D_i^r} - \omega_{ij} \frac{\lambda_i S_i}{p_i^{out} D^r} \right) \eta_{S,out} \right)$$
$$S_i = \prod D_{ij}^{\omega_{ij}} \quad \text{(technology)}$$

Not analytical anymore (due to λ_i), and moreover **not Cobb Douglas**!

back

Active/inactive links

Linear schedules are:

- they are identically zero for some links $E_{i,0} \subset E$ - **inactive links**
- for the **active links** ($\notin E_{i,0}$), $\exists$ a matrix B_i and a vector $B_{i,\varepsilon}$ s.t.:

$$D_i = B_i \begin{pmatrix} \vdots \\ \omega_{ij} p_i - p_j \\ \vdots \end{pmatrix} + \varepsilon_i B_{i,\varepsilon} \quad B_i \textbf{ symmetric positive definite}$$

back

Price impacts-line

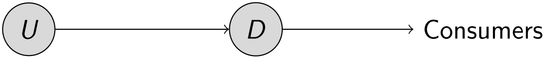
$$\Lambda_0 = \begin{pmatrix} \Lambda_0^{out} & 0 \\ 0 & \Lambda_0^{out} \end{pmatrix} = \begin{pmatrix} B_c & 0 \\ 0 & \left(\frac{1}{n_1 B_1} + \frac{1}{n_2 B_2}\right)^{-1} \end{pmatrix}$$

$$\Lambda_1 = \begin{pmatrix} \Lambda_1^{out} & 0 \\ 0 & \Lambda_1^{out} \end{pmatrix} = \begin{pmatrix} \left(\frac{1}{B_c} + \frac{1}{n_0 B_0}\right)^{-1} & 0 \\ 0 & n_2 B_2 \end{pmatrix}$$

$$\Lambda_2 = \left(\frac{1}{B_c} + \frac{1}{n_1 B_1} + \frac{1}{n_0 B_0}\right)^{-1}$$

back

Markups and markdowns



Rewrite best reply problem as:

$$\max_{p^{out}, p^{in}, z} D^r p^{out} - p^{in} S^r - C(z)$$

subject to:

$$(\lambda) \quad D^r = z \quad \lambda: \text{ marginal value of } \textit{output}$$
$$(\mu) \quad S^r = z \quad \mu: \text{ marginal value of } \textit{input}$$

FOCs:

$$\text{markup: } M = \frac{p^{out} - \lambda}{p^{out}} = -\frac{D^r}{p^{out}(D^r)'} \tag{1}$$

$$\text{markdown: } m = \frac{\mu - p^{in}}{p^{in}} = \frac{S}{p^{in}(S^r)'} \tag{2}$$

In S&D equilibrium:

- U has larger **markup**;
- D has larger **markdown**.

Why markup increasing upstream

Inverse demand of sector 0: $p_0(Q_0)$

First order conditions in market 1 give inverse demand:

$$p_1 = \left(p_0' \frac{Q_1}{n} + p_0 \frac{Q_1}{n}\right) - \frac{Q_1}{n}$$

The markup of firms in sector 1 are then:

$$\begin{aligned}\mu_1 =& \eta_{p_1} = -\left(\frac{p_0' \frac{Q_1}{n}}{p_0' \frac{Q_1}{n} + p_0 - \frac{Q_1}{n}}(\eta_{p_0'} + 1) + \frac{p_0}{p_0' \frac{Q_1}{n} + p_0 - \frac{Q_1}{n}}\eta_{p_0}\right) \\ =& \underbrace{\frac{-p_0' \frac{Q_1}{n}}{p_0' \frac{Q_1}{n} + p_0 - \frac{Q_1}{n}}}_{>0}(\underbrace{\eta_{p_0'}}_{>0} + 1) + \underbrace{\frac{p_0}{p_0' \frac{Q_1}{n} + p_0 - \frac{Q_1}{n}}}_{>1}\mu_0 > \mu_0\end{aligned}$$

back

Price impact in terms of the network

In a tree M can be written as $D^{1/2}(I - L)D^{1/2}$, where:

- L is the (weighted) **adjacency matrix** of the **line graph**: is indexed by **links** and has a 0 where **links do not share a node**.

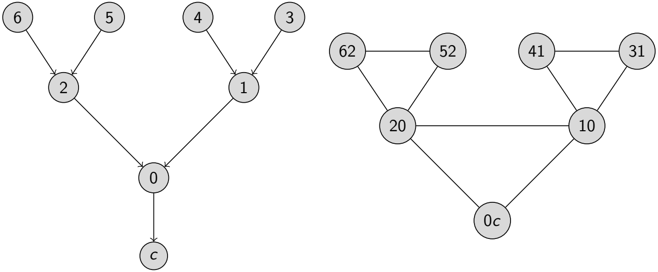


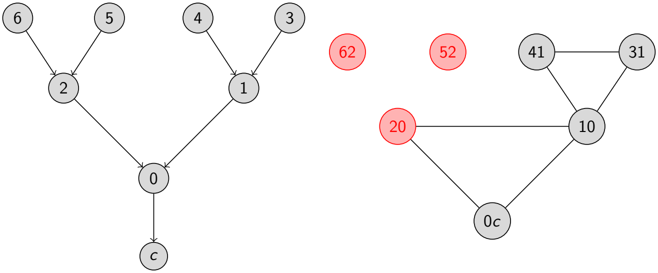
Figure: Right: the line graph of the tree on the Left.

In a tree M^{-1} is the analogous of a **bilateral Leontief inverse**:

- in equilibrium $M^{-1} = D^{-1/2}(I - L)^{-1}D^{-1/2}$.
- entry i, j of M^{-1} measures the **number of links** between i and j (properly normalized and weighted)

Moreover, price impact matrix Λ_i is **diagonal**, and:

- consider **reduced** line graph (cancel links corresponding to sector i);
- price impact on input $j \rightarrow$ **number of self-loops** of (i, j) in the **reduced** line graph.



back

Nash bargaining

Methodology adapted and simplified from Acemoglu and Tahbaz-Salehi (WP):

- upstream firm U and downstream D;
- fixed labor supply: $L = 1$;
- linear technology: $\Phi_i(q_i) = q_i$
- prices determined by **Nash products** on each link:

$$\max_{p_U} \pi_D(p_D, p_U)^{1-\delta} \pi_U^{\delta}(p_U)$$

The surplus is split according to:

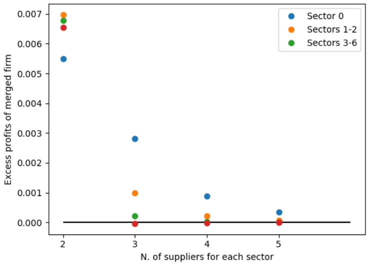
$$\delta(p_0 - p_1) = (1-\delta)p_1$$
$$A_c - B_c p_0 = 1$$

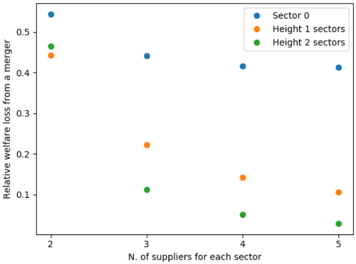
so D has larger **markups** and **profits** if and only if $\delta < \frac{1}{2}$.

Choice of δ is imposing relative market power structure. Back

Impact of mergers - Trees

- profitability of mergers is larger at the root (Left);
- welfare impact of mergers is too (Right).





Back

Horizontal mergers are harmful - Idea of proof

Best reply (slope) (p.s.d-)increasing in n_i:

$$BR_i(B_{-i}, n_i) \geq BR_i(B_{-i}, n_i - 1)$$

Max fixed point pre-merger B^*, after merger B^*_m.

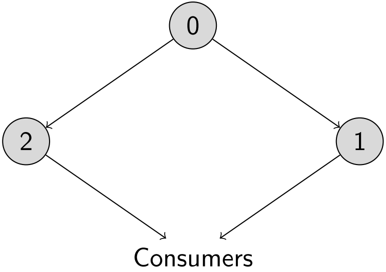
$$B^*_m = BR(B^*_m, n_i - 1) \leq BR(B^*_m, n_i)$$

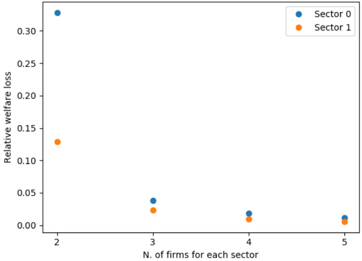
Now iterating $BR(\cdot, n_i)$ we get an increasing sequence, that converges to an equilibrium $D^* \leq B^*$.

Hence $B^*_m \leq B^*$.

Back

Revenues are not sufficient statistics





In the network above assume $\omega_{10} = \omega_{20} = \omega_0 = 2$, then:

- all sectors have the **same revenues**;
- mergers in sector 0 have a much larger impact.

Back

Local S&D Equilibrium

Formally: the game $\mathcal{G}$ remains identical, but now

- firm α in sector i, when optimizing, takes prices of *other* sectors p_{-i} as given.

That is, in computing payoffs, solve the market clearing conditions **only** for p_i, p_i^{in}:

$$\textbf{for links } (k \to i) \quad \sum_\beta D_{i\beta,k}(p_i, p_i^{in}, \varepsilon_j) = \sum_\alpha S_{k\alpha}(p_k, p_k^{in}, \varepsilon_k)$$

$$\textbf{for links } (i \to j) \quad \sum_\beta D_{j\beta,i}(p_j, p_j^{in}, \varepsilon_j) = \sum_\alpha S_{i\alpha}(p_i, p_i^{in}, \varepsilon_i)$$

finding $p_i^L(\varepsilon, p_{-i})$, $(p^L)_i^{in}(\varepsilon, p_{-i})$ and then

$$\pi_{i\alpha}^L = \mathbb{E}\left(p_i^L S_{i\alpha} - \sum_j p_j^L D_{i\alpha,j} - \sum_j \ell_{i\alpha,j}\right)$$

Definition of symmetric linear equilibrium remains the same.

Back

Global vs Local - Idea of proof

The market clearing system coefficient matrix can be written:

$$M = \begin{pmatrix} M_i & S_i \\ S_i' & M_{-i} \end{pmatrix} \quad \text{and is p.d.}$$

Residual demand coefficients:

- under local competition: $\Lambda_i^{local} = M_i$
- under global competition: $\Lambda_i^{global} = M_i - S_i' M_{-i}^{-1} S_i \leq M_i$

Hence $BR_i^{global}(B_{-i}) \leq BR_i^{local}(B_{-i})$, then use strategic complementarity.

Back

Matrix Inversion Lemma - Woodbury Identity

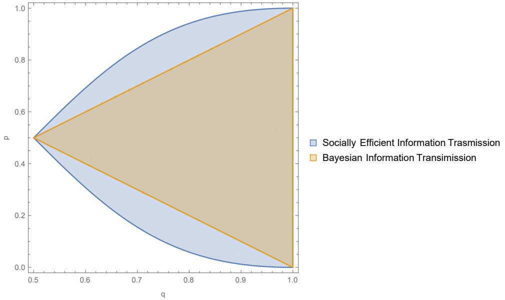
Suppose A is $n \times n$, U is $n \times k$, V is $k \times n$, and $k \ll n$. Then:

$$(A + UV)^{-1} = A^{-1} - A^{-1}U(I_k + VA^{-1}U)^{-1}VA^{-1}$$

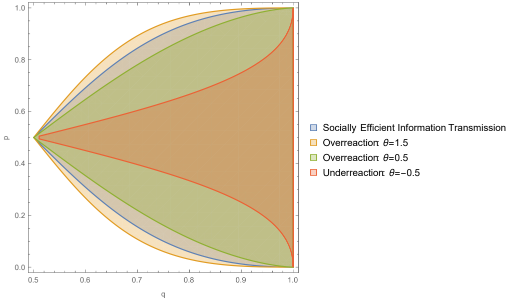
and in particular, to compute the correction is sufficient to invert the $k \times k$ matrix $I_k + VA^{-1}U$.

back

Efficient information transmission as a function of parameters

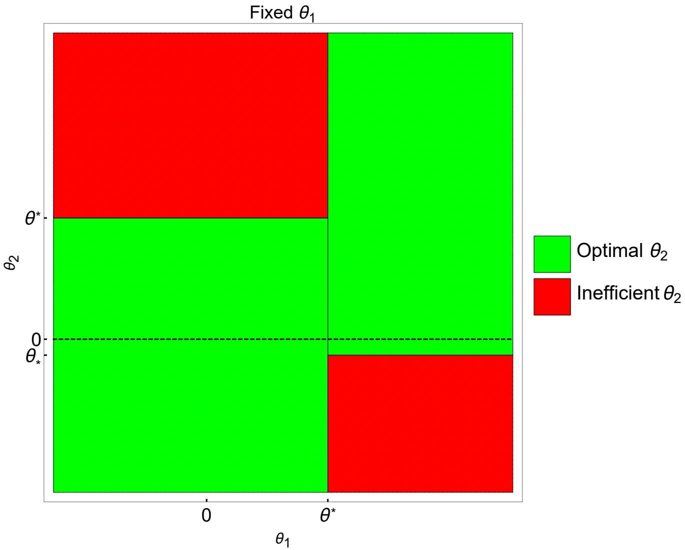


Efficient information transmission as a function of parameters

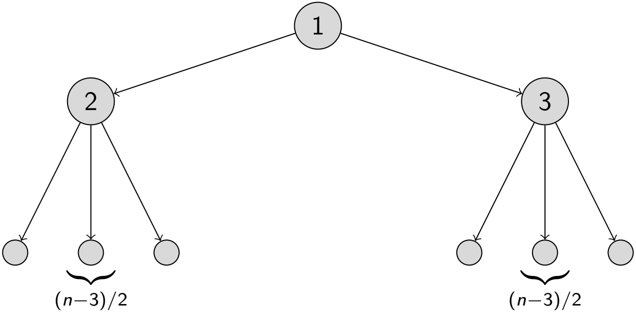


back

Fix θ_1: when does Mr.2 benefit from Mr.1? A numerical example



Example



β means consumers discount more heavily **longer paths**:

- in static model consumer prefers a negative shock to 2;
- in dynamic model prefers shock to 1.

back

Why smaller volatility?

Let's look at expression for covariances in the static model:

$$\mathrm{Cov}_{stat}(y_i, y_j) = \sum_k \sigma_k^2 m_{ki} m_{kj}$$

this is high when:

- there are nodes k such that there are many paths from k to both i and j;
- since $\sum_k m_{ki}=1$, to be very high must be that these nodes are few (asymmetric contribution) and have high volatility.

and dynamic model:

$$\mathrm{Cov}_{dyn}(i, j) = \sum_n \alpha^{2n} \sum_k \sigma_k g_{ki}^{(n)} g_{kj}^{(n)}$$

similar to above, but

- now only paths from k to i and j of **same length** matter.
- covariance always smaller.

back